General physics Ed. 1st

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BOOK I

GENERAL PHYSICS

PREFACE TO THE FIRST EDITION

In bringing out this volume for the advanced students of science, I have followed the same plan as I adopted in writing my Elements of Physics for the Intermediate students and I hope that this volume will also receive a hearty support from the students for whom it is meant.

The need of a treaties like the present volume has been suggested to me by some of my B. Sc. students who find it a very difficult task to manage the whole course of Physics as prescribed for the B. Sc. examination by the Calcutta University within the period of two years and it is with the desire to remove this long-felt want of the students I have ventured to bring out this book.

In this book I have dealt with the more important question and have purposely avoided those that are of a simpler nature Moreover, to enable the students to get a clear grasp of the subject numerous problems have been worked out and scattered throughou the whole book in their proper places.

In compiling this book I have freely consulted all the text book as well as help-books on Physics which are in the market and take this opportunity to acknowledge my indebtedness to them.

My best thanks are due to Professors Birendra Nath Sar and Upendra Chandra Ghosh for their usual encouragement valuable suggestions.

Finally, I thank my student and colleague Mr. Gobordhan Bil Thakur B. Sc. for the trouble he took to go through the pro before they are finally sent to the press.

PHYSICAL LABORATORY
Krishnath College.
BERHAMPORE,
September, 1928.

J. N. Roy

Hence, the total work done in moving through x

$$= \int_0^x m\omega^2 x\,dx = \tfrac{1}{2}m\omega^2 x^2$$

or $P.E. = \tfrac{1}{2}m\omega^2 x^2 = \tfrac{1}{2}m\omega^2 . a^2 \sin^2 \omega t$, since $x = a \sin \omega t$

$$\therefore \text{Total Energy} = K.E. + P.E. = \tfrac{1}{2}ma^2\omega^2 (\cos^2 \omega t + \sin^2 \omega t)$$
$$= \tfrac{1}{2}m\omega^2(a^2 - x^2) + \tfrac{1}{2}m\omega^2 x^2 = \tfrac{1}{2}ma^2\omega^2.$$

Thus the total energy of the particle executing a simple harmonic motion is always constant, and is independent of x *i.e.* the displacement of the body.

Thus we see that energy is never lost but is transformed from one form into another. This is a confirmation of the **Principle of Conservation of Energy.**

26. Variation of Kinetic and Potential energies of a particle executing a S. H. M.: The variation of Kinetic or Potential Energy with time can be graphically represented very easily.

Since Kinetic Energy at any instant t is equal to $\tfrac{1}{2}ma^2\omega^2 \cos^2 \omega t$, and Potential Energy at the same instant t is equal to $\tfrac{1}{2}ma^2\omega^2 \sin^2 \omega t$, the Kinetic Energy curve is easily drawn by plotting values of energy as obtained from the term containing $\cos^2 \omega t$ against ωt *i.e.* t or θ. The Potential energy curve is similarly drawn with the values of $\sin^2 \omega t$ against ωt *i.e.* t or θ.

For plotting these curves, the values of t may be taken as 0, T/4, T/2, 3T/4, etc. and those for θ may be taken as 0°, 90°, 180°, 270°, etc.

If we study these curves we will see that when the potential energy is maximum, the kinetic energy is zero, and when the potential energy is zero, the kinetic energy is maximum, and at any instant the sum of two forms of energies is constant.

27. Expression for period involving the mass of the particle: we have acceleration $\frac{d^2x}{dt^2} = -\omega^2 . x$ where x = displacement ... (1)

$$\text{or } \frac{d^2x}{dt^2} + \omega^2 x = 0 \quad \ldots \quad (2)$$

The solution of this equation is $x = a \cos(\omega t - \epsilon)$, where a is the amplitude, $(\omega t - \epsilon)$ is the phase at the instant t, ϵ is the epoch and $\omega = 2\pi/T$, T being the period of vibration.

From (1) we can write $\omega^2 = -\frac{1}{x} \cdot \frac{d^2x}{dt^2}$ ... (3) which is acceleration per unit displacement; that is

$$\left(\frac{2\pi}{T}\right)^2 = \text{acceleration per unit displacement}$$

$$\text{or } T = \frac{2\pi}{\sqrt{\text{accn. at unit displacement}}}$$

Again, equation (1) may be written as $\frac{m.d^2x}{dt^2} = -\mu x$ where m is the mass of the particle and μ is a constant and is equal to the restoring force at unit displacement.

$$\text{Hence, } \frac{m.d^2x}{dt^2} + \mu x = 0 \quad \text{or} \quad \frac{\mu}{m} \ x = -\frac{1}{x} \cdot \frac{d^2x}{dt^2} \quad \ldots \quad (3)$$

Comparing (2) and (3), we have

$$\omega^2 = \frac{\mu}{m} \quad \text{or} \quad \left(\frac{2\pi}{T}\right)^2 = \frac{\mu}{m} \quad \text{or} \quad T = 2\pi\sqrt{\frac{m}{\mu}} \quad \ldots \quad (4)$$

This restoring force μ may be due to gravity, elasticity, magneti action etc. From (4)

$$n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{\mu}{m}}, \text{ where } n \text{ is the frequency.}$$

28. Resisted Simple Harmonic Motion: All actual vibrating systems are subject to a resisting force which causes decay of amplitude or damping.

Let us consider the resisting force introduced by friction and let the resisting force be proportional to the velocity or the relative velocities of the rubbing substances.

If we take its magnitude as $2k$ per unit velocity, the equation for oscillation becomes

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + \omega^2 x = 0$$

since the resisting force is always opposed to the direction of motion.

The solution of this equation is

$$x = ae^{-kt} \cos(qt - e), \text{ where } q = \sqrt{\omega^2 - k^2}$$

The equation represents an oscillation of frequency $\frac{1}{2\pi}\sqrt{\omega^2 - k^2}$.

29. Illustrations of Simple Harmonic Motion:

(A) The motion of a Simple Pendulum is S. H. M.

The simple pendulum is a small heavy particle suspended by a weightless thread and swings in a vertical plane under the action of gravity. In practice, every simple pendulum is really a compound pendulum for its mass is not a point-mass.

[The action of a compound pendulum will be dealt with in a subsequent chapter.]

Let the pendulum (Fig. 10) of mass m be displaced through an angle θ from its mean position A. At the displaced position B, the weight mg of the bob is resolved into two components $mg\cos\theta$ along BD, the direction of the string keeping it taut, and $mg\sin\theta$ along BE tending to restore the bob to its position of rest.

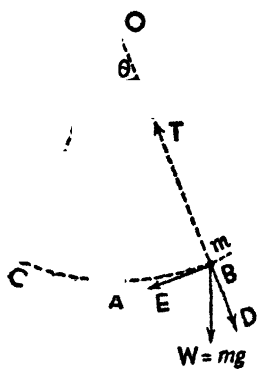


Fig. 10

The effective acc. of the bob $= g\sin\theta$

$$= g\left(\theta - \frac{\theta^3}{\lfloor 3} + \frac{\theta^5}{\lfloor 5} - \ldots\ldots\right) = g\theta$$ for small angular displacements (not exceeding 4°).

If l be the length of the pendulum and θ the angle which the direction of the length of the pendulum at this displaced position makes with the direction when the pendulum is at rest, then the displacement of the bob which is approximately a straight line $= l\theta$, when θ is small,

Now, $\dfrac{\text{Displacement}}{\text{Acceleration}} = \dfrac{l\theta}{g\theta} = \dfrac{l}{g}$ = a constant.

Thus we see that in the motion of the pendulum, the acceleration is proportional to the displacement and therefore the motion is S. H. M.

Note : If T be the time period of the pendulum, then

$$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} = 2\pi\sqrt{\frac{l}{g}}$$

Note that we have neglected θ^3 and higher powers of θ and so the above relation is true only for small angles and the motion of the pendulum is isochronous.

(B) The motion of a heavy body suspended by an elastic filament is Simple Harmonic.

Let a body of mass m suspended by an elastic filament be in equilibrium, and at this position the downward pull on the body due to gravity is just counterbalanced by the tension of the stretched elastic filament.

If the body be displaced downwards, the tension will increase and the body will have a tendency to move up towards the position of rest. Again if the body be displaced upwards the tension will diminish and the body will tend to move down towards the position of rest.

Thus we see that when the body is displaced along the length of he filament through a distance within the elastic limit, it will

be acted on by a force which will be directed towards the position of equilibrium and proportional to the displacement of the body from the position of rest.

Let x be the vertical displacement of the body upwards or downwards.

If μ be the force of restitution for unit displacement, then the change of tension $= \mu x$.

$\therefore$ the acc. of the body directed towards the position of rest

$$= \frac{\mu x}{m}. \quad \text{Hence, } \frac{\text{Displacement}}{\text{Acceleration}} = \frac{m}{\mu} = \text{constant.}$$

$$\left[\textbf{Note: } T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} = 2\pi\sqrt{\frac{m}{\mu}}\right]$$

30. Graphic Representation of a S. H. M.; Displacement Curve: We know that as a particle revolves round the circle of reference, the displacement of the foot of the perpendicular drawn from the particle on any fixed diameter is not constant but passes through a cycle of changes. The displacement curve is that curve which shows the amounts of displacements at different instants of time.

In Fig. 11. let the particle P revolve round the circle of reference in the anti-clockwise direction and in such a way that the foot N of the perpendicular drawn from the moving particle P on the vertical diameter through ON executes an up-and-down S. H. M. with O as its mean position.

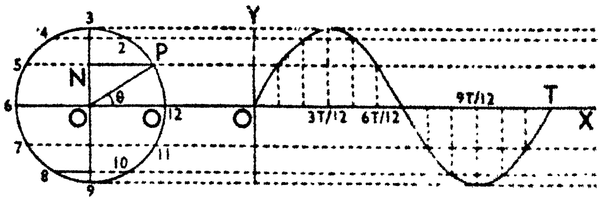


Fig. 11

To obtain the **time-displacement curve** of the motion of N, let us divide the circle into twelve equal parts and draw straight lines (dotted) perpendicular to the vertical diameter ON through each of the points of division 0, 1, 2, 3, etc.

Taking O as the origin, let us plot time along the axis OX and corresponding displacement along OY.

From OX cut off a length OT representing the periodic time T. Divide the distance OT into twelve equal parts and erect ordinates from each of these points of division to intersect the

dotted lines at points shewn in the figure. These points shewing the corresponding values of time and displacement are joined by a continuous curve known as the **time-displacement curve.**

Since the displacement at any instant is expressed by $a \sin \theta$ or $a \sin \omega t$, the curve is called the Sine Curve, being such that the ordinate at any point is proportional to the sine of the angle which is also proportional to the abscissa and the equation of the curve is therefore expressed by $y = a \sin \omega t$, where y is the displacement of the particle at the instant t.

31. Composition of two collinear Simple Harmonic Motions : By Graphical Method :—If two simple harmonic motions occur in the *same line* their resultant is also a simple harmonic motion, which may be obtained by the usual law of vector addition as follows :

Case I. *Composition of two simple harmonic motions in the same line, having equal periods but different amplitudes and phases.*

Let two simple harmonic motions having same time-period but different amplitudes and phases be executed on the line X'OX (Fig. 12). The circle of reference of one S. H. M. is APB and that of the other

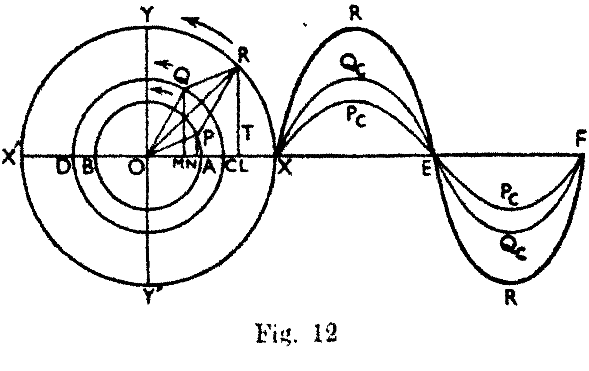


Fig. 12

is CQD. The amplitudes of the two S. H. M.'s are respectively OA and OC. Let P and Q be the positions at a certain instant of time, of the two points rotating anti-clockwise on their circles of reference with the same period as that of each S. H. M. The phases of the two S. H. M.'s at the said instant are $\angle$POA and $\angle$QOC respectively, so that the phase difference between the two S. H. M.'s is $\angle$QOP. The corresponding instantaneous displacements of the two S. H. M.'s are ON and OM along the same straight line X'OX. Then the resultant displacement is equal to their sum by the principle of superposition. To get this, complete the parallelogram OPRQ and from R draw a perpendicular RL on X'OX.

Now OQ is equal and parallel to PR, therefore their projections OM and NL upon X′OX must be equal.

$\therefore$ ON + OM = ON + NL = OL which is the resultant displacement at the instant under consideration. If we, then, imagine that the point R revolves uniformly in a circle of radius OR in the direction of P and Q with same period as that of P and Q, the projection of OR on X′OX will be the resultant displacement at any instant. Evidently the resultant motion is simple harmonic having amplitude equal to OR and period same as that of either component of two S. H. Ms. The angle ROX is the instantaneous phase angle.

From the triangle PRO,

$$\begin{aligned} OR^2 &= OP^2 + PR^2 - 2OP.PR \cos OPR \\ &= OP^2 + PR^2 - 2OP.PR \cos (180^\circ - POQ) \\ &= OP^2 + PR^2 + 2OP.PR \cos POQ. \end{aligned}$$

$$\therefore \text{ Amplitude } OR = \sqrt{OP^2 + PR^2 + 2OP.PR \cos POQ}.$$

If a perpendicular RS be drawn from R on OP (produced) then

$$\tan ROP = \tan ROS = \frac{RS}{OS} = \frac{RP \sin RPS}{OP + PS}$$

$$= \frac{OQ \sin POQ}{OP + RP \cos RPS} = \frac{OQ \sin POQ}{OP + OQ \cos POQ}.$$

From this we can find the angle ROP which gives the phase difference between the component S. H. M. of N and that of the resultant S. H. M. of L.

The sine curves P_c and Q_c are respectively time-displacement curves for particles P and Q, while the curve R is the resultant curve of the motions of the two particles.

Case II. *Two S. H. Ms. of the same period, same amplitude and same phase.*

The displacement and time curves of the two motions are drawn and since the period, the phase and the amplitude are the same in both the cases, the resultant curve is obtained by simply adding up the ordinates of the two curves at the same instant whether above or below the straight line representing the time. The nature of the motion will remain the same, but the amplitude will only be doubled.

Case III. *Two S. H. Ms. of the same phase but of different amplitudes and periods.*

Let two simple harmonic curves for the two motions be separately drawn with the same axis of reference.

Since the periods of the two motions are different, the curves will intersect the abscissa at different points. The method of effecting the composition is to get the algebraic sum of the ordinates

of the component curves at the same instant. The ordinates are to be added if they are in the same direction and their difference taken if they are in the opposite directions. So the resultant curve will be obtained by drawing a curve through the extremities of the ordinates representing the algebraic sum of the ordinates of the component curves and will intersect the abscissa at points where the algebraic sum is zero. The resultant curve will be periodic but not simple harmonic.

Case IV. *Two S. H. Ms. of equal periods and amplitude, but of opposite phase.*

Since the two motions differ in phase by 180°, the ordinates of the component curves at any instant will be equal and opposite in directions. So the algebraic sum of the ordinates at any instant is zero and therefore the resultant displacement and consequently the resultant motion will be zero.

But if the amplitude of the motions be slightly greater than that of the other, the resultant motion will be a S. H. M., but of amplitude smaller than that of either of the motions, and the ordinates of the curve for the resultant motion will be situated in the same direction as those for the curve for which the ordinates are greater.

Case V. *Two S. H. Ms. of nearly the same period.*

If these two motions are compounded together as before, the resultant curve as obtained by the algebraic sum of the ordinates of the component curves at any instant, will present certain irregularities regarding the amplitudes which alternately wax and wane, becoming maximum when the component vibrations are exactly in the same phase and minimum when they are in opposite phases.

The waxing and waning of the amplitude in the resultant motion is the cause of formation of *beats*.

31a. Analytical treatment of Composition of two collinear S. H. Ms. having same period but differing in amplitude and phase.

Let the two S. H. Ms. be represented by the displacement equations,

$$x_1 = a \sin \omega t \; ; \quad x_2 = b \sin (\omega t + \delta)$$

where x_1 and x_2 are the displacements at the instant t and a and b are the amplitudes of the two motions and δ, the phase difference.

The resultant displacement $x = x_1 + x_2 = a \sin \omega t + b \sin (\omega t + \delta)$

$$= \sin \omega t(a + b \cos \delta) + b \sin \delta \cos \omega t$$

If $A \cos \gamma = a + b \cos \delta$; $A \sin \gamma = b \sin \delta$

Then, squaring and adding, we have

$$A^2 = a^2 + b^2 + 2ab \cos \delta \; ; \quad \tan \gamma = \frac{b \sin \delta}{a + b \cos \delta}$$

Thus $x = A \cos \gamma \sin \omega t + A \sin \gamma \cos \omega t = A \sin (\omega t + \gamma)$

The resultant motion is a S. H. M. of the same period with an amplitude equal to the value of A.

Special Cases: (1) If $\delta = 0$, then $A = a + b$ *i.e.* the resultant amplitude is the sum of the component amplitudes. (Compare Case II. Art. 31)

(2) If $\delta = \frac{\pi}{2}$, $A^2 = a^2 + b^2$. or $A = \sqrt{a^2 + b^2}$

(3) If $\delta = \pi$, $A = a \sim b$ *i.e.*, the resultant amplitude is the difference of the component amplitudes.

(4) If $\delta = \pi$ and $a = b$, then, $A = 0$, and the two S. H. Ms. destroy each other. (Compare Case IV. Art. 31)

31b. When the periods are also different, so that they are in ratio $m : n$, the equations of two simple harmonic motions may be expressed as :

$$x_1 = a \sin n\theta,$$
$$\text{and } x_2 = b \sin (m\theta + \delta).$$

Resultant displacement is given by $x = a \sin n\theta + b \sin (m\delta + \delta)$. The above expression is periodic when the periods are commensurable. But in general, no useful reduction of it is possible. It represents what is called a compound harmonic motion.

Note: The converse of the above process also holds good. A S. H. M. may be resolved into two, by resolving its radius vector into two vectors according to the law of resolution of vectors, each vector denoting a component S. H. M.

32. Composition of two Rectangular S. H. Motions (*i.e.* two S. H. Ms. acting at right angles to each other):

The resultant of two S. H. Ms. which a particle is made to perform simultaneously along directions perpendicular to each other is a curve, lying in the same plane as the two component motions and its form depends on the amplitude, period or frequency and phase difference between the two motions. We shall now describe **Graphical method** of finding the resultant curves for a few of important cases.

Case I. *When periods (or frequencies) of the two motions and their phases are the same, but their amplitudes are different.*

Draw two straight lines AB and CD in directions perpendicular to each other, of lengths equal to twice the amplitudes of the two S. H. Ms. performed along axes XX′ and YY′ parallel to AB and CD. With AB and CD as diameters draw reference circles C_1 and C_2. Let P_1 and P_2 be the tracing points rotating anti-clockwise on these circles. As the periods of two motions are the same the

points P_1 and P_2 describe their circumferences or same fractions of their circumferences in the same time. Hence, if each circumference be divided into any equal number of parts, the points P_1 and P_2 will travel over their respective part in the same time.

Divide each circumference into any, say 8 equal parts indicated (Fig. 13) by numerals from 1 to 8, the starting points of the particles being marked zero. Then draw straight lines through points bearing the same numbers in the two circumferences, and parallel to AB or XX′ and CD or YY′ respectively. Lines parallel to XX′ will interset lines parallel to YY′ at certain points. The lines through the starting points A, C (or 0, 0) on circles C_1, C_2, cut at the point Q.

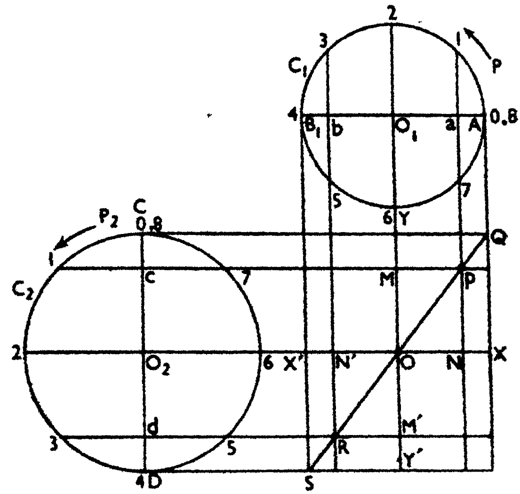


Fig. 13

Now OQ is the diagonal of rectangle OXQY, in which OX and OY equal respectively to O_1A and O_2C, represent the displacements of two S. H. M.'s at the start. Hence, Q is the point of resultant displacement for 0, 0 positions of the particles. When the tracing point of circle C_1, comes to number 1, the tracing point of circle C_2 comes to same number 1, since period is the same. Lines through 1, 1 points on circles C_1, C_2 cut at point P. Again OP is the diagonal of the rectangle ONPM, in which ON and OM equal respectively to O_1a and O_2c represent the displacements of two motions, after 1/8th of the common period. Hence, P denotes the position of the resultant displacement after 1/8th of the period. Thus points of intersection of lines drawn through same numbers on C_1, C_2 will give the positions of resultant disp'acements.

Lines through numbers 2, 3, 4 of two circles intersect at O, R and S respectively. When the points Q, P, O, R and S are joined, a **straight line** is obtained as the path along which resultant motion will take place. The lines through numbers 5, 6, 7 and 8 of two circles C_1, C_2 will cut at points R, O, P and Q. The resultant motion is thus a to and fro motion about the point O along QS. As will be evident from the figure, the line QS is the diagonal of the rectangle

with sides $2a$ and $2b$, where a and b are the amplitudes of two motions. Hence, the amplitude of resultant motion of the particle given by OQ or OS is equal to $\sqrt{a^2+b^2}$ and the **straight line** SQ is inclined to XX′ by an angle equal to $\tan^{-1}b/a$.

Case II. *When the time-periods are the same, amplitudes are different, and phase difference is π.*

Draw as in Fig. 14, two circles of reference C_1 and C_2 with diameters AB and CD, parallel to axis X′X and Y′Y along which two motions take place, the lengths AB and CD being twice the amplitudes of the two motions. Divide the circles into eight equal parts indicated by numerical figures from 1 to 8 the starting points being marked 0, 0. Let P_1 and P_2 be the tracing points rotating anti-clockwise on the reference circles. Since the phase difference is π, the starting point of P_1 and P_2 are A and D respectively. Draw lines through A and D parallel to YY′ and XX′. They intersect at point S. Draw straight lines in same way through points bearing the same numerical figures on C_1 and C_2. These lines are found to intersect at R, O, P and Q. Joining these points of intersection, we get a **straight line** QS along which resultant motion takes place about the point O. This line is inclined in the opposite direction, (Refer Case I) and is a diagonal of the rectangle with sides $2a$ and $2b$, where a and b are amplitudes of two motions. The amplitude of resultant motion given by OQ or OS is equal to $\sqrt{a^2+b^2}$ and angle of inclination of the line to XX′ axis is $\tan^{-1}\left(-\frac{b}{a}\right)$.

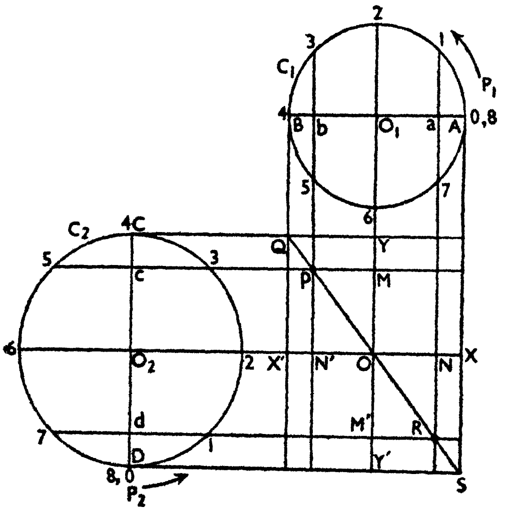


Fig. 14

Case III. *When the periodic times are the same, amplitudes and the phase difference is $\pi/4$.*

Constructions and subdivisons of the reference circles and drawing of the lines are done exactly as above with the only difference that in

this case shift the zero of the starting position of the tracing point P_2 of the second circle of reference C_2 (Fig. 15) by one eighth of its path, the first motion being ahead of the second motion by $\pi/4$. Then join smoothly the points of intersection of the straight lines drawn through the points of C_1 and C_2 bearing same numbers, parallel to CD or YY′ and AB or XX′ respectively. An **oblique ellipse** will be obtained as the resultant path of the motion of the particle.

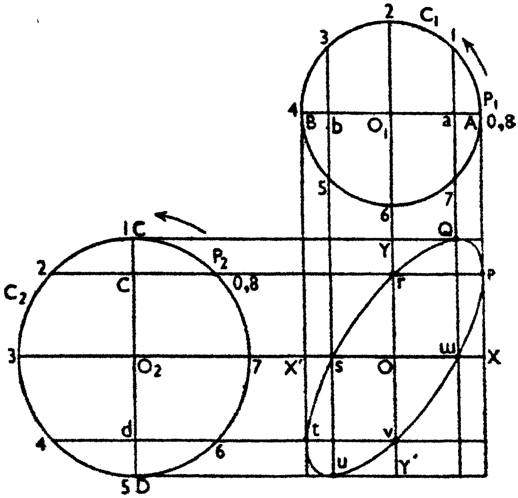


Fig. 15

Case IV. *When the time periods are the same, amplitudes different and the phase difference is $\pi/2$ or 90°.*

Proceed exactly as in the previous cases, but here shift the zero or starting position of the tracing point in the second circle of reference by one quarter of its path (which subtends 90°at the centre) ahead or behind its original position for no phase difference (Case I) so that the phase difference between two motions is $\pi/2$.

Draw lines and join the points of intersections, as in the above cases., An **ellipse** (Fig. 16) will be obtained as the path of the resultant motion having its axes coincident with the directions of the constituent motions.

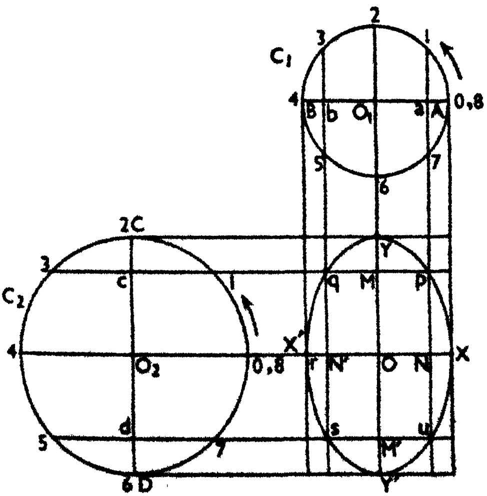


Fig. 16

Case V. *When periods are the same, amplitudes are equal and the phase difference is $\pi/2$.*

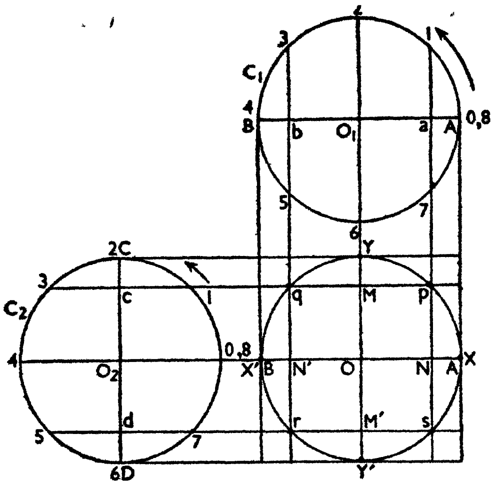


Fig. 17

In this case draw both the circles of reference C_1 and C_2 of the same diameters and proceed exactly as in Case IV. A **circle** will be obtained as the path of the resultout motion. (Fig. 17)

33. Composition of two rectangular S. H. Motions having different periods or frequencies.

Case I. *When amplitudes are different, phase difference at start is zero and the periods different (say in ratio 2 : 1).*

In this case the graphical construction for the resultant motion can be performed as in case I, with the only difference that here divide the circles of reference C_1 (circle having diameter AB) and C_2 (Fig. 18) into number of parts having the same ratio as that of the periods of the two S. H. Motions. Let, for example, the motions along XX′ (or AB) and YY′ (or CD) have periods as 2 : 1 ; then divide C_1 into say 8 parts and C_2 into 4 parts respectively. Then indicate different parts by numbers from 0, 1, 2 etc. to 8 for C_1, and from 0, 1, 2 etc. to 8 for C_2, the starting points being 0, 0 at A and C of the circles C_1, C_2 respectively.

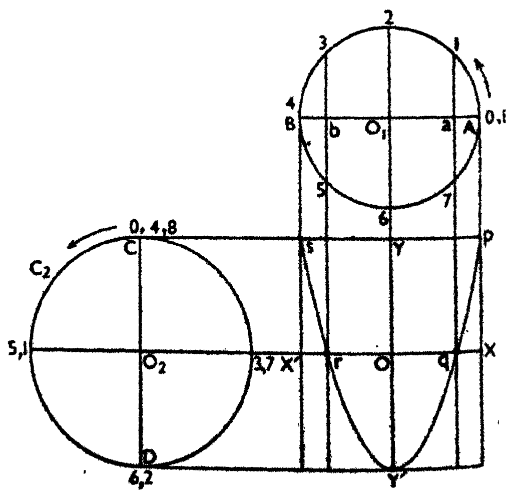


Fig. 18

After constructions have been done, draw lines through similar numbers of C_1 and C_2, parallel to CD and AB. Join the points of intersection and a **parabola** will be obtained as the path of the resultant simple harmonic motion.

N. B. Note that any phase difference set up between two S. H. Motions can not remain constant, since due to a difference in periods of the two motions, there will be continuous change of phase difference. It is therefore customary to consider resultant motion introducing a phase difference at the start of the two motions.

Case II. *When amplitudes are different, periods are in ratio 2 : 1 and initial phase difference is* $\pi/3$.

Draw circles of reference C_1, C_2 as before and divide them into 12 and 6 equal parts respectively (Fig. 19) each part of C_1 subtending an angle $\pi/3$ at the centre of the circle O_1. The starting points P_2 of C_2 which is marked 0 as before is ahead of the starting point P_1 of C_1, by one part, since each part subtends angle $\pi/3$ at the centre. Then proceeding as before, the curve shown in **figure 19** will be obtained as the path of the resultant motion.

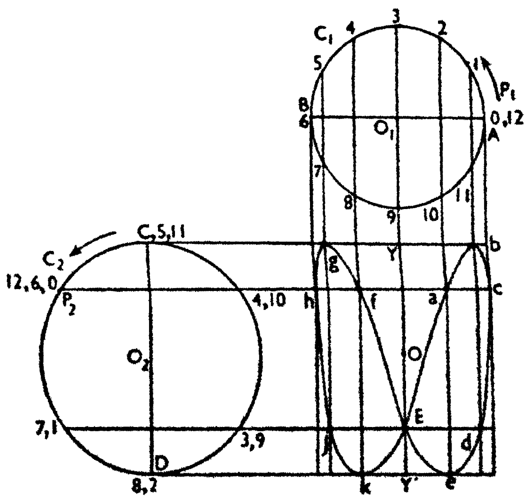


Fig. 19

Case III. *When amplitudes are different, periods are in the ratio 2 : 3 and initial phase difference is zero.*

Here divide the circles of reference into 8 and 12 equal parts respectively and proceed as above. The curve shown in figure 20 will be obtained as the path of the resultant motion.

34. General Rules for the composition of two S. H. Ms. at right angles to one another and having different amplitudes, phases and periods :

If the amplitudes of the two motions are different, draw two circles one for each S. H. M. having diameters perpendicular to each other and lengths proportional to the amplitudes of the corresponding S. H. Ms.

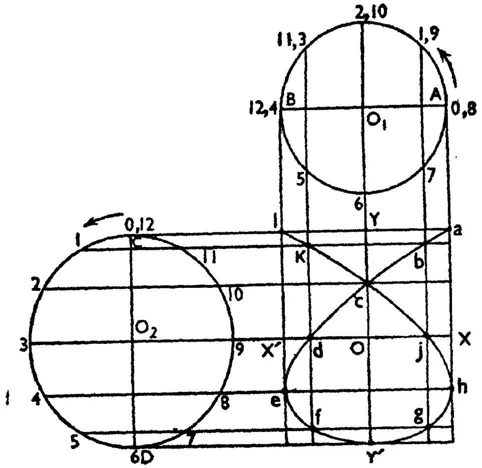


Fig. 20

The difference in the amplitudes of the two motions does not alter the shape of the curve but it only affects the size of the curve.

Let the periods of the horizontal and vertical motion be in the ratio $m : n$. Then,

(1) Divide each quadrant of the circles for the two motions into km and kn parts respectively, k being any integer and its value so chosen that each quadrant is divided into a whole number of equal parts.

(2) Determine the initial position of the particles from the initial phase difference.

(3) Draw straight lines through the points of division in both the circles so that they intersect and form rectangles inside the bigger rectangle formed on the diameters of the two circles.

(4) Determine the initial position of the particle on the rectangular diagram and then pass along the diagonals of the smaller rectangles turning so as to touch the side when obstructed until the initial position is reached.

35. Analytical treatement of composition of two S. H. Ms. at right angles to each other, having same period but different amplitudes and phases :

Let the two simple harmonic motions be along the axes of co-ordinates XOX′ and YOY′, and let their amplitudes be a and b respectively. Suppose at a certain instant, θ (or ωt) is the phase of S. H. M. along XX′ and $\theta+\delta$ (or $\omega t+\delta$) is that of the other S. H. M. along YY′. The phase difference is evidently δ. If x and y be displacements for these motions respectively at certain instant, the displacement equations can be expresed as

$$x=a \sin \omega t \qquad \ldots \qquad \ldots \qquad (1)$$

$$y=b \sin (\omega t+\delta) \qquad \ldots \qquad \ldots \qquad (2)$$

From (1) $\sin \omega t = \frac{x}{a}$; $\therefore \cos \omega t = \sqrt{1-\sin^2 \omega t} = \sqrt{1-\frac{x^2}{a^2}}$

Again from (2) $\frac{y}{b} = \sin(\omega t + \delta) = \sin \omega t \cos \delta + \cos \omega t \sin \delta$

$$= \frac{x}{a} \cos\delta + \sqrt{1-\frac{x^2}{a^2}}.\ \sin\delta.$$

or $\frac{y}{b} - \frac{x}{a}\cos\delta = \sqrt{1-\frac{x^2}{a^2}}.\ \sin\delta.$

Now squaring both sides

$$\frac{y^2}{b^2} + \frac{x^2}{a^2}\cos^2\delta - \frac{2xy}{ab}\cos\delta = \left(1-\frac{x^2}{a^2}\right)\sin^2\delta$$

or $\frac{y^2}{b^2} + \frac{x^2}{a^2}\sin^2\delta + \frac{x^2}{a^2}\cos^2\delta - \frac{2xy}{ab}\cos\delta = \sin^2\delta$

or $\frac{y^2}{b^2} + \frac{x^2}{a^2}\left(\sin^2\delta + \cos^2\delta\right) - \frac{2xy}{ab}\cos\delta = \sin^2\delta$

or $\frac{y^2}{b^2} + \frac{x^2}{a^2} - \frac{2xy}{ab}.\ \cos\delta = \sin^2\delta.$ (3)

This is the equation to an **ellipse**. The resultant motion is thus in general, along an elliptical path.

36. Some special cases derived from above general case.

Case I. When $\delta = 0$, (or $2n\pi$, when n is an integer odd or even) *i.e.* when there is no phase difference, $\sin\delta = 0$ and $\cos\delta = 1$; so that substituting these values in equation (3), we have,

$$\frac{y^2}{b^2} + \frac{x^2}{a^2} - \frac{2xy}{ab} = 0$$

or $\left(\frac{y}{b} - \frac{x}{a}\right)^2 = 0$, or $\frac{y}{b} = \frac{x}{a}$ $y = \frac{b}{a}x.$

This is the equation to a **straight line** passing through the origin (*i.e.*, the mean position) and inclined to the direction of the first motion at an angle $\tan^{-1}\frac{b}{a}$. The resultant amplitude is $\sqrt{a^2+b^2}$.

Case II. When $\delta = \pi$ {or $(2n+1)\pi$}, *i.e.* when phase difference is π. Here $\sin\delta = 0$ and $\cos\delta = -1$, so that equation (3) becomes

$$\frac{y^2}{b^2} + \frac{x^2}{a^2} + \frac{2xy}{ab} = 0$$

or $\left(\frac{y}{b} + \frac{x}{a}\right)^2 = 0$, or $y = -\frac{b}{a}x.$

This also represents **straight line** passing through the origin but inclined to the direction of the first motion at an angle $\tan^{-1}\left(-\frac{b}{a}\right)$. The resultant amplitude is given by $\sqrt{a^2+b^2}$.

Note :—If in the above two cases amplitudes are also equal *i.e.* $a=b$, then each line will be inclined at an angle of 45° to the axes XX′ & YY′.

Case III. When $\delta=\pi/2$ (or any odd multiple of $\pi/2$), *i.e.* when phase difference is $\pi/2$; In this case $\sin\delta=1$ and $\cos\delta=0$, so that equation (3) reduces to

$$\frac{y^2}{b^2}+\frac{x^2}{a^2}=1.$$

This is an equation to an **ellipse,** whose axes coincide with the directions of the component S. H. motions. The resultant path is therefore an ellipse.

Note :—If in addition, in the above case $a=b$, the equation becomes $x^2+y^2=a^2$ and represents a **circle** of radius a. Thus the resultant path is circular whose radius is equal to the amplitude of either S. H. motion.

Case IV. When $\delta=\pi/4$, *i.e.* when the phase difference between the two motions is $\pi/4$; Here $\sin\delta=\frac{1}{\sqrt{2}}$, $\cos\delta=\frac{1}{\sqrt{2}}$, so that equation (3) reduces to

$$\frac{y^2}{b^2}+\frac{x^2}{a^2}-\frac{2xy}{ab}\frac{1}{\sqrt{2}}=\frac{1}{2} \quad \text{or} \quad \frac{y^2}{b^2}+\frac{x^2}{a^2}-\frac{\sqrt{2}.xy}{ab}=\frac{1}{2}$$

This is the equation to an **oblique ellipse.**

37. Direct Methods : The students may obtain the resultant motions in the simple cases without deducing the general equation, as follows.

Let the displacement equations for two S. H. Ms. be, as before $x=a\sin\omega t$ and $y=b\sin(\omega t+\delta)$.

Case I. When $\delta=o$ *i.e.* when there is no phase difference, then $x=a\sin\omega t$, $y=b\sin\omega t$

$\therefore$ $\frac{y}{x}=\frac{b}{a}$ or $y=\frac{b}{a}.x$ which is an equation to a straight line passing through the origin and inclined to XX′ axis at an angle $\tan^{-1}\frac{b}{a}$.

Case II. When $\delta=\pi$, *i.e.* phase difference is π ; Here $x=a\sin\omega t$ and $y=b\sin(\omega t+\pi)=-b\sin\omega t$; $\therefore$ $\frac{y}{x}=-\frac{b}{a}$ or $y=\frac{-b}{a}.x$ which

is again equation to a straight line inclined to the XX′ axis at an angle $\tan^{-1}\left(-\frac{b}{a}\right)$.

Case III. When $\delta=\pi/2$ *i.e.*, phase difference is $\pi/2$; Then $x=a\sin\omega t$; $y=b\sin(\omega t+\pi/2)=b\cos\omega t$.

so that $\frac{x}{a}=\sin\omega t$ and $\frac{y}{b}=\cos\omega t$. Hence, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\sin^2\omega t+\cos^2\omega t$

$$\text{or}\quad \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

This is an equation to an ellipse with its axes coinciding with XX′ YY′ axes.

Case IV. When $\delta=\pi/2$ and $b=a$.

Here $x=a\sin\omega t$; $y=a\sin(\omega t+\pi/2)=a\cos\omega t$.

So that $x^2+y^2=a^2(\sin^2\omega t+\cos^2\omega t)=a^2$. Which is an equation to a circle with radius equal to amplitude of either motion.

38. Composition of two rectangular S. H. motions having different amplitudes, phases and periods.

Let the amplitudes be a and b, periods in ratio 2 : 1, phase difference δ. Then at any instant displacement equations for motions along XX′ & YY′ axes are given by

$$x=a\sin\omega t\ldots\ldots(1)\quad y=b\sin(2\omega t+\delta)\ldots\ldots(2)$$

From (1) $\sin\omega t=x/a$ $\therefore$ $\cos\omega t=\sqrt{1-\sin^2\omega t}=\sqrt{1-x^2/a^2}$.

From (2) $\frac{y}{b}=\sin(2\omega t+\delta)=\sin 2\omega t\cos\delta+\cos 2\omega t\sin\delta$

$$=2\sin\omega t.\cos\omega t.\cos\delta+\sin\delta(\cos^2\omega t-\sin^2\omega t)$$

$$=2\frac{x}{a}\sqrt{1-\frac{x^2}{a^2}}.\cos\delta+\sin\delta(1-2\sin^2\omega t)$$

$$=2\frac{x}{a}\left(1-\frac{x^2}{a^2}\right)^{\frac{1}{2}}\cos\delta+\sin\delta\left(1-\frac{2x^2}{a^2}\right)$$

$$\text{or}\quad \frac{y}{b}-\sin\delta+\frac{2x^2}{a^2}.\sin\delta=\frac{2x}{a}\left(1-\frac{x^2}{a^2}\right)^{\frac{1}{2}}\cos\delta$$

$$\text{or}\quad \left[\left(\frac{y}{b}-\sin\delta\right)+\frac{2x^2}{a^2}\sin\delta\right]^2=\frac{4x^2}{a^2}\left(1-\frac{x^2}{a^2}\right)\cos^2\delta.$$

$$\text{or}\quad \left(\frac{y}{b}-\sin\delta\right)^2+\frac{4x^2}{a^2}\sin\delta\left(\frac{y}{b}-\sin\delta\right)+\frac{4x^4}{a^4}\sin^2\delta$$

$$\frac{4x^2}{a^2}\left(1-\frac{x^2}{a^2}\right)\cos^2\delta=\frac{4x^2}{a^2}\cos^2\delta-\frac{4x^4}{a^4}\cos^2\delta$$

$$\text{or} \quad \left(\frac{y}{b} - \sin\delta\right)^2 + \frac{4x^4}{a^4}\left(\sin^2\delta + \cos^2\delta\right)$$

$$+\frac{4x^2y}{a^2b}\sin\delta - \frac{4x^2}{a^2}\sin^2\delta - \frac{4x^2}{a^2}\cos^2\delta$$

$$\text{or} \quad \left(\frac{y}{b} - \sin\delta\right)^2 + \frac{4x^2}{a^2}\left[\frac{y}{b}\sin\delta - (\sin^2\delta + \cos^2\delta)\right] + \frac{4x^4}{a^4} = 0$$

$$\text{or} \quad \left(\frac{y}{b} - \sin\delta\right)^2 + \frac{4x^2}{a^2}\left(\frac{x^2}{a^2} + \frac{y}{b}\sin\delta - 1\right) = 0 \quad\ldots\ldots\ldots(3)$$

This represents the general equation to a curve (Fig. 19) having two loops, for any difference in phase and amplitude.

Certain special cases :—

Case I. If $\delta = 0$ or phase difference be zero., $\sin\delta = 0$, equation (3) becomes $\frac{y^2}{b^2} + \frac{4x^2}{a^2}\left(\frac{x^2}{a^2} - 1\right) = 0$

which is the equation to the loop as in figure 20.

Case II. If $\delta = \frac{\pi}{2}$, then $\sin\delta = 1$, so that equation (3) becomes

$$\left(\frac{y}{b} - 1\right)^2 + \frac{4x^2}{a^2}\left(\frac{x^2}{a^2} + \frac{y}{b} - 1\right) = 0$$

$$\text{or} \quad \left(\frac{y}{b} - 1\right)^2 + \frac{4x^2}{a^2}\left(\frac{y}{b} - 1\right) + \left(\frac{2x^2}{a^2}\right)^2 = 0$$

$$\text{or} \quad \left\{\left(\frac{y}{b} - 1\right) + \frac{2x^2}{a^2}\right\}^2 = 0$$

$$\text{or} \quad \frac{2x^2}{a^2} = -\left(\frac{y}{b} - 1\right) \quad \text{or} \quad x^2 = -\frac{a^2}{2b}\left(y - b\right).$$

which represents two **coincident parabolas.**

39. Uniform circular motion is equivalent to two S. H. Ms. at right angles to each other having same period and amplitude, but differing in phase by $\pi/2$:—

Suppose a point moving uniformly in a circle (Fig. 21) start from A and let it be at position P at any instant t. Draw perpendiculars PN and PM to diameters A′A and BB′ of the circle, at right angles to each other. The vector OP is then equivalent to the vectors ON and OM. Since P is revolving round the circle, N performs a S. H. M. along the diametral line A′A, while M performs a S. H. M. along the diametral line BB′. At any instant of time, the vector OP denoting the distance of the point P from O will be equivalent to the vecto

denoting the displacement of N and M from O. Evidently the circular motion, of P is equivalent to the S. H. Ms. of N and M along the diameters A′A and BB′.

Again, when the point P is at A, N is also at A, but M is at O the centre of the circle. In other words when N is at one extreme end of A′A, M is at mean position at O on BB′. Similarly when P is at B, N is at mean position while M is at one extreme end. Hence, motion of N differs from that of M, in phase by $\pi/2$.

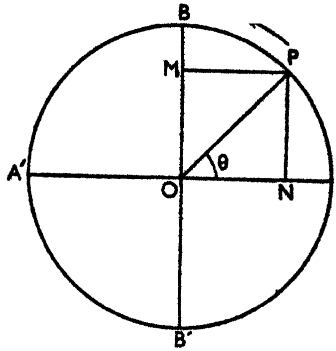


Fig. 21

39a. Alternative method :

Let θ be the phase angle of N at any instant t, so that $\angle POA = \theta$. If a be the amplitude of N, the displacement of N along A′A is given by $x = a \cos \theta$

When N is moving along A′A, M will be moving along BB′. As the phase of M is to be measured from the instant P passes through B, the phase of M is obviously equal to the reflex angle BOP or $\theta + 3\pi/2$. As the phase is not altered due to a change by 2π, the phase of M is also given by $\theta + \frac{3\pi}{2} - 2\pi = \theta - \frac{\pi}{2}$. Hence, the phase of M lags behind the phase of N by $\pi/2$. The displacement equation for M is then given by $y = a \cos(\theta - \pi/2) = a \sin \theta$.

where a = amplitude of M

Thus a uniform circular motion may be resolved into tw S. H. M's in perpendicular directions having the same amplitude and period but a phase difference of $\pi/2$.

40. Lissajous' Figures :

The figures or curves obtained by the composition two Simple Harmonic motions in perpendicular directions first produced by Lissajous by optical method by reflecting beam of light from two mirrors in turn attached to two forks ating at right angles to one another. Their forms depend up he phase difference and also on the ratio of the frequencies of component vibrations. These figures (Fig. 22) are known as Lis' figures.

41. Demonstration of Lissajous' Figures :

(A) Mechanical Method

Blackburn's Pendulum :—The instrument (Fig. 23) of a vertical frame-work with a peg P at the top. cone filled with fine

silver sand fits into a conical hole in a metal mass M which is suspended by three short strings and attached to a ring C from which two separate strings pass up through the corners A and B of the frame-work and their ends are tied to the peg P which can be turned to regulate the length below A and B.

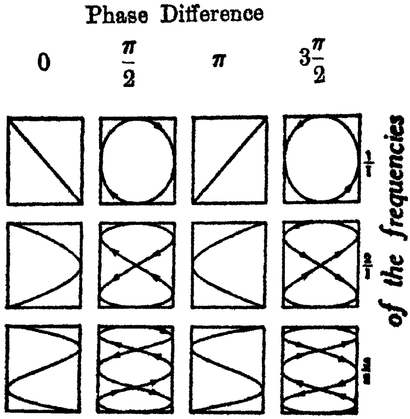


Fig. 22

By means of the ring C, a portion of the string is separated off from the portion ABC.

If the funnel is set swinging in the vertical plane containing A and B, the part CM will swing, with a period equal to that of a pendulum of length CM.

If again the funnel moves in a plane perpendicular to AB, it will swing with a period equal to that of a pendulum of length DM, D being the middle point of AB.

By adjusting the lengths CM and DM and causing the funnel to move obliquely by burning out the thread tied to the small upright fixed on the right, the sand will trace Lissajou's figures on a piece of paper placed below the funnel.

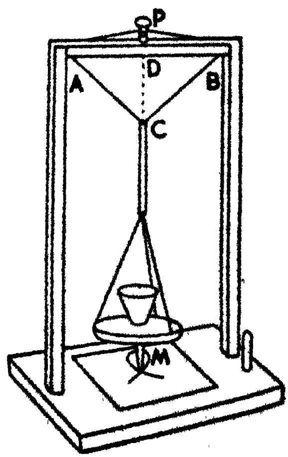


Fig. 23

(B) Optical method. A strong beam of light produced by a convergent lens L is allowed to fall on a mirror M, attached to one of the prongs of a tuning fork F_1 in such a way that after reflection from mirror M it is again reflected in the mirror M_2 attached to a second fork F_2 vibrating at right angles to the plane of vibration of the first fork F. (Fig. 24)

Wh[illegible] forks vibrate in directions at right angles to one [illegible] motion of the beam of light will trace out

figures on a screen whose shapes depend upon the frequencies and the phases of the forks. These are Lissajou's figures.

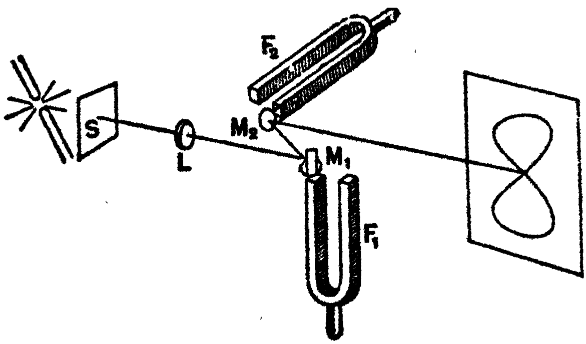


Fig. 24

42. Composition of two circular motions in opposite senses into a S. H. M.: Let the two circular motions one along ABA′ and the other along A′BA (Fig. 25) pass simultaneously, the points A and A′, the extremities of the diameter AA′ of the circle ABA′. Since a circular motion can be resolved into two S. H. Ms. at right angles to one another *i.e.* along AA′ and BB′ differing in phase by $\frac{\pi}{2}$, then, when the two motions pass A and A′ simultaneously, the displacement along AOA′ for both the motions are equal and opposite, since OA = OA′ and those along BOB′ for both the motions are zero.

Again, when the motion in the direction ABA′ reaches the point P, the other motion in the direction A′BA will in the same time reach the point P′.

The displacements for motion along ABA′ are OM (*i.e.*, QP), and OQ along AOA′ and BOB′ respectively and those for the motion in the direction A′BA are OM′ *i.e.* QP′, and OQ along AOA′ and BOB′ respectively.

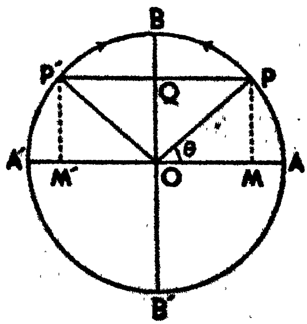


Fig. 25

Since QP and QP′ are equal and opposite in directions the resultant displacement due to the two circular motions in opposite directions is therefore equal to the sum of the component displace-

ments along BOB′ *i.e.* equal to 2OQ since they are in the same direction.

Again since the components of the circular motion are S. H. Ms., the resultant motion along BOB′ is then a S. H. M. of double the amplitude.

42a. *Two opposite circular vibrations can be compounded into a S. H. M. of double the amplitude.*

Let $X = a \cos \omega t$; $Y = a \sin \omega t$ be a circular vibration *i.e.*, $X^2 + Y^2 = a^2$. Changing the sign of ω we have $X = a \cos \omega t$; $Y = -a \sin \omega t$ as the other circular vibration is in the opposite direction.

Here X and Y represent the displacements along X and Y axes respectively.

Adding the X-components and Y-components, we have for the resultant a S. H. M. of double the amplitude.

That is, $X = 2a \cos \omega t$., $Y = 0$

42b. *A. S. H. M. can be resolved into two equal circular motions in opposite senses.*

Let $X = a \cos \omega t$ be the S. H. M.

This can be written in the form

(1) $X = \frac{1}{2}a \cos \omega t$, $Y = \frac{1}{2}a \sin \omega t$

(2) $X = \frac{1}{2}a \cos \omega t$, $Y = -\frac{1}{2}a \sin \omega t$

The first and the second pair represent two circular motions of amplitude $\frac{1}{2}a$ in opposite senses.

QUESTIONS

1. Define a S. H. Motion explaining the meanings of the terms *period amplitude* and *phase*. [C. U. 1946, '50, '52

2. Show that when a particle moves with uniform speed around a circle, the motion of the foot of the perpendicular drawn from the position of the particle on a diameter of the circle is simple harmonic. [C. U. 1954

3. Define a S. H. Motion and state under what conditions a particle will execute such a motion. Prove that the acceleration of a particle executing S. H. M. is directed towards the centre and is proportional to its displacement.

4. Deduce the equation for the simple harmonic motion of a particle.

Two simple harmonic motions having the same period but differing in phase and amplitude are acting in the same direction on a particle. Show that the resultant motion is simple harmonic and deduce the expression for the resultant amplitude and phase. [C. U. 1950

5. Define simple harmonic motion and show that if the displacement of particle executing such a motion from a fixed point is, given by $x = a \cos (\omega t + \epsilon)$, the total energy at any instant t is $\frac{1}{2}m\omega^2 a^2$ ergs, in being the mass of the particle.

Find graphically or otherwise the resultant of two simple harmonic motions of same frequency and of different amplitudes and right angles to each other and having a phase difference of $\pi/2$. [C. U. 1957]

6. A particle is subjected simultaneously to two S. H. vibrations of same period but of different amplitudes and phases in perpendicular directions. Find an expression for the resultant motion and show that the path traced by the particle is an ellipse.

For what conditions, the path may be a circle and a straight line? [C. U. 1946, '50, '52]

Indicate a method of experimentally demonstrating the result obtained. [C. U. 1946, '48]

7. Show that uniform motion in a circle is equivalent to two S. H. Motions at right angles to each other. [C. U. 1953]

8. Find expressions for the velocity, acceleration and period of a particle executing Simple Harmonic Motion. [C. U. 1948]

EXAMPLES

1. A. S. H. M. has a period of 2 secs. and an amplitude of 1 metre. What are the velocity and the acceleration corresponding to a displacement of ·5 metre?

We have $x = a \sin \omega t = a \sin \frac{2\pi t}{T}$

$$\omega = 2\pi/T = 2 \times 3{\cdot}14/2 = 3{\cdot}14 \text{ rad/sec.}$$

velocity $v = a\omega \cos \omega t = 1 \times 3{\cdot}14 \sqrt{1 - \frac{x^2}{a^2}}$

$$= 1 \times 3{\cdot}14 \sqrt{1 - \frac{1}{4}} \qquad \left[\because \frac{x}{a} = \frac{{\cdot}5}{1} = \frac{1}{2}\right]$$

$$= 1 \times 3{\cdot}14 \times \frac{\sqrt{3}}{2} = 3{\cdot}14 \times {\cdot}866 = 2{\cdot}72 \text{ m/sec.}$$

Acceleration $f = -\omega^2 . x = -(3{\cdot}14)^2 \times {\cdot}5 = -9{\cdot}87 \times {\cdot}5 = -4{\cdot}93$ m/sec^2.

2. A particle moving with a Simple Harmonic Motion has a period of ·001 sec. and amplitude 5 cm. Find its acceleration when it is ·2 cm. from its mean position and its maximum velocity. [C. U. 1915]

In any Simple Harmonic Motion we know that $x = a \sin \omega t$, where x = displacement of the particle from the mean position.

ω = angular velocity $= \frac{2\pi}{T}$, where T is the period. a = amplitude and t = time since it has passed through its mean position in the positive direction.

Then velocity v = Rate of change of displacement $= \frac{dx}{dt} = \frac{d(a \sin \omega t)}{dt}$

$$= a\omega \cos \omega t$$

Maximum velocity = Maximum value of $a\omega \cos \omega t = a\omega = \frac{2\pi}{T} a$.

$\therefore$ The maximum velocity $= 2 \times \frac{22}{7} \times \frac{1}{{\cdot}001} \times 5$ cm. per sec.

$$= 3{\cdot}142 \times 10^4 \text{ cms/sec.}$$

Now, acceleration = Rate of change of velocity $= \frac{dv}{dt} = \frac{d(a\omega \cos \omega t)}{dt}$

$$= -a\omega^2 \sin \omega t = -\omega^2 . x = -\left(\frac{2\pi}{T}\right)^2 x.$$

$\therefore$ the acceleration of the given particle at a distance of ·2 cm. from the mean position $= -\left(\frac{2 \times 22/7}{{\cdot}001}\right)^2 \times {\cdot}2$ cm. per sec².

$= -4 \times \frac{22}{7} \times \frac{22}{7} \times \frac{1}{10^{-6}} \times \cdot 2$ cm. per sec². $= -7 \cdot 9 \times 10^{6}$ cm. per sec².; since it has a negative sign, the acceleration is towards the mean position.

3. A point performs simple harmonic vibration in a line 4 cms. long. Its velocity when passing through the centre of the line is 12 cms. per sec. Find the period.

Amplitude of vibration $= \frac{4}{2}$ cm. $= 2$ cm. Velocity of the point when it passes through the centre of the line $=$ the maximum velocity it can have $a\omega = \frac{2\pi a}{T} = \frac{4\pi}{T}$

$\therefore \quad \frac{4\pi}{T} = 12, \quad \therefore \quad T = \frac{4\pi}{12} = \frac{\pi}{3} = \frac{22}{7} \times \frac{1}{3} = 1 \cdot 05$ sec.

4. A body moves on a circle of radius 10 cms. with a uniform linear speed of 20 cms. per sec. Find (a) the angular velocity (b) the period (c) the position of the body with reference to the y axis $\pi/8$ sec. after passing the middle point in the positive direction.

(a) $\omega = \frac{v}{r} = \frac{20}{10} = 2$ radian/sec. $= 2 \times \frac{180}{\pi} = 114°39'$ per sec.

(b) $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$ secs. $= 3 \cdot 14$ secs.

(c) $y = a \sin \omega t = 10 \sin \frac{2\pi t}{T} = 10 \sin \left(\frac{2\pi}{\pi} \cdot \frac{\pi}{8}\right) = 10 \sin 45° = 7 \cdot 07$ cms.

5. A particle starting at the end of its swing performs horizontal S. H. M. of amplitude 12 cms. and frequency of 40 vibrations per minute. What is the displacement at the end of 2 seconds? [C. U. 1948]

Let x be the required displacement

Then $x = a \cos \omega t = a \cos \left(\frac{2\pi}{T} \cdot t\right) = a \cos (2\pi . n . t)$

Here $a = 12$ cms.; $n = \frac{40}{60} = \frac{2}{3}$; $t = 2$ secs.

$\therefore \quad x = 12 \cos (360 \times \frac{2}{3} \times 2) = 12 \cos 480° = 12 \cos 60 = 6$ cms.

6. If the period of S. H. M. is 12 seconds and amplitude 10 cm., what are the phase and the displacement at a time 14 secs. after a passage of the particle through its extreme positive elongation. [C. U. 1952]

We know that $x = a \cos \frac{2\pi}{T} t = 10. \cos. \frac{2\pi}{T} \cdot 2 = 10. \cos \frac{\pi}{3} = 5$ cms.

$$\text{Phase} = \frac{2\pi}{12} \times 2 = \frac{\pi}{3}, \quad \text{or} \quad \frac{2\pi}{12} 14 = \frac{7\pi}{3}.$$

7. A body having a mass of 4 grams executes simple harmonic vibration. The force acting on the body when the displacement is 8 cms. is 24 gms. weight. Find the period. If the maximum velocity is 500 cms. per sec., find the amplitude and maximum acceleration.

We have $F = \mu x \qquad \mu = \frac{F}{x} = \frac{24 \times 981}{8} = 3 \times 981$ dynes.

Again, $T = 2\pi \sqrt{\frac{m}{\mu}} = 2 \times \frac{22}{7} \sqrt{\frac{4}{3 \times 981}} = \cdot 2319$ secs.

But we know that maximum velocity $= \omega a = 500$ cms. per sec.

$$\therefore \quad a = \frac{500}{\omega} = \frac{500}{2\pi} = \frac{500 \times \cdot 2319 \times 7}{2 \times 22} = 18 \cdot 45 \text{ cms.}$$

Again maximum acceleration $=\omega^2 a=\left(\frac{2\pi}{T}\right)^2 \times 18{\cdot}45=\left(\frac{2\times 3{\cdot}142}{{\cdot}2319}\right)^2=13550$ cms. per sec^2.

8. A light elastic string has an unstretched length of 8 cms.; when a weight is hung from it, its length becomes 14 cms. Calculate the periodic time of oscillation of the weight if displaced vertically.

Original length of string $=L$ cm., Weight attached to lower end $=mg$ dynes. Tension of string acting upwards $=T$, Young's modulus of the string $=Y$.

Then $Y=\frac{\text{stress}}{\text{strain}}$ $\therefore$ stress $=Y\times$ strain.

If $\alpha=$ area of cross-section of the string

$$\text{stress}=\frac{T}{\alpha}\text{ ; strain}=\frac{l}{L}\text{ where } l=\text{elongation.}$$

$$\frac{T}{\alpha}=Y\cdot\frac{l}{L} \quad \text{or} \quad T=Y.\frac{l\alpha}{L}$$

But $mg=T$; $\qquad mg=\frac{Y.l.\alpha}{L} \qquad \frac{mg}{l}=\frac{Y.\alpha}{L}.$

If the string be pulled down a little through distance x, the tension in the string acting upwards will clearly be

$$=\frac{Y(l+x).\alpha}{L}=\frac{mg}{l}(l+x).$$

Since, downward force $=mg$, the resultant upward force acting on the string will now be, $\frac{mg}{l}(l+x)-mg=\frac{mg.x}{l}$.

Thus resultant upward force $=\frac{mg.x}{l}$.

Now, acceleration $=\frac{\text{force}}{\text{mass}}=\frac{mgx}{l.m}=\frac{gx}{l}=\mu.x$ (where $g/l=\mu$ a constant).

i.e. acceleration is $\propto$ displacement.

$\therefore$ Oscillations are S. H. in nature; time period t is given by

$$t=2\pi\sqrt{\frac{1}{\mu}}=2\pi\sqrt{\frac{1}{g/l}}=2\pi\sqrt{\frac{l}{g}}.$$

In the given problem $l=14-8=6$ cms. $\quad g=981$ cm.-sec^2.

$$\therefore\ t=2\pi\sqrt{\frac{6}{981}}=\frac{2\times 3{\cdot}14\sqrt{6\times 981}}{981}=\frac{2\times 3{\cdot}14\times 76{\cdot}7}{981}=0{\cdot}49\text{ sec.}$$

9. A mass of 40 gms. hung from an elastic cord stretches it by 5 cms. What will be its period of oscillation if drawn down a little and then set free.

Ans. ·45 secs. nearly.

10. A particle P moves with a constant speed on a circle of radius 10 cms. making 3 complete revolutions per sec. Discuss the motion of the projection Q of the particle P on the diameter of the circle and find the expression for the velocity of Q at any instant. Calculate the position and the velocity of Q at the end of 1st, 2nd and 3rd second. [C. U. 1927]

Velocity of Q at any instant *i.e.* $v = \omega a \cos \omega t$.

We have $\omega = \frac{2\pi}{T}$ and since in the case $T = \frac{1}{3}$ sec. $\qquad v = 6\pi . 10 \cos 6\pi t$.

This angle $6\pi t$ (in radians) is a multiple of 2π for all integral values of t and since when $t = 1$, 2 or 3 secs., $\cos 6\pi t$ *i.e.* $\cos 2\pi.(3t) = 1$.

Thus v, the velocity of Q at the end of 1st, 2nd and 3rd sec. is the same and equal to 60π or 188·5 cms. per sec.

11. The period of a simple harmonic motion is $2\pi/p$ and its amplitude is A. Prove that the displacement can be expressed in the form $A \cos (pt - \alpha)$ and find the velocity.

12. A test tube of mass 6 gm. and of external diameter 2 cms. is floated vertically in water by placing 10 gm. of mercury at the bottom of the tube. The tube is depressed a small amount and then released. Find the time of oscillation.

Mass of the tube and mercury $= 6 + 10 = 16$ gms.
External radius of the tube $= 1$ cm.

$\therefore$ Area of cross-section of the tube $= \pi r^2 = \pi \times 1^2 = \pi$ sq. cms.

Let the tube be depressed through a distance $= x$ cms.
Then, volume of water displaced $= \pi \times x$.c.c.
Mass of this water $= \pi.x$. gm. [$\because$ density of water $= 1$ gm./c.c.]
Upward thrust experienced by the tube
$= \text{weight of water displaced} = \pi.x.g$ dynes

Now, acceleration $= \frac{\text{force}}{\text{mass}} = \frac{\pi.xg}{16} = \mu.x$ where $\mu = \pi g/16$ (a constant)

$\therefore$ Acceleration is $\propto$ displacement and the motion is simple harmonic. Hence time-period t is given by

$$t = 2\pi\sqrt{\frac{1}{\mu}} = 2\pi\sqrt{\frac{1}{\pi g/16}} = 2\pi\sqrt{\frac{16}{\pi g}} = 2\sqrt{\frac{\pi 16}{g}} = 2\sqrt{\frac{3{\cdot}14 \times 16}{981}} = {\cdot}452 \text{ secs.}$$

13. Two pendulums of lengths 20 and 80 cms. begin to vibrate together in the same direction through their mean positions simultaneously. If the amplitudes are in the ratio of 1 : 2 respectively when they will be again in the same phase?

[4·9 secs. ($g = 980$ cm./sec^2.)]

14. Prove that a body falling from the surface of the earth through a hole passing through the earth's centre will travel with simple harmonic motion, if frictional effects are inappreciable and the density of the earth is uniform. Find the period of the motion if the value of 'g' at the surface is 981 cm. per sec^2. and radius of the earth $R = 6{\cdot}38 \times 10^8$ cms.

[T = 84·4 min.]

CHAPTER III

MOMENT OF INERTIA

43. Motion of Translation: If a body moves along a straight line so that motion of all the particles constituting the body is exactly the same, it is said to have *motion of translation.*

The kinetic energy of translation of the body of mass m moving uniformly with a velocity v is equal to $\frac{1}{2}mv^2$.

44. Motion of Rotation: A body is said to have a motion of rotation when it moves in such a way that all the particles of which the body is composed, describe concentric circles round a fixed point or an axis about which the body rotates.

Let a body rotate about an axis passing through O (Fig. 26) perpendicular to the plane of the paper, and let OP represent the position at any time t of a line revolving in the plane of the figure about the axis through O.

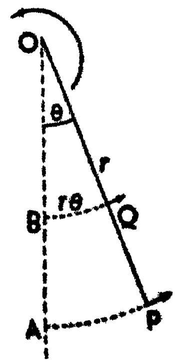


Fig. 26

Let OA be its position at time $t=0$. If θ be the angular displacement of the line OP then $d\theta/dt=\omega=$ angular velocity, $d^2\theta/dt^2=d\omega/dt=$ angular acceleration.

Any point Q on the line OP has a linear velocity and acceleration.

Linear displacement of $Q=r\theta$, where $r=OQ$.

Linear velocity $= \dfrac{d}{dt}(r\theta)=r\dfrac{d\theta}{dt}=r\omega$,

Linear acceleration $= \dfrac{d}{dt}(r\omega)=r\dfrac{d^2\theta}{dt^2}=r\phi$,

where $\phi=$ angular acceleration.

Thus the linear motion of any point in the body is correlated with the angular motion of the body.

45. Energy Consideration and Moment of Inertia: The total kinetic energy of a revolving body is the sum of the separate kinetic energies of each of the particles of which it is made. Let a particle of mass m move with uniform speed v in a circle of radius r and let the angular velocity of the particle be $\omega(=v/r)$, then the kinetic energy of this particle is $\frac{1}{2}mv^2$ or $\frac{1}{2}m\omega^2r^2$. Since ω is constant for all the particles of the revolving body at a given instant, the kinetic energy of the whole body is $\frac{1}{2}\omega^2\Sigma mr^2$ or $\frac{1}{2}I\omega^2$, where I represents the summation Σmr^2 over the whole body and is termed the **Moment of Inertia** of the body about the axis of rotation.

Thus **moment of inertia** *of a body about an axis may be defined as the sum of the product of the masses of the particles of the body and the square of their distances from the given axis.*

46. Rotational couple and Moment of Inertia : We know that the mass of any body is the measure of the force required to produce unit acceleration when the movement is of pure translation, and so the moment of inertia of a rigid body rotating about a fixed axis is a measure of the couple which would produce unit acceleration of the angular velocity.

Let m be the mass of any particle situated at a distance ...... the axis and let ϕ be the angular acceleration. Then $r\phi$ is the linear acceleration and therefore the force acting on the particle for producing this acceleration is $mr\phi$. The moment of this force about the axis of rotation is $mr\phi.r$ or $mr^2\phi$.

Since the rigid body is made up of a number of particles of masses m_1, m_2, m_3, etc. situated at distances r_1, r_2, r_3, etc. from the axis of rotation, then the sum of the moments of all the forces acting on the particles is equal to the couple which produces the acceleration of the body.

That is $m_1r_1{}^2.\phi + m_2r_2{}^2.\phi + m_3r_3{}^2.\phi + \ldots = \text{couple} = C$ (say).
or $\phi\Sigma mr^2 = \text{couple} = C$.

The expression for the couple C is usually written as

$$C = I\,\frac{d\omega}{dt} = I.\,\frac{d^2\theta}{dt^2} = I\phi$$

If $\phi = 1$ (unity) $I = C$. *Hence,* **moment of inertia** *of a body about an axis is the moment of the couple for producing unit angular acceleration.*

Again if I denotes the moment of inertia, then $I = \Sigma mr^2 = MK^2$. where M is the mass of the body and K, the *radius of gyration i.e.* the distance from the axis of rotation of the point, at which the whole mass M of the body is supposed to be concentrated.

47. Physical significance of Moment of Inertia : The kinetic energy of the translatory motion of a body of mass M and moving with a velocity v is given by $\frac{1}{2}Mv^2$, whereas the kinetic energy of the rotational motion of the same body is $\frac{1}{2}I\omega^2$.

Comparing these two expressions we find the angular velocity ω as the rotational analogue of the linear velocity v, and I corresponds to M, and represents the effects of mass and its space distribution in rotational dynamics.

Note : The moment of inertia of a body plays exactly the same part in the rotational motion as mass (inertia) does in the translational

motion. The angular acceleration that a force of given moment can produce in a body, free to rotate about an axis, is governed not by the mass of the body, but by its moment of inertia being inversely proportional to it. Again the moment of inertia of a body does not depend only on the mass of the body, but also on the manner of distribution of mass in a body relative to the axis of rotation, and the position of the axis.

We know that heavier *i.e.* more massive a body is, the more difficult it is to change its linear velocity. Small variations in the forces can produce very little changes in the linear velocity of the massive bodies. A body of larger mass is more stable than a body of smaller mass. Similarly in rotational motion, bodies with larger moments of inertia are more stable than those having smaller moments of inertia; since even appreciable variations in the moments of the force would be able to produce relatively small changes in the angular velocity of a body with large moment of inertia. It is for this reason that in big factories and mills huge fly wheels of very large diameter are used for stabilising the rotation of the whole machinery. In the fly-wheels larger part of the total mass is near the rim, so that very large moment of inertia may be obtained.

48. Table showing the relation between rectilinear and rotational dynamics :

Rectilinear Motion		Rotational Motion	
Displacement (linear),	x	Displacement (angular),	θ
Velocity (linear),	$\frac{dx}{dt}$, v	Velocity (angular),	$\frac{d\theta}{dt}$, ω
Acceleration,	$\frac{d^2x}{dt^2}$, f	Acceleration ,,	$\frac{d^2\theta}{dt^2}$, $\frac{d\omega}{dt}$, ϕ
Mass	M	Moment of Inertia,	I, MK^2
Momentum (linear)	Mv	Momentum, (angular)	$I\omega$
Force	$M.\frac{d^2x}{dt^2}$, Mf	Torque,	$T=I\frac{d\omega}{dt}=I\phi$
Kinetic Energy,	$\frac{1}{2}Mv^2$	Kinetic Energy,	$\frac{1}{2}I\omega^2$

49. Units and dimensions of Moments of Inertia : The oment of inertia of a body is expressed in gm-(cm)^2 in C. G. S. and lb-(ft)^2 in F. P. S. units. If M be the mass and K the dius of gyration $I=K^2$, $\therefore$ dimensions of $I=ML^2$.

50. Radius of Gyration : If a rigid body be made to rotate bout a fixed axis with angular velocity ω, its kinetic energy is to $\frac{1}{2}I\omega^2$ where I is the moment of inertia of the body.

If the whole mass of the body M be supposed to be concentrated a point situated at a distance K from the axis of rotation and move ith the same angular velocity, it possesses the same kinetic energy.

This distance K is called the Radius of Gyration.

Thus $\frac{1}{2}I\omega^2 = \frac{1}{2}\Sigma mr^2.\omega^2 = \frac{1}{2}MK^2.\omega^2$ [$\because$ $\Sigma m = M$] whence $\Sigma r^2 = K^2$

The dimension of K is L. For a given body it depends on the position of the axis of rotation.

51. Theorem of Perpendicular Axes : The theorem applies to a laminer body, *i. e.* a body in the form of a thin plate. *It states that if the moments of inertia of a lamina about two axes* OX *and* OY, *in its own plane at right angles to each other are* I_x *and* I_y *respectively, the moment of inertia of the lamina* I_z *about an axis* OZ *passing through the point of intersection of those two axes and perpendicular to the plane of the lamina is given by* $I_z = I_x + I_y$.

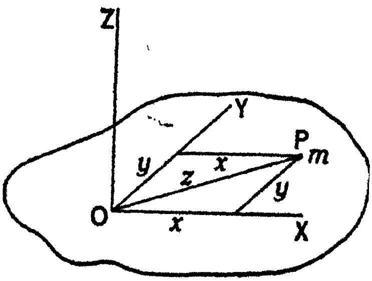


Fig. 27

In figure 27, OX, OY are two mutually perpendicular axes intersecting at O, in the plane of the lamina. OZ is another axis perpendicular to the lamina and passing through the point O.

Consider a particle of the lamina of mass m at P at distances x, y and z from the axes OY, OX and OZ respectively.

Then $I_z = \Sigma mz^2 = \Sigma m(x^2 + y^2)$ [$\because$ $z^2 = x^2 + y^2$] $= \Sigma my^2 + \Sigma mx^2 = I_x + I_y$. Thus the theorem is proved.

52. Theorem of Parallel Axes : *The theorem states that the moment of inertia of a lamina about any axis* CD (Fig. 28) *in its own plane is equal to the sum of the moment of inertia of the lamina about a parallel axis* AB *through its centre of mass* G, *and the product of the mass of the lamina into the square of the distance between the two axes.*

If I_{CD} and I_{AB} represent the moments of inertia about the axes CD and AB respectively, and if M be the mass of the lamina and h the distance between the axes AB and CD, then, by the theorem $I_{CD} = I_{AB} + Mh^2$.

Consider a particle of the lamina of mass m at a point P. Drop a perpendicular PQR to AB and CD meeting them at Q and R respectively.

Let $PQ = x$, and we have $QR = h$.

The moment of inertia about the axis CD is given by

$$I_{CD} = \Sigma m(x+h)^2 = \Sigma m(x^2+h^2+2xh) = \Sigma mx^2 + \Sigma mh^2 + \Sigma 2mx.h$$
$$= \Sigma mx^2 + h^2\Sigma m + 2h\Sigma mx. \quad [\because h \text{ is constant}]$$

The term $\Sigma mx^2 = I_{AB}$; the second term $h^2\Sigma m = Mh^2$, where M = mass of the lamina. The last term is equal to $2h$ times Σmx which is the moment of the mass of the whole lamina about AB. But, since AB passes through the centre of mass of the lamina the moment of the whole mass of the lamina with respect to AB is zero, *i.e.* $\Sigma mx = 0$, and hence $2h\Sigma mx = 0$.

Thus we have,

$$I_{CD} = I_{AB} + Mh^2$$

Thus the theorem is proved.

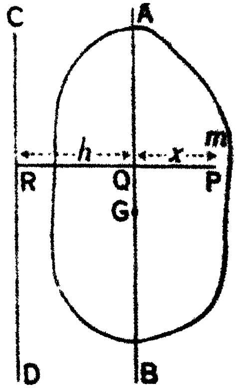


Fig. 28

53. Calculation of Moment of Inertia : The value of the moment of inertia in many cases may be determined by simple integration.

Thus, if dm represents an infinitesimal part of the whole mass and is situated at a distance r from the axis of rotation then,

$$I = \int dm.r^2$$

The limits of integral being suitably chosen to cover the whole of the body.

54. Moment of Inertia of a thin uniform rod or bar :

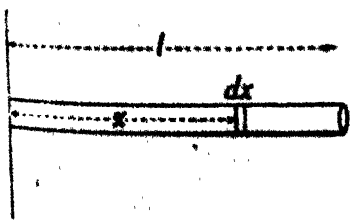


Fig. 29

(A). Moment of inertia of a thin uniform rod (or bar) about an axis through one end perpendicular to its length :

Consider a short element dx of a thin uniform rod at a distance x from the axis (Fig. 29) which is perpendicular to the length and passes through the end of the rod.

Let the mass of the rod be M and let l be its length.

short element $= \frac{M}{l}.dx$

Its moment of inertia about the given axis $= \frac{M}{l} . dx . x^2$

Integrating this expression between the limits $x = 0$ and $x = l$, the moment of inertia of the rod,

$$= \int_0^l \frac{M}{l} x^2 . dx = \frac{M}{l}\left[\frac{x^3}{3}\right]_0^l = \frac{M}{l}\left[\frac{l^3}{3} - 0\right] = \frac{1}{3} M l^2 .$$

Then $Mk^2 = \frac{1}{3} Ml^2$

$\therefore$ Radius of gyration (K) about the same axis $= \frac{1}{\sqrt{3}} . l$.

(B). Moment of inertia of a thin uniform rod about an axis passing through its centre and perpendicular to its length :

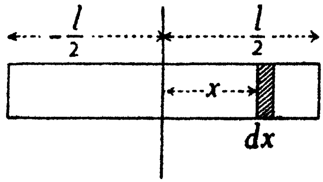


Fig. 30

Consider a thin uniform rod of mass M and length l. Let dx be a short element of length dx at a distance x from the axis through its length. (Fig. 30).

Then the mass of the element $dx = \frac{M}{l} . dx$.

Moment of inertia of the element about the given axis $= \frac{M}{l} dx . x^2$.

Integrating this expression between the limits $x = 0$ and $x = l/2$ and multiplying the result by 2 in order to include each half of the rod, the moment of inertia of the whole rod

$$= 2\int_0^{l/2} \frac{M}{l} . x^2 . dx = \frac{2M}{l}\left[\frac{x^3}{3}\right]_0^{l/2} = \frac{2M}{l}\left[\frac{l^3}{24} - 0\right] = \frac{1}{12} Ml^2$$

[**Alternately**, taking limits as $x = -l/2$ and $x = +l/2$, the

$$\text{moment of inertia} = \int_{-l/2}^{l/2} \frac{M}{l} . x^2 . dx = \frac{M}{l}\left[\frac{x^3}{3}\right]_{-l/2}^{l/2}$$

$$\frac{M}{l}\left[\frac{l^3}{24} + \frac{l^3}{24}\right] = \frac{1}{12} Ml^2 \text{]}$$

The radius of gyration about the given axis (K) $= l/\sqrt{12}$

55. Moment of inertia of a thin rectangular lamina of sides a and b, about an axis through its centre of mass and perpendicular to the lamina :

Let the rectangular (Fig. 31) lamina of mass M be divided into strips of breadth dx parallel to the axis through its centre G and parallel to the side a and let one of them be at a distance x from this axis.

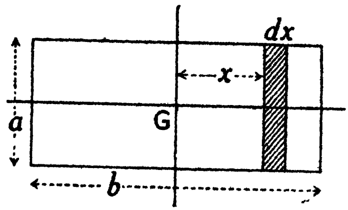


Fig. 31

The mass of the strip $= \mathrm{M}\dfrac{dx}{b}$. Its moment of inertia about an axis through the centre and parallel to the side $a = \mathrm{M}\dfrac{dx}{b}x^2$

$\therefore$ Moment of inertia of the whole lamina about this axis, as in Art. 54 (b).

$$= 2\int_0^{\frac{b}{2}} \mathrm{M}\frac{dx}{b}x^2 = \mathrm{M}\frac{b^2}{12}$$

Similarly, the moment of inertia of this rectangular lamina about an axis through its centre and parallel to the side $b = \dfrac{\mathrm{M}a^2}{12}$.

Therefore, the moment of inertia of the rectangular lamina of sides a and b about an axis through its centre of mass and perpendicular to the plane of the lamina is, according to the principle of perpendicular axes $= \dfrac{\mathrm{M}b^2}{12} + \dfrac{\mathrm{M}a^2}{12} = \mathrm{M}\left(\dfrac{a^2+b^2}{12}\right)$

The radius of gyration about the axis through G perpendicular to its plane $= \sqrt{(a^2+b^2)/12}$.

Corollary : Moment of inertia of a **rectangular bar** of length a, breadth b and thickness c, about an axis through its centre of mass and parallel to the side c (thickness) $= \dfrac{a^2+b^2}{12}$ M, where M is the mass of the bar.

For, the rectangular bar may be supposed to be made of rectangular lamina of equal thickness and of sides a and b. The moment of inertia of each lamina about an axis perpendicular to its plane (*i.e.* parallel to the side c) and through its centre of mass $= \dfrac{a^2+b^2}{12}.m$, where $m =$ mass of each lamina. Evidently, moment of inertia of the bar about an axis parallel to side c and through its centre of mass $= \Sigma\dfrac{a^2+b^2}{12}m = \dfrac{a^2+b^2}{12}\Sigma m = \dfrac{a^2+b^2}{12}$ M.

Similarly, if the axis be parallel to side b, moment of inertia will be $=\frac{a^2+c^2}{12}$M.

56. Moment of inertia of a disc about an axis through its centre and perpendicular to its plane :

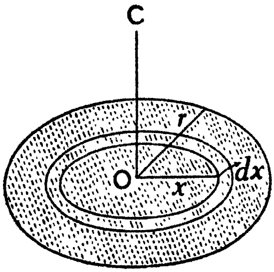


Fig. 32

Let M be the mass of the disc (Fig. 32) of radius r. Then mass of the disc per unit area is $M/\pi r^2$. Consider in it a concentric ring of width dx and of radius x.

Then mass of the ring $=\frac{2\pi x.dx.M}{\pi r^2}$

Its moment of inertia about an axis through the centre and perpendicular to the plane $=\frac{2\pi x.dx}{\pi r^2}M.x^2=\frac{2M}{r^2}x^3.dx$

Moment of inertia of the whole disc about this axis

$$=\int_0^r \frac{2M}{r^2}.x^3dx=\frac{2M}{r^2}\int_0^r x^3.dx=\frac{2M}{r^2}\left[\frac{x^4}{4}\right]_0^r=\frac{2M}{r^2}.\frac{r^4}{4}=\frac{Mr^2}{2}$$

Radius of gyration $(K)=r/\sqrt{2}$

56a. Moment of inertia of a disc about a diameter :

Let I be the required moment of inertia about a diameter, then I is also the moment of inertia about a perpendicular diameter.

Therefore, by the principle of perpendicular axes the moment of inertia of the disc about an axis through its centre and perpendicular to its plane $=I+I=\frac{Mr^2}{2}\quad\therefore\quad I=\frac{Mr^2}{4}$.

Here, radius of gyration $(K)=r/2$.

56b. Moment of inertia of a circular disc about a tangent :

If I be the required moment of inertia, then by the theorem of parallel axes and from the above case, $I=Mr^2/4+Mr^2=5Mr^2/4$.

57. Moment of inertia of a flat ring about an axis through its centre and perpendicular to its plane :

A flat annular ring or disc (Fig. 33) is just a circular disc from which a co-axial circular portion has been removed producing a

concentric circular hole in it. Let r and R be the inner and the outer radius of the ring respectively, and let M be its mass.

Then the face area of the ring = face area of the disc of radius R − face area of the disc of radius r, *i.e.* area of the hole $= \pi R^2 - \pi r^2 = \pi(R^2 - r^2)$

Hence, mass per unit area of the flat ring $= \dfrac{M}{\pi(R^2 - r^2)}$.

Now, consider a small annular ring of inner radius x and outer radius $x + dx$. The face area of this ring $= 2\pi x.dx$

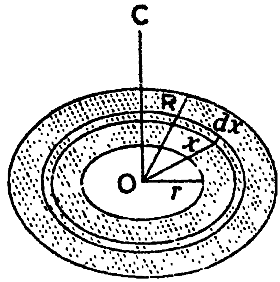


Fig. 33

And its mass $= \dfrac{M}{\pi(R^2 - r^2)}.2\pi x.dx = \dfrac{2Mx}{(R^2 - r^2)}.dx.$

The moment of inertia of this ring about the given axis

$$= \frac{2Mx}{(R^2 - r^2)}.dx.x^2$$

∴ Moment of inertia of the whole flat ring about this axis

$$= \int^{R} \frac{2M}{(R^2 - r^2)}.x^3.dx \quad \frac{2M}{(R^2 - r^2)}\left[\frac{x^4}{4}\right]^{R}$$

$$= \frac{2M}{(R^2 - r^2)}\left[\frac{R^4 - r^4}{4}\right] = \frac{2M}{(R^2 - r^2)} \times \frac{(R^2 + r^2)(R^2 - r^2)}{4}$$

$= \frac{1}{2}M(R^2 + r^2)$. Radius of gyration $(K) = \sqrt{\dfrac{R^2 + r^2}{2}}$

Corollary : **(1)** Moment of inertia of a flat ring about a diameter by 56a above $= \frac{1}{4}M(R^2 + r^2)$

(2) Moment of inertia of a thin circular ring (*i.e.* of negligible width) about an axis through its centre and perpendicular to its plane.

Moment of inertia in case of a flat ring $= \frac{1}{2}M(R^2 + r^2)$. For thin ring put $R = r$, radius of the ring. Then the required moment of inertia

$$= \tfrac{1}{2}M(r^2 + r^2) = Mr^2.$$

(3) Moment of inertia of a thin circular ring about a diameter $= \frac{1}{2}Mr^2$.

(4) Moment of inertia of a flat ring about a tangent is equal to $\frac{1}{4}M(R^2 + r^2) + MR^2 = \frac{1}{4}M(5R^2 + r^2)$. (By parallel axes theorem).

(5) Moment of inertia of a thin ring about a tangent $= \frac{1}{2}Mr^2 + Mr^2 = \frac{3}{2}Mr^2$. (By parallel axes theorem).

58. Moment of inertia of a solid circular cylinder about an axis passing through its centre of gravity and perpendicular to its length :

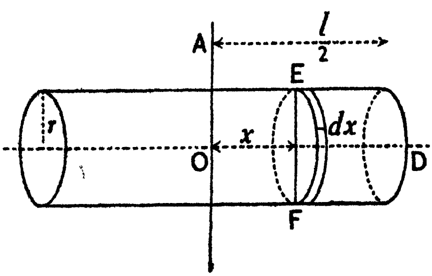


Fig. 34

Let, M l and r be the mass, length and radius of cross section, of the cylinder, respectively. Let AO be the axis through its centre O and perpendicular to its own axis along OD.

Consider a very thin disc EF (Fig. 34) of the cylinder, of thickness dx at a distance x from the axis of rotation through AO, and perpendicular to the axis of the cylinder. Then, the mass of the disc $= \frac{M}{l}.dx$, since M/l is mass per unit length.

The moment of inertia of the disc about a diameter EF parallel to AO $= \frac{1}{4}.\frac{M}{l}.dx \times r^2$ where r is also the radius of the disc. Now, as the disc is thin, EF may be considered as an axis passing through the centre of the disc.

Then, the moment of inertia of the disc about the axis AO parallel to EF, by the theorem of parallel axes, is

$$= \frac{1}{4}\frac{M}{l}dx.r^2 + \frac{M}{l}dx.x^2.$$

The moment of inertia of the whole cylinder about AO can be found by integrating the above expression between the limits $x=0$ and $x=l/2$, and multiplying by 2 to include both sides.

$\therefore$ Moment of inertia of the cylinder about an axis through AO

$$= 2\int_0^{l/2}\left(\frac{1}{4}\frac{M}{l}r^2.dx + \frac{M}{l}.x^2.dx\right) = \frac{1}{2}\frac{M}{l}r^2\int_0^{l/2} dx + \frac{2M}{l}\int_0^{l/2} x^2.dx.$$

$$= \frac{1}{2}\frac{M}{l}r^2\Big[x\Big]^{l/2} + \frac{2M}{l}\left[\frac{x^3}{3}\right]^{l/2}$$

$$= \frac{1}{2}\frac{M}{l}.r^2.\frac{l}{2} + \frac{2M}{l}\frac{l^3}{24} = \frac{M}{4}r^2 + \frac{M}{12}l^2 = M\left(\frac{l^2}{12} + \frac{r^2}{4}\right)$$

Radius of gyration (K) $= \sqrt{\frac{l^2}{12} + \frac{r^2}{4}}$

58a. Moment of inertia of a solid circular cylinder about its own axis :

The adjoining figure represents a circular cylinder having r as the radius of the cross-section with a co-axial cylindrical shell having a cross-section of radius x and width dx.

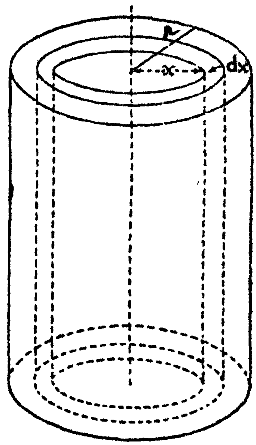


Fig. 35

The sectional area of the cylinder $= \pi r^2$ and that of the shell $= 2\pi x.dx$.

Let M be the mass of the cylinder. Therefore the mass of this

$$\text{shell} = \frac{M}{\pi r^2}\,2\pi x.dx = \frac{2M}{r^2}.xdx$$

The moment of inertia of the shell about the given axis

$$= \frac{2M}{r^2}.x.\,dx.x^2 = \frac{2M}{r^2}x^3dx$$

The moment of inertia of the cylinder about this axis

$$\int_0^r \frac{2M}{r^2}.x^3.dx = \frac{2M}{r^2}\left[\frac{x^4}{4}\right]_0^r = \frac{2M}{r^2}.\frac{r^4}{4} = \frac{Mr^2}{2}.$$

Radius of gyration (K) of the cylinder $= \frac{r}{\sqrt{2}}$.

Alternative method : The cylinder may be supposed to be divided into *thin discs* perpendicular to its own axis. The moment of inertia of each disc about the axis of the cylinder $= \frac{1}{2}mr^2$, where m is the mass of each disc. Hence, moment of inertia of the whole cylinder $= \Sigma\frac{1}{2}mr^2 = \frac{1}{2}r^2\Sigma m$ (since r is same for all discs) $= \frac{1}{2}Mr^2$, where M is mass of the cylinder.

58b. Moment of inertia of a hollow cylindrical shell about its own axis :

Let the internal and external radii of the shell be r and R respectively. The shell may be supposed to be divided into a large number of *flat rings* each of mass m. Then moment of inertia of each flat ring about the axis of the cylinder $= \frac{1}{2}m(R^2 + r^2)$

$\therefore$ Moment of inertia of the whole shell is $= \Sigma\frac{1}{2}m(R^2 + r^2)$
$= \frac{1}{2}(R^2 + r^2)\Sigma m = \frac{1}{2}M(R^2 + r^2)$, where M = mass of the shell.

Radius of gyration about the axis $= \sqrt{(R^2 + r^2)/2}$.

59. Moment of inertia of a solid sphere about a diameter :

Let M be the mass and r the radius of the sphere of which AB is the diameter. (Fig. 36).

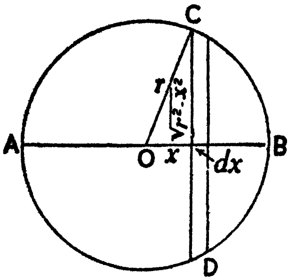


Fig. 36

The volume of the sphere $=4\pi r^3/3$. Then the mass of unit volume of the

$$\text{sphere}=\frac{M}{4\pi r^3/3}=\frac{3M}{4\pi r^3}.$$

Consider a thin circular slice CD at a distance x from the centre, and of thickness dx and having its plane perpendicular to the diameter AB.

The radius of the slice $=\sqrt{r^2-x^2}$

The volume of the slice $=\pi(r^2-x^2).\,dx.$

The mass of the slice $=\pi(r^2-x^2).\,dx.\,\dfrac{3M}{4\pi r^3}$ The moment of inertia of the slice about the diameter AB which is perpendicular to the slice and passes through its centre

$$=\frac{3\pi M(r^2-x^2)}{4\pi r^3}.\ \frac{(r^2-x^2)}{2}dx$$

$\therefore$ Moment of inertia of the sphere about AB, considering two halves of it, OB and OA

$$=2\times\frac{3\pi M}{8\pi r^3}\int_{x=o}^{x=r}(r^2-x^2)^2.dx.=\frac{3M}{4r^3}\int_{x=o}^{x=r}(r^4-2r^2x^2+x^4)dx$$

$$=\frac{3M}{4r^3}\left[r^4x-\frac{2}{3}r^2x^3+\frac{x^5}{5}\right]=\frac{3M}{4r^3}\left[r^5-\frac{2}{3}r^5+\frac{r^5}{5}\right]$$

$$=\frac{3M}{4r^3}\cdot\frac{8r^5}{15}=\frac{2}{5}Mr^2.$$

The radius of gyration about the axis $(K)=\sqrt{\dfrac{2}{5}}.r$

Corollary : Moment of inertia of a solid sphere about any tangent. By the theorem of parallel axes, the required moment of inertia $Mr^2/5+Mr^2=7Mr^2/5.$

60. Moment of inertia of a hollow sphere (thin-walled) about any diameter :

Let ACBD represent a section of the spherical shell of mass M and radius R, through its centre O. It is required to find its moment of inertia about the diameter AB. (Fig. 37).

Surface area of the shell $= 4\pi R^2$

$\therefore$ Mass per unit area of the shell $= \dfrac{M}{4\pi R^2}$.

Let us consider a very thin circular slice of the shell lying between two planes EG and FH perpendicular to AB and at distance x and $x+dx$ from the centre O.

The slice is simply a ring of radius $y(=EN)$ and width EF (not NQ which is equal to dx).

Since width is very small, area of the ring $= 2\pi y \times EF$.

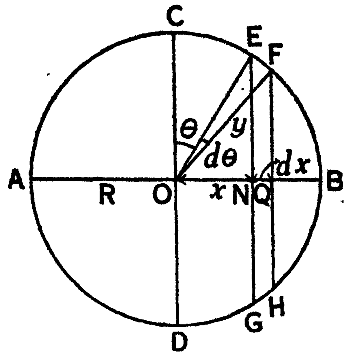


Fig. 37

$$\text{Its mass} = 2\pi y \times EF \times \frac{M}{4\pi R^2}$$

Join OE and OF and suppose $\angle COE = \theta$ and $\angle EOF = d\theta$.

Then $y = EN = OE \sin(90-\theta) = OE \cos\theta = R\cos\theta$, and $x = R\sin\theta$

Differentiating x with respect to θ, $dx = R\cos\theta.d\theta = y.d\theta$.

and $EF = OE \times d\theta = Rd\theta$.

(Since arc = radius × angle subtended by the arc)

The area of the ring $= 2\pi y \times EF = 2\pi y \times Rd\theta$

$$\text{Mass of the ring} = 2\pi y \times Rd\theta \times \frac{M}{4\pi R^2} = 2\pi.yd\theta.R\frac{M}{4\pi R^2}$$

$$= \frac{2\pi dx.R.M}{4\pi R^2} = \frac{Mdx}{2R} \qquad [\because \; yd\theta = dx]$$

Moment of inertia of the ring about the diameter AB

$$= \frac{Mdx}{2R} \times y^2 = \frac{Mdx}{2R}(R^2 - x^2) \qquad [\because \; y^2 = R^2 - x^2]$$

Hence, moment of inertia of the sphere about AB, considering two halves OB and OA

$$2\int^{R} \frac{M\,dx}{2R}(R^2 - x^2) = \frac{M}{R}\int^{R} (R^2 - x^2)dx = \frac{M}{R}\left\{\int^{R} R^2 dx - \int^{R} x^2 dx\right\}$$

$$= \frac{M}{R}\left[R^2x - \frac{x^3}{3}\right]_o^R = \frac{M}{R}\left[R^3 - \frac{R^3}{3}\right] = \frac{M}{R}\times\frac{2}{3}R^3 = \frac{2}{3}MR^2$$

Radius of gyration $(K) = \sqrt{2/3}.R$

61. Perpendicular Axes Theorem of Three-dimensional body :

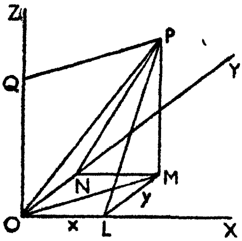


Fig. 38

Let a particle of mass m be at point P having co-ordinates (x, y, z) so that $OL = x$, $ML = y$ and $PM = z$ (Fig. 38). Draw PL, PN and PQ perpendiculars to the three axes OX, OY and OZ respectively.

If I_x, I_y and I_z be the moments of inertia of particle about these three mutually perpendicular axes OX, OY and OZ, then

$$I_x = \Sigma m.PL^2 = \Sigma m(y^2 + z^2) \quad \ldots \quad (1)$$

$$I_y = \Sigma m.PN^2 = \Sigma m(z^2 + x^2) \quad \ldots \quad (2)$$

$$I_z = \Sigma m.PQ^2 = \Sigma m.OM^2 = \Sigma m(x^2 + y^2) \quad (3)$$

$$I_o = \Sigma m.PO^2 = \Sigma m(x^2 + y^2 + z^2) \quad \ldots \quad (4)$$

From above, $I_x + I_y + I_z = 2\Sigma m(x^2 + y^2 + z^2) = 2I_o$ ... ... (5)

Here I_o is the summation $\Sigma m(x^2 + y^2 + z^2)$ or Σmr^2, the moment of inertia about the origin O, where $PO^2 = r^2 = x^2 + y^2 + z^2$.

62. Applications of the above Theorem :

(*a*) *Moment of inertia of a thin spherical shell about a diameter :*

Let I_x, I_y and I_z be its moment of inertia about three mutually perpendicular diameters. Then from symmetry $I_x = I_y = I_z$.

Now, since all parts of the shell are equidistant from the centre of the shell, $I_0 = Mr^2$, where M is the mass of the shell and r its radius.

$$\therefore \quad I_x + I_y + I_z = 3I_x = 2I_o = 2Mr^2$$

$$\therefore \quad I_x = \tfrac{2}{3}Mr^2.$$

(*b*) *Moment of inertia of a solid sphere about a diameter :*

Divide the sphere into thin concentric shells and consider one of radius x and thickness dx. If ρ be the density of the sphere, the mass of the shell is equal to $4\pi x^2.dx.\rho$.

Hence, the moment of inertia of the sphere about the centre

$$I_0 = \int_0^r 4\pi x^2.dx.\rho.x^2 = \int_0^r 4\pi\rho.x^4.dx = 4\pi\rho\left[\frac{x^4}{5}\right]_0^r$$

$$= \frac{4\pi\rho}{5}.r^5$$

If $I_x = I_y = I_z$, where each is the moment of inertia of the solid sphere about a diameter, then by relation (5) of Art. 61

$$3I_x = 2I_o \text{ or } I_x = \frac{2}{3}I_o = \frac{2}{3}\cdot\frac{4\pi\rho}{5}r^5 = \frac{4}{3}\pi r^3\rho.\frac{2}{5}r^2 = \frac{2}{5}Mr^2$$

Since $4\pi r^3\rho/3 = M$ the mass of the solid sphere.

63. Routh's Rule : The rule states that if the axis about which the moment of inertia is required be an axis of symmetry passing through the centre of mass of the body, the moment of inertia in simple cases is generally given by,

Mass of the body $\times \dfrac{\text{Sum of squares of perpendicular semi-axes}}{3,\ 4 \text{ or } 5}$ in which the number in the denominator will be 3, 4 or 5 depending on whether the body is a rectangle, ellipse or circle, or ellipsoid or sphere.

Thus in case of a solid sphere of mass M and radius r, the moment of inertia about a diametrical axis $= M\dfrac{r^2+r^2}{5} = \dfrac{2}{5}Mr^2$.

64. Kinetic energy of rotation of body : Suppose a body is rotating about an axis passing through its centre of mass and the energy of rotation of the body is the kinetic energy of the body.

Let the body be made up of a number of particles having masses m_1, m_2, m_3, etc. and situated at distances r_1, r_2, r_3, etc. from the axis of rotation.

Then the total kinetic energy or the energy of rotation

$= \frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2\omega^2 + \frac{1}{2}m_3r_3^2\omega^2 + \text{etc.}$

$= \frac{1}{2}\omega^2\Sigma mr^2$ [Since the linear velocity of the

$= \frac{1}{2}\omega^2 I$ particle is $r\omega$ where ω is the angular

$= \frac{1}{2}MK^2\omega^2$. velocity.]

$= \frac{1}{2}I\omega r^2$.

65. Total energy of a moving body : When a body is rotating and at the same time moving with its centre of mass in the forward direction with a certain velocity, the total energy of the body is equal to the energy of translation plus the energy of rotation of the body.

Energy of translation $= \frac{1}{2}Mv^2$, and energy of rotation $= \frac{1}{2}MK^2\omega^2$, where M is the mass of the body ; v the velocity of the centre of mass ; ω the angular velocity and K the radius of gyration of the body.

Hence, total kinetic energy of the body $= \frac{1}{2}Mv^2 + \frac{1}{2}MK^2\omega^2$.

66. Acceleration of a body rolling down an inclined plane : Suppose a spherical body of (Fig. 39) mass M and radius r roll down an inclined plane freely, the inclination of the plane to the horizontal being θ. As the plane is assumed to be rough, there is no slipping and hence, no work is done by friction.

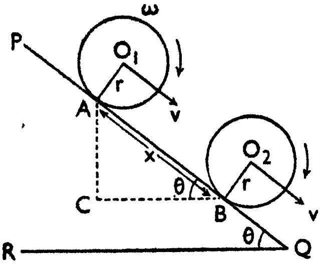


Fig. 39

Let v = velocity acquired by the body after traversing a distance AB along the plane,

$\therefore$ The vertical distance of descent AC = AB $\sin\theta$. = S. $\sin\theta$, where S = AB (say).

Then, potential energy lost by the body = MgS. $\sin\theta$.

This is evidently equal to the kinetic energy gained by the body. Again, K. E of rotation of the body $= \frac{1}{2}I\omega^2$, where ω is its angular velocity about a diametral axis through its centre.

Now, since the centre of mass has a linear velocity v, the kinetic energy of translation $= \frac{1}{2} Mv^2$.

Thus total kinetic energy gained $= \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2$

$$= \frac{1}{2}MK^2\omega^2 + \frac{1}{2}Mv^2 = \frac{1}{2}Mv^2(K^2/r^2 + 1) \quad \left[\because \ \omega = \frac{v}{r}\right].$$

But gain in K. E. = loss in its P. E.

$$\therefore \ \frac{1}{2}Mv^2(K^2/r^2+1) = Mg.\sin\theta.S \quad \text{or} \quad v^2 = \frac{2r^2}{K^2+r^2}\, g \sin\theta.S$$

In Kinetic relation $v^2 = 2f.S$ for a body starting form rest, where f = acceleration ; hence, in above case acceleration is given by

$$f = \frac{}{K^2+r^2}\, g\sin\theta.$$

Thus for a given inclination, acceleration is proportional to $r^2/(K^2+r^2)$, g being constant.

It follows therefore that greater the value of K compared to r the smaller the acceleration of the body rolling down the plane and hence, the greater is the time it takes in rolling down along it. If K be smaller compared to r, it will roll down faster.

67. Illustrative examples : A solid sphere for which $K^2 = 2r^2/5$ will roll down faster than a disc of same radius for which $K^2 = r^2/2$: and similarly, a disc will roll down faster than a loop of same radius r for which K^2 is r^2.

K^2 for a hollow sphere about its diameter $= 2r^2/3$. and that for solid sphere of same radius $= 2r^2/5$. Hence, K^2 for a hollow sphere about its diameter is greater than that for a solid sphere of the same

radius and mass. Thus a solid and a hollow sphere of same mass and radius can be distinguished from each other by allowing them to roll down an inclined plane. The one rolling down faster will be the solid sphere and the other rolling slowly will be the hollow sphere. A hollow and solid cylinder can be similarly identified.

68. Angular Momentum : Torque : Let m be the mass of a particle revolving along the circumference of a circle of radius r, and let the linear velocity of the particle at any instant be v.

The momentum of the particle at the instant $= mv = m\omega r$.

[Since $v = \omega r$, where ω is the angular velocity.]

Then the moment of this momentum about an axis passing through the centre and perpendicular to the plane of the circle

$$= \omega mr.r = \omega mr^2.$$

For all particles constituting the body, the **angular momentum** or the **moment of momentum** of the body, as it is called, is

$$= \omega.\Sigma mr^2 = \omega . \text{I}.$$

But we know that the moment of inertia of the body is equal to the couple or torque for producing unit angular acceleration.

Therefore, if ϕ be the angular acceleration

$$\phi\Sigma mr^2 = \text{torque}, \quad \frac{d\omega}{dt}\Sigma mr^2 = \text{torque}, \quad \text{I}\frac{d\omega}{dt} = \text{torque} \quad \ldots \quad (1)$$

That is, torque acting on the body is equal to the rate of change of moment of momentum or of angular momentum of the body.

The equation (1) may be written as

$$\text{I}\phi = c, \text{ where } \phi = \frac{d\omega}{dt}, \text{ and } c = \text{torque, or I}.\ d\omega = c.\ dt$$

$$\text{I}\omega = \int_0^t c.\ dt.$$

If the time is extremely short and the couple great, $\int_0^t c.\ dt$ is called an **angular impulse.**

69. Units and dimensions of angular momentum : In C. G. S. units, angular momentum is expressed in gm. (cm.)^2/sec. and in F. P. S. units it is expressed in lb. (ft.)^2/sec.

Angular momentum $= \text{I}\omega$

$$\text{Its dimensions are} = \text{ML}^2 \times \frac{1}{\text{T}} = \text{ML}^2\text{T}^{-1}$$

since the dimension of I is ML^2, and that of ω is $\frac{1}{\text{T}}$

70. Angular Simple Harmonic Motion : When a body rotates about an axis under a couple which is proportional to the

angular displacement from a certain position, the body executes an angular S. H. M.

If the couple acting on the body be related to its angular displacement by the equation, couple $= \tau\theta$

the period of the angular S. H. M. $t = 2\pi\sqrt{\frac{I}{\tau}}$

where I is the moment of inertia of the body and τ the couple for unit twist.

71. Torsional Oscillation : If a body is suspended in such a way that its displacement about a given axis produces a couple tending to produce further displacement, then this couple is proportional to the displacement and the body is in equilibrium when the opposing couple is equal in magnitude to the displacing couple.

Since the restoring couple c is proportional to the angular displacement, we have

Torque $c = \tau\theta$, where τ is the couple for unit twist.

But $\quad c = I.\frac{d\omega}{dt} = I.\frac{d^2\theta}{dt^2} \quad \therefore \quad I\frac{d^2\theta}{dt^2} = -\tau\theta$

The sign is $-$ve, since the displacing and restoring couples act in opposite directions.

Thus the equation of motion is given by $\frac{d^2\theta}{dt^2} + \frac{\tau}{I}\theta = 0 \qquad (1)$

The angular acceleration is proportional to the angular displacement and so the motion of the body is Simple Harmonic and the solution of the equation (1) is $\theta = \theta_0 \sin\frac{2\pi t}{T}$, where θ_0 is the maximum amplitude and T, the period of oscillation

$$\therefore \quad \frac{d\theta}{dt} = \frac{2\pi}{T}.\theta_0 \cos\frac{2\pi t}{T}; \quad \frac{d^2\theta}{dt^2} = -\frac{4\pi^2}{T^2}\theta_0 \sin\frac{2\pi t}{T}$$

By substituting the values of θ and $\frac{d^2\theta}{dt^2}$ in (1) we have the period given by

$$T = 2\pi\sqrt{\frac{I}{\tau}}$$

72. Determination of Moment of Inertia of a body : A rectangular bar of known moment of Inertia I_1 is suspended by a fine metal wire and made to oscillate about an axis passing through

the middle point of the bar and perpendicular to it. This axis is evidently along the suspension wire.

The period of oscillation is given by $T_1 = 2\pi\sqrt{\frac{I_1}{\tau}}$, ...(1) where τ is the couple for unit twist of the suspension wire.

A second body whose moment of inertia I_2 is to be determined is tied up to the rectangular bar and the combination made to oscillate.

The period of oscillation of the combined system is given by

$$T_2 = 2\pi\sqrt{\frac{I_1 + I_2}{\tau}}, \quad T_2^2 = \frac{4\pi^2(I_1 + I_2)}{\tau}. \text{ From (1) } T_1^2 = \frac{4\pi^2 I_1}{\tau}$$

$$\therefore \quad \frac{T_2^2}{T_1^2} = \frac{I_1 + I_2}{I_1} \quad i.e. \quad I_2 = \frac{T_2^2 - T_1^2}{T_1^2} \cdot I_1$$

Knowing T_1, T_2 and I_1 the value of I_2 can be found out.

QUESTIONS

1. Explain moment of inertia, radius of gyration and moment of momentum and write down their dimensions and the units in which they are measured. [C. U. 1944, '46, '51, '56, '58]

2. Find an expression for the moment of inertia of a thin rod about an axis passing through its ends at right angles to the length of the rod. [C. U. 1940, '58]

3. State and explain the theorem of perpendicular axes.

Derive expressions for the moment of inertia of a circular disc about (i) an axis perpendicular to its plane passing through the centre and (ii) about a diameter.

4. Define the moment of inertia of a body about a given axis and the radius of gyration.

Find the moment of inertia of a uniform rod of length b and mass M about a transverse axis through one end. [C. U. 1958]

5. Calculate the moment of inertia of a circular cylinder about its own axis. What is the radius of gyration about that axis? [C. U. 1946]

6. Calculate the moment of inertia of a thin circular ring of mass M and radius a about an axis in the plane of the ring.
   (i) When the axis passes through the centre.
   (ii) When it is at a distance from the centre. [C. U. 1951]

7. Given the moment of inertia of a body about an axis through its centre of gravity, determine its value about any other parallel axis. [C. U. 1944]

8. Prove that the period of torsional vibration of a rod suspended by a wire is given by $T = 2\pi\sqrt{I/\tau}$ where I = moment of inertia and τ = restoring torque.

How can this method be used for finding out the moment of inertia of any body? [C. U. 1941]

9. Calculate the moment of inertia of solid sphere about a diameter.

You are given two spheres of same mass and size, one being hollow and made of a substance of higher density, while the other is solid but made of a substance of lower density. Explain how you will identify the hollow one. [C. U. 1954]

EXAMPLES

1. Shew that the kinetic energy of a thin rod of length l and mass m per unit length rotating about an axis through the middle point and perpendicular to its length with angular velocity 'ω' is $\frac{1}{24} m\omega^2 l^3$. [C. U. 1944]

Kinetic energy of rotation $= \frac{1}{2}I\omega^2$. Here $I = \frac{1}{12}.Ml^2 = \frac{1}{12}ml \times l^2 = \frac{1}{12}ml^3$

$$\therefore \text{ K. E.} = \frac{1}{24}.ml^3\omega^2.$$

2. A rod, weighing 10 lb and of length 3 ft. revolves 50 times in one minute about one of its ends. Find its kinetic energy. [D. U. 1947]

$$\text{K.E.} = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}I.(2\pi n)^2 = \tfrac{1}{2}.\tfrac{1}{3}Ml^2.(2\pi n)^2 \quad [\because \quad I = \tfrac{1}{3}Ml^2]$$

$$= \tfrac{1}{6} \times 10 \times 3^2 \times (2 \times \tfrac{22}{7} \times \tfrac{50}{60})^2 = 411{\cdot}5 \text{ foot poundals} = 12{\cdot}8 \text{ ft.-pounds.}$$

3. A hoop 3 feet in diameter weighs 2 lbs. Find the kinetic energy of the hoop when it is rolling on a horizontal road at a linear speed of 7 miles per hour. [D. U. 1945]

$$\text{K. E.} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 \text{ and } I = mr^2$$

$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}mr^2.\frac{v^2}{r^2} = mv^2 = 2\left(\frac{7 \times 1760 \times 3}{60 \times 60}\right)^2 = 210{\cdot}8 \text{ ft.-poundals.}$$

4. A sphere of mass 50 gms. and diameter 2 cms. rolls without slipping with a velocity of 5 cms./sec. Calculate the total kinetic energy in ergs.

Mass of the sphere $(M) = 50$ gms. Its radius $(r) = 1$ cm. Moment of inertia of a solid sphere about a diameter $(I) = \frac{2}{5}Mr^2$

$$\therefore \quad I = \tfrac{2}{5} \times 50 \times 1^2 = 20 \text{ gm./cm}^2.$$

The centre of mass of the sphere moves with a velocity $(v) = 5$ cms./sec.

Total K. E. of the sphere = K. E. of rotation + K. E. of translation

$$= \tfrac{1}{2}I\omega^2 + \tfrac{1}{2}Mv^2 = \tfrac{1}{2} \times 20 \times \left(\frac{v}{r}\right)^2 + \tfrac{1}{2} \times 50 \times (5)^2 = \tfrac{1}{2} \times 20 \times \frac{5^2}{1^2} + 625 = 250 + 625 = 875 \text{ Ergs.}$$

CHAPTER IV

ACCELERATION DUE TO GRAVITY

73. Acceleration due to Gravity : Galileo first demonstrated that at any given place, all bodies, irrespective of their mass and nature falling freely in vacuum, will have the same acceleration. This acceleration is termed acceleration due to gravity, since it is caused by the gravitational attraction on the body by the earth towards its centre. It is represented by the letter "g".

The value of g changes from place to place being maximum at the poles and minimum at the equator. It also decreases with height above earth's surface. Inside the earth, it gradually decreases as the distance from the earth's centre decreases, being finally zero at the centre of the earth. For all practical purposes, g is taken to be 32 ft./sec^2. in F. P. S. system, and 981 cm./sec^2. in the C. G. S. system.

As bodies fall very quickly to the surface of the earth when dropped freely, due to rather large value of g, direct measurement of

"g" with any great accuracy is not possible. The value of g is therefore found indirectly by simple or compound pendulum.

74. Simple Pendulum : The definition and equation of simple pendulum have been already given in article 29(A). The motion of a simple pendulum is simple harmonic and isochronous, *i.e.*, the periodic time is independent of its amplitude, for small swings and is given by the expression $t=2\pi\sqrt{l/g}$, where l=length of the pendulum *i.e.* the distance between its point of suspension and the centre of gravity of the bob.

Squaring the above expression for t, we have

$$t^2=4\pi^2.\frac{l}{g} \quad \text{whence} \quad g=4\pi^2.\frac{l}{t^2}.$$

Hence, knowing l and t, the value of g at a given place can be ound out. As a simple pendulum is only an *ideal* one which can ıardly ever be obtained in practice, method of simple pendulum is ıot very accurate.

Note : A pendulum whose time-period is two seconds, is called a second's pendulum. Its equation is $2=2\pi\sqrt{l/g}$. Or $1=\pi\sqrt{l/g}$.

75. Compound Pendulum : While studying the period of oscillation of a simple pendulum, we have defined it to be a single heavy particle suspended by a weightless thread. Such a pendulum is a heavy spherical bob suspended by a thin string, the centre of gravity of which being coincident with the centre of the bob.

If the bob be fairly heavy and the suspending thread thick the centre of gravity of the pendulum will not coincide with the centre of the spherical body.

It is for these reasons that the value of 'g' has been determined accurately by oscillating a compound pendulum *i.e.* a rigid body about a fixed axis.

A compound pendulum is a rigid body suspended so as to be free to rotate about a horizontal axis.

If the centre of gravity of the body dose not lie on the axis, the body will oscillate to and fro about its position of equilibrium. Since the particles in the rigid body are situated at different distances from the axis of rotation, the periods of oscillations will be different for different particles and it will be possible to find in the body of any shape whatsoever two points asymmetrically situated with respect to its centre of gravity about which the body oscillates in one and the same period.

The distance between these two points is equal to the length of an equivalent simple pendulum.

To determine the value of 'g' by a compound pendulum, the length of the equivalent simple pendulum is first determined and then the value of g is calculated from the expression $T=2\pi\sqrt{\frac{L}{g}}$, where T is the time period and L, the length of the equivalent simple pendulum.

75a. Mathematical Treatment : Suppose a rigid body of mass m is capable of free vibration about a horizontal axis at O and let G be the centre of gravity of the body. (Fig. 40)

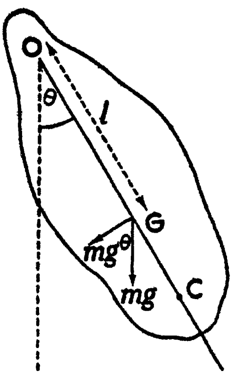


Fig. 40

Let θ be the angular displacement of the body at time t, from the equilibrium position as indicated by the broken line.

Let $OG=l$.

The restoring couple
or torque $=mgl\sin\theta$
$=mgl\theta$, when θ is small.

Thus the restoring couple for unit twist $=mgl$ and the period T is given by

$$T=2\pi\sqrt{\frac{I}{mgl}}$$

If R be the radius of gyration about a parallel axis through G, the centre of gravity, we have $I=ml^2+mR^2$

$$\therefore\quad T=2\pi\sqrt{\frac{ml^2+mR^2}{mgl}}=2\pi\sqrt{\frac{l^2+R^2}{gl}}$$

Comparing this expression with the period of oscillation of a simple pendulum, $T=2\pi\sqrt{\frac{L}{g}}$, we find that the length of the equivalent simple pendulum having the same period as the body is given by

$$L=\frac{l^2+R^2}{l}=l+\frac{R^2}{l}$$

If the line OG be produced to a point, say C on OG produced such that $GC=\frac{R^2}{l}$, the length OC gives the *length of the equivalent simple pendulum.*

The point O is called the **centre of suspension** and the point C is called the **centre of oscillation.**

These two points are interchangeable : When the body is suspended at O or C the period of oscillation remains the same.

Let T_1 and T_2 be respectively the periods of oscillation when the body is suspended from O and C, and let l_1 and l_2 be respectively the distances of O and C from the centre of gravity. Then we have,

$$T_1 = 2\pi\sqrt{\frac{l_1^2+R^2}{l_1 g}} \; ; \quad T_2 = 2\pi\sqrt{\frac{l_2^2+R^2}{l_2 g}}$$

But $l_2 = GC = \dfrac{R^2}{l_1}$ $\quad = 2\pi\sqrt{l_2^2+R^2} \quad = 2\pi\sqrt{R^2 + l_2}$

$$= 2\pi\sqrt{\frac{\frac{R^2}{l_1}+l_1}{g}} = 2\pi\sqrt{\frac{l_1^2+R^2}{l_1 g}} \quad \text{That is } T_2 = T_1$$

For a compound pendulum, we have $T = 2\pi\sqrt{\dfrac{l^2+R^2}{lg}}$

In fig. 41, $CK_1 = l_1$ and $CK_2 = l_2$

$$\therefore \quad T = 2\pi\sqrt{\frac{l_1^2+R^2}{l_1 g}} = 2\pi\sqrt{\frac{l_2^2+R^2}{l_2 g}}$$

since the period at the two knife-edges K_1 and K_2 is the same.

We have again, $\dfrac{T^2}{4\pi^2} l_1 g = l_1^2 + R^2$; $\dfrac{T^2}{4\pi^2} l_2 g = l_2^2 + R^2$

$$\frac{T^2}{4\pi^2}(l_1 - l_2)g = l_1^2 - l_2^2 \quad \text{or} \quad \frac{T^2}{4\pi^2} = l_1 + l_2$$

$\therefore \quad T = 2\pi\sqrt{\dfrac{l_1+l_2}{g}}$ Here $l_1 + l_2 = L$, the length of the equivalent simple pendulum.

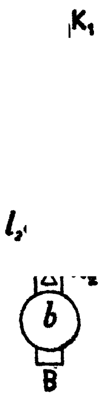


Fig. 41

76. Kater's Reversible Pendulum :

The principle of reversible pendulum was employed by Kater in 1817. It consists of a metal rod AB with a heavy lens-shaped bob C fixed at its lower end.

Two steel knife-edges K_1 and K_2 are fixed near the ends of the rod with their edges turned towards each other and are used for supporting the pendulum for oscillation. (Fig. 42)

The heavy bob C is fixed to the rod but the two weights D and E can slide along and may be clamped to it.

The larger weight D is moved about until the periodic time of the pendulum is nearly the same when suspended from either K_1 or K_2. Then it is screwed in position. The fine adjustment is made by moving the smaller weight E by means of a micrometer screw until the times of oscillations about both the knife-edges are exactly the same.

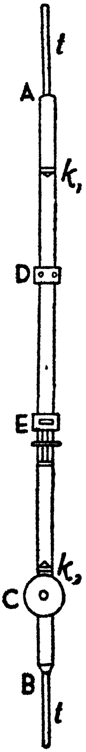


Fig. 42

Now the distance between the knife-edges is equal to the length of the equivalent simple pendulum and thus the value of g is calculated from the expression

$$t = 2\pi\sqrt{\frac{L}{g}}$$

In Kater's pendulum the adjustment of pendulum to exact equality of periods about both the knife-edges is extremely tedious. It is not absolutely necessary and can also be avoided as follows.

Let t_1 and t_2 be the periods at the two knife-edges and l_1 and l_2, the distances of the knife-edges from the centre of gravity.

$$\therefore \quad t_1 = 2\pi\sqrt{\frac{l_1^2 + R^2}{gl_1}}\,;\ t_2 = 2\pi\sqrt{\frac{l_2 + R^2}{gl_2}}$$

$$\therefore \quad \frac{gt_1^2}{4\pi^2} = \frac{l_1^2 + R^2}{l_1}\,;\ \frac{gt_2^2}{4\pi^2} = \frac{l_2^2 + R^2}{l_2}$$

$$\therefore \quad \frac{g}{4\pi^2}(t_1^2 l_1 - t_2^2 l_2) = l_1^2 - l_2^2$$

$$\therefore \quad \frac{4\pi^2}{g} = \frac{t_1^2 l_1 - t_2^2 l_2}{l_1^2 - l_2^2} = \frac{1}{2}\left\{\frac{t_1^2 + t_2^2}{l_1 + l_2} + \frac{t_1^2 - t_2^2}{l_1 - l_2}\right\}$$

Since t_1 and t_2 are nearly equal, the term $\dfrac{t_1^2 - t_2^2}{l_1 - l_2}$ is small compared with $\dfrac{t_1^2 + t_2^2}{l_1 + l_2}$ and can therefore be neglected

$$\therefore \quad \frac{4\pi^2}{g} = \frac{1}{2}\left\{\frac{t_1^2 + t_2^2}{l_1 + l_1}\right\}.$$

The length $l_1 + l_2$ is the distance between the knife-edges and can be determined accurately. Hence, t_1 and t_2 being known, g can be found out.

77. Conical Pendulum: A simple conical pendulum is essentially a simple pendulum which is given such a motion tha

the bob describes a horizontal circle and the suspension string generates a conical surface. The length of the pendulum is the distance between the point of suspension and the centre of gravity of the bob.

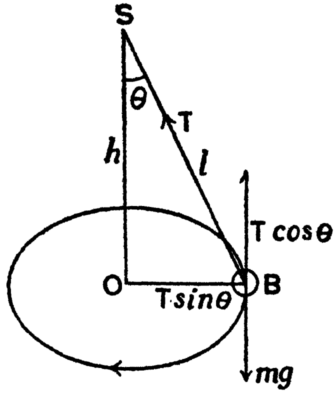


Fig. 43

Let m be the mass of the bob, v its linear velocity and r the radius of the circle described by it. Let $\angle OSB = \theta$ and h = height OS of the cone. (Fig. 43)

The forces acting on the bob at B are its weight mg, the centrifugal force mv^2/r, and the tension T of the string.

The weight mg is balanced by vertical component $T\cos\theta$ of tension T of the string, its horizontal component $T\sin\theta$ provides the centripetal force mv^2/r towards O.

Thus $T\sin\theta = mv^2/r$ and $T\cos\theta = mg$

$$\frac{T\sin\theta}{T\cos\theta} = \frac{mv^2/r}{mg} \quad \text{or} \quad \tan\theta = \frac{v^2}{rg} \quad \text{or} \quad \frac{r}{h} = \frac{v^2}{rg} \quad \left[\because \tan\theta = \frac{r}{h}.\right]$$

or $v^2 = r^2 g/h$ or $\omega^2 r^2 = r^2 g/h$ [$\because v = \omega r$ where ω = angular velocity]

$$\omega^2 = g/h \quad \text{or} \quad \omega = \sqrt{\frac{g}{h}}$$; Then, time period of the pendulum

is given by $t = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{h}{g}} = 2\pi\sqrt{\dfrac{l\cos\theta}{g}}$ [$\because h = l\cos\theta$]

It is to be noted that the time period is the same as that of a simple pendulum of length h, the axial height of the cone. If θ be very small, $\cos\theta$ becomes nearly equal to unity ; then $h = l$, so that the time period is almost independent of θ, and it will remain same whether the bob moves in a liner or a circular path.

QUESTIONS

1. Explain how the value of g at any point on the surface of the earth can be determined with Kater's pendulum. [C. U. 1983]

2. Distinguish clearly between a simple and a compound pendulum. Define the terms 'centres of suspension and oscillation' and show that they are interchangeable.

3. The bob of a simple pendulum of length l moves with angular speed ω in a horizontal circle of radius r. Derive an expression for the period of rotation of the bob. [C. U. 1954]

EXAMPLES

1. A metal disc oscillates in its own plane about an axis passing through a point on its edge. What is the length of the equivalent simple pendulum?

Let the disc of radius r oscillate about an axis through a point P on its edge, so that plane of oscillation is vertical and the axis is horizontal. (Fig. 45)

The time-period is given by $t=2\pi\sqrt{\dfrac{I}{mg.l}}$

where I = moment of inertia of the disc about the axis through P.
m = its mass, $l=r$ = effective length.

Now by principle of parallel axes I = moment of inertia of the disc about an axis through O, *i.e.* C. G. of the disc $+m\times$ (distance between two parallel axes)$^2=\dfrac{mr^2}{2}+mr^2=\dfrac{3mr^2}{2}$

$\therefore t=2\pi\sqrt{\dfrac{3mr^2/2}{mg.r}}=2\pi\sqrt{\dfrac{\frac{3}{2}r}{g}}$, which is same as that for a simple pendulum of length l equal to $\frac{3}{2}r$. Hence, the length of the equivalent simple pendulum is $\frac{3}{2}$ times the radius of the disc.

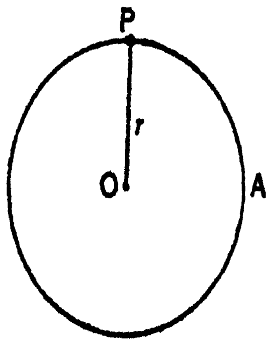


Fig. 44

2. A heavy spherical bob of diameter 10 cm. suspended by a very fine wire. If the distance from the point of suspension to the centre of the bob be 1 metre, calculate the length of equivalent pendulum.

Period $t=2\pi\sqrt{I/mgl}$. where I = moment of inertia of the sphere about an axis through the point of suspension; l = distance between the point of suspension and the c. g. of the sphere, m = mass of the sphere.

Here $l=100$ cm. radius $r=5$ cm. $I=\frac{2}{5}mr^2+ml^2$ by theorem of parallel axes

$$t=2\pi\sqrt{\frac{\frac{2}{5}mr^2+ml^2}{mg.l}}=2\pi\sqrt{\left(\frac{2}{5}\frac{r^2}{l}+l\right)\Big/g}$$

Length of the equivalent simple pendulum

$$=\frac{2r^2}{5l}+l=\frac{2}{5}\times\frac{25}{100}+100=100\cdot1 \text{ c.m.}$$

CHAPTER V

GRAVITATION

78. Introduction: The discovery of the Laws of Gravitation by Newton is, it is said, associated with the popular story of the fall of an apple from a tree. From his observations and other experimental results Newton concluded that all bodies fall to the earth with the same acceleration and that the attraction by the earth on any body is proportional to its mass.

Newton developed this simple law and extended it to the doctrine of universal gravitation.

This law has been regarded as the most perfect generalisation of experience with such a vast amount of confirmatory evidence that any want of exactness of the law in certain cases may easily be ignored.

Before Newton, Kepler formulated certain laws regarding the movements of the planets round the sun, purely from astronomical considerations.

79. Kepler's Laws:

(1) Every planet moves in an ellipse of which the sun occupies one focus.

(2) The radius vector drawn from the sun sweeps out equal areas in equal times.

(3) The squares of the times taken to describe their orbits by two planets are proportional to the cubes of the major-axes of the orbits.

Kepler's three laws were put forward as pure facts of experience. It was reserved for Sir Isaac Newton to reduce the three laws into a single one, the so-called *Law of Gravitation.*

From the first and the second law, it can be mathematically deduced that the central acceleration is inversely proportional to the square of the distance of the planet from the sun.

From the third law we infer that for all the planets the central forces, at any instant, are inversely proportional to the squares of their respective distances from the sun and also that these forces are proportional to the masses of the respective planets.

This is proved in the way described below.

The orbits of the planets round the sun are considered circular and the centripetal force F acting on a planet towards the centre is given by $F=m\omega^2r$ where m is the mass of the planet and r, the radius of the orbit.

For a planet of mass m_1 and radius r_1, the force

$$F_1 = m_1\omega_1^2 r_1 = m_1.\frac{4\pi^2}{T_1}.r_1$$

For a second planet of mass m_2 and radius r_2, the force

$$F_2 = m_2\omega_2^2 r_2 = m_2\frac{4\pi^2}{T_2^2}.r_2 \quad \therefore \quad \frac{F_1}{F_2} = \frac{m_1}{m_2}.\frac{T_2^2}{T_1^2}.\frac{r_1}{r_2}$$

But from Kepler's third law we have $T^2 \propto r^3$

So, for the two planets we have $\dfrac{T_1^2}{T_2^2} = \dfrac{r_1^3}{r_2^3}$

$$\therefore \quad \frac{F_1}{F_2} = \frac{m_1}{m_2}.\frac{r_2^3}{r_1^3}.\frac{r_1}{r_2} = \frac{m_1}{m_2}.\frac{r_2^2}{r_1^2} \; i.e.\; F \propto \frac{m}{r^2}.$$

This is the *Law of Gravitation* which follows as a necessary corollary of the Kepler's laws.

79a. General discussion : The motion of the moon on its orbit round the earth is an accelerated motion and the acceleration is directed towards the centre of the earth.

If we compare the acceleration of the moon with that of a falling body at the surface of the surface of the earth, we find that the acceleration of the moon is $\frac{1}{3600} = \frac{1}{60^2}$ of that of a falling body at the earth's surface.

Again, since the distance of the moon from the centre of the earth is 60 times the earth's radius, the comparison leads to the result that the attractive force of the earth varies inversely as the square of the distance.

So Newton concluded that the force which attracts bodies at the surface of the earth is the same as that which guides the moon and the planets in their orbits.

The force of gravitation, unlike **magnetic** and **electric** forces is practically independent of the nature of the intervening **medium.**

But since the mass of a body varies with its velocity, and the distance between the masses depends upon the circumstances of the observer measuring it, Newton's Law of Graviation can not be said to be universally true.

Again, it does not depend on the nature of the material bodies, neither upon the properties of the bodies nor upon their temperature. Except for the smallest distances and for one or two outstanding phenomena Newton's Law of Gravitation is very approximately true.

To explain this slight divergence Einstein's theory of relativity becomes necessary.

Considering the fact that the law regarding the force of gravitation is independent of the nature of the masses of the bodies and of the medium intervening them, it may be said that the *law is universally* true for all practical purposes.

80. Laws of gravitation : We know that every material particle in this universe attracts every other material particle according to the laws known as the Laws of Gravitation. *The laws state that the force of attraction between any two bodies is directly proportional to the product of the masses of the two bodies, and inversely proportional to the square of the distance between them.*

If m and m' be the masses of the two bodies and d, the distance between them, then the force of attraction F between the bodies is $F \propto \frac{mm'}{d^2}$ or $F = G\frac{mm'}{d^2}$, where G is a constant and is called the **constant of Gravitation.** It is also sometimes referred to as the astronomical unit of force.

If $m = 1$, $m' = 1$ and $d = 1$ cm., Then $G = F$.

This constant of gravitation G is the force of attraction between two bodies of unit masses when they are placed one centimetre apart. The value of G is about $6{\cdot}65 \times 10^{-8}$ C. G. S. units.

In the cases of electric and magnetic fields the constant for the force is eliminated by suitable choice of units in air.

But in the case of gravitational field, the constant cannot be eliminated since the units of force and mass are defined independently of the law of gravitation.

81. Gravity : The force of attraction between the earth and any other material body is known as *terrestrial gravitation* or simply *gravity.*

The **weight** of a body is the force with which the body is attracted towards the centre of the earth and its value depends on the force of gravity at the place. Since the force of gravity increases from the equator to the poles, the weight of the body necessarily increases from the equator to the poles.

81a. Causes of variation of weight on earth :

(*A*) *The daily rotation of the earth about its axis* : The shape of the earth is not exactly spherical but flattened at the poles, and the centrifugal force generated due to the daily rotation of the earth decreases from the equator to the poles of the earth. Consequently, the apparent weight of the body due to gravity increases from the equator to the poles.

A body placed on the surface of the earth shares the diurnal motion of the earth and is consequently acted on by a **centrifugal force** of value $\frac{mv^2}{r}$ or $m\omega v$ acting against the force of gravity, where m is the mass of the body, v, velocity of that part of the earth's surface on which the body is placed, r the radius of the circle

described by the surface round the earth's axis, and ω the angular velocity of the earth.

So the apparent weight of the body is equal to the excess of the true weight over the centrifugal force at the place.

$\therefore$ $mg = W' = W - m\omega v$ where W' is the apparent weight and W, the true weight of the body.

As the angular velocity of the earth is constant, the centrifugal force depending on the velocity of the body, decreases from the equator to the poles and so from the above expression we see that the weight mg of a body increases from the equator to the poles.

(B) The flattening of the earth at the poles. Due to this, the equatorial radius of the earth is greater than the polar radius and consequently by the law of gravitation, the force of gravity is greater at the poles than at the equator. So the weight of a body increases as it is taken near the poles.

82. Cavendish's Method of determination of gravitation constant :

The apparatus called the torsion balance and used by Cavendish is shown diagrammatically in figure 45. A long cross-bar was suspended from the ceiling of a room and could be turned about a vertical axis by an arrangement manipulated from outside. Two long, thin rods suspended from the ends of the bar carried two large and equal lead balls. Directly below the mid-point of the cross-bar, was a torsion head which also could be rotated from outside, from which a light rod slightly longer than the cross-rod was suspended by a fine wire of silvered copper. Two inclined wires tied the ends of the light rod to a short vertical rod in the middle which was attached to the suspension wire. This strengthened the light rod without affecting its moment of inertia. Two small lead balls were suspended by short wires from two ends of the light rod such that the centres of the four balls

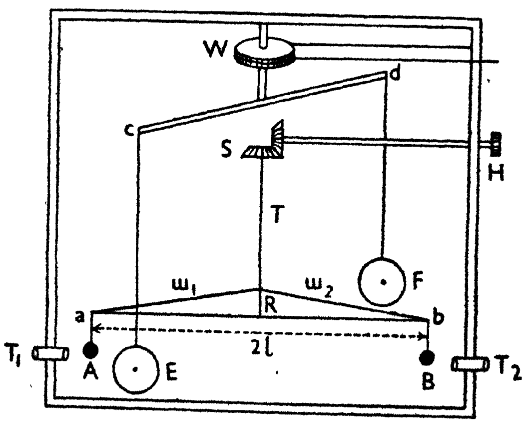


Fig. 45

lay in the same horizontal plane. Each end of the torsion rod carried a vernier which could move over a fine ivory scale fixed to vertical stands, without touching the scale. The large balls could be placed either in the positions E, F or E′, F′. The rods and the balls were enclosed in a glass enclosure.

When the heavy balls were fixed in the positions E and F on the opposite sides of the small balls and at equal distance from the two small balls A and B, the attractive forces between the heavy and the small balls formed a couple tending to turn the rod so that the suspending wire was twisted in the clockwise direction. The angle of twist was measured with the help of scale and vernier arrangement. The heavy balls were then moved into the positions E′ and F′ and the angle of twist determined keeping distance between large and small balls same as in the positions E, F. The mean angle of twist was then calculated.

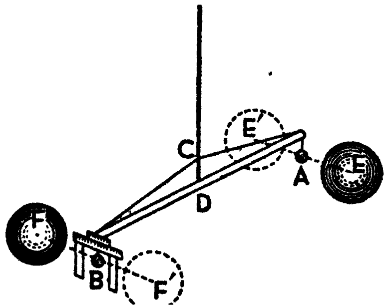


Fig. 46

The couple due to the forces of attraction between each pair of balls (one large and another small) which turned the rod was balanced by the torsional couple of the suspending wire.

Let M be the mass of each of the large balls and m the mass of each of the small balls, d the distance between the centres of the large and small balls when the mean twist or deflection was α and let $2l$ be the length of the rod.

Then force of attraction between M and m $= G\frac{Mm}{d^2}$.

The moment of the couple tending to rotate the rod $= 2Gl \times Mm/d^2$.
The torsional couple exerted by the suspension wire when twisted through an angle $\alpha = c\alpha$, when c is the couple for unit twist.

$$2Gl.\frac{Mm}{d^2} = c\alpha$$

$$\text{or} \quad G = \frac{cd^2}{2lMm}\alpha. \qquad \cdots(1)$$

To determine c, the attracting large balls were removed and the suspended system was allowed to execute torsional oscillation of

period given by $T=2\pi\sqrt{\frac{I}{c}}$, where I the moment of inertia of the system, could be evaluated from knowledge of the mass and dimensions of the system. Then knowing I and T, c was determined.

All the quantities on the right hand side of the expression (1) being known, the value of G was calculated and found to be equal to 6.6579×10^{-8} c.g.s. units. Hence, two small spheres each of mass 1 gram. placed with their centres at a distance of 1 cm. apart attract each other with a force of 6.6579×10^{-8} dynes.

Note: The deflection of the rod carrying small balls or the twist of its suspension wire could also be found by observing by a telescope the graduations of a scale as seen by reflection from a small plane mirror fixed to the centre of the rod.

Note: In the expression (1) c the torsional couple for unit twist may also be obtained from the relation $c=\frac{\eta\pi r^4}{2L}$

where η = rigidity of the material of the wire
r = radius of the wire
L = length of the wire.

Precautions: Cavendish kept his apparatus in a closed room which he did not enter while making his experiment. The temperature of the room was kept constant.

To avoid electrostatic attraction from outside, the apparatus was kept inside a guilt covered glass case.

Errors in Cavendish's Experiment:

(1) The suspension wire was thick and the deflection was small.

(2) Measurement of the angle of deflection by vernier was not capable of great accuracy.

(3) In the large apparatus temperatures could not be kept constant.

(4) The counter gravitational couple reduced the deflection.

83. Boys' improved method: Prof. Vernon Boys modified Cavendish's experiment to some extent by using a very fine quartz fibre in place of the torsion wire of Cavendish. The two smaller balls were made of gold and suspended from the ends of a short bar and the two heavy balls were of lead and placed in such a way that the suspended masses were at two different levels.

In Boys' experiment the above errors were avoided in the following ways.

The error (1) was avoided by using a fine quartz fibre of great strength. The error (2) was avoided by measuring the deflection by means of a mirror and illuminated scale. The error (3) was avoided by reducing the size of the apparatus to prevent the action of air

currents. The error (4) was avoided by placing the pairs of masses M and m at different levels.

Boys' Experiment : In the apparatus devised by Boys, a small mirror strip EF (Fig. 47) which acts as the torsion rod is suspended from a torsion head by a long, fine quartz fibre S inside a glass tube. From the ends of the mirror strip are suspended two small gold balls A and B of equal mass by quartz fibres of unequal length, so that one gold ball was about 15 cm. below the level of the other. Two large lead balls C and D of equal mass and diameter are suspended outside the glass tube from two points of the revolving lid of another co-axial tube, such that the entre of C is in level with the centre of A, and the centre of D in level with that of B. The distances between the centres of one pair of near balls is exactly same as that for the other pair. To measure deflection, telescope and scale method is adopted, a half-millimetre scale being placed at a distance of about seven metres from the small mirror strip.

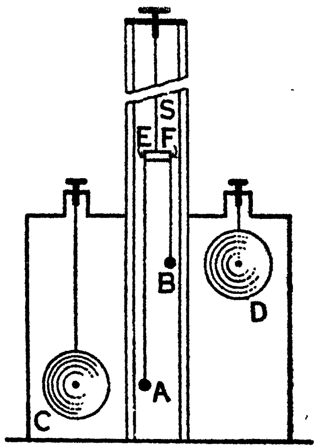


Fig. 47

To perform the experiment, the lid is rotated so that the large lead balls are on the opposite sides of the two gold balls but not in line with the mirror strip and its position is adjusted for the largest angle of deflection. The lid is then turned and adjusted so that the lead balls being on the other sides of the gold balls in similar positions produces the largest angle of deflection. The mean of the two deflections is then noted. Let its value be θ radians.

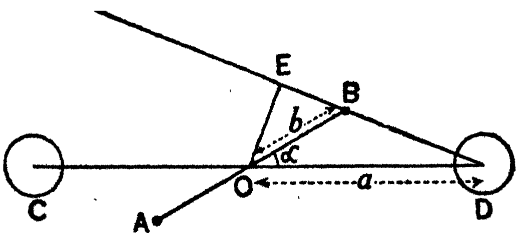


Fig. 48

Let A, B, C and D (Fig. 48) be the positions of the four balls for maximum deflection θ. Let b be half the length of the mirror strip and O its middle point. Take OD $= a$, $\angle$BOD $= \alpha$. Draw OE perpendicular from O on BD produced.

In the triangle OBD, BD $= \sqrt{a^2 + b^2 - 2ab \cos \alpha}$.

Again, $\frac{\sin \alpha}{\sin \text{BDO}} = \frac{\text{BD}}{\text{OB}} = \frac{\text{BD}}{b}$ $\quad\therefore\quad \sin \text{BDO} = \frac{b \sin \alpha}{\text{BD}}$

From the right-angled triangle OED

$$\text{OE} = \text{OD} \sin \text{ODE} = a \sin \text{BDO} = \frac{ab \sin \alpha}{\text{BD}}.$$

Attractive force between two balls of each pair

$$= \text{G}.\frac{\text{M}m}{\text{BD}^2}, \text{ where M} = \text{mass of each lead ball}$$

$$m = \ldots \quad \ldots \quad \ldots \text{ gold ball}$$

$$\therefore \text{ Deflecting couple} = \frac{\text{GM}m}{\text{BD}^2} \times 2\text{OE} = \frac{\text{G.M}m}{\text{BD}^2} \times \frac{2ab \sin \alpha}{\text{BD}}$$

$$= \frac{2\text{GM}m\ ab \sin \alpha}{\text{BD}^3} = \frac{2\text{GM}m\ ab \sin \alpha}{(a^2 + b^2 - 2ab \cos \alpha)^{\frac{3}{2}}}.$$

If c be the torsional couple per unit deflection, then when the maximum deflection is θ, the restoring couple $= c\theta$; then for equilibrium

$$\frac{2\text{GM}m\ ab \sin \alpha}{(a^2 + b^2 - 2ab \cos \alpha)^{\frac{3}{2}}} = c\theta \quad \text{or} \quad \text{G} = \frac{c\theta(a^2 + b^2 - 2ab \cos \alpha)^{\frac{3}{2}}}{2\text{M}.m\ ab \sin \alpha}.$$

The value of the couple c is determined in the same way as in Cavendish's experiment. The value of α can be found from the geometry of the apparatus. Hence, the value of G can be easily found out from the above expression. The value of G obtained by Boys was $6{\cdot}6576 \times 10^{-8}$ C. G. S. units.

84. Gravitation Constant and the Density of the Earth :

To determine the density of the earth we are to equate the weight of the body to the force of attraction between it and the earth.

Let m be the mass of the body placed on the earth of radius R and density ρ.

The force of attraction between it and the earth

$$= \text{G}\frac{\frac{4}{3}\pi\text{R}^3.\rho.m}{\text{R}^2}, \text{ then, } mg = \text{G}.\tfrac{4}{3}.\pi\text{R}.\rho.m \quad \therefore \quad \rho = \frac{3g}{4\pi\text{RG}}. \quad \ldots \quad \ldots \quad (1)$$

The value of ρ, the density of the earth has been found by Cavendish to be equal to $5{\cdot}45$ gm. per c. c.

From a knowledge of the density of the earth and its radius, its mass M is obtained from $\text{M} = 4\pi\text{R}^3\rho/3$.

85. Value of g on the surface of the earth and at a point inside the earth :

If a body of mass m be situated on the surface of the earth.

we have $mg = G\frac{Mm}{R^2}$ (1) where G is the gravitational constant, M, the mass of the earth, R, the radius of the earth and g, the acc. due to gravity at the surface of the earth.

Again if the same body be taken inside a mine of depth d we have $mg' = G.\frac{M'm}{(R-d)^2}$ ... (2) where M$'$ is the mass of the solid sphere of radius $(R-d)$ and g', the acceleration due to gravity at a depth d.

$$\therefore \quad \frac{g'}{g} = \frac{R^2M'}{(R-d)^2M} = \frac{R^2.\frac{4}{3}.\pi(R-d)^3D}{(R-d)^2.\frac{4}{3}\pi R^3D} = \frac{R-d}{R}$$

where D is supposed to be the uniform density of the earth.

$$\therefore \quad g' = g.\frac{R-d}{R} \text{ or } g' < g.$$

Thus the value of acceleration due to gravity inside a mine is less than that at the surface of the earth.

But Airy found by his experiment in a colliery in Durham that the weight of a body and hence, the value of g inside a mine was greater than that at the surface of the earth.

This increase of g with the depth is due to the fact that the central portions of the earth are much denser than the surface crust.

Note : It is to be noted that the value of g is maximum at the surface of the earth and decreases according to Inverse Square Law above the surface of the earth but below it, it decreases with the decrease of distance of the point from the centre of the earth.

86. Experimental determination of the density of the earth : The experimental methods for finding gravitational constant and hence, the density of the earth fall mainly into two classes. The first is known as mountain method in which certain natural large mass such as mountain is chosen, the centre of gravity and mass of the mountain being known from necessary survey and mineralogical investigation. The attraction of the earth on a plumb line is compared with the attraction of the selected mountain on the same plumb line. The second class refers to the laboratory class of experiments in which the attraction between finite masses is ascertained. Then knowing G the gravitation constant, the mean density of earth can be found out.

(*A*) *Theory of mountain method* : Let a plumb bob be suspended near the bulkiest portion of a mountain H so that centres of gravity of the bob and the mountain lie on a horizontal line. The bob is attracted by the mountain in a horizontal direction, and also by the remaining part of the earth E in a vertical direction. So that the cord supporting the bob assumes the direction of the resultant of the two forces and it becomes inclined to the vertical line by an angle θ.

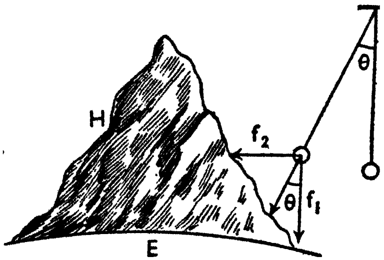


Fig. 49

$$\text{Evidently } \tan\theta = \frac{\text{Hor. Force due to mountain}}{\text{Vertical force due to earth}}$$

If m and m' be the mass of the mountain and that of the plumb bob, d the distance between their centres of gravity and G the gravitation constant, horizontal force $= G\ mm'/d^2$ ... (1)

Again, if M be the mass of the rest of the earth and R the distance of the bob from the centre of earth, the vertical force $= GMm'/R^2$ ... (2)

$$\text{Then } \tan\theta = \frac{G.mm'}{d^2} \div \frac{GMm'}{R^2} = \frac{R^2 m}{d^2 M} \qquad (3)$$

Bouguer first conducted experiment to measure the deflection of the plumb line and the next experiment for it was due to Maskelyne. In both the experiments the deflection of the plumb line was obtained from the apparent shift of a star viewed by a telescope. The mass of the earth M was determined from relation (3) using known values of R, d and m. Knowing M and the volume of the earth its mean density was readily found out.

(*B*) *Airy's Mine experiment* : Airy suggested that the mean density of the earth could be determined by observing the difference in the rates of oscillation of a pendulum at the top and bottom of a mine denoted by the points A and B in Fig. 50. Suppose ρ is the mean density of that part of the earth whose surface passes through the bottom of the mine of depth h. Let ρ' be the mean density of the matter in the annular shell of the earth of thickness h.

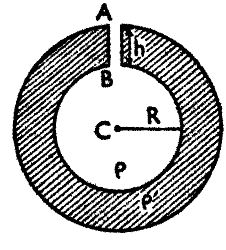


Fig. 50

If F_A and F_B be the forces of gravity and g_A and g_B the accelerations due to gravity at A and B respectively, then

$$F_B = -\frac{G.\frac{4}{3}\pi R^3\rho}{R^2} = -\frac{4}{3}G\pi R\rho \tag{1}$$

where G = gravitation constant
R = radius of the earth inside the annular shell

Similarly $F_A = -G\left[\frac{\frac{4}{3}\pi R^3\rho}{(R+h)^2} + \frac{\frac{4}{3}\pi\{(R+h)^3 - R^3\}\rho'}{(R+h)^2}\right]$

$$= -\frac{4}{3}\pi G\left[\frac{R^3\rho + (R^3 + 3R^2h - R^3)\rho'}{R^2\left(1+\frac{2h}{R}\right)}\right]$$

[neglecting h^2 and higher powers of h]

$$= -\frac{4}{3}\pi G\left[R\left(1+\frac{2h}{R}\right)^{-1}.\rho + 3h\left(1+\frac{2h}{R}\right)^{-1}.\rho'\right]$$

$$= -\frac{4}{3}\pi G\left[R\left(1-\frac{2h}{R}\right)\rho + 3h\left(1-\frac{2h}{R}\right)\rho'\right]$$

$$= -\frac{4}{3}\pi G[(R-2h)\rho + 3h\rho'] \quad \ldots \tag{2}$$

$\left[\text{neglecting } \frac{h^2}{R}\right]$

Then $\frac{F_A}{F_B} = \frac{g_A}{g_B} = \frac{(R-2h)}{R} + \frac{3h\rho'}{R\rho}$ ... ... (3)

$$1 - \frac{g_A}{g_B} = 1 - \frac{R-2h}{R} - \frac{3h}{R}.\ \frac{\rho'}{\rho}$$

$$= \frac{2h}{R} - \frac{3h}{R}.\ \frac{\rho'}{\rho}$$

or $\left(1-\frac{g_A}{g_B}\right).\ \frac{R}{3h} = \frac{2}{3} - \frac{\rho'}{\rho}$

$$\rho = \frac{\rho'}{\frac{2}{3} - \left\{1-\frac{g_A}{g_B}\right\}\frac{R}{3h}} = \frac{\rho'}{\frac{2}{3} - \left\{1-\left(\frac{T_B}{T_A}\right)^2\right\}\frac{R}{3h}} \quad \ldots \tag{4}$$

where T_A and T_B are the periods of the pendulum at A and at B respectively. The value of ρ' was obtained by determining the densities of different specimens of rocks at different levels down to the bottom of the mine. Then knowing all quantities of the right hand side of realtion (4), ρ could be found out.

87. Gravitational intensity and Potential:

Any point situated in the space surrounding a gravitating particle will experience a definite attracting force.

The space surrounding the particle within which certain forces act in such a way that a force of definite magnitude and direction is associated with every point in the space, is called a *field of force*, or simply *field*.

Thus the **Gravitational attraction** or intensity at a point is the force which would act on a particle of unit mass placed at the point.

Gravitational Potential : If a particle of unit mass be moved from one point to another in the gravitational field against the force of attraction a certain amount of work is to be done.

The amount of work done may be positive or negative according to the direction of the movement and is called the difference in gravitational potential between the points.

To get an absolute value of potential at any point, the zero position is taken to be that at infinity.

Thus the gravitational potential at any given point is the work done in bringing a unit mass from infinity up to the point.

If the distance between the particles which attract one another be increased, work will have to be done on the particles but if the distance between them be dminished work will be done by the particles. The amount of work done on the particles will be maximum and positive or negative according as the particles are moved from the position of contact to infinite distance apart or moved from an infinite distance apart into contact with one another.

It follows that the two particles possess a certain amount of potential energy when not in contact as they can be made to do work during their approach.

Difference between Gravitational Potential and other kinds of potential (such as Magnetic and Electrical).

(1) Gravitational potential is *negative* in sign whereas magnetic and electric potentials are *positive* in sign.

(2) Gravitational potential is independent of the intervening medium, whereas magnetic and electrical potentials depend on it.

88. Relation between Intensity or Attractive force (per unit mass) and Potential:

If a force of attraction *i.e.* intensity F in a given direction acts at any point in the field, then the work done in moving unit mass through an infinitesimal distance ds against this force of attraction is Fds.

But this work is the difference of potential dV between the points ds distance apart. $\quad -dV = Fds$ or $F = -\dfrac{dV}{ds}$ $\quad\cdots(1)$

Again, $dV = Fds$ or $V = \int Fds$

where the limits for s apply to the two points considered.

From the relation (1), we find that the gravitational potential at a point may be defined as that whose negative space rate of variation gives the gravitational intensity at that point.

Hence if we know V, then differentiating it with respect to distance we can find intensity F; conversely if F be known, its space integral will give the value of the potential V.

89. Potential at a point due to a point mass :

If a unit mass be placed at a distance r from a very small particle of mass m, then we have for the force of attraction between the bodies as equal to $G.\dfrac{m}{r^2}$.

If the body of unit mass be moved through an infinitely small distance dr against this force of attraction, then the work done $= G.\dfrac{m}{r^2}dr$.

90. Mathematical treatment :

Let a point mass m be at O Fig. 51 and let A and B be two points on a straight line through O, and at distances r and r_1 from O. Now suppose P is a point on the straight line at distance x from O. Then gravitational intensity (*i.e.*, force on unit mass) at P

$$= G.\frac{m}{x^2}.$$

If Q be a point very close to P and at a distance dx from it, the gravitational intensity may be taken same along PQ. Then work done in bringing unit mass from Q to P *i.e.*, the otential difference across PQ is given by

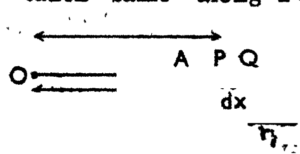


Fig. 51.

$$dV = G.\frac{m}{x^2} \times dx$$

($\because$ work done = Force × displacement)

Then potential difference between A and B is given by

$$\int_B^A dV = \int_{r_1}^{r} G.\frac{m}{x^2}\,dx \text{ or } V_A - V_B = -Gm\left[\frac{1}{x}\right]_{r_1}^{r} = -Gm\left(\frac{1}{r}-\frac{1}{r_1}\right)$$

If the point B be at infinity, then the work done in bringing unit mass from infinity to A, *i.e.*, the potential at the point A is given by

$$V_A = -Gm\left(\frac{1}{r}-\frac{1}{\alpha}\right) = -Gm\left(\frac{1}{r}-0\right) = -\frac{Gm}{r}.$$

91. Elementary Proof :

Let us consider that a body of unit mass is moved in a straight line successively from A to B, B to C and finally to a point (Fig. 52) N representing a point at an infinite distance from the body of mass m situated at O, where AB, BC...LM, MN are equal small intercepts. Let the distances of the points A, B, C,...M, N from O be r, r_1, r_2 ......r_{n-1}, r_n respectively.

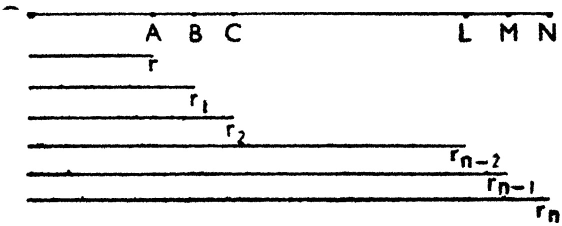


Fig. 52

Then force of attraction at $r = \frac{G.m \times 1}{r^2}$ ... (1)

(

The average force along AB = geometric mean of (1) and (2)

$$\sqrt{\frac{Gm}{r^2}\times\frac{Gm}{r_1^2}} = \frac{Gm}{rr_1}$$ Work done in moving unit mass

$$\text{from A to B} = \frac{Gm}{rr_1}(r_1 - r) = G\left(\frac{m}{r}-\frac{m}{r_1}\right)$$

$$\text{,, B to C} = \frac{Gm}{r_1r_2}(r_2 - r_1) = G\left(\frac{m}{r_1}-\frac{m}{r_2}\right)$$

$$\text{to M} = \frac{Gm}{r_{n-2}r_{n-1}}(r_{n-1} - r_{n-2}) = G\left(\frac{m}{r_{n-2}}-\frac{m}{r_{n-1}}\right)$$

$$\text{from M to N} = \frac{Gm}{r_{n-1}.r_n}(r_n - r_{n-1}) = G\left(\frac{m}{r_{n-1}}-\frac{m}{r_n}\right)$$

Adding all these, the total work done in carrying the unit mass from a point at a distance r from the body of mass m to distance r_n

$$= G\left(\frac{m}{r} - \frac{m}{r_n}\right).$$

If $r_n = \infty$, then $m/r_n = 0$; hence the total work done in moving the body of unit mass from a point at a distance r from the body of mass m to infinity $= \frac{Gm}{r}$. It is positive in sign.

Hence, the gravitational potential at a given point at a distance r is equal to the work done in bringing unit mass to the point from an infinite distance and is therefore expressed by $\frac{Gm}{r}$ with the negative sign *i.e.* by $-\frac{Gm}{r}$.

92. Potential at a point outside a thin uniform Spherical Shell :

Let P be a point outside a spherical shell of radius a, at a distance r from O, the centre of the spherical shell. Let the surface density, *i.e.* mass per unit area, of the shell be σ. Join OP. (Fig. 53)

Consider a circular ring CD of the shell of small width, in a plane perpendicular to OP, so that centre of the ring is on OP at N.

Suppose the radius CN of the ring subtends angle θ at O and the width CE of the ring subtends an angle $d\theta$ at O.

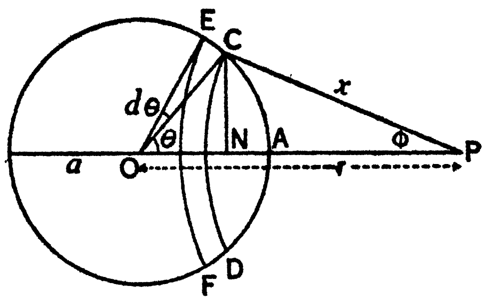


Fig. 53

The radius of the ring $CN = a \sin\theta$; width of the ring $CE = a d\theta$. Area of the ring = its circumference along CD × its width $= 2\pi a \sin\theta \times a d\theta = 2\pi a^2 \sin\theta . d\theta$.

$\therefore$ Its mass $= 2\pi a^2 \sin\theta d\theta . \sigma$.

If $CP = x$, every point of the ring is at a distance x from P, and therefore, the potential at P due to the ring

$$= \frac{-G.2\pi a^2 \sin\theta . d\theta . \sigma}{x} \quad \ldots \ldots \ldots \quad (i)$$

Now in the triangle OCP, $x^2 = a^2 + r^2 - 2ar \cos\theta$.

Differentiating the above expression.

$$2x.dx = 0 + 0 + 2ar \sin\theta d\theta \quad (\because a \text{ and } r \text{ are constants}).$$

$$\therefore \sin\theta . d\theta = \frac{xdx}{ar}.$$

Substituting the value of $\sin\theta d\theta$ in (*i*) the potential at P due to the ring $= -\dfrac{G2\pi a^2.xdx\sigma}{ar.x} = -G\dfrac{2\pi a\sigma}{r}.dx$......(*ii*)

$\therefore$ Integrating this between the limits $x = AP = r - a$, and $x = BP = r + a$, the potential due to the whole shell at P

$$= \int_{x=r-a}^{x=r+a} -G\frac{2\pi\sigma a}{r}.dx = -G\frac{2\pi\sigma a}{r}\Big[x\Big]_{r-a}^{r+a}$$

$$= -G\frac{2\pi\sigma a}{r}\Big[(r+a)-(r-a)\Big] = -G\frac{2\pi\sigma.a}{r}\times 2a = -G\frac{4\pi a^2\sigma}{r}$$

Now $4\pi a^2$ is the surface area of the whole shell, and therefore $4\pi a^2\sigma$ is equal to its mass M. Then potential at P due to the shell $= -G.\dfrac{M}{r}$.

But this would be also potential at P due to a particle of same mass as M placed at O.

Therefore for an external point, the mass of the whole spherical shell behaves as though it were concentrated at its centre.

93. Potential at a point inside a thin uniform spherical shell : Let P be a point inside (Fig. 54) the shell and let $OP = r$, and $CP = x$. Proceeding with same construction as in the previous case, the potential at P due to the ring CD

$$-G.\frac{2\pi\sigma.a}{r}dx \quad \text{[Equation (ii) in above case]}.$$

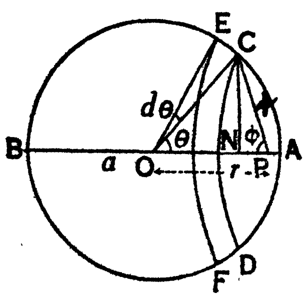


Fig. 54

Integrating this between the limits, $x = AP = a - r$ and $x = PB = a + r$, the potential due to the whole shell at P

$$= \int_{x=a-r}^{x=a+r} -\frac{G2\pi\sigma.a}{r}dx = -\frac{G2\pi\sigma.a}{r}\Big[x\Big]_{a-r}^{a+r}$$

$$= \frac{G2\pi\sigma.a}{r}\Big[(a+r)-(a-r)\Big] = -\frac{G2\pi\sigma\times 2r}{r}$$

$$= G.4\pi a.\sigma = -\frac{G4\pi a^2\sigma}{a}$$

Now, $4\pi a^2.\sigma = M$, the mass of the shell.

Potential at any point inside the shell $= -G\dfrac{M}{a}$.

This is independent of r and is the same as at a point on the shell. Again, since the above value of potential has been obtained for a point P, *anywhere* inside the shell, it follows that the potential at all points inside a spherical shell has the same constant value which is equal to the potential of the shell itself.

94. Force of attraction (per unit mass) or Intensity at any point outside a thin uniform spherical shell: With the same construction (Fig. 54) and notation as before (Art. 92) the force of attraction at P due to an element of the ring of area ds of mass $ds\sigma$, along $PC = \frac{G.ds\sigma}{x^2}$.

Resolving this force at P into two components along PO and perpendicular to it, it is seen that component perpendicular to PO is cancelled by an equal and opposite component of force at P due to a dimetrically opposite and equal element at D. Consequently, the resultant of the forces due to both these elements act along PO. The whole ring can be supposed to be divided into similar pairs of diametrically opposite elements, for each pair of which the components perpendicular to PO are neutralised, the effective components being along PO.

$\therefore$ For an element the effective component of force at P along $PO = \frac{Gds\sigma}{x^2}.\cos\phi$, where $\angle CPO = \phi$.

Hence, force at P due to the ring

$$= G.\frac{\text{area of the ring} \times \sigma}{x^2}\cos\phi \qquad \ldots(i)$$

$$= \frac{2\pi a^2 \sin\theta d\theta.\sigma}{x^2}\cos\phi$$

In the triangle COP, $a^2 = x^2 + r^2 - 2xr\cos\phi$

$$\therefore \quad \cos\phi = \frac{x^2 + r^2 - a^2}{2xr}$$

$\therefore$ Force at P due to the ring

$$= G.\frac{2\pi a^2 \sin\theta.d\theta.\sigma}{x^2}.\frac{x^2 + r^2 - a^2}{2xr}$$

$$= G\frac{\pi a^2.\sigma}{x^2}\,\frac{x.dx}{ar}\,\frac{x^2 + r^2 - a^2}{xr}$$

$$\left[\text{Since } \sin\theta.d\theta = \frac{xdx}{ar}. \text{ See Art. 92}\right]$$

$$= \pi a\sigma \;\; x^2 + r^2 - a^2 .dx. \qquad \ldots(ii)$$

Integrating this between limits $x = PA = r - a$, and $x = PB = r + a$, the force at P due to the shell

$$= \int_{x=r-a}^{x=r+a} G.\frac{\pi a\sigma}{r^2}\ \frac{x^2+r^2-a^2}{x^2}.dx.$$

$$= \frac{G\pi a\sigma}{r^2}\left\{\int_{r-a}^{r+a} dx + (r^2-a^2)\int_{r-a}^{r+a}\frac{dx}{x^2}\right\}$$

$$= \frac{G\pi a\sigma}{r^2}\left\{\Big[x\Big]_{r-a}^{r+a} + (r^2-a^2)\Big[-\frac{1}{x}\Big]_{r-a}^{r+a}\right\}$$

$$= \frac{G\pi a\sigma}{r^2}\left[\Big\{(r+a)-(r-a)\Big\} + (r^2-a^2)\Big\{-\frac{1}{r+a}+\frac{1}{r-a}\Big\}\right]$$

$$= \frac{G\pi a\sigma}{r^2}[2a+2a] = \frac{G4\pi a^2\sigma}{r^2} = G\frac{M}{r^2}.$$

This would be the force at P due to a particle of same mass as M, at O. Hence, mass of the spherical shell behaves to an external point as if it were concentrated at the centre.

95. Force at any point inside the shell : With the same construction (Fig. 54) and procedure as in Art. 93, the force at P due to the ring CE as in equation (*ii*) above $= G.\frac{\pi a\sigma}{r^2} . \frac{x^2+r^2-a^2}{x^2}.dx.$

Integrating this between limits $x = PA = a - r$ and $x = PB = a + r$, the force at P due to the shell

$$= \int_{x=a-r}^{x=a+r} G\frac{\pi a\sigma}{r^2} . \frac{x^2+r^2-a^2}{x^2}.dx$$

$$= G.\frac{\pi a\sigma}{r^2}\left\{\int_{a-r}^{a+r} dx + (r^2-a^2)\int_{a-r}^{a+r}\frac{dx}{x^2}\right\}$$

$$= \frac{G\pi a\sigma}{r^2}\left\{\Big[x\Big]_{a-r}^{a+r} + (r^2-a^2)\Big[-\frac{1}{x}\Big]_{a-r}^{a+r}\right\}$$

$$= G.\frac{\pi a\sigma}{r^2}\left[\{(a+r)-(a-r)\} + (r^2-a^2)\Big\{-\frac{1}{a+r}+\frac{1}{a-r}\Big\}\right]$$

$$= G.\frac{\pi a\sigma}{r^2}\left[2r + (r^2-a^2).\frac{a+r-a+r}{-(r^2-a^2)}\right] = G.\frac{\pi a\sigma}{r^2}[2r-2r] = 0.$$

Hence, force inside a spherical shell at every point is zero.

96. Alternative method by Calculus :

Case I. Point outside the shell :—We have Intensity (F) = space rate of variation of potential $= -\frac{dV}{dr}$, where r is the distance of the point P and V, the potential at P.

But Potential at any point outside the shell is given by

$$V = -G.\frac{M}{r}. \qquad F = \frac{dV}{dr} = -GM.\left(\frac{1}{r}\right) = G.\frac{M}{r^2}$$

Case II. Point inside the shell.

In this case $V = -G.\frac{M}{a}$, where a = radius of the spherical shell.

$$\therefore \quad F = \frac{dV}{dr} = -GM.\frac{1}{dr}\left(\frac{1}{a}\right) = 0 \quad [\because \ a \text{ is constant}]$$

That is, the force at any point inside the shell is zero.

N. B. To use the the above method it will be necessary first to deduce the expression for potential at the relevent point.

97. Force inside a solid sphere : Let P be any point inside the sphere of radius a at a distance r from its centre and let ρ be the density of the solid sphere. (Fig. 55)

The whole sphere may be supposed to be made up of shells concentric with the solid sphere and that the shell whose radii are greater than the radius r of the shell passing through the point P will exert no force at P, since P lies inside these shells ; but each of the shells whose radius is equal to or less than r will exert a force at P as if its mass were concentrated at the centre, since P lies outside these shells.

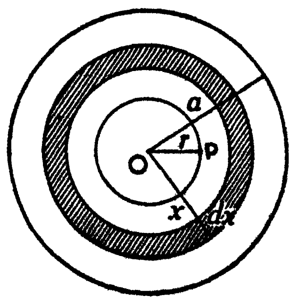


Fig. 55

Therefore, the force or intensity at P due to the shells having radii equal to and less than r and total mass equal to the mass of the solid sphere of radius r

$$= G.\frac{\frac{4}{3}\pi r^3\rho}{r^2} = \tfrac{4}{3}\pi G r\rho = \tfrac{4}{3}\pi a^3.\rho.\frac{r}{a^3}.\ G = G\frac{M}{a^3}.r.$$

where ρ is the volume density and a the radius of the sphere.

That is, the force or intensity at any point inside a solid sphere is proportional to its distance from its centre.

98. Potential inside a solid sphere : The potential at a point P inside the sphere at a distance r from the centre due to the inner sphere (Fig. 55).

$$= G.\frac{M}{r} = -G.\frac{\frac{4}{3}\pi r^3\rho}{r} = -G\tfrac{4}{3}\pi r^2\rho. \quad \ldots(1)$$

The potential at P due to the outer thick shell is calculated by considering a thin concentric shell in it of radius x and thickness dx.

The volume of the thin shell $=4\pi x^2 dx$.

Its mass $=4\pi x^2 dx\rho$.

$\therefore$ The potential at P due to this thin shell

$$= -G.\frac{4\pi x^2.dx.\rho}{x} = -G.4\pi x.dx\rho.$$

$\therefore$ the potential due to the outer thick shell

$$= -\int_r^a G.4\pi x dx.\rho = -G.4\pi\rho\left[\frac{x^2}{2}\right]_r^a = -G.2\pi\rho(a^2-r^2) \quad \ldots(2)$$

where a is the radius of the sphere.

Hence, the potential due to the solid sphere at an internal point P is equal to the sum of (1) and (2), *i.e.*,

$$= -G.\tfrac{4}{3}\pi r^2\rho - G.2\pi\rho(a^2-r^2) = -G.\tfrac{2}{3}\pi\rho(3a^2-r^2)$$

$$= -G.M.\frac{3a^2-r^2}{2a^3}, \text{ where } M = \tfrac{4}{3}\pi a^3\rho$$

99. Potential at a point outside a solid sphere :

The sphere which is of uniform volume density may be supposed to be made up of a number of uniform thin concentric shells. The potential at the outside point P due to any of these shells may be deduced by assuming the mass of the shell to be concentrated at the common centre. Then, adding up for all the shells, the potential at the point P

$$= -\left\{G\frac{m_1}{r} + G\frac{m_2}{r} + \ldots\ldots\right\} = -G.\frac{1}{r}\Sigma m = -G\frac{M}{r}.$$

100. Force at a point outside a solid sphere :

As before, the mass of the sphere, for an outside point may be considered as concentrated at its centre.

Hence, the force at the P, distant r from the centre of the sphere $= G.\frac{M}{r^2}$.

QUESTIONS

1. State and explain Newton's law of gravitation. What celestial evidence led to the formation of the law? [C. U. 1945, '51]

Is the law universally correct? Give reasons for your answer. [C. U. 1951]

2. Describe an accurate method of finding out the gravitational constant in the laboratory. [C. U. 1941, '43, '45, '50 '51 '55]

3. Explain what is meant by gravitational potential at a point and state how it differs from other kinds of Potential with which you are familiar. [C. U. 1942, '47, '53]

4. Find an expression for the gravitational potential due to a homogeneous sphere at a point outside it. [C. U. 1942]

5. Find an expression for the potential at a point inside a thin uniform shell. Hence, show that the force at the point is zero,

6. Find an expression for the gravitational potential due to a thin hollow sphere of uniform density at a point outside it. [C. U. 1947, '53]

Shew also that the gravitational potential at any external point due to the spherical shell of uniform density is the same as if the whole mass is concentrated at the centre.

7. Show how from the value of gravitational constant and other known quantities the mean density of the earth can be calculated. [C. U. 1943]

8. Explain how you would calculate the mass of the earth. [C. U. 1955]

EXAMPLES

1. Shew that the acceleration due to gravity at the bottom of a mine of depth 5 miles is $\frac{799}{800}$ of its value at the surface assuming the earth to be of uniform density throughout and the radius of the earth 4000 miles.

Since the bottom of the mine is 5 miles below the surface of the earth, a sphere may be considered inside the surface of the earth, and concentric with it, with a radius of 3995 miles and its surface passing through the bottom of the mine.

Now if a body of mass m be placed on the bottom of the mine *i.e.* on the surface of the inner sphere of radius r_2 the force exerted on it is equal to

to $mg_2 = \frac{G.\frac{4}{3}\pi r_2{}^3 \rho m}{r_2{}^2} = G\cdot\frac{4}{3}\pi r_2 \rho m$

As the force at any point inside a spherical shell is zero, the body being situated inside the outer shell will experience no force due to it.

Again, the force exerted on the body when it is placed on the surface of the earth of radius r_1 is equal to $mg_1 = \frac{\frac{4}{3}G\pi r_1{}^3 \rho.m}{r_1{}^2} = G.\frac{4}{3}\pi r_1 \rho m$

Here g_1 and g_2 denote the acc. due to gravity on the surface of the earth and at the bottom of the mine respectively

$$\therefore\ \frac{g_2}{g_1} = \frac{r_2}{r_1} = \frac{3995}{4000} = \frac{799}{800}.$$

2. If the earth were a solid sphere of iron of radius 6·37 million metres and density 7·86 grams per c. c., what would be the value of gravity at its surface, taking the gravitation constant to be $6\cdot658 \times 10^{-8}$ C. G. S. Units. [C. U. 1932]

Let m be the mass of a body placed on the surface of the earth and ρ its density. Then $mg = G.\frac{Mm}{R^2} = G.\frac{4}{3}\pi R.\rho.m$ Since $M = \frac{4}{3}\pi R^3 \rho$.

$$\therefore\ g \text{ (acc. due to gravity)} = \frac{G.4\pi R.\rho}{3}$$

$$\text{or } g = \frac{6\cdot658 \times 10^{-8} \times 4 \times 22 \times 6\cdot37 \times 10^{8} \times 7\cdot8}{7 \times 3} = 1396 \text{ cms. per sec}^2.$$

3. Determine the constant of gravitation from the following data.

Acc. due to gravity at the earth's surface is 980 cms. per sec^2. Mean density of earth is 5·527 *gm.* per c.c.

Radius of the earth is 3960 miles. [D. U. 1946]

The constant of gravitation really means the force of attraction between two bodies of unit masses when they are placed one centimetre apart from each other.

Let m be the mass of a body and M that of the earth. If R be the radius of the earth, then the force of attraction on the body by the earth $=G\frac{Mm}{R^2} = mg$ *i.e.* the weight of the body, where g is the acc. due to gravity.

But since $M=\frac{4}{3}\pi R^3\rho$, where ρ is the mean density of the earth,

$$mg=G.\tfrac{4}{3}\pi R\rho.m$$

$$\text{or}\quad G=\frac{3g}{4\pi R\rho}\quad [R=3960 \text{ miles}=3960\times1760\times3\times12\times2{\cdot}54 \text{ cm.}]$$

$$=\frac{3\times980\times7}{22\times4\times1760\times3960\times3\times12\times2{\cdot}54\times5{\cdot}527}=6{\cdot}6\times10^{-8} \text{ C. G. S. units.}$$

4. Assuming the earth to be a sphere of uniform density 5·5 gm. per c.c. and of radius $6{\cdot}4\times10^8$ cm., calculate the value of g on its surface, G being $6{\cdot}7\times10^{-8}$. [C. U. 1937]

$$\text{We have}\quad g=G\frac{4\pi R.\rho}{3}=\frac{6{\cdot}7\times10^{-8}\times4\times22\times6{\cdot}4\times10^8\times5{\cdot}5}{3\times7}$$

$$=988{\cdot}6 \text{ cms. per sec.}^2$$

5. If the earth were a solid sphere of iron of radius 6·37 million metres and of density 7·86 gms. per c c., what would be the value of gravity (g) at its surface, taking the gravitation constant to be $6{\cdot}58\times10^{-8}$ C. G. S. units ?

[C. U. 1952]

[Ans. 1380 cm./secs^2]

6. Taking the value of G to be $6{\cdot}8\times10^{-8}$, find the mass of the earth. The value of g=981 cm. per secs.^{2}, radius of the earth=6400 Km. [C. U. 1941]

We know that, $M=\frac{gR^2}{G}$ Here $R=6400\times1000\times100$ cms.

$$=64\times10^7 \text{ cms.}$$

$$\text{or } M=\frac{981\times(64\times10^7)^2}{6{\cdot}8\times10^{-8}} \text{ gms.} =\frac{981\times4096\times10^{14}}{6{\cdot}8\times10^{-8}}=59\times10^{26} \text{ gms. (approx.)}$$

7. Given $G=6{\cdot}7\times10^{-8}$ C. G. S. units, the radius of the earth$=6{\cdot}4\times10^8$ cm., and g=981 cm./sec.^{2}; calculate the mean density of the earth. [C. U. 1943, '50]

Use the formula $\rho=\frac{3g}{4\pi RG}$. [Ans. 5·46 gm./c.c.]

8. A short, straight and frictionless tunnel is bored through the earth from one point of the surface to another. Assuming the earth to be a sphere of radius 4,000 miles, shew that an object will travel along the tunnel under gravitational forces and will again reach the opposite surface in about 42 minutes. Acceleration due to gravity is 978 cm. per sec.^{2} [C. U. 1946]

Acceleration 'f' *i.e.*, force on a unit mass at a point inside a solid sphere of radius r up to the surface, is equal to

$$f=\frac{G.4\pi r^3\rho.\times1}{3r^2}=\tfrac{4}{3}G\pi\rho r \quad i.e\ f \propto r$$

$\therefore\ f=-\mu x$ where x is the displacement and μ, is a constant.

The motion is simple harmonic, and $T=\frac{2\pi}{\sqrt{\mu}}$ where T is the time-period.

In our case $f=978=\mu R$ (R = radius of the earth). $\therefore$ $T=2\pi\sqrt{\frac{R}{978}}$ sec.

But R = 4000 miles $=4\times 1{\cdot}61\times 10^8$ cms.

$$\therefore \quad T=2\pi\times 10^4\sqrt{\frac{6{\cdot}44}{978}}=5100 \text{ secs. (aprox.)}$$

$\therefore$ The required time $=\frac{T}{2}=2550$ secs. $=42$ m. 30 secs.

9. Assuming that the whole variation of the weight of a body with its position on the earth's surface is due to the rotation of the earth, find the difference in the weight of a gram at the equator and at the pole. Radius of the earth $=6.4\times 10^8$ cm.

10. A very narrow tunnel is bored from the surface to the centre of a uniform sphere. Shew that at a distance r from the centre the intensity of force is $\frac{4}{3}G\pi r\rho$ where G is gravitational constant. [C. U. 1948]

$$F=G.\tfrac{4}{3}\frac{\pi r^3\rho}{r^2}=\frac{4}{3}G.\pi r\rho.$$

11. The potentials of two homogeneous spherical shells of the same surface density at their respective centres are in the ratio 3 : 4. If the two shells coalesce into a single one, the surface density remaining unchanged, what is the potential at an internal point of this shell ? [C. U. 1913]

Let r_1 and r_2 be the radii of the two shells and r, that of the shell formed by coalescence.

Since the surface density remains unaltered, the mass of the shell formed by coalescence will be the same as the sum of the masses of the two shells formed before coalescence.

That is, we have $4\pi r_2{}^2\sigma+4\pi r_1{}^2.\sigma=4\pi r^2\sigma$, where σ is the surface density.

$$\therefore \quad r_1{}^2+r_2{}^2=r^2.$$

The potential at the centre of the shell of radius $r_1=4\pi G r_1\sigma$

" " " " " $r_2=4\pi G r_2\sigma$

Therefore, we have $\frac{4\pi G.r_1\sigma}{4\pi G.r_2\sigma}=\frac{3}{4}$, or $\frac{r_1}{r_2}=\frac{3}{4}$ or $\frac{r_1{}^2}{r_2{}^2}=\frac{9}{16}$

$$\frac{r_1{}^2+r_2{}^2}{r_2{}^2}=\frac{9+16}{16}=\frac{25}{16} \qquad \frac{r^2}{r_2{}^2} \qquad \frac{r}{r_2}=\frac{5}{4}$$

$\therefore$ $r_1 : r_2 : r=3 : 4 : 5$

Now if V_1, V_2 and V denote respectively the potentials at the centres of the shells of radii r_1, r_2 and r,

then $V_1 : V_2 : V=4\pi G.r_1\sigma : 4\pi G.r_2\sigma : 4\pi G r.\sigma=r_1 : r_2 : r=3 : 4 : 5.$

12. The potentials of two homogeneous spherical shells at internal points are in the ratio of 3 : 4. Find the ratio of their radii. [C. U. 1914]

13. A particle is suspended from a point by means of an inextensible string of length 120 cm. It is projected horizontally with a velocity of 30 cm. per second. Find the height to which it will rise. [C. U. 1925] [Ans. ·45 cm.]

14. Calculate the mass of the sun given that the distance between the sun and the earth is 1.49×10^{13} cm., and $G=6{\cdot}66\times 10^{-8}$ c.g.s. units.

Take the year to consist of 365 days.

15. Two small balls of mass m each are suspended side by side by two equal threads of length l. If the distance between the upper ends of the threads be a, find through what angle the threads are pulled out of the vertical by the attraction of the balls. [*Ans.* $\theta=\tan^{-1}mG/(a-x)^2g$]

CHAPTER VI

ELASTICITY

101. Elasticity : When a system of forces act on a body the body is deformed to a greater or less extent, and due to this altered condition of the body, a force is called into play tending to restore the body to its original state. The restoring force called into play owing to the deformation of the body is termed the **stress** and the deformation or change produced in the body is called the **strain.**

Thus a body which offers a stress tending to restore the body to its original state when it is strained, is said to be *elastic* or to possess *elasticity.*

The ratio of stress to strain is constant for a particular material and for a particular temperature, and is known as the **coefficient or modulus of elasticity.**

If the body remains in the deformed state after the forces (external) cease to act, it is said to be *plastic.*

It is important to note here that the magnitude of the restoring force is not always equal to that of the deforming force. When a lump of a soft material such as a piece of cork or clay is strained by a force, the restoring force called into play by the deformation, is much smaller than the deforming force, for the piece of cork or clay does not regain its original state when the deforming force ceases to act. The restoring force becomes equal and opposite to the deforming force when the strained body is in a state of equilibrium *i.e.,* when the restoring force is such as to be just sufficient to prevent any further strain. In this equilibrium state only, the stress or the restoring force is equal and opposite to the deforming force and hence, the stress is measured in terms of the deforming force.

101a. Elastic and Plastic bodies : The body which recovers its size and shape fully when the deforming force is removed, is said to be perfectly **elastic.**

The body which retains completely its altered shape and size after the deforming forces cease to act, is called perfectly **plastic.**

Stress is measured as the force exerted per unit area of the body.

Strain is generally measured as deformation or distortion in the body per unit dimension.

102. Different kinds of Stress : Stress is generally divided into three classes.

(1) *Tensile or Longitudinal stress.*—It is the stretching force acting per unit area of the section of the solid acting in the direction of its length.

(2) *Compressive or Volume stress.*—It is the uniform pressure acting per unit area on the faces of a cube.

(3) *Shearing or Tangential stress.*—It is the force acting tangentially per unit area on the body.

103. Different kinds of Strain : Strain produced in a body by the application of external forces acting on it may be divided into three kinds according as they consist of a change in length and in volume or a change in shape only.

(1) Longitudinal or tensile strain : It is the change of length per unit length in the direction of the length of a rod or wire whose length is very large in comparison with its dimensions.

Longitudinal strain $= \frac{l}{L}$, where L is the original length of the rod and l the change produced in this length.

(2) Volume strain : The change in volume without any change in shape may be produced in all the three states of matter when a body in any one of these states is subjected to uniform pressure acting normally everywhere on its surface. This strain is measured by the change in volume per unit volume of the body.

If V be the original volume of the body and v, the change produced in this volume, then *Volume Strain* $= \frac{v}{V}$.

(3) Shearing strain : The shape of the body may be altered in various ways *viz.*, (1) by the tension or stretching, (2) by flexure or bending, (3) by torsion or twisting.

(1) If weights are attached to the end of a wire, the other end being held fast, the wire will elongate and when the weights are removed it regains its original length provided it is not strained beyond the *elastic limit*. The strain in this case is the **longitudinal strain** or **tensile strain**.

(2) If one end of a rod is clamped by a vice and if the other end supports a load, the rod becomes bent.

(3) If one end of a wire be kept fixed and the other end twisted the shape is altered.

Again, if a force be applied horizontally to the upper surface of a block of a material (say India rubber) in the form of a parallelopiped fixed down to a horizontal bed all the upper layers remain undistorted and unaltered in dimension while they are displaced tangentially relative to each other. The substance is said to be

sheared and the stress produced in the body is called the *shearing stress*. The body is changed in shape but not in volume and the shearing strain thus produced is measured by the angle of deformation or shear.

104. Hooke's Law : Up to the elastic limit the stress acting on a body is proportional to the strain produced in the body and the relation between the stress and the strain is represented by a straight line. The term **elastic limit** means the largest deformation which does not leave permanent distortion.

105a. Elastic Limit : The elastic limit is also defined by the load, in kilograms at which the body, say a wire, just ceases to return to its original form, when the load is removed.

If a body be deformed more than the elastic limit, then on the removal of the stress, it does not regain its original form but acquires a permanent set. If the stress be still further increased the strain begins to increase more quickly than the stress until the *yield point* is reached and the body (the strained wire, say) breaks.

105b. Behaviour of a strained wire : The *stress-strain* curve shows how the strain changes with the stress.

At O (Fig. 56) the stress is zero and so the strain is zero. From O up to the point A, Hooke's Law holds as the strain increases proportionately with the gradual increase of stress. The point A corresponds to the *elastic limit*. With further increase of stress the strain increases at a faster rate until the point B is reached. The point B is called the *yield point*.

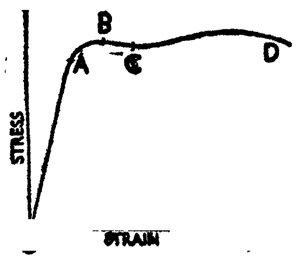


Fig. 56

The wire then goes on increasing in length up to C without any increase of load.

Beyond C the stress (load) increases up to D where contraction of the section begins, rupture taking place shortly thereafter. The ordinate of D gives the breaking stress.

According to the positions of various points the bodies may be classified as *ductile, plastic* or *brittle*.

The stress *i.e.*, the force per unit area for which the wire breaks is termed the *Breaking Stress* and the *Breaking Weight* of the wire is the product of its breaking stress and the area of its cross section.

If the deforming force be continued for a long time, the shape of the body will be permanently altered even though the stress be

less than the limit of elasticity. This phenomenon is referred to as *elastic fatigue.*

By Hooke's Law, $\frac{stress}{strain}$ = a constant called Modulus or Coefficient of Elasticity.

That is, the ratio of a stress to the strain (of a given kind) which the stress produces is called the **coefficient** or the **Modulus of Elasticity.**

105c. Elastic Constants : There are four kinds of elastic constants of a solid. *Young's Modulus* or *Tensile Elasticity*, *Bulk Modulus* or *Volume Elasticity*, *Shear Elasticity* or *Simple Rigidity*, and *Possion's Ratio* are the four elastic constants.

106. Definition of various Moduli : Young's Modulus or Coefficient of Tensile elasticity : Young's Modulus may be defined as the ratio of the longitudinal stress to the longitudinal strain, the stress being within elastic limit.

Let a force F act along the length L of a wire of cross section A and produce an elongation l, within elastic limit.

Then longitudinal stress $= \frac{F}{A} = \frac{F}{\pi r^2}$, where r is the radius of the wire. The longitudinal ~~stress~~ Strain $\frac{l}{L}$.

$$\therefore \text{ Coefficient or Young's Modulus, } Y = \frac{\frac{F}{A}}{\frac{l}{L}} = \frac{F \times L}{l \times A} = \frac{FL}{\pi r^2 l} = \frac{mgL}{\pi r^2 l}$$

dynes per sq. cm.

Here m is the mass of the load and g, the acc. due to gravity.

107. Experimental determination of Young's modulus : Vernier Method : Two wires A,B (Fig. 57) of the same material and size are taken and their ends are fixed close together to the same support C. One of the wires A is stretched by a constant load W to keep it taut while the free end of the other wire B carries a hanger to receive weights F for producing different amounts of extension. The wire which is stretched tight carries a graduated brass scale S over which slides a vernier V carried by the other wire and the elongation produced is measured by the position of the vernier zero on the scale correctly up to one-tenth of a millimetre. The object of using two wires of the same material

and of the same length is to eliminate the effect of any change of temperature, for both these wires would be affected equally by any change of temperature. Again any yield in the support would depress both the wires equally and consequently there will be no change in the elongation reading.

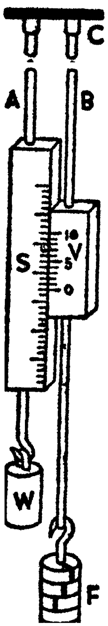


Fig. 57

The wires are at first just stretched by loads and the initial vernier reading is taken. These loads which may be from 2 to 5 kgms. depending on smaller or larger thickness of the wires are called **dead loads**. In calculating results the dead load is taken as zero load. Weights are then gradually placed on the hanger attached to the wire carrying the vernier keeping the stretching force of the other wire the same as before and the elongations due to different loads increased in step by 1 or ·5 kgm. are then read off from the scale and vernier.

Two sets of elongations, one for the increasing and the other for the decreasing loads are taken. If the two sets of elongations do not agree, the mean of the elongations for the same load in the two observations, is taken.

The results are then tabulated and a graph (Fig. 58) drawn with elongations as abscissa and loads as ordinates. From the graph the elongation for any particular load can be easily obtained. The length of the wire is carefully measured and the diameter is also measured crosswise at different positions along the length of the wire and from which the mean radius is determined. Then, using the formula $Y = \frac{FL}{\pi r^2 l} = \frac{mgL}{\pi r^2 l}$, where m is mass of the load and g the acc. due to gravity, the Young's Modulus of the material of the wire is determined. In this experiment care should be taken not to overload the experimental wire carrying the vernier beyond its elastic limit.

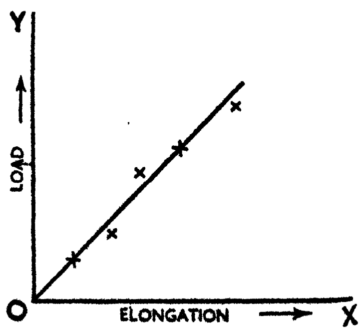


Fig. 58

107a. Searle's apparatus for determining Young's Modulus:

Two wires of the same material are suspended from the same rigid support (Fig. 59) each of which carries at its lower end a brass rectangular frame. The frames are joined to each other by cross pieces

C_1 and C_2 with the help of hinges so that there is no twisting of the suspended wires, and that one frame may be displaced relatively to the other in a vertical direction. One end of a plane metal strip. which carries a spirit level L is pivoted on a cross-rod in the frame ABCD while its other end rests upon a micrometer screw S operated by the divided head S′ working in the frame EFGH. From the hanger attached to ABCD suitable load is suspended to make the wire attached to ABCD straight. Sufficient weight is also put on the scale pan supported by the frame EFGH to keep the wire attached to it stretched and hence straight. The micrometer screw is then worked to bring the bubble of the spirit level to the centre. The load on the scale pan is then increased by 1 or ·5 kgm. so that the wire carrying the scale pan is stretched and the air bubble displaced. By turning the screw the bubble is again brought to the central position. The elongation of the experimental wire is thus found from the readings of the screw head along the linear scale fixed to the frame EFGH. Then proceeding as in the previous method the Young's modulus of the material of the wire can be determined.

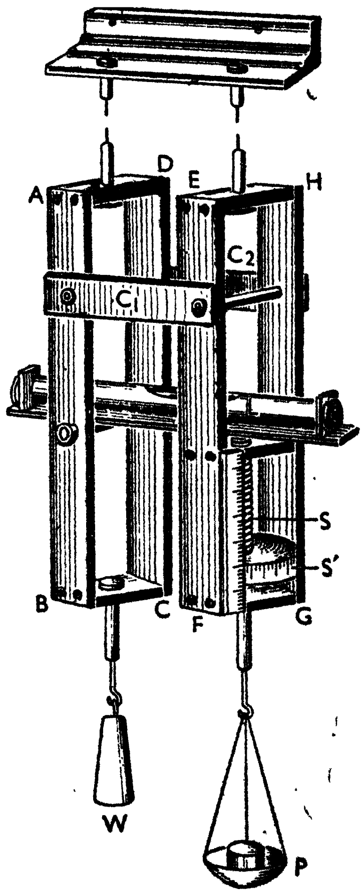


Fig. 59

108. Bulk Modulus or Coeff. of Volume Elasticity :

The ratio of stress to strain in the body which undergoes a change in volume only (without any change in shape), is known as *Bulk Modulus.* Here the stress acting on the body is the pressure and the strain is the change in volume per unit volume of the body.

If by the pressure p, *i.e.*, the force per unit area on the body, a change in volume, v is produced in the body of volume V, then the Bulk Modulus E is given by $E=\dfrac{p}{\dfrac{v}{V}}=\dfrac{pV}{v}$.

The determination of Bulk Modulus of solids and liquids presents great difficulties, but in gases where Hooke's Law does not hold good, the ratio of the increase of stress to that of strain expresses the bulk modulus in any given condition.

The Bulk modulus of any liquid can be determined by using Peizometer or by determining the velocity of sound inside the liquid by the relation $V=\sqrt{\frac{E}{D}}$, where V is the velocity of sound and E the volume elasticity and D, the density of the liquid.

The Bulk modulus of a gas may be determined under two different conditions. If the temperature of the gas remains constant during the change of volume, the bulk modulus determined in this case is called the *isothermal elasticity* and is found to be numerically equal to P, the pressure of the gas. But if the change of volume in the gas takes place so rapidly that no heat is allowed to pass out or enter into the gas to or from outside, the modulus in this case is called the *adiabatic elasticity* and is found to be equal to γ times the pressure of the gas, γ being the ratio of the sp. heat of the gas at constant pressure to that at constant volume.

109. Compressibility : It is the ratio of the compression to the pressure producing it.

The compression is expressed as v/V, where v is the diminution in volume produced in the body of volume V by the pressure p.

Thus, Compressibility $=\dfrac{\frac{v}{V}}{p}=\dfrac{v}{pV}$. But we have, volume elasticity or Bulk modulus $=\dfrac{pV}{v}$. Therefore volume elasticity is reciprocal of compressibility.

110a. Isothermal elasticity of a gas is numerically equal to the pressure of the gas : Let V be the volume of a certain mass of gas under a pressure P and let v be the diminution in volume by an increase of pressure p.

Then, since the temperature of the gas remains unchanged, according to Boyle's law, we have

$$(P+p)(V-v)=PV\ ; \text{ or } PV-Pv+pV-pv=PV.$$

$$\text{or } Pv=pV \qquad [v \text{ and } p \text{ being very small quantities the product } pv \text{ is neglected}].$$

$$\therefore P=\frac{pV}{v}$$

or $P = \frac{p}{\frac{v}{V}} = E_i$ (Isothermal elasticity), or $E_i = P$

110b. Alternative proof by Calculus : Let p and v be pressure and volume respectively of a gas at a certain temperature. Suppose a change of pressure by dp produces a change in volume of the gas by dv, temperature remaining constant, then Bulk modulus $E = \frac{dp}{dv} = \frac{vdp}{dv}$

Since temperature is constant, *i.e.*, change is isothermal then

$pv = K$ (a constant)

Differentiating $pdv + vdp = 0$; $vdp = -pdv$

or $\frac{vdp}{dv} = -p$

$\therefore$ Isothermal (Bulk) Elasticity $E_i = p$

The negative sign in the above expression indicates that with the increase of pressure the volume diminishes.

111a. Adiabatic elasticity is γP or γ times the isothermal elasticity : The relation between pressure P and volume V of a gas under adiabatic condition is expressed by $PV^{\gamma} =$ Constant, where γ is the ratio of the sp. heat of gas at constant pressure to that at constant volume.

If the pressure increases by p and consequently the volume diminishes by v, we have $PV^{\gamma} = (P+p)(V-v)^{\gamma}$

$= (P+p)(V^{\gamma} - \gamma v V^{\gamma-1} + \cdots) = PV^{\gamma} - \gamma P v V^{\gamma-1} + pV^{\gamma} - \gamma v p V^{\gamma-1} + \cdots$

or $pV^{\gamma} - \gamma P v V^{\gamma-1}$, since p and v are very small, the term $\gamma p v V^{\gamma-1}$ and terms containing higher powers of p and v are neglected.

$\therefore$ $pV = \gamma P v$ or $\frac{pV}{v} = \gamma P$, Here $\frac{pV}{v} = E_A$ (adiabatic elasticity)

$\therefore$ adiabatic (Bulk) elasticity $E_A = \gamma P$.

Thus we see that the *adiabatic elasticity* of a gas is $\gamma \times$ pressure of the gas, or γ times the *isothermal elasticity*.

111b. Alternative Proof by Calculus :

As in 110b, Bulk modulus $E = \frac{v.dp}{dv}$.

Under adiabatic conditions the relation between the pressure p and volume v of a gas is expressed by $pv^{\gamma} = C$..

Differentiating, we have $\gamma p v^{\gamma-1}.dv + v^{\gamma}.\ dp = 0$

or $\quad -v^{\gamma} dp = \gamma p v^{\gamma-1} dv$, or $-\dfrac{v^{\gamma}.dp}{dv.v^{\gamma-1}} = \gamma p$, or $\dfrac{vdp}{dv} = -\gamma p$

$\therefore$ E_A (adiabatic bulk elasticity) $= \gamma$ times $p = \gamma$ times E_i
$= \gamma$ times isothermal elasticity.

112. Rigidity Modulus: When a body suffers a shearing strain by the application of a tangential force on its surface, *the modulus of rigidity is defined as the ratio of the tangential force per unit area to the angular deformation produced.*

Let the upper face AB of the rectangular block (Fig. 60) or a cube be sheared through the distance AA′ or BB′ by a force P acting uniformly and tangentially over the upper face AB, the lower face remaining fixed. The rectangular block undergoes only a change of form but no change of volume.

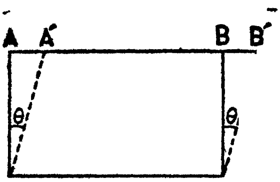


Fig. 60

The block suffers a shearing strain which is measured by the angle θ, called the angle of shear.

Let the perpendicular distance between the upper and the lower face be equal to b while AA′ or BB′ is equal to x.

$\therefore$ The shearing strain $\theta = \tan\theta$ (since θ is small)

$$= \frac{x}{b}.$$

Let the area of the upper face of the block be s.

$\therefore$ The shearing stress $= \dfrac{P}{s} = p$

Thus the **Modulus of Rigidity** η is given by,

$$\eta = \frac{\text{shearing stress}}{\text{shearing strain}} = \frac{P/s}{\theta} = \frac{p}{\theta} = \frac{p}{x/b} = \frac{pb}{x}$$

113. Twisting of a cylinder: Consider a cylinder fixed at one end and twisted at the other end by a couple whose axis coincides with the axis of the cylinder. (Fig. 61).

The restoring couple is due to the shear elasticity of the cylinder and is called the shearing couple.

To calculate the moment of the shearing couple, let us consider an elementary ring of radius x and width dx situated at a distance l from the fixed end.

Let an element at P on the ring in the undisplaced position be shifted to P′ through θ. Then the linear shift PP′ $= x\theta$ and the shearing strain $= \frac{x\theta}{l}$. Hence, the shearing stress $= \frac{\eta x\theta}{l}$ where η is the rigidity of the material of the cylinder.

Then the shearing force on the whole ring $= \frac{\eta x\theta}{l} \times 2\pi x.dx$ since $2\pi x dx$ is the area of the whole ring.

The moment of this force about the axis of the cylinder

$$= 2\pi x dx \frac{\eta x\theta}{l}.x = \frac{\eta.2\pi\theta}{l} x^3 dx$$

∴ The moment of the shearing couple G for the whole cylinder of radius r

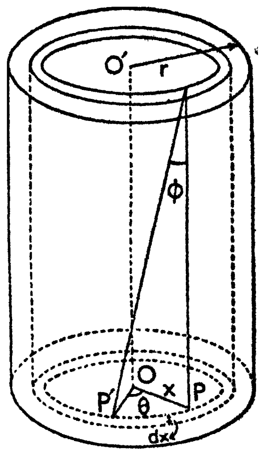


Fig. 61

$$= \int_0^r \eta.\frac{2\pi\theta}{l} x^3 dx = \frac{\pi\eta r^4\theta}{2l}.$$

If $\theta = 1$ radian, we have, twisting couple per unit twist of the cylinder (or wire) $= \frac{\pi\eta r^4}{2l}$.

This twisting couple, per unit twist of the wire, is also called the torsional rigidity of the cylinder or wire.

114. Determination of the Rigidity Modulus of a wire: Statical Method: The wire for which the rigidity is to be determined is fixed at the top, (Fig. 62) the free end of it being firmly attached to a metal cylinder. Two strings are wound round the cylinder in such a way that they leave the cylinder at the opposite extremities of a diameter in two opposite directions, pass over two pulleys and carry two scale pans on which loads are to be placed.

As loads of equal mass are placed in the pans, the cylinder rotates and the pointer fixed to the top of the cylinder also moves showing deflection or rather twist on a circular scale graduated in degrees and placed above the cylinder.

Let l be the length of the string from the support to the top of the cylinder, m, the mass of the load (including the scale-pan) and θ, the deflection or twist produced for the load m.

The couple exerted by the string on the cylinder $= 2mg$R and since this couple is balanced by the couple G exerted by the wire, we have $G = 2mgR$, where R is the radius of the cylinder. But $G = \pi\eta r^4\theta/2l$, where η is the rigidity and r, the radius of the wire.

$$\therefore \quad \frac{\eta\pi r^4\theta}{2l} = 2mgR, \text{ or } \eta = \frac{4mRgl}{\pi\theta r^4}$$

whence η can be found out.

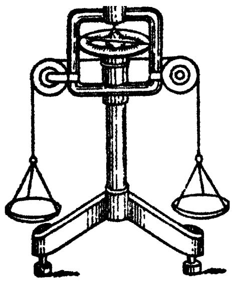


Fig. 62

114a. Rigidity modulus by Torsional Pendulum (Dynamical method) :

The arrangement called the torsion pendulum consists (Fig. 63) in its simplest form, of a long uniform wire PO hung vertically from a rigid support. The lower end of the wire is fixed to a massive disc D (or cylinder) of brass so kept that the axis of the wire coincides with that of the disc (or cylinder). To perform an experiment a fine vertical line is marked on one side of the disc or cylinder. The line is focussed and viewed by a telescope. The cylinder is then turned through a small angle, clockwise or anticlockwise so that the wire supporting it is twisted. Due to shear elasticity the wire tends to untwist itself as the twisting force is removed. As the velocity does not at once become zero, torsional oscillations are set up. The period of oscillation can be measured by observing the movements of the mark on the side of the disc or cylinder through the telescope.

The period of oscillation $T = 2\pi\sqrt{\frac{I}{C}}$, where I is the moment of inertia of the cylinder, C the torsional couple for unit twist. But we know $C = \pi\eta r^4/2l$, where the symbols have usual significance.

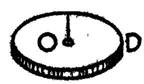


Fig. 63

$$T = 2\pi\sqrt{\frac{2lI}{\pi\eta r^4}}, \text{ or } T^2 = 4\pi^2\frac{2lI}{\pi\eta r^4}, \text{ or } \eta = \frac{8\pi l.I}{T^2 r^4}$$

The value of I can be calculated from the mass and dimensions of the cylinder. Then knowing all quantities of the right hand side, η can be found out.

115. Poisson's Ratio :

It is the ratio of the lateral strain to the longitudinal strain produced in a wire when it is stretched by a force along its length. If l be the elongation produced by the force in length L of the wire and d, the change produced in the diameter D of the wire then,

$$\text{Poisson's Ratio } \sigma = \frac{d/D}{l/L} = \frac{dL}{Dl}.$$

116. Complementary stresses due to shear :

It can be proved that a shear stress in a given direction cannot exist without an equal shear stress existing at right angles to it.

Consider a rectangular solid of sides a, b and c, shown in fig. 64. Let F_1, F_1 be the forces applied which tend to displace the upper face with respect to the lower. The area of each of these faces is ab, so that the shearing stress is equal to F_1/ab. Suppose F_2, F_2 be the shearing forces at right angles to F_1, F_1. The corresponding stress is then equal to F_2/bc. In the position of equilibrium, moment of all the forces about any point in their plane, should be zero, so that, $F_1.c = F_2.a$.

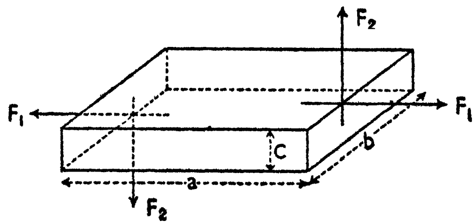


Fig. 64

Dividing by abc, $\frac{F_1}{ab} = \frac{F_2}{bc}$; but F_1/ab and F_2/bc are stresses at right angles to each other.

Therefore the stresses are equal.

117. Shear is equivalent to compression and extension :

We know that when a cube is sheared its volume is unchanged but its shape is altered, the thickness of the cube remaining same.

In the figure 65 in which ABCD is the section of a unit cube the diagonal DB has been increased to length DB′ and at the same time diagonal AC has been shortened to A′C.

Since the amount of shear is extremely small, this extension and compression may be expressed in terms of the angle $\theta(=\angle ADA' = \angle BCB')$ and if BE and A′F be drawn perpendicular to the diagonals

DB′, AC, the triangles AFA′ and BEB′ will be right angled 45° and triangles.

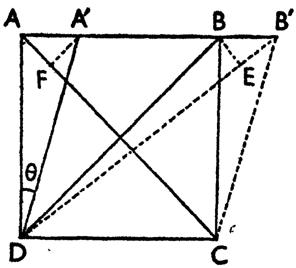


Fig. 65

Then $EB' = BB' \cos 45 = BB'/\sqrt{2}$

If $AB = l$, $BD = DE = l\cos 45 = l\sqrt{2}$

$\therefore$ Extension along the diagonal BD

$$\frac{EB'}{DB} = \frac{BB'}{\sqrt{2}} \times \frac{1}{l\sqrt{2}} = \frac{BB'}{2l}$$

But $\dfrac{BB'}{l} = \theta$, Extension $= \dfrac{\theta}{2}$.

Similarly, the compression along AC

$$= \frac{AF}{AC} = \frac{AA'}{\sqrt{2}} \cdot \frac{1}{AC} = \frac{l\theta}{\sqrt{2}.l\sqrt{2}} \quad \frac{l\theta}{2l} \quad \frac{\theta}{2} \quad [\because AA' = BB' = l\theta]$$

Thus a simple shear θ is equivalent to compression and extension at right angles to each other, each of value $\theta/2$.

118. Relation between elastic constants, Y(Young's modulus), η (rigidity modulus), k (bulk modulus) and σ (poisson's ratio) :

Let a unit cube of a substance be under the action of tangential stresses as shown in the figure 66(a).

The result will be to distort the cube so that the face ABCD becomes a rhombus A′B′CD, as in the figure 66(b).

There is only a change of shape but the size remains the same since the area ABCD is equal to that of A′B′CD. The body is unchanged in dimension in a direction perpendicular to ABCD.

The forces applied to the cube must be in equilibrium among themselves *i.e.*, they must have no tendency to set the cube in motion either linear or rotational.

The forces T, T acting on the faces EFBA and HGCD have no tendency to communicate linear motion to the cube, on the other hand they constitute a torque which tends to set the cube in rotation, and so to produce equilibrium, an equal and opposite torque T, T on the faces FBCG and EADH must be applied.

Thus the forces T, T, T, T acting on the above faces are sufficient to produce a shear.

The strain in this case is called a shear or a shearing strain and is measured by the angular deformation θ.

Then according to Hooke's law, we have $\eta = S/\theta$, where S is the tangential stress and θ the strain.

A shear may be regarded as a combination of an extension together with a contraction perpendicular to the extension.

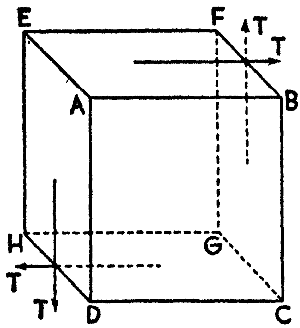


Fig. 66(a)

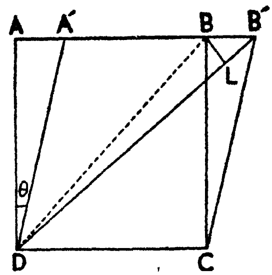


Fig. 66(b)

For in the figure 66(b) the diagonal DB becomes of length DB′ strain and if BL be drawn perpendicular to DB′, then the extension along DB is

$$\frac{DB'-DB}{DB}=\frac{LB'}{DB}=\frac{BB'\cos 45}{\sqrt{2}BC}=\frac{BB'}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}BC}=\frac{BB'}{2BC}$$

$$=\frac{\theta}{2}=\frac{1}{2}\cdot\frac{S}{\eta}.$$

Now the strain in DB may be regarded as compounded of a part due to the stretching force S in DB *i.e.*, $\frac{S}{Y}$ and a part due to a compressive force S in CA *i.e.*, $\sigma\frac{S}{Y}$, where σ is the Poisson's Ratio.

[**Note** : Young's modulus $Y=\frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}=\frac{S}{\text{Longitudinal strain}}$

∴ Longitudinal strain $=$

Again Poisson's Ratio $\sigma=\frac{\text{Lateral contraction strain}}{\text{Longitudinal strain}}$

∴ Lateral contraction strain $=\sigma\cdot\frac{S}{Y}$]

$$\therefore\ \frac{1}{2}\frac{S}{\eta}=\frac{S}{Y}+\sigma\frac{S}{Y}=\frac{S}{Y}(1+\sigma)\ \textit{i.e.},\ Y=2\eta(1+\sigma). \qquad (1)$$

If the unit cube be subjected to a uniform normal pressure P over each face, the strain in this case is $\frac{\delta V}{V}$, where δV is the diminution in volume and V the original volume.

But by Hooke's Law $\frac{\text{Stress}}{\text{Strain}} = \text{Bulk modulus K}$. $\therefore \frac{\delta V}{V} = \frac{P}{K}$.

Let each side of the unit cube become $1-\alpha$, where α is the contraction. Then the altered volume of the cube $=(1-\alpha)^3 = 1-3\alpha$ (approx.). $\therefore \frac{\delta V}{V} = 3\alpha = \frac{P}{K}$; $\therefore \alpha = \frac{P}{3K}$.

Again, in the direction perpendicular to one pair of faces the pressure P on these faces produces a compression $\frac{P}{Y}$ and the pressures on the other two pair of faces produce stretches each equal to

$$\frac{\sigma P}{Y}. \quad \therefore \frac{P}{3K} = \frac{P}{Y} - \frac{2\sigma P}{Y} = \frac{P}{Y}(1-2\sigma)$$

$$\therefore \quad Y = 3K(1-2\sigma) \qquad (2)$$

From (1) and (2) we have $\sigma = \frac{3K-2\eta}{6K+2\eta}$; $Y = \frac{9K\eta}{3K+\eta}$.

Again, from (1) and (2), $Y = 2\eta(1+\sigma)$; $Y = 3K(1-2\sigma)$ *i.e.*, $2\eta(1+\sigma) = 3K(1-2\sigma)$. Here η and K are positive quantities,

or $\frac{3K}{2\eta} = \frac{(1+\sigma)}{(1-2\sigma)}$; when $\frac{3K}{2\eta}$ is $+ve$, $\frac{1+\sigma}{1-2\sigma}$ is also $+ve$.

If σ is $>\frac{1}{2}$, the numerator will be positive, but the denominator will be negative. But if σ is <-1, the denominator will be positive while the numerator will be negative.

Hence, the fraction will be + only when σ lies between $\frac{1}{2}$ and -1.

119. Dimensions of Young's modulus, Bulk modulus and Rigidity modulus :

$$\text{Young's modulus} = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

$$= \frac{\text{Force per unit area}}{\text{Elongation per unit length}} = \frac{F/A}{l/L} = \frac{MLT^{-2}}{L^2} \div \frac{L}{L} = ML^{-1}T^{-2}$$

Since the dimensions of force F are MLT^{-2} and dimension of area A is L^2.

As the longitudinal strain is measured by the ratio of the change in length to the original length, it has therefore no dimension.

$$\text{Bulk modulus} = \frac{\text{Compressive stress}}{\text{Compressive strain}} = \frac{p}{dv/v} = \frac{F/A}{dv/v}$$

$$= \frac{MLT^{-2}/L^2}{L^3/L^3} = ML^{-1}T^{-2}.$$

$$\text{Rigidity modulus} = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{F/A}{\theta} = MLT^{-2}/L^2 = ML^{-1}T^{-2}$$

since the shearing strain θ has no dimension as it is the ratio of one length to another length.

$$\text{Poisson's Ratio} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{d/D}{l/L} = \frac{L/L}{L/L} = 1$$

So Poisson's Ratio has no dimension.

Hence, all the three moduli have the same dimensions, as the stress in all cases is expressed as a certain number of units of force per unit area and the strain has no dimension since it is the ratio of two quantities of the same kind.

QUESTIONS

1. What do you understand by stress, strain and elastic limit ? Explain the terms :—Young's modulus, Bulk and Rigidity modulus, and Poisson's ratio. Derive an expression shewing the relation between them. Hence, shew that Poisson's ratio lies between ½ and −1. [C. U. 1942, '45, '47, '52, '53, '54, '55]

2. What is coefficient of elasticity ? [C. U. 1952]
Write down the dimensions of Young's modulus and Rigidity modulus. [C. U. 1943, '53, '54]
Define an isotropic body. [C. U. 1949]

3. Describe a method of finding experimentally the modulus of rigidity of a solid and give the theory of the method. [C. U. 1943, '45, '55]

4. Obtain the relation between Young's modulus, modulus of rigidity and Poisson's ratio. [C. U. 1954]

5. Explain Hooke's law. What do you mean by "elastic limit" and "breaking strain" ? [C. U. 1956]

6. Explain the terms (a) Young's modulus (b) Shear Modulus and (c) Poisson's ratio.

A rod of diameter 1 inch and length 100 inches has Young's modulus 30×10^6 lbs/sq. inch and Poisson's ratio 0·3. What is the strain along the length ? [C. U. 1958]

EXAMPLES

1. What will be the elongation produced in a steel wire one metre long and sq. mm. cross section when it is stretched with the weight of a kilogramme, Young's modulus for steel being 2×10^{12} dynes per square centimetre ? [C. U. 1912]

$$\text{Young's modulus} = \frac{\text{Stress}}{\text{Strian}} = \frac{\text{force per unit area}}{\text{elongation per unit length}}$$

Let l be the elongation produced. 1 metre = 100 cm., 1 sq. mm. = ·01 sq. cm. and wt. of a kilogramme = 1000×980 dynes. $\therefore$ $2\times10^{12} = \dfrac{\dfrac{1000\times980}{\cdot01}}{\dfrac{l}{100}}$

$$\therefore\quad l=\frac{1000\times980\times100}{\cdot01\times2\times10^{12}}\text{ cms.} = \cdot0049\text{ cms.}$$

2. Taking the modulus of steel wire as 1800×10^{6} gram-wt. per sq. cm. find the elongation of such a wire 10 metres long and 1 sq. mm. in cross sectional area when stretched by weight of 10 kilos. [C. U. 1910]
(wt. of 10 kilos = $10\times1000\times980$ dynes) (Ans. = ·55 cm.)

3. Calculate in C. G. S. units the value of Young's modulus from the following data : [C. U. 1918] [D. U. 1944]
Mean extension for 6 Kgm. = 0·537 mm.
Radius of the wire = 0·675 mm.
Length of the wire = 250 cms. (Ans. $1{\cdot}99\times10^{12}$ dynes per sq. cm.)

4. A steel wire of length 2 metres and of diameter 0·52 mm. is suspended from an unyielding support and loaded with 5 kilogrammes. An elongation of 2·31 mm. is observed. Calculate the coefficient of longitudinal elasticity of steel. [C. U. 1935]

$$\text{We have } Y=\frac{F.\ L}{\pi r^2 l}=\frac{5\times1000\times981\times200}{\pi\times(\cdot026)^2\times\cdot231}\text{ dynes per sq. cm.}$$
$$=\frac{5\times1000\times981\times200\times7}{22\times(\cdot026)^2\times\cdot231}$$
$$=2\times10^{12}\text{ dynes per sq. cm.}$$

5. A copper wire 2 metres long and 0·5 mm. in diameter, supports a mass of 10 kilogrammes. It is stretched by 2·38 mm. Calculate the Young's modulus of the wire. [C. U. 1953]
[Ans. $4{\cdot}2\times10^{12}$ dynes/cm^2]

6. You are given 200 c. c. of air at a pressure due to 760 mm. of mercury. On increasing the pressure by that due to 1 mm. of mercury without change of temperature, the volume is observed to decrease by 0·263 c. c. Find the coefficient of volume elasticity of the gas. [C. U. 1952]

$$\text{Volume elasticity} = \frac{p}{\frac{v}{V}} = \frac{\cdot1\times981\times13{\cdot}6\times200}{\cdot263} = 1{\cdot}0146\times10^{6}\ \text{dynes/cm}^2.$$

The sum may also be worked out from the relation, pressure = isothermal elasticity.

7. Compare the density of water at the surface and at the bottom of a lake 100 metres deep, given that the compressibility is $\frac{1}{20000}$ per atmosphere and that the density of mercury is 13·6. [C. U. 1928]

Compressibility (C) is the reciprocal of Bulk modulus.

Therefore, the compressibility C is equal to $\frac{v_1 - v_2}{v_1}$, where v_1 and v_2 are the volumes of unit mass at the surface and at the bottom of the lake and P, the pressure in atmosphere at the bottom of the lake.

Then $C = \dfrac{\frac{v_1 - v_2}{v_1}}{p} = \dfrac{1 - \frac{v_2}{v_1}}{p}$. Therefore $C.p = 1 - \frac{v_2}{v_1}$ or $\frac{v_2}{v_1} = \frac{d_1}{d_2} = 1 - C.p$

where d_1 and d_2 are the densities at the surface and at the bottom of the lake ; or $\frac{d_1}{d_2} = 1 - \frac{1}{22000} \times \frac{100 \times 100}{13{\cdot}6} \times \frac{1}{76} = 1 - \frac{10}{22 \times 13{\cdot}6 \times 76}$

$= 1 - {\cdot}00044 = {\cdot}99956.$

8. A metal wire one metre long. and of 1 sq. mm. cross section is stretched with the weight of 2 kilograms. Find the elongation of the wire. Young's modulus for the wire $= 2 \times 10^{12}$ dynes/sq. cm. [C. U. 1954]

Young's modulus $= \frac{\text{stress}}{\text{strain}} = \frac{\text{force per unit area}}{\text{elongation per unit length}} = \frac{mgL}{\pi r^2 l}$

Here, l is the elongation, L = 1 metre = 100 cms., 1 sq. mm = ·01 sq. mm ; wt. of 2 kilogrames = 2000 × 980 dynes.

$\therefore$ elongation $l = \frac{mgL}{\pi r^2 Y} = \frac{2000 \times 980 \times 100}{{\cdot}01 \times 2 \times 10^{12}} = {\cdot}0098$ cm.

9. A metal rod having coefficient of linear expansion $= 12 \times 10^{-6}$ per deg. C. has its temperature raised by 10°C. Calculate the linear compressive stress required to prevent expansion of the rod, assuming $Y = 2{\cdot}0 \times 10^{12}$ C.G.S. units. [C.U. 1956]

Let l be the original length of the rod and l' be the length of the rod when its temperature is raised by 10°C [C. U. 1956]

Therefore, the expansion $l' - l = l \times 1{\cdot}2 \times 10^{-5} \times 10 = l \times 1{\cdot}2 \times 10^{-4}$

But Young's modulus $Y = \frac{\text{stress}}{\text{strain}} = \frac{\text{stress}}{l \times 1{\cdot}2 \times 10^{-4}/l}$

$\therefore$ Linear compressive stress $= Y \times 1{\cdot}2 \times 10^{-4}$

$= 2 \times 10^{12} \times 1{\cdot}2 \times 10^{-4}$

$= 2{\cdot}4 \times 10^{8}$ dynes/sq. cm.

CHAPTER VII

MOLECULAR PHENOMENA IN LIQUIDS

120. Molecular forces of cohesion and adhesion : If a rod or tube of glass be dipped into water and is then taken out, a drop of water will be left hanging to the end of the rod or the tube.

The definite shape of the drop leads us to conclude that its surface must act as an elastic system.

In the same way, if a clean metal ring be dipped into soap solution and then withdrawn, a film of the liquid will remain stretched across the ring.

In both these cases the effects are due to the **cohesion** of the liquid, which term implies the attraction between the different particles of the same liquid. The term **adhesion** is used to denote attraction between a solid and a liquid.

121. Molecular theory of Surface Tension : The existence of this force of cohesion in a liquid is easily explained by the molecular hypothesis in which it is assumed that in a liquid the molecules exert on one another an attractive force. This force is appreciable when the molecules are within a *short distance* between one another which is called the **range of molecular attraction.** For a liquid molecule well inside the liquid, this range of molecular attraction is a sphere, while for a molecule near the surface, the range is a small portion of the lower hemisphere.

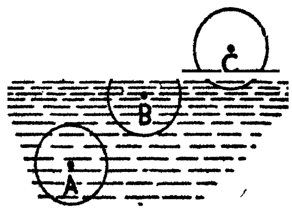


Fig. 67

Let us consider a molecule A (Fig. 67) situated well inside the liquid. This molecule exerts equal attraction simultaneously in all directions and is itself uniformly attracted by the molecules all around it and therefore no unbalanced resultant force, tending to move it in any direction is exerted on it.

But the molecule B which is very near the suface is subjected to greater force from the lower side than from the upper side.

Again the molecules, on the surface itself, experience only the forces of attraction of those molecules within the range of molecular attraction. The resultant of these unbalanced forces is directed towards the interior of the liquid.

But if the molecule be at C, a point just above the surface, the downward force becomes practically nil and the molecule is then free to move as a molecule of gas or vapour.

Thus the assumption of intermolecular forces of attraction leads to the conclusion that all the molecules in or very near the surface must experience an inward force and tend to move towards the interior of the liquid and so the surface of the liquid tends to contract and behave like a stretched elastic membrane. Thus the force along the surface which tends to make the surface to contract is known as **surface tension.**

122. Experiment to show that liquid surface behaves as an elastic membrane : If a circular loop of a stiff wire be dipped in some soap solution a soap film is formed on the loop of the wire. An endless *i.e.*, closed loop of thread placed on the film will exhibit an irregular shape. If then the portion of the film inside the loop is pierced with a pencil so as to break that part of the film, the loop of thread is stretched outwards on all sides and is found to assume a circular shape. [Fig. 68(a)]

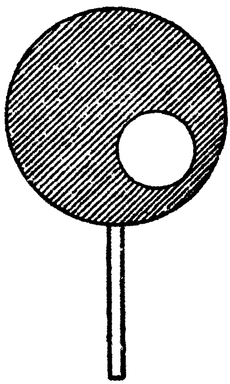


Fig. 68(a)

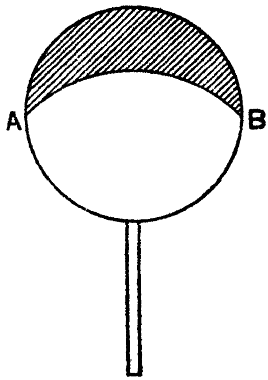


Fig. 68(b)

Now it is known that the circle is a curve that encloses the maximum area for a given length of the perimeter *i.e.*, the boundary line. Hence, the loop of thread is drawn by the surrounding film on breaking the film within the loop of thread so that the area inside the loop of thread is maximum and that of the surrounding film minimum. This shows that a film of liquid tends to contract and occupy least area just as a stretched elastic membrane would tend to do.

The above experiment can be done in another way. After making the circular loop of wire, a piece of thread is loosely tied at two points A and B on the wire loop. A thin film of soap

solution is formed upon the wire loop by dipping it momentarily in soap solution. The attached thread will lie across the film in an irregular manner dividing the film into two portions. If one portion be pierced by a pencil and broken, the thread will be found to be at once pulled towards the other portion and assume the form of an arc of a circle [Fig. 68(b)]. The unbroken portion of the film thus contracts to a minimum possible area behaving (as in previous experiment) as a stretched elastic membrane. It should be noted that the force pulling the thread is quite different from the force of gravity acting on it. If the wire loop with soap film be held vertically as in fig. 68(b) and the lower portion of the film is broken, the thread is pulled upwards though the force of gravity pulls the thread downwards.

123. Energy considerations : When we consider the molecules situated near or on the surface, their potential energy is greater than when they were in the interior of the liquid, for work must be done by the molecules against the resultant downward pull towards the interior of the liquid while moving up towards the surface of the liquid and therefore the molecules gain potential energy at the expense of their kinetic energy.

We know that any system of bodies arranges itself in such a manner that the potential energy of the system is minimum. So, to diminish the total surface energy, the area of the surface should be diminished and thus the tendency of the surface to contract is explained. The molecules on the surface lying side by side cling to one another and endow the surface with properties somewhat resembling those of a stretched membrane. Hence, the existence of *surface tension* is proved.

124. Definition and Measurement of Surface Tension : Factors on which it depends : The molecules on the surface of a liquid are acted upon by an unbalanced resultant force arising out of intermolecular attraction, towards the interior of the liquid. This force gives rise to what is called the surface tension of the liquid.

The *tension in dynes* exerted across unit length of any line imagined in the surface of the liquid is a measure of the *surface tension* of the liquid.

The surface tension depends not only on the liquid itself but also on the medium on the other side of the surface of the liquid.

A mass of any liquid when relieved from the action of gravity would assume a spherical form which has the least area for a given volume. A drop of oil when introduced into a mixture of water and alcohol of the same density as the oil, will assume a

spherical form since the buoyancy of the liquid counterbalances the action of gravity.

The value of the surface tension of any liquid depends on the **temperature** and for all liquids it *decreases* with the **rise** of temperature. It vanishes at the **critical temperature** at which there is no definite line of separation between gas and liquid.

The presence of impurities in a liquid reduces the surface tension. This is illustrated in the movement of a few pieces of camphor when thrown in water.

125. Dimensions of Surface Tension : Surface tension is the force per unit length of a line in the surface of the film acting at right angles to it. Hence, its dimensions are,

$$\frac{\text{Force}}{\text{Length}} = \frac{ML}{T^2} \div L = \frac{M}{T^2} = MT^{-2}$$

126. Phenomena due to Surface Tension :

(1) Movements of small fragments of **camphor** when placed on the surface of water is explained by the existence of surface tension of the liquid. As soon as camphor is placed on the surface of water it begins to dissolve slowly in water ; but if the camphor dissolves a little faster at one of its sides than at the other, the surface tension of the solution of camphor in water at the side in which it dissolves quickly is reduced and the greater surface tension at the other side draws the fragments away and thus causes the movement.

(2) The spherical shape assumed by a drop of mercury when laid upon a table is due to the fact that mercury does not wet the table. The shape of the drop of mercury may be partly due to the gravitational attraction exerted on each of its molecules and partly to the elastic properties of the surface caused by the molecular attraction. If the drop be small, the gravitational force is considerably diminished and the greater molecular attraction causes the drop to assume a spherical form which has the smallest area compared with any other shape of equal volume.

(3) The floating of a needle and the phenomenon of insects walking on the surface of water without sinking, may be explained by the existence of surface tension of the liquid.

(4) Rise of liquid column in a capillary tube.

(5) Formation of bubbles and soap films.

(6) Formation of ripples in water.

(7) Spreading of liquid over the surface of a solid or another liquid.

(8) Action of rain coat and umbrella.

(9) The rise of oil in wicks of oil lamps, the rise of sap in plants, the rise of ink in the narrow slit of a pen and the soaking up of ink by the blotting paper.

127. Spreading of a liquid over the surface of a solid or another liquid : When a drop of liquid is spread on a horizontal solid, the shape of the liquid will be determined by the condition that the total potential energy due to gravitation and surface tension is to be as small as possible.

If the drop is small, the potential energy due to gravity is negligible compared with that due to surface tension and so the drop takes the shape in which the potential energy due to surface tension will cause the small drops to assume spherical forms.

Drops of mercury formed on a horizontal surface, dew-drops and rain-drops are examples of this phenomenon.

When a liquid, say A is placed on another liquid B it may spread or contract and gather itself up into a drop.

In each case we have to consider three surface tensions, T_1 between the liquid A and the air, T_2 between the liquid B and the air, and T_{12} between the liquids A and B.

When the drop spreads on the liquid *i.e.*, the surface of contact between A and B is increased by S, the energy due to surface tension between these two liquids is increased by $T_{12} \times S$. Again the increase in the energy due to the surface tension between A and air is $T_1 \times S$.

But the diminution of energy due to surface tension between B and the air is $T_2 \times S$.

Hence, the total increase in the potential energy

$$= (T_1 + T_{12} - T_2)S,$$

Now if $T_2 > T_1 + T_{12}$ *i.e.*, if $(T_1 + T_{12} - T_2)S$ is negative, the liquid A will spread out into a thin film and cover B.

If T_2 be less than $T_1 + T_{12}$, a triangle can evidently be obtained so that the three forces T_1, T_{12} and T_2 can be represented by its three sides. Such a triangle is referred to as *Neumann's triangle.* Values of the three forces found experimentally, show that for any two liquids no such triangle can be constructed. Evidently one liquid when placed on another liquid always spreads over it and never can collect in form of drops, provided both the liquids are pure and free from grease and oily matters.

128. Calming of Waves by Oil : We know that the greater the contamination of water surface, the more is its surface tension reduced. When oil is poured over troubled waters, the surface of water becomes contaminated.

When the wind acts on the portion of the contaminated surface blowing it forward, the motion of the surface film will heap t

oil in front and so reduce the surface tension, leaving behind a clean part of water with increased surface tension. Thus the clean part of water with greater surface tension will pull back the heaped oily surface so as to hinder the formation of waves or surfaces with strong curvature.

129. Electrification of a surface alters its surface tension : It is found that the surface tension of mercury in contact with dilute solution of sulphuric acid depends upon the difference of potential between the solution and mercury. It becomes maximum when this difference is zero.

If this difference of potential be varied by any means and in either direction the surface tension diminishes.

Experimental proof : A little mercury is poured into a beaker containing a solution of salt or acid.

A glass tube having one end drawn out to a tapering point is held vertical with the fine end dipping into the solution and a little mercury is poured into the other end.

Now the mercury in the glass tube will not flow through the capillary part at the lower end but is prevented from running out by surface tension. A difference of potential between the mercury in the capillary tube and that in the beaker is maintained and suitably adjusted. The effect will be to make the mercury meniscus in the capillary tube rise, shewing that its surface tension is increased.

130. Capillarity or Capillary Action : If a very narrow glass tube be dipped in water or any other liquid which *wets it*, it will be observed that the liquid instead of remaining at the same height inside and outside of the tube, will rise not only against the outer wall of the tube but also to a considerable height inside the tube terminating in a **concave surface.** This phenomenon of rise of liquid in a tube of fine bore is what is known as **capillarity.** This rise or *capillary ascension* in the tube depends on the nature of the liquid and on the diameter of the tube. *Capillary depression* of a liquid also takes place when a tube of fine bore is immersed in the liquid such as mercury which *does not wet it.* It is to be noticed that the liquid inside the tube terminates in a **convex surface.**

The above capillary phenomena depends upon the molecular actions which take place between the particles of the liquid itself and between the liquid and the solid containing it. The capillary elevation or depression of the liquid inside a tube is due to the angle of contact between the material of the tube and the liquid so that the surface tension of the liquid will either raise the liquid

up or lower it down the tube according as the angle of contact is smaller or greater than 90°.

[Note :—The surface tension when expressed in grams is known as Capillary constant.]

131. Rise or Fall of liquid in a narrow tube : The above phenomena can easily be explained by the molecular theory.

Let the horizontal surface of the liquid meet the surface of the plate T at the point O. (Fig. 69).

The adhesive force between the liquid and the plate pulls the liquid molecule at O in the direction OG and the cohesive force between the molecules of the liquid pulls it in the direction OL towards the interior of the liquid.

Thus the liquid molecule at O will be acted on by the resultant force in the direction OR and therefore the surface of the liquid will set itself in such a direction as to be at right angles to the resultant forces acting on the molecules lying on it. Here *b* represents the position of the liquid surface.

In Fig. 69 (Left side) the liquid wets the plate and the adhesive force being greater than the force of cohesion, the surface of the liquid at O rises up against the side of the plate and presents a concave surface.

In Fig. 69 (Right side) the liquid such as mercury does not wet the plate and in this case the adhesive force being smaller than the cohesive force, the surface of the liquid at O is depressed and presents a convex surface upwards.

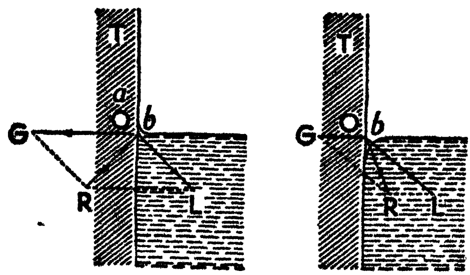


Fig. 69

132. Angle of contact : If a plate of glass be vertically dipped in water or any other liquid which wets it, it will be found that where the liquid touches the glass it is drawn up above the level of the liquid and if the same plate of glass be immersed vertically in mercury or any other liquid which does not wet it, the liquid is seen to be depressed down at the point where it touches the plate (Fig. 69). The angle which the tangent to the liquid surface at the point where it meets the plate makes with the wall of the plate inside the liquid is called the **angle of contact.**

The value of the angle of contact depends on the nature of the solid, liquid and the third material in contact with both of

If the liquid wets the solid the angle of contact will be less than 90°, but if the liquid does not wet the solid this angle is greater than 90°.

In the case of water, alcohol, chloroform etc, angle of contact with glass is nearly zero.

133. Energy of a liquid film : Let a film of any liquid be formed in the closed space ABCD of the frame-work ABCD made of thick wires, of which (Fig. 70) the bounding lines AB, BC and CD are fixed and the wire EF parallel to BC is movable. The surface tension of the film acts all along the wire EF and acts at right angles to the wire EF and in the upward direction. Since the film has two surfaces the resultant surface tension acting on the wire EF will be due to both the surfaces. Let T be the tension or the force exerted on unit length of EF due to one face of the film. Then the total upward force due to tension on EF is equal to 2T.EF.

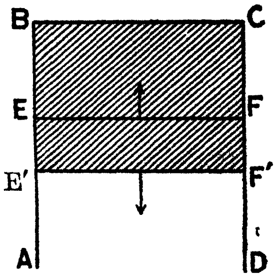


Fig. 70

If the film be stretched by pulling EF down to the position E′F′ by an external force, the work done against the force, due to surface tension, 2T.EF is equal to 2T.EF.EE′.

Then potential energy $E_p = 2T.EF.EE' = T.A$, where $2EF.F'F' = A$, A being the total area of two sides of the film on the rectangle

EE′F′F. $T = \frac{E_p}{A}$

Thus the surface tension of a liquid film is equal to the potential energy per unit area of the film.

Note : The above relation is not quite correct as the above quantity of work does not represent the whole of the energy expended when a fresh surface is formed. When the surface of the film is enlarged it becomes cooled unless any heat is supplied to it. If the enlargement takes place isothermally, an equivalent amount of heat flows in from outside to keep the temperature constant. So when the surface area of the film is increased by one square centimetre, the **total surface energy** is equal to the work done in overcoming the surface tension plus the work equivalent of heat absorbed and is given by

$$E = T + h$$

where E is the total surface energy and h, the amount of heat energy absorbed per unit area.

The quantity of energy that is numerically equal to the surface tension is known as the **free surface energy.**

The above equation may be written as $E = T - \theta \frac{dT}{d\theta}$. from thermodynamic considerations, where $h = \theta \frac{dT}{d\theta}$, For all liquids, the **surface tension** **reases with rise of temperature** and hence $\frac{dT}{d\theta}$ is negative so that $E = T + \theta \frac{dT}{d\theta}$. Evidently E is greater than T.

Example : At 15°C or 288°A, the surface tension of water is 74 dynes per cm. and $\frac{dT}{d\theta} = -0{\cdot}148$ $\therefore$ $E = 74 + (288 \times {\cdot}148) = 74 + 48 = 117$ ergs per sq. cm.

134. Excess Pressure inside a spherical bubble or drop :

Case I. Let us consider a bubble of gas assumed to be spherical, in equilibrium **when totally submerged in a liquid.**

Let r be the radius of the bubble and let p be the excess of pressure of gas inside the bubble over the external pressure and let T be the surface tension of the film.

Let the bubble undergo a slight increase in size so that the radius changes from r to $r + dr$ and the excess pressure changes from p to $p + dp$.

Then the work done in pushing out the surface through dr

$$= \text{force} \times \text{distance} = p \times 4\pi r^2.dr$$

Again the work done by the surface tension in stretching the surface isothermally $= T.d(\text{area}) = T.d(4\pi r^2) = 8\pi Tr\ dr$ ergs.

Since the bubble is in equilibrium, $p \times 4\pi r^2 dr = 8\pi Tr\ dr$

$$\text{or} \quad p = \frac{2T}{r}.$$

Case II. Case of a bubble surrounded by a gas :

Let p be the excess of pressure inside the bubble over the external pressure and let R be the radius of the bubble and T, the surface tension.

Consider the bubble ABCD to be divided by an imaginary partition AC round the circumference of which the film meets at right angles and exerts forces due to the surface tension on the boundary of the partition AC in directions normal to the bounding line and tangential to the surface as shown by the small arrows. (Fig 71)

The portion AC has an area equal to πR^2 and therefore the force on the partition due to the excess of pressure p is equal to $\pi R^2.p$ and directed along OE normal to AC.

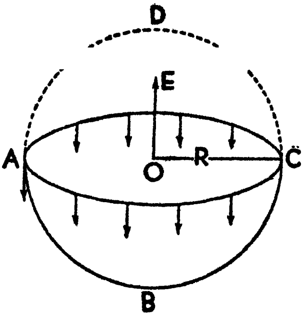


Fig. 71

The force due to the surface tension acting along the circumference of length $2\pi R = 2T \times 2\pi R = 4T\pi R$, since the film has two surfaces. Then for the equilibrium of the upper hemisphere, these two forces are equal.

That is, $4T\pi R = \pi R^2 p$ *i.e.*, $p = \dfrac{4T}{R}$

The energy of the soap bubble neglecting the compression of air inside it, is equal to $2T.4\pi R^2 = 8T\pi R^2$, for the film has two surfaces. The above relation is obtained from the knowledge that the surface tension in a film is the energy per unit area of the film.

135. Experimental determination of surface tension by soap bubble : A soap bubble D is blown at the end of a glass tube having the shape shown in the figure 72 by dipping it into a soap solution and blowing gently through the top arm which can be closed by a stop-cock A. The remaining limb of the tube is connected to a special type of manometer for observing small changes of pressure.

The radius R of the bubble is measured by a travelling microscope and the surface tension is determined by the formula

$$p = \frac{4T}{R}, \quad \text{or} \quad h\rho g = \frac{4T}{R}, \text{ whence } T = \frac{h\rho g R}{4}$$

where h is the difference of level of the liquid in the manometer and ρ, the density of the liquid. Then knowing h, ρ, g and R, T can be found out.

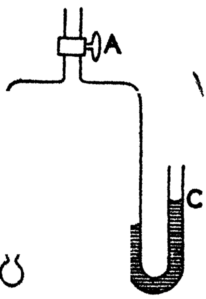


Fig. 72

136a. Determination of the Surface Tension of water by a Capillary Tube :

Theory : Let a capillary tube (Fig. 73) of radius r be dipped in water in a vessel, which is found to rise through a height h above the general surface. Let the force exerted by the liquid on the inside of the tube due to surface tension be everywhere directed inwards along the tangent to the liquid surface at the point of contact between the liquid and the solid and make an angle θ with the side of the tube. If the surface tension be T dynes per cm. the total force on the whole line of

length $2\pi r$ is $2\pi r$T, since the surface of the liquid meets the inside wall of the tube along the circumference of a circle of radius r. As this force acting on the inside of the tube makes an angle θ with the vertical, the horizontal components of this force for opposite sections of the line of contact neutralise each other and the vertical components acting in the same direction are only effective. Therefore, the total force exerted by the liquid on the wall of the tube is $2\pi r\text{T}\cos\theta$.

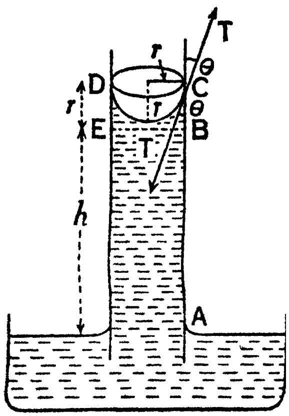


Fig. 73

By third law of motion, the wall of the tube exerts on the liquid an equal force $2\pi r\text{T}\cos\theta$ in opposite direction *i.e.*, in vertically upward direction. It is this force which supports the weight of the column h of the liquid in the tube.

Now, weight of the liquid column in the tube = cross-sectional area × height × density × $g = \pi r^2 \times h \times \rho \times g$.

Therefore $\quad 2\pi r\text{T}\cos\theta = \pi r^2.h.\rho.g \quad \ldots \quad \ldots \quad (1)$

$$\text{T} = \frac{rh\rho.g}{2\cos\theta}$$

In case of water and glass θ is very small so that $\cos\theta = 1$.

$$\text{Hence,} \quad \text{T} = \frac{rh.\rho.g}{2}$$

The above relation is not correct, since in considering the height or the volume of the liquid column, the liquid above the plane touching the lower part of the meniscus has been ignored. Since the angle of contact is practically zero, the meniscus is hemispherical in shape having same radius r.

$\therefore$ Volume of the liquid above the lower part of the meniscus = volume of liquid in cylinder of height r and radius r − volume of liquid in hemisphere of radius $r = r.\pi r^2 - \frac{1}{2} \times \frac{4}{3}\pi r^3 = \pi r^3 - \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^3$.

$\therefore$ Total volume of liquid rising in the tube $= \pi r^2 h + \frac{1}{3}\pi r^3 = \pi r^2\left(h + \frac{r}{3}\right)$.

Then the relation (1) is given by $2\pi r\text{T}\cos\theta = \pi r^2\left(h + \frac{r}{3}\right).\rho.g$

$$\text{T} = \frac{r.\rho g}{2}(h + \tfrac{1}{3}r) \text{ [taking } \theta = 0 \text{ so that } \cos\theta = 1] \qquad (2)$$

Experiment : A glass tube (Fig. 74) of fine bore is washed thoroughly with nitric acid and potash solution so as to free it from dirt and grease. It is then clamped vertically above a glass beaker previously cleansed, in such a way that the lower end of the capillary tube dips in the liquid (water) contained in the beaker. A long pointer is fixed vertically to the capillary tube by a rubber band so that its lower end just touches the surface of water in the beaker. By means of a travelling microscope the reading corresponding to the bottom of the meniscus is obtained. The beaker is then removed and the reading for the lower end of the pointer taken. Then the difference in these readings gives the height of water in the tube above the surface of water in the beaker. To determine the radius of the tube, the capillary tube is cut off at the place where the meniscus stands and the fragment is mounted vertically on the bed of the microscope and the diameters determined in crosswise directions and their mean value is taken.

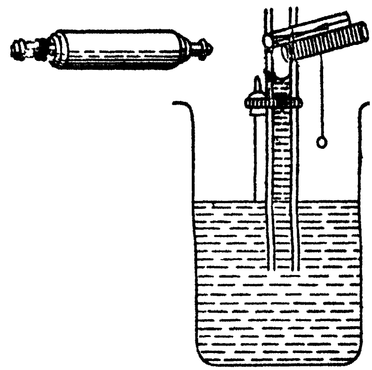


Fig. 74

Then, by the formula $T=\frac{r\rho g}{2}\left(h+\frac{1}{3}r\right)$, the value of surface tension T is determined.

Sources of Errors and Precautions :

(1) The bore of the tube should be as uniform as possible.
(2) The tube should be thoroughly washed and made free from dirt and grease.
(3) Distilled water should not be used as it may contain grease which will vitiate the results to a great extent.
(4) The tube must be vertical.
(5) The temperature correction for the density of the liquid should be taken into consideration.
(6) The curvature correction for capillarity should be considered.
(7) Readings by microscope should be accurately taken.

136b. Jurin's Law : The relation between the height h to which a liquid rises or falls in the tube and radius r of its bore, known as *Jurin's Law* is given by, $h \times r =$ constant.

If h and r are plotted for different tubes on a graph paper, the curve will be a hyperbola.

137. Jager's method of measuring Surface Tension : If a spherical bubble be produced in a liquid, the excess of pressure p inside the bubble is given by $p = 2T/r$ where r is the radius of the bubble and T, the surface tension.

Jager utilised this relation to determine the surface tension.

The figure 75 shows the arrangement of the apparatus. The reservoir A is connected with a pressure pump and with a glass tube bent twice at right angles and provided with a tap T and a liquid manometer M. The vertical portion BC of the bent tube has a narrow orifice of radius r at the bottom C and is placed inside a liquid contained in a vessel.

The tap T being opened, bubble is formed at C by carefully forcing some air in BC, in which some liquid has risen due to capillary action. The bubble is of nearly hemispherical shape with radius equal to that of the orifice at C. The pressure at C is slowly increased until the bubble breaks off with a corresponding change in the manometer reading.

Another bubble begins to form and the manometer indicates a pressure difference h which is maximum when the bubble is completely formed.

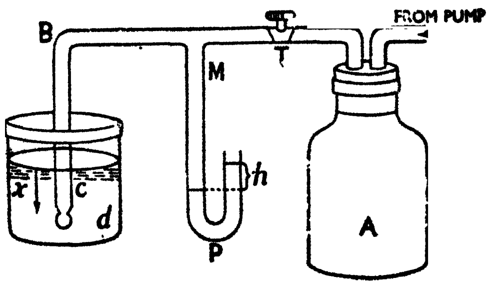


Fig. 75

The pressure difference h is noted when the bubbles form at the rate of one every second or even slower.

The maximum pressure p_1 inside the bubble is given by $p_1 = h\rho g + \pi$, where π is the atmospheric pressure and ρ the density of the manometer liquid.

The pressure p_2 just outside the bubble is given by

$p_2 = xdg + \pi$ where d is the density of the liquid whose surface tension is to be determined and x, the depth of C below the surface of the liquid.

In this case the bubble formed has got only one surface, so the excess of pressure $p = \frac{2T}{r}$ (instead of $4T/r$), where r is the radius of the bubble and T, the surface tension.

But $p = p_1 - p_2 = g(\rho h - dx) = \frac{2T}{r}$ $\therefore$ $T = \frac{gr}{2}(\rho h - dx)$

The value of r can be obtained by a microscope having micrometer eye-piece.

The method is best suited for measuring the relative values of T at different temperatures rather than determining it absolutely.

138. Quincke's method of determining the surface tension of mercury : We have noticed that a depression is produced when a glass tube of fine bore is immersed in mercury or in a liquid which does not wet it. So observation of depression is not possible with a microscope.

To determine the surface tension of mercury, Quincke used a large drop of it on a clear horizontal plate of glass. The drop is so large that the central part of its upper surface is practically plane.

Let an imaginary vertical plane divide the drop into two halves at the section AB and a thin slice be cut from one half of the drop by two parallel vertical planes CD and EF perpendicular to the plane AB and at a distance δ apart. [Fig. 76(a)].

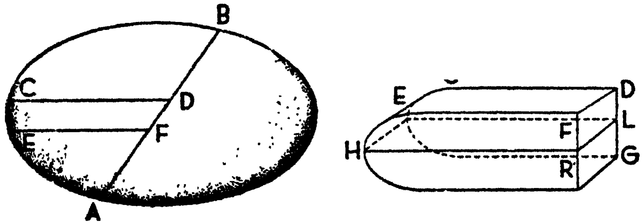


Fig. 76(a) Fig. 76(b)

Consider a horizontal plane HRL which passes through the point H where the surface of the drop is vertical and meets the vertical plane AB. [Fig. 76(b)].

Let us now consider the equilibrium of the upper portion CDLRH of the slice under the action of the forces acting on it.

We are now to take into account the horizontal forces in the direction HR or EF.

At H, the force due to surface tension is vertical and has, therefore, no horizontal component.

At the end FR, the only horizontal forces are the hydrostatic pressure over the face FDLR due to the liquid to the right of the vertical plane FL acting from *right to left* and the surface tension pull over FD acting from *left to right* due to the surface of the bubble on the right of FL.

The hydrostatic thrust over FDLR is

$$FR \times FD \times (\tfrac{1}{2}FR)\rho g \text{ dynes} = h\delta.\frac{h}{2}.\rho g = \frac{h^2 \rho g.\delta}{2},$$ where h is the depth FR or DL and ρ the density of mercury.

Here the average hydrostatic thrust is taken, as the pressure varies from O at F to ρgh at R.

The surface tension pull = T.δ. Therefore for equilibrium,

$$T\delta = \frac{h^2 \rho g \delta}{2} \quad \text{or} \quad T = \frac{h^2 \rho g}{2}.$$

138a. Experimental determination : A bubble of mercury about two inches in diameter is formed on a piece of glass plate supported on a small table provided with levelling screws. The screws are adjusted until the upper surface of the bubble is horizontal.

To determine h a vernier microscope is mounted with its axis horizontal near the bubble. A piece of plane glass plate C inclined at a angle of 45° to the axis of the microscope (Fig. 77) is arranged between the bubble and the microscope. Light from an electric lamp A (adjusted in the horizontal plane through the axis of the microscope) is focussed by the lens L and the plate glass acting as a mirror, on the edge of the bubble. The light is reflected and passes through it and enters the microscope. On focussing the microscope a thin bright horizontal line is seen at the vertical portion of the periphery of the bubble.

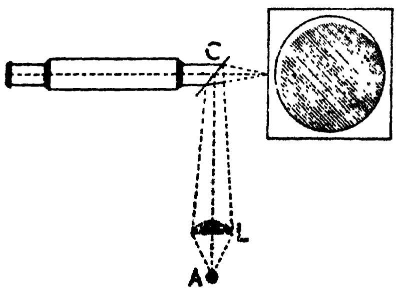


Fig. 77

The microscope is then raised or lowered until the image of this line coincides with the horizontal cross-wire. The position of the microscope is noted.

The microscope is then raised and focussed on some lycopodium scattered on the upper surface of the bubble and the new position of the microscope noted.

The difference in readings of the two positions of the microscope gives the value of h.

The surface tension of mercury is determined by the expression $T = \frac{h^2 \rho g}{2}$, h, ρ and g being known.

QUESTIONS

1. Explain surface tension from the view of molecular structure of matter. [C. U. 1942, '49, '51, '53]

2. Describe an accurate method of measuring the surface tension of a liquid and give the theory underlying it. [C. U. 1942, '44, '45 '48, '51, '53, '55]

Find out the dimensions of surface tension. [C. U. 1944]

3. What do you understand by surface tension, surface energy and angle of contact? [C. U. 1945, '57]

4. Explain the relation between the surface tension and energy per unit area of a liquid film. [C. U. 1941, '52, '53]

5. What is capillary action? Explain why water rises in a narrow tube and why oil spreads over water. [C. U. 1945]

Can the liquid be depressed in any case? If so, why? Explain the form of upper free surface in the capillary tube. [C. U. 1948]

6. Find an expression for the excess pressure inside a spherical air bubble of radius r inside a liquid of surface tension T. [C. U. 1957]

7. How does surface tension vary with temperature and the state of electrification of the surface? [C. U. 1951]

EXAMPLES

1. Calculate the capillary constant of distilled water from the following data.

Tube	Radius	Height
No. 1	·0178 cm.	8·12 cm.
No. 2	·0264 cm.	5·57 cm.
No. 3	·0420 cm.	3·43 cm.

The density of water is taken as unity. [C. U. 1917]

The capillary constant *i.e.*, the surface tension T is given by the equation

$T=\rho g\left(\frac{rh}{2}+\frac{r^2}{6}\right)$; where ρ is the density of the liquid.

(1) For the first tube $T=981\left[\frac{\cdot0178\times8\cdot12}{2}+\frac{(\cdot0178)^2}{6}\right]$

$=70\cdot92$ dynes nearly.

(2) For the second tube $T=981\left[\frac{\cdot0264\times5\cdot57}{2}+\frac{(\cdot0264)^2}{6}\right]$

$=72\cdot2$ dynes nearly.

(3) For the third tube $T=981\left[\frac{\cdot042\times3\cdot43}{2}+\frac{(\cdot0420)^2}{6}\right]$

$=70\cdot96$ dynes nearly.

$\therefore$ Mean value of $T=(70\cdot92+72\cdot20+70\cdot96)/3=71\cdot36$ dynes nearly.

2. A ring is cut from a platinum tube 8·5 cm. internal and 8·7 cm. external diameter and supported horizontally from the pan of a balance so that it comes in contact with water in a glass vessel. It is found that an extra weight of 3·97 gm. is required to pull the ring away from water. Calculate the surface tension of water. [C. U. 1942]

Let r_1 and r_2 be the internal and external radii of the ring and T, the surface tension.

The total force exerted on the ring due to surface tension $=2\pi(r_1+r_2)$. T

Therefore, $2\pi(r_1+r_2).T=3\cdot97\times981$

or $T=\frac{3\cdot97\times981\times7}{2\times22\times(4\cdot25+4\cdot35)}=\frac{3\cdot97\times981\times7}{2\times22\times8\cdot6}=72\cdot06$ dynes per cm.

3. A soap bubble 2 mm. diameter is blown at the end of a tube which is connected at the other end to a U-tube manometer containing oil of sp. gr. 0·8. Find the difference in level in the two limbs of the manometer. The surface tension of the soap solution is 25 dynes per cm. [D. U. 1946]

Using the formula $h\rho g=\frac{4T}{r}$, $h=\frac{4T}{r\rho g}=\frac{4\times 25\times 10}{1\times \cdot 8\times 981}=1{\cdot}275$ cm.

4. A bubble 1·5 cms. in radius is blown out of a liquid of surface tension 30 dynes/cm. Calculate the excess of pressure inside the bubble.

From the formula $p=\frac{4T}{r}=\frac{4\times 30}{1{\cdot}5}=80$ dynes.

5. Calculate the excess pressure inside a spherical soap bubble of diameter one inch blown with a soap solution of surface tension 25 dynes per cm. [C. U. 1957]

In the case of a soap bubble surrounded by air, the excess pressure $p=\frac{4T}{r}$, where T is the surface tension of the soap solution and r, the radius of the bubble.

$$=\frac{4\times 25}{1{\cdot}25}=80 \text{ dynes since } r=\tfrac{1}{2} \text{ inch}=\frac{2{\cdot}5}{2}=1{\cdot}25 \text{ cm. nearly.}$$

6. An air bubble situated just below the surface of water has a diameter 1 cm. What is the pressure within the bubble if the atmospheric pressure on the surface$=10^6$ dynes/cm^2. and surface tension$=70$ dynes/cm. [D. U. 1945]

Since the air bubble is inside water it has only one surface and the excess of pressure inside the bubble is given by $p=\frac{2T}{r}$.

$\therefore$ The pressure P within the bubble is given by $P=\pi+g\rho d+\frac{2T}{r}$, where d is the distance of the bubble below the surface and π, the atmospheric pressure. In this case $d=r$ (radius of the bubble).

7. A small drop of water has a radius equal to r cms. ; calculate the decrease in the surface energy of the drop when the radius is diminished by a small quantiy δ cm. due to evaporation.

Calculate the radius of the largest drop that can evaporate at 0°C., without heat being communicated to it.

Surface energy of water at 0°C$=117$ ergs/cm.
Latent heat of evaporation at 0°C$=606$ cal per gram. [C. U. 1949]

$E=T.A$; $dE=T.dA=T.d(4\pi r^2)=8\pi r T.dr=8\pi r T.\delta$.

Here T is the surface tension or energy per unit area and δ, the change in radius.

The thermal energy needed to evaporate a layer of thickness $\delta=4\pi r^2.\delta.\rho.L$ where L is the latent heat of vaporisation.

Decrease in surface energy due to evaporation$=8\pi r E\delta$

Here $E=T=$ the surface energy per unit area. ρ for water$=1$ gm/c.c.

$$4\pi r^2\delta\,\rho.\,L.\,J=8\pi r E.\delta \qquad \therefore r=\frac{2E}{LJ}=\frac{2\times 117}{606\times 4{\cdot}2\times 10^7}=9{\cdot}2\times 10^{-9} \text{ cm.}$$

7. The two arms of an U-tube have diameters 10 mms. and 1 mm. The tube is partially filled with water and is held with the arms vertical. Find the difference in the levels of water in the two limbs if the surface tension of water is 72 dynes per cm. [C. U. 1955]

From the formula $T = \frac{rh\rho g}{2}$

We have $h = \frac{2T}{r\rho g} = \frac{4T}{d\rho g}$ where d is the diameter of the tube.

Let h_2 and h_1 be respectively the heights of the levels of water in the narrower and wider limbs of the U-tube and d_2 and d_1 their respective diameters.

Then, for the narrower limb, $h_2 = \frac{4T}{d_2 \rho g}$

For the wider limb, $h_1 = \frac{4T}{d_1 \rho g}$

Then the difference in the levels of water in the two limbs is given by

$$h_2 - h_1 = \frac{4T}{\rho g}\left(\frac{1}{d_2} - \frac{1}{d_1}\right) = \frac{4T}{g}\left(\frac{1}{d_2} - \frac{1}{d_1}\right) \text{; Since } \rho \text{ for water} = 1 \text{ gm/cc.}$$

$$\text{or } h_2 - h_1 = \frac{4 \times 72}{981}(10-1) = \frac{4 \times 72 \times 9}{981} = 2{\cdot}64 \text{ cms.}$$

CHAPTER VIII

VISCOSITY

139. Rate of flow of a liquid : A liquid is usually regarded as perfectly mobile and almost incompressible and as such the same amount of it flows across every section of the tube in a certain definite time.

The rate of flow of a liquid is, therefore, defined and measured as the volume or mass of the liquid which flows across any section in unit time.

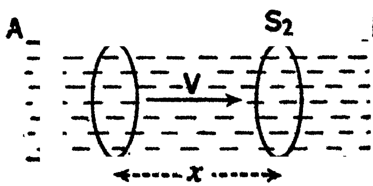


Fig. 78

Let the velocity of flow of the liquid in a tube be V, in a direction at right angles to two sections S_1 and S_2 (Fig. 78) each of area α and distance x apart, and let the liquid flow from S to S in time t, then $V.t = x$.

$\therefore$ Volume of liquid flowing through the length S_1S_2 in time t = volume of the cylinder $S_1S_2 = x \times \alpha = Vt \times \alpha$.

$\therefore$ Rate of flow of liquid $= \dfrac{Vt \times \alpha}{t} = V \times \alpha$. c.c./sec.

= Velocity of liquid × area of cross-section of the tube.

If ρ = density of the liquid, then mass of liquid flowing through S_1S_2 in time $t = Vt \times \alpha \times \rho$.

$\therefore$ **Rate of flow of liquid** $= \dfrac{Vt \times \alpha \times \rho}{t} = V.\alpha.\rho$ **gm/sec,**

= Velocity of flow × cross-section × density.

140. Viscosity: If a liquid flows over a fixed horizontal plane, the layer of liquid in contact with the plane is at rest, while the other layers at different distances from the plane move relatively to one another, the velocities of layers becoming greater and greater as the distance increases from the fixed plane.

Again, if different liquids such as water, milk, honey, treacle etc., are allowed to flow through a glass tube successively, the rates of flow will be different in different cases, water and milk will be seen to flow more readily than honey and treacle.

The property of a fluid by virtue of which it retards or opposes the relative motion between its different layers and thus produces a variation of velocity, in different layers, is called viscosity.

141. Stream-Line and Turbulent Motion: The motion of the liquid in the tube may be either *steady* or *unsteady*. When the motion is steady, the liquid moves in straight lines parallel to the axis of the tube and this kind of motion is known as *orderly* or **stream-line motion** (Fig. 79*a*). When the motion is unsteady and the velocity exceeds a critical value, the motion becomes irregular and **turbulent**, the particles of the liquid moving from side to side of the tube, and eddies are formed in the liquid (Fig. 79*b*).

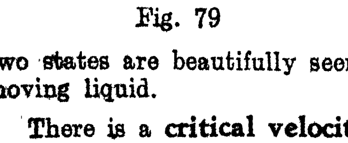


Fig. 79

The motions of the liquid in the two states are beautifully seen if a coloured jet be injected into the moving liquid.

There is a **critical velocity** of the flow of a liquid, below which the motion of a fluid is *orderly* but above which the motion is *turbulent*. It is inversely proportional to the radius of the tube and the density of the liquid, and directly proportional to the viscosity of the liquid.

142. Coefficient of Viscosity : When contiguous layers of a material fluid are moving relatively to each other, forces are called into play which destroy their relative motion. The more quickly moving layer is retarded by the more slowly moving layer which is itself accelerated by the action of the former. Thus any horizontal layer in a fluid is acted on, from both above and below, by two opposite tangential forces one in the direction of motion of the liquid, while the other opposite to it. These two forces are due to the property of the liquid called **viscosity** which is really a kind of frictional resistance between the particles of a liquid or a gas when its different parts are moving with different velocities.

142a. Velocity Gradient : Let us consider two horizontal layers in the moving fluid at a distance d apart and let v_1 be the velocity of the particles in the upper layer from left to right and v_2 that of the particles in the lower layer from right to left and let v_1 be greater than v_2. Since the velocity does not change abruptly from layer to layer, the change of the velocity per unit distance between the layers is $\frac{v_1 - v_2}{d}$. This quantity is called the **Velocity Gradient.**

The fluid above the upper plane is moving more quickly than the fluid below. So the fluid above the upper plane is acted on by a tangential retarding force from right to left and the fluid below it is similarly acted on by an equal accelerating tangential force from left to right. Let either of the tangential forces be denoted by F and let it act on the area A of the plane.

Again, since either of this tangential stress, as expressed by $\frac{F}{A}$, does not depend on the actual velocity of the parts either above or below the plane, it may be said to be proportional to the velocity gradient $\frac{v_1 - v_2}{d}$; we have therefore $\frac{F}{A} \propto \frac{v_1 - v_2}{d}$

or $\frac{F}{A} = \eta . \frac{v_1 - v_2}{d}$, where η is a constant depending on the nature c the fluid. The constant η is termed the **coefficient of viscosity** c

$$\eta = \frac{Fd}{(v_1 - v_2)A}$$

If V be the velocity with which the liquid in the upper plan is moving relatively to that in the lower plane, *i.e.* $V = v_1 - v_2$, the

Coefficient of Viscosity $\mu = \frac{Fd}{VA} = \frac{F}{\frac{V}{d}.A}$

If the velocity gradient $V/d = 1$, and $A = 1$, then $\eta = F$.

Definition : **Thus, the coefficient of viscosity of a fluid may be defined quantitatively as the tangential force per unit area per unit velocity gradient.**

143. Variation of Viscosity with temperature : In the case of liquids the coefficient of viscosity decreases with the rise of temperature but in the case of gases it increases with the rise of temperature, being directly proportional to the square root of the absolute temperature of the gas.

According to Thorpe and Rodger, the coefficients of viscosity of a larger number of liquids agree very closely with the empirical formula given by $\eta = \dfrac{C}{(1+bt)^n}$, where η is the coefficient of viscosity at the temperature t, and C, b and n are constants depending on the nature of the liquid.

If has been found that the viscosity of water at 80°C is only about one-third of its value at 10°C.

144. Importance of knowledge of Viscosity : The knowledge of viscosity of fluids is extremely important in the study of motion of both large bodies and ultra-atomic particles in a viscous medium.

The knowledge of viscosity of liquids is also useful for practical purposes. The preparation of lubricant as used in machines, and of fountain pen ink requires some knowledge of the viscosity of different liquids.

145. Dimensions and Units of Viscosity : Coefficient of viscosity is defined as the tangential force per unit area per unit velocity gradient.

Hence, the dimensions of viscosity are as follows :—

$$\eta = \frac{F}{A} \Big/ \frac{V}{d} = \frac{MLT^{-2}}{L^2} \Big/ \frac{L}{T \times L} = \frac{MLT^{-2}}{L^2} \Big/ \frac{1}{T} = \frac{M}{LT} = ML^{-1}T^{-1}$$

since the dimensions of the tangential force per unit area *i.e.*, $\dfrac{F}{A}$ are $\dfrac{MLT^{-2}}{L^2}$ and those of velocity gradient $\dfrac{V}{d}$ are $\dfrac{L}{T} \Big/ L = \dfrac{1}{T}$.

The coefficient of viscosity is expressed in *gm. cm. sec., unit.*

146. Deduction of the formula for flow of liquid through a Capillary tube : Poiseuille's Equation : If a liquid flows through a long straight capillary tube, the volume of the liquid escaping per sec. is according to Poiseuille, given by $V = \dfrac{\pi p a^4}{8\eta l}$, where p = pressure difference between the ends of the tube ; a = radius of the tube ; l = length of the tube ; η = coeff. of viscosity.

Consider the motion of a thin shell of liquid (Fig. 80) between the cylinders of radius r and $r+dr$ having the same axis as the capillary tube. Let the velocities of the liquid on these surfaces be v and $v+dv$.

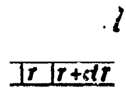


Fig. 80

The viscous drag on the inner cylinder will retard the motion of the liquid whereas the difference of pressure between its ends will accelerate the motion of the liquid. When these two forces are equal the flow of the liquid will be steady.

The viscous drag on the inner cylinder per unit area $=\eta.\dfrac{dv}{dr}$, where $\dfrac{dv}{dr}$ is the velocity gradient and η = coefficient of viscosity. Let l be the length of the capillary tube and therefore that of the cylinder considered.

Then the surface area of the cylinder $=2\pi rl$.

$\therefore$ The total drag on the cylinder $=-2\pi rl.\eta\dfrac{dv}{dr}=\text{F}$ (say.)

The negative sign indicates that v decreases with r, and hence dv/dr is negative.

In the steady state, the difference of thrusts or pressures between the ends of the cylinder = total viscous drag.

If the pressure difference be p, then $\text{F}=\pi r^2.p$

or, $-2\pi rl\eta.\dfrac{dv}{dr}=\pi r^2.p.$ $\therefore$ $-l\eta.\dfrac{dv}{dr}=\dfrac{pr}{2}$ or $-l\eta.dv=\dfrac{p}{2}rdr.$

By integration we have,

$$-l\eta.v=\frac{pr^2}{4}+\text{C} \quad .. \quad (1), \text{ where C is a constant.}$$

Now the velocity of the liquid in contact with the tube is zero and therefore $v=0$ when $r=a$.

$$0=\frac{pa^2}{4}+c \quad \text{or} \quad \text{C}=-\frac{pa^2}{4}$$

Hence, by substituting for C in (1) we have,

$$-l\eta v=\frac{pr^2}{4}-\frac{pa^2}{4}=-\frac{p}{4}(a^2-r^2) \qquad v=\frac{p}{4l\eta}(a^2-r^2).$$

This equation gives us the magnitude of the velocity at a distance r from the axis of the tube.

To determine the quantity of the liquid which flows through the tube per second, we shall again refer to the thin shell.

The area of cross-section of this shell $=2\pi r.dr$ and the velocity of the liquid in the shell $=\frac{p}{4l\eta}.(a^2-r^2)$. Therefore the total volume V of the liquid issuing per sec. is given by

$$V=\int_0^a \frac{p}{4l\eta}.(a^2-r^2)2\pi r dr.=\frac{p\pi}{2l\eta}.\int_0^a (a^2 r-r^3)dr$$

$$=\frac{p\pi}{2l\eta}\left[\frac{a^2r^2}{2}-\frac{r^4}{4}\right]_0^a=\frac{p\pi}{2l\eta}\left[\frac{a^4}{2}-\frac{a^4}{4}\right]=\frac{p\pi}{2l\eta}.\frac{a^4}{4}=\frac{p\pi a^4}{8l\eta}. \quad \therefore \quad \eta=\frac{\pi p.a^4}{8l.V}$$

This equation is not quite complete as a correction due to the kinetic energy of the issuing liquid is necessary. In case of a long tube of fine bore, the result is practically true under a pressure difference sufficiently small for the liquid to drop from the outlet end. The above equation is known as **Poiseuille's Equation.**

146a. By method of dimensions: Poiseuille's formula for the volume V of a liquid flowing per unit time is given by

$$V=K\eta^x a^y \left(\frac{p}{l}\right)^z \quad \dots \quad \dots (1)$$

where K = a numerical constant with no dimension; x, y and z may have any value positive or negative, fractional or integral.

p/l = pressure gradient, a = radius of the bore of the tube.

Dimension of V $\cdot \frac{\text{volume}}{\text{time}}=L^3T^{-1}$; dimension of $\eta=ML^{-1}T^{-1}$

Dimension of $a=L$; dimension of $\frac{p}{l}=\frac{MLT^{-2}/L^2}{L}=ML^{-2}T^{-2}$.

Hence, relation (1) in dimensional form becomes,

$$L^3T^{-1}=(ML^{-1}T^{-1})^x.L^y.(ML^{-2}T^{-2})^z=M^{x+z}.L^{-x+y-2z}.T^{-x-2z}$$

Equating the dimensions of similar terms of both sides we have

$$x+z=0;\quad -x+y-2z=3;\quad -x-2z=-1$$

Solving these we have $x=-1$, $y=4$ and $z=1$

Substituting for x, y and z in (1) $V=\frac{K.pa^4}{\eta l}$ c.c. per sec. ... (2)

The value of K has been experimentally determined to be equal to $\pi/8$.

Hence, the expression (2) becomes $V=\frac{\pi pa^4}{8\eta l}$ (Poiseuill'es Equation)

147. Determination of the coefficient of Viscosity of a liquid : The method used in this experiment depends on the measurement of the volume of a liquid escaping from a uniform capillary tube under a given difference of pressure.

According to Poiseuille, the volume v of the liquid which escapes in T seconds is expressed by

$$v = \frac{\pi p T a^4}{8 l \eta}$$

$$\text{or} \quad \eta = \frac{\pi p T a^4}{8 l v}$$

where p is the difference of pressure between the ends of the tube a, the radius and l, the length of the tube, η the coefficient of viscosity of the liquid.

Experiment : The apparatus (Fig. 81) used in this experiment consists of a wide cylindrical glass vessel having two openings one at the bottom and the other at the side near the bottom.

The opening at the side is fitted with a cork through which is passed horizontally a long capillary tube B and through the cork fitted into the opening at the bottom is also passed a wider glass tube C extending about $\frac{2}{3}$rd up the vessel and serves as an overflow tube. The liquid is introduced into the vessel, by a supply tube D at such a rate that as soon as it comes up to the upper end of the overflow tube, it escapes through it and so the head of the water always remains the same. In this way a constant flow is maintained in the tube. The liquid flows out through the capillary tube and is collected in a weighed beaker E during a known interval of time. The volume of this liquid is determined by dividing the mass of this liquid by its density at the temperature of the experiment. The difference of pressure p is determined by measuring the difference in height h of the liquid between the upper end of the central overflow tube and the end of the capillary tube by a cathetometer. The value of pressure p in dynes is obtained by multiplying the height h by g, the acceleration due to gravity and d, the density of the liquid ; *i.e.* $p = h \times d \times g$.

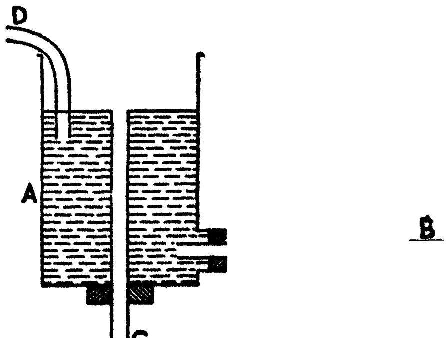


Fig. 81

If the bore of the tube be uniform, the radius is determined by cutting off a portion of the tube and placing it on the bed of a microscope. If the bore is not uniform, a certain quantity of mercury is sucked in nearly filling the whole length of the tube. The length of the thread of mercury is accurately measured and its mass determined by taking it on a weighed cup or watch glass.

$$\text{Then} \quad \pi a^2 \times L.d = m\,; \quad a = \sqrt{\frac{m}{\pi L d}}$$

where a is the radius of the tube, L, the length of the thread of mercury filling the tube, d, the density and m, the mass of mercury in the tube.

Thus by substituting the values of v, T, p, a and l in the expression $\eta = \dfrac{\pi T p a^4}{8 l v}$, the Coefficient of viscosity η is determined.

The coeff. of viscosity of a liquid **decreases** with the rise of temperature ; whereas in gases the viscosity varies directly as the absolute temperature of the gas but is independent of the pressure.

148. Relation between Viscosity and Friction : Viscosity or the coefficient of viscosity is the tangential force per unit area required to maintain a relative velocity of unity between two parallel planes in the fluid at unit distance apart.

Friction is also a tangential force which is called into play between the surfaces of contact of two bodies when we attempt to slide one body over another and it always acts in the direction in which it can resist motion. This force is called **Static Friction**

Viscosity depends on the nature of the substances between whose layers the tangential force is exerted.

Friction also depends on the nature of substances whose surfaces are in contact.

149. Difference in Friction between (i) solid and solid and (ii) solid and liquid :

When we attempt to slide one solid body over another a tangential force, known as the *force of friction* is exerted between the surfaces of contact of the two bodies, tending to resist motion.

When the body is on the point of sliding the friction that is exerted is called the *limiting friction.*

Again when a solid moves through a liquid, the portions of the liquid next the solid move with the same velocity as the solid while portions of the liquid at a greater distance away are at rest. So a relative motion between the layers of the liquid takes place and consequently due to viscosity (a sort of frictional resistance) a force acts on the solid tending to resist its motion

In the case of friction between a solid and a solid the friction is independent of the area of the surfaces in contact but in the case of friction (viscous resistance) between a solid and a liquid it depends on the area of the surfaces in contact.

150. Fluid in Motion : Bernoulli's Theorem : When the particles of a fluid in motion which follow one another at a fixed point, possess the same density and velocity, pressure remaining constant, the motion of the fluid is called steady. Under these ideal conditions which can seldom be obtained in practice, the motion is said to be stream-line-motion and a stream line may be defined as the actual path of a particle of the fluid in motion. A tube of flow is such that its walls are formed of stream lines within a fluid in steady flow. In tubes of uniform cross-section all the stream lines are parallel to the axis of the tube. Generally the stream lines are curved and are such that at any instant the tangent at any point on it indicates the direction of motion at that point.

151. Mathematical relation : At each point in a fluid of density ρ flowing steadily there will be generally certain pressure p due to gravity, the magnitude of which will vary from point to point. If z be the gravitational hydrostatic head then this pressure is equal to $p=z\rho g$. A fluid element having volume δv and mass $\rho\delta v$ possesses the potential energy $\rho\delta v.g.z$ under this pressure. If again h is the height of the element above a certain datum gravitational level the gravitational potential energy is $\rho h.g.\delta v$. The kinetic energy of the element δv moving with velocity V is $\frac{1}{2}\rho V^2.\delta v$. For ideal fluid the energy cannot be used up by friction. Hence, during motion the sum of the potential and kinetic energies of the element must remain same, *i.e.*

$$\delta v.\rho z g + \delta v.\rho h g + \tfrac{1}{2}\delta v.\rho V^2 = \text{constant} \quad \ldots(1)$$

$$z + h + \tfrac{1}{2}\frac{V^2}{g} = \text{constant}$$

$$\text{or} \quad \frac{p}{\rho g} + h + \tfrac{1}{2}\frac{V^2}{g} = \text{constant} \quad \ldots(2) \quad \left[\because z = \frac{p}{\rho g}\right]$$

Denoting the term $\frac{1}{2}V^2/g$ as velocity head, which is the height through which the fluid particles would have fallen freely in order to attain the velocity V, the following theorem called **Bernoulli's theorem** can be stated :

At every point in a stream-lined tube of fluid the sum of the pressure head $p/\rho g$, the velocity head $V^2/2g$, and the gravitational head (h), is constant.

Obviously from the relation (2) it follows that greater velocities V correspond to diminished pressures p and vice versa.

QUESTIONS

1. Define the coefficient of viscosity and its dimensions. [C. U. 1943, '55, '58]

2. Describe a method of determining the coefficient of viscosity of a liquid. [C. U. 1958]

3. Describe the way in which the different parts of a viscous liquid move when flowing through a fine tube. What change takes place if the motion is increased. [C. U. 1943, '52]

4. Explain what is meant by the coefficient of viscosity of a liquid and how it varies with temperature. Discuss the points of difference in friction between (i) solid and solid and (ii) solid and liquid. [C. U. 1954]

EXAMPLES

1. In an experiment with Poiseulle's apparatus the following figures were obtained :

Volume of water issuing per minute $=7{\cdot}08$ c.c.
Head of water $=34{\cdot}1$ cm. Length of tube $=56{\cdot}45$ cm.

Radius of tube $={\cdot}0514$ cm. Find the coefficient of viscosity. We have from Art. 147. [C. U. 1943]

$$\eta=\frac{\pi p\mathrm{T}a^4}{8l\mathrm{V}}=\frac{3{\cdot}14\times34{\cdot}1\times981\times60\times({\cdot}0514)^4}{8\times56{\cdot}45\times7{\cdot}08}={\cdot}01377.$$

2. In a certain experiment on the flow of a liquid through a capillary tube the following data were obtained : volume of liquid coming out per minute $=15$ c.c. ; head of liquid $=30$ cm. ; length of tube $=25$ cm. ; radius of tube $=1$ mm.

Calculate the coefficient of viscosity of the liquid, its density being $2{\cdot}3$ gms./c.c. [C. U. 1954]

According to Poiseuille the volume V of a liquid which escapes in T seconds is expressed by

$$\mathrm{V}=\frac{\pi p\mathrm{T}a^4}{8l\eta} \qquad \eta=\frac{\pi p\mathrm{T}a^4}{8l\mathrm{V}}$$

where p is the difference of pressure between the ends of tube of radius a. l. the length of the tube and η the coefficient of viscosity of the liquid.

Hence, we have

$$=\frac{3{\cdot}14\times30\times2{\cdot}3\times981\times60\times({\cdot}1)^4}{8\times25\times15}={\cdot}424 \text{ dynes sec./cm.}^2 \text{ (Poise).}$$

CHAPTER IX

OSMOSIS

152. Diffusion : It is really the spreading of one fluid in another. If two liquids which are miscible are introduced into a vessel so that the denser liquid is below and the lighter above, and if the liquids possess the capability of mixing in any proportion it will be found that a process of self-mixing is going on, some of the lighter liquid travelling down and mixing with the heavier liquid and *vice versa.* This process is known as **Diffusion**.

The rate at which liquids diffuse is extremely small as compared with the rate at which gases diffuse and the rates of diffusion of different substances are different.

The diffusion of one liquid into another is observed when water is placed slowly and carefully on copper sulphate solution taken in a beaker so as to avoid currents in the liquids. A sharp line of demarcation is noticed between water and the liquid solution. But after a short time the blue coloration will be seen to spread upwards until after a considerable time the mixture will assume a uniform coloration.

Graham first studied the diffusion of liquids by filling a wide-mouthed bottle with a salt solution. The bottle was then placed in a larger vessel filled with water above the top of the open bottle. After some days some amount of salt had diffused into water.

From Graham's experiment it has been found that :—

(1) Solution of different salts diffuse at different rates.

(2) The rate of diffusion increases with temperature.

(3) Solution containing sugar etc. known as crystalloids diffuse much faster than colloids (solution containing albumen etc.)

(4) The amount of a solute which passes from layer to layer in unit time is proportional to the difference in strength between the layers.

153. Fick's Law : The results of Graham were given in a simple mathematical law by Fick known as Fick's Law.

This law is exactly similar to the law of conduction of heat through a solid and is given by $Q = K.\frac{c_1 - c_2}{d}$, where Q is the amount of salt passing across unit area per second from a plane of higher concentration c_1 to that of lower concentration c_2 and K, a constant known as **diffusibility**.

The rate of diffusion in any direction is proportional to the concentration gradient of the solute in that direction.

It means that the amount of the solute passing through a slab of the solution of area A and thickness x having concentrations c_1 and c_2 at the two faces in a given time is proportional to the **concentration gradient** $\frac{c_1 - c_2}{x}$.

If d_m is the mass of the salt passing in time dt

$\frac{d_m}{dt} = K.\ A.\ \frac{dc}{dx}$ ... (1) where K is the coefficient of diffusion.

Dimensions of K—

From (1) $[MT^{-1}] = [K][L^2.ML^{-3}.L^{-1}]$ $\therefore$ $[K] = [L^2T^{-1}]$

since c, the concentration is the mass per unit volume and has the dimension ML^{-3}.

154. Diffusion of Gases : Like liquids gases also diffuse into one another but the rate of diffusion in gases is very rapid in comparison with that in liquids.

Two jars one containing Hydrogen and the other containing Carbon dioxide are placed mouth to mouth in the vertical position, the jar containing Hydrogen being at the top. It will be seen that after some time the heavy Carbon dioxide gas diffuses upwards into the lighter Hydrogen gas and *vice versa,* in opposition to gravity.

The rates of diffusion depend on the nature of the gases. If v_1 and v_2 are the rates of diffusion of the two gases and d_1 and d_2 their corresponding densities, then

$$\frac{v_1}{v_2} = \sqrt{\frac{d_2}{d_1}}$$

Thus, **the rates of diffusion two gases are inversely proportional to the sq. root of their densities.** This is Graham's law of diffusion for gases.

155. Osmosis : If two miscible liquids are separated by a membrane, diffusion which takes place through the membrane is called **Osmosis.**

If a pig's bladder is filled with alcohol, closed and placed in water, the liquids mix through the bladder which will gradually swell due to the greater rate of osmotic diffusion of water and finally burst.

If instead, the bladder contains water and is immersed in alcohol, water will flow out of the bladder and consequently the bladder will shrink.

A membrane which allows some substances but not others to pass through it, is called a *semi-permeable membrane.* This selective

transmission of a liquid through a semi-permeable membrane is called **Osmosis.**

Osmosis is confined to the crystalloids since colloids will not pass through a membrane.

156. Osmotic Pressure ; Its measurement : The subject of osmosis was systematically studied by Pfeffer. In his experiments he used a membrane of copper ferrocyanide which was found to be permeable to water but not to many substances such as sugar which can be dissolved in water.

If two solutions having different concentrations are separated by a membrane, the flow will take place from the weaker to the stronger solution until the concentrations of the solutions are equalised. At this stage the flow will cease.

The membrane was prepared by filling a porous pot with cupric sulphate solution and immersing it in a dilute solution of potassium ferrocyanide. The two solutions met in the walls of the pot and the precipitate formed made the membrane.

The pot was washed and then filled with a solution of sugar and its mouth fitted up with a well-fitting cork provided with a hole for a long vertical tube.

The pot with the solution was then immersed in pure water which gradually passed through the membrane into the pot and the solution was found to rise in the vertical tube. The pressure due to the head of the liquid in the tube prevents the inflow of water and is known as the **osmotic pressure** of the solution in the pot. There is then equilibrium *i.e.* water passes out at just the rate at which it enters.

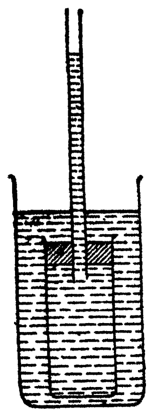


Fig. 82

Let h be the height of the column of liquid, ρ, the density of the liquid and g, the acceleration due to gravity, then the osmotic pressure of the solution $= h.\rho.g$.

The amount of this pressure depends on the nature of the solution, its strength and its temperature.

If the solution be concentrated the osmotic pressure involved requires the use of a membrane of copper ferrocyanide prepared in a way different from that of Pfeffer.

For measurement of high osmotic pressure closed U-tube manometers containing Nitrogen are used.

If the solution be concentrated, the pressure will be great and to measure the pressure mercury manometer is suitable.

157. Laws of Osmotic pressure: Pfeffer found experimentally a quantitative relation between the osmotic pressure of a solution and its concentration. The experimental result of Pfeffer led to the discovery of three laws known as laws of Osmosis. These laws hold good for weak solutions which are non-conductors of electricity. Vant Hoff, the discoverer of the laws showed a remarkable parallelism between the osmotic properties of solutions and the properties of gases. The Laws are as follows :

Law 1. For a given temperature, the osmotic pressure is proportional to the concentration of the solution ; in other words the osmotic pressure is proportional to the mass of substances dissolved per unit volume of the solution.

If m grams of substance be dissolved in v volume of water, the osmotic pressure p is given by the equation

$p = k.\ \frac{m}{v}$, where k is a constant depending on temperature.

The above relation is similar to Boyle's Law for gases which gives a relation between the pressure and volume of a given mass of a gas at a constant temperature.

Law 2. For a given concentration of the solution the osmotic pressure is proportional to the absolute temperature (T).

i.e., $p \propto T$, when $\frac{m}{v}$ is constant

Again $p \propto \frac{m}{v}$, when T is constant

$\therefore\ p \propto \frac{m}{v}T$, when $\frac{m}{v}$ and T both vary

Hence, $pv = kmT = RT$ where R is a constant which depends on m only.

The expression $pv = RT$ (R = gas constant) gives also a relation between the pressure, volume and temperature of a given mass of gas and the condition of the gas is known when they are known.

A gram-molecule of sugar dissolved in the volume v (22·4 litres) exerts an osmotic pressure of 76 cms. at 0°C, just as one gram-molecule of a gas occupying a volume of 22·4 litres exerts the normal pressure of 76 cms. at 0°C, provided the solution is dilute.

The constant R has the same value as the corresponding constant for a gram-molecule of a perfect gas. ($R = 8{\cdot}21 \times 10^7$ ergs).

In the case of a solute in a dilute solution the above relation has been found to be true. So **the solute is said to behave like a gas.** It can be verified by the following example.

According to Pfeffer's experimental data a solution of cane-sugar of 1 gram-molecule or 342 grams, in 34200 c. c. (*i.e.* 1 gm. in 100 c. c.) exerts an osmotic pressure of ·649 atmosphere at 0°C.

$$\text{Then } R = PV/T = \frac{\cdot 649 \times 76 \times 13\cdot 6 \times 981 \times 34200}{273} = 8\cdot 2 \times 10^7 \text{ ergs.}$$

Law 3. Equal volumes of solutions (of non-electrolytes) which have same osmotic pressure contain the same number of gramme-molecules, at the same temperature.

Compare this with Avogadro's Law which states that gases at the same pressure contain the same number of molecules per c. c.

Solutions exerting the same osmotic pressure are called *isotonic.*

The osmotic pressure of solution (non-electrolyte) is equal to the pressure that would be exerted by the solute were it capable of existing as a gas at the temperature and volume of the solution. Solutions of electrolytes exert a greater osmotic pressure than solutions of non-electrolytes.

158. Effusion : The flow of a gas through fine pores in a thin plate is called effusion.

159. Crystalloids : Colloids : *Crystalloids* are substances such as metallic salts, sugar *etc.* which dissolve and diffuse easily.

Colloids are substances such as glue, gelatine, starch *etc.* which dissolve and diffuse more slowly, and not in definite proportions as crystalloids.

Crystalloids when dissolved in water are found to lower the freezing point and raise the boiling point while colloids scarcely show this effect.

160. Dialysis : It is the process by means of which colloids are separated from crystalloids. The principle of separation depends on the fact that solutions of crystalloids pass through a bladder or parchment membrane much faster than colloids such as gum or albumen.

161. Difference between a perfectly rigid body and a perfect fluid :

Rigid body : A perfectly rigid body is one in which the particles of matter always retain the same position with respect to one another. It has therefore a definite size and shape.

Perfect fluid : It is a substance such that its shape can be altered by any tangential force, however small, if applied long enough and of which portions can be easily separated from the rest of the mass and between different portions of which there is a tangential force of the nature of friction.

QUESTIONS

1. What do you understand by osmosis, diffusion and dialysis. [C. U. 1944, '47, '51, '53, '55, '58]

2. State the laws of osmosis and describe an arrangement for measuring osmotic pressure. [C. U. 1944, '51, '55]

3. Write a short account of the relation between osmotic pressure and the concentration, pointing out any similarities which have been observed between this phenomenon and the property of gases. [C. U. 1947, '53]

EXAMPLES

1. Calculate the strength of a cane-sugar solution whose osmotic pressure at 27°c. is one atmosphere. Molecular weight of sugar = 342 gms. and $R = 8.4 \times 10^7$ ergs/deg C.

We have, $PV = RT$ or $V = \frac{RT}{P}$, where V is the volume of the solution containing 1 gm-mol. *i.e.* 342 gms. of sugar.

The V c.c. of the solution contain 342 gms. of sugar

$\therefore$ 1000 c.c. „ „ $\frac{342 \times 1000}{V}$ gms. of sugar

Hence, strength of the sugar solution $= (342 \times 1000)/V = 342 \times 1000 \times P/RT$

$$= \frac{342 \times 1000 \times 76 \times 13.6 \times 981}{8.4 \times 10^7 \times 300} = 13.76 \text{ gms./litre.}$$

CHAPTER X

PRODUCTION OF HIGH VACUUM AND MEASUREMENT OF LOW PRESSURE

162. Gaede's High-Vacuum Rotary Pump: The principle of the pump is to isolate from the main bulk a quantity of air and to force it out by compressing it in a suitable way.

The pump a vertical section of which is shown in (Fig 83) consists of a stout, thick-walled metal cylinder A which is kept fixed, and a solid metal cylinder B inside A which can rotate freely about an axis parallel to that of A. The solid cylinder is mounted eccentrically with respect to the hollow cylinder, and it is made to revolve in such a way that it always remains in contact with the hollow cylinder at a fixed point C. A has on either side of this fixed point, two openings M and N, of which M is connected to the vessel to be evacuated and N is fitted with a valve immersed in an oil bath (not shown in the diagram) for preventing external air from passing into the hollow cylinder.

Two thick rectangular steel vanes or plates E, F separated by a steel spring S are mounted in a slot cut **radially** right through the solid cylinder. The outer extremities of E, F are rounded properly so as to bear smoothly in perfect air-tight contact with the inside walls of A, during the rotational motion of B. The outer edges of E, F are always held evenly pressed against the walls of A by the steel spring. As stated above, B is always in smooth intimate contact with A at the point C. To minimise the friction between the steel vanes and the cylinders, lubrication is provided for. The outlet valve opens outwards through the oil bath.

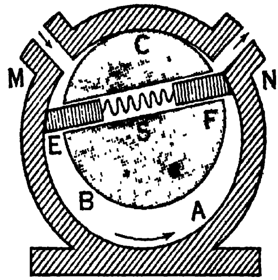


Fig. 83

Let us consider the working of the pump from the position indicated by Figure 83. It will be seen that as B rotates in the direction of the arrow (here anti-clockwise), the air in the annular space between B and A and limited by E and F, is compressed into gradually smaller space and finally expelled out of the exit valve at N. When this goes on, the space behind E *i.e.*, between E and M gradually increases so that air from the vessel to be evacuated rushes in to occupy this space. This air is again isolated and entrapped inside the space between F and exit valve at N, when during rotation of B, F passes down across the inlet tube M. During the latter part of the rotation this air is again compressed as before and finally expelled out through the outlet tube. During each rotation of B, the above processes of (i) isolation of a quantity of air, (ii) its compression and ultimate (iii) expulsion through the outlet tube, are repeated and the vessel from which air is drawn becomes highly evacuated in a comparatively short time.

With this pump a high vacuum with pressure as low as ·01 *mm*. can be obtained.

Advantages over other pumps :

(1) As there is no valve in the tube M, air even under a very low pressure can enter into the cylinder and may be drawn out from the receiver. This advantage is not obtained in other pumps which are provided with valves and which require a certain amount of pressure for lifting them.

(2) The action of this pump is continuous, whereas in other pumps the air is drawn during one stroke and forced out in the next stroke.

(3) In this pump there is practically no limiting stage of exhaustion but in other pumps a limiting stage is reached when the amount of air forced out by the pump from a vessel becomes equal to the amount of air entering the pump by leakage.

Note: Gaede's pump can be used both as an exhaust pump and as a blower. A vacuum having as low a pressure as ·01 *mm.* of mercury may be obtained by it. When used as a blower it can produce a pressure of one atmosphere.

163. Diffusion Pump: The process of gaseous diffusion has been utilised in the mercury diffusion pump for producing a high degree of vacuum in a short time. There are many forms of diffusion pumps. The original form of Gaede pump being of complicated glass mechanism is now seldom used. In all these pumps a preliminary reduction of pressure is at first made to different degrees by some auxiliary pump.

Figure 84 represents one of the two pumps designed by **Volmer** and arranged in series to form a two stage pump.

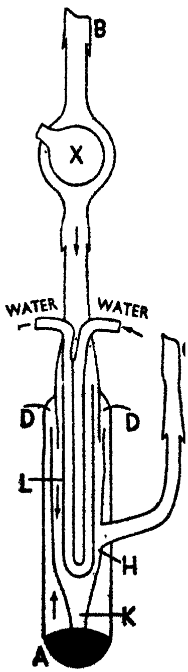


Fig. 84

In the pump, A is the boiling chamber in which clean, pure mercury freed from grease is heated and kept boiling steadily but not violently. This is done when necessary fore-vacuum *i.e.*, preliminary reduction of pressure, has been provided by the auxiliary rotary pump connected at C. The vapour thus formed passes upwards between the outer walls of the boiler and passes downwards through the jet D over the condenser L through which cold water is being circulated. The vapour which is thus condensed returns to the boiler through the narrow tube K.

The downward stream of vapour while passing through the jet D entrains the air molecules diffusing from the vessel connected at B, which is to be evacuated, and ejects it through the tube H to the auxiliary pump connected at C.

A vapour trap is provided at X to arrest any mercury vapour issuing upwards from the boiler A.

It is to be noted that for the good working of this pump a higher fore-vacuum is required and that is attained by the auxiliary pump connected at C.

Gaede shewed that the volume of the gas which diffuses into the mercury vapour and is entrained depends upon the width of the

aperture and the rate of evacuation depends on the partial pressure of the gas and not on the total pressure of the gas and vapour. This pump is usually used for manufacturing **electric bulbs, thermionic valves** and for running *X-ray tubes.* This pump produces a pressure of 10^{-6} *mm.* of Hg or less.

164. Measurement of Low pressure : Mc.Leod's Pressure Gauge : The adjoining figure illustrates the gauge for measuring low pressure.

A vertical tube ABCD (Fig. 85) with a bulb between B and C is sealed at the top. The portion AB of the tube is of small bore and graduated in c.c. A branch at C leads to another tube FG of the same bore as AB to avoid capillary effect and is connected to the vessel in which the pressure is to be measured.

The lower end D of the tube ABCD is connected to the mercury cistern E provided with a tap by a flexible tubing.

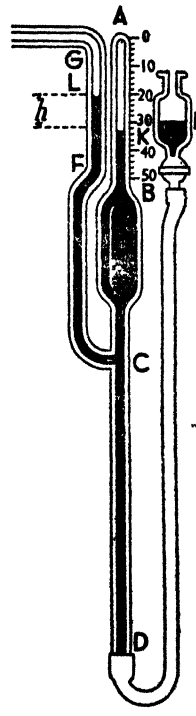


Fig. 85

To measure the pressure of the gas enclosed in the vessel, the level of mercury is adjusted so as to be lower than C and the space above mercury in the tubes is filled with the gas at the same pressure as that in the vessel.

The cistern E is then raised slowly until mercury seals the gas in ABC. Let V be the volume of the bulb and the tube AB. On further raising the cistern the levels of mercury in the tubes FG and AB stand at L and K, the gas being entrapped in the tube AB.

Let V′ be the volume of the entrapped gas in the portion AK and let h mm. be the difference in levels at K and L.

Let p be the pressure of the gas of volume V, to be measured. Then the pressure of V′ volume of the gas is equal to $p+h$.

Then, if there be no change in temperature during the observation we have, according to Boyle's Law,

$$pV=(p+h)V'=pV'+hV' \text{ or } p(V-V')=V'h. \qquad p=\frac{V'h}{V-V'}$$

Thus p, the low pressure of the gas can be measured knowing V, V′ and h.

QUESTIONS

1. Write a short note on Rotary Pump. [C. U. 1945, '48, '49]
2. Write a short note on the production and measurement of low pressure. [C. U. 1942, '47, '51]

Give a general account of high vacuum pumps and describe any one of them. [C. U. 1956]

SUPPLEMENTARY EXAMPLES

1. A solid cone of weight w whose height is h and radius of the base r, stands on a horizontal plane. Find the work expended in upsetting it. [C.U. 1910]

[The C. G. of the cone is on the axis at a distance $\frac{1}{4}h$ from the base of the cone.] [Work done = Force × displacement of C. G.]

The cone is tilted and the new position of the C. G. is taken where it is just vertically above the point of contact of the cone with the plane.

$$\left[\text{Ans. } w\left(\sqrt{r^2+\frac{h^2}{16}}-\frac{h}{4}\right)\right]$$

2. A pendulum which beats once in 2 seconds at A, gains two beats an hour at B. Compare the weights of the same substance at the two places. [C. U. 1916]

[At A the number of beats per hour is 1800 and at B the number of beats per hour is 1802. The period at A is 4 secs. and the period at B is $\frac{3600}{901}$ secs.]

$$\text{Then, } 4=2\pi\sqrt{\frac{l}{g_1}}\;;\; \frac{3600}{901}=2\pi\sqrt{\frac{l}{g_2}}.$$

Here g_1, and g_2 are the accelerations due to gravity at A and B respectively.

$$\text{Therefore } \sqrt{\frac{g_2}{g_1}}=\left(\frac{901}{900}\right), \quad \therefore \quad \frac{mg_2}{mg_1}=\frac{wt.\text{ of the body at B}}{wt.\text{ of the body at A}}=\left(\frac{901}{900}\right)=1{\cdot}002$$

Here m is the mass of body.

3. The potentials of two homogeneous spherical shells at internal points are in the ratio 3 : 4. Find the ratio of their radii. [Ans. 3 : 4]

4. Find the kinetic energy acquired by a body of given mass in falling through a given distance from rest, the original position of the body from the attracting sphere of mass M being also given. [C. U. 1916]

We know that Work done = Force × distance moved.

If M be the mass of the body, the force on a body of mass m at a distance r from the body is $\frac{GMm}{r^2}$.

Then the work done in falling through $dr = GM\frac{dr}{r}$

The work done in falling from a point at a distance r_1 to the point at the distance r_2 *i.e.* through r_2-r_1

$$=GMm\int_{r_1}^{r_2}\frac{dr}{r_2}=GMm\left[-\frac{1}{r}\right]_{r_1}^{r_2}=GMm\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=GMm.\frac{r_2-r_1}{r_1r_2}$$

Here r_1, r_2 and r_2-r_1 are known from the conditions of the problem.

CHAPTER XI

UNITS AND DIMENSIONS OF UNITS

165. Units : To express a physical quantity we must fix on a certain definite amount known as the unit, in terms of which the whole quantity can be expressed and the expression for the quantity involves two components *viz.* (1) a numerical factor which is the ratio of the quantity to the unit and (2) the unit with which the quantity is compared. Thus when we say that the length of a rod is 6, it means nothing, but when we say that its length is 6 feet, we mean that the length has been compared with a foot, which is here the unit and the ratio has been found to be 6.

The ratio between the quantity and the selected unit is called the numerical value or a measure of the quantity.

Suppose that we are to measure a definite length l and that we adopt as our unit length L, the numerical value (n) of the length to be measured will be $n = l/\mathrm{L}$ where n may be any number, whole or fractional.

The numerical value n of a length l is given in terms of unit length L by the equation $n = l/\mathrm{L}$.

To find the numerical value n' when the unit length is L'.
Now $n = l/\mathrm{L}$ and $n' = l/\mathrm{L}'$ $\therefore$ $l = n\mathrm{L} = n'\mathrm{L}'$ or $n' = n\,\mathrm{L}/\mathrm{L}'$
The value of L/L' is called the *change-ratio.*

For measuring different kinds of physical quantities we must have different units and so to avoid complications three **Fundamental Units** and other units, derived from these three, called **Derived Units**, have been chosen to express a large number of physical quantities.

Units may be chosen arbitrarily and so any complete system in which certain Fundamental Units are used and in which all the other units are derived from them, is called the **absolute system** of units.

The three Fundamental Units are generally taken as (1) the **unit of length** ; (2) the **unit of mass** ; (3) the **unit of time.**

In C. G. S. system the units of length, mass and time are the centimetre, gramme and second respectively and the other units derived from them are spoken of as the C. G. S. system of units. In F. P. S. system the foot and pound replace the centimetre and gramme of the C. G. S. system, unit of time being same *i.e.* second.

166. Dimensions : The precise way in which the fundamental units appear in the unit of a quantity is called the dimension of the quantity.

The value of any derived unit depends on the values of the fundamental units from which it is derived. Thus if we write L for the unit of length and A for the unit of area, then the unit of area A may be expressed as L^2 and similarly the unit of volume may be expressed as L^3. Thus the powers to which the fundamental units are to be raised to obtain any derived unit are called the **dimension** of that unit. Thus the area is of second dimension in length and the volume, the third dimension in length.

166a. Some Important Dimensional Equations :

$$[\text{Area}] = [\text{Length}]^2 = L^2$$

$$[\text{Volume}] = [\text{Length}]^3 = L^3$$

$$[\text{Density}] = \frac{[\text{Mass}]}{[\text{Volume}]} = \frac{M}{L^3} = ML^{-3}$$

$$[\text{Velocity}] = \frac{[\text{Length}]}{[\text{Time}]} = \frac{L}{T} = LT^{-1}$$

$$[\text{Acceleration}] = \frac{[\text{Velocity}]}{[\text{Time}]} = \frac{LT^{-1}}{T} = LT^{-2}$$

$$[\text{Force}] = [\text{Mass}] \times [\text{Acceleration}] = MLT^{-2}$$

$$[\text{Pressure}] = \frac{[\text{Force}]}{[\text{Area}]} = \frac{MLT^{-}}{L^2} = ML^{-1}T^{-}$$

$$[\text{Energy}] = [\text{Force}] \times [\text{Distance}] = MLT^{-2} \times L = ML^2T^{-2}$$

166b. Dimensional Equations : Any equation referring to the units only is known as Dimensional Equation.

The ordinary equation for velocity is written by $v = \frac{l}{t}$, where v is the velocity and l, the length covered in time t.

The symbols in the equation are merely numeric. The dimensional form of the equation for v should be $[V] = \frac{l[L]}{t[T]}$ where V, L and T represent respectively the units of Velocity, Length and Time.

If v, l and t are each equal to the corresponding unit, then,

V (velocity) $= \frac{L}{T} = LT^{-1}$, where L and T are the units of length and time respectively. Thus the unit of velocity is of one dimension in length, and minus one dimension in time.

167. Use of Dimensional Equations : A knowledge of dimensions of different physical units may be conveniently used to solve physical problems.

(1) The time of swing t of a simple pendulum depends only its length and on the value of g. How does the time of swing depend upon the length and the value of g?

i.e. To prove that $t = 2\pi\sqrt{\frac{l}{g}}$

Let $t \propto l^x g^y$, where x and y are the unknown powers fractional or integral, positive and negative to which l and g are to be raised.

Then the dimensional equation is given by

$T \propto L^x(LT^{-2})^y$, or $T \propto L^{x+y}T^{-2y}$ {where LT^{-2} is the dimension of g, the acceleration due to gravity.

Hence, $x + y = 0$, $-2y = 1$ or $y = -\frac{1}{2}$ and $x = \frac{1}{2}$,

Therefore $t \propto l^{\frac{1}{2}} g^{-\frac{1}{2}}$, $t \propto \sqrt{\frac{l}{g}}$ or $t = k\sqrt{\frac{l}{g}}$

The value of the constant k which has no dimension has been found out to be 2π when the angle of swing is small; $t = 2\pi\sqrt{\frac{l}{g}}$

(2) The surface tension (T) of a liquid film is the energy (E) per unit area of the film, *i.e.* $T = \frac{E}{A}$.

Shew that this expression is of proper dimensions.

The dimension of T which is force per unit length of film is

$$\frac{MLT^{-2}}{L} = MT^{-2}$$

The dimension of T as obtained from energy consideration is

$$\frac{F}{A} = \frac{\text{Energy}}{\text{Area}} = \frac{M(LT^{-1})^2}{L^2} = \frac{ML^2T^{-2}}{L^2} = MT^{-2}$$

Thus the expression $T = \frac{E}{A}$ is of proper dimensions.

(3) The velocity (V) of sound in a gas is a function of its pressure (P) and density (D). Prove that $V \propto \sqrt{\frac{P}{D}}$

Let $V \propto P^x D^y$, where x and y are unknown powers, $+ve$ or $-ve$ fractional or integral. The dimension of V is LT^{-1}.

The dimension of P is $\frac{MLT^{-2}}{L^2} = MT^{-2}L^{-1}$.

The dimension of D is $\frac{M}{L^3} = ML^{-3}$.

Therefore, the dimensional equation of the above relation is given by $LT^{-1} = (MT^{-2}L^{-1})^x (ML^{-3})^y$
$= (M^x L^{-x} T^{-2x})(M^y L^{-3y}) = M^{x+y} L^{-x-3y} T^{-2x}$

Hence, $x+y=0$ and $-x-3y=1$ and $-2x=-1$

or $x=\frac{1}{2}$ and $y=-\frac{1}{2}$. Therefore, $V \propto P^{\frac{1}{2}} D^{-\frac{1}{2}}$ or $\sqrt{\frac{P}{D}}$.

(4) The frequency (n) of a vibration of a stretched string is a function of the tension (W), the length (l) and the mass per unit length (m). Prove $n \propto \frac{1}{l}\sqrt{\frac{W}{m}}$.

Let $n \propto l^x . W^y . m^z$

The dimension of n is T^{-1} ... ... (1)

The dimension of W *i.e.*, force is MLT^{-2}

The dimension of m is $\frac{M}{L} = ML^{-1}$

The dimensional equation is therefore,

$(T^{-1}) = L^x.(MLT^{-2})^y.(ML^{-1})^z = L^{x+y-z}. T^{-2y}. M^{y+z}$.

Hence, $x+y-z=0$, $y+z=0$, $-2y=-1$, or $x=-1$, $y=\frac{1}{2}$, $z=-\frac{1}{2}$

Therefore, the relation (1) becomes $n \propto \frac{1}{l}\sqrt{\frac{W}{m}}$

168. Limitations of the Dimensional Equation : The gravitational force between two bodies depends on their masses and distance D between them.

Thus we have $F = M^x M^y D^z$; $MLT^{-2} = M^{x+y} L^z T^0$ from which $x+y=1$ and $z=1$ and we have the absurd statement, $-2=0$ which are powers of time dimension (T) of two sides of the equation.

This breakdown is not due to any imperfection in the method but to our ignorance of the mechanism of gravitation.

169. Conversion of the magnitude of a physical quantity from one system to another :

(1) Express the acceleration due to gravity in terms of the mile and hour as the units of length and time, its value being 32 when the foot is the unit of length and the second is the unit of time.

Since the actual value of the acceleration due to gravity is the same whatever be the units used, we have the relation

$$n_1[L_1T_1^{-2}] = n[LT^{-2}]$$

Here n_1 and n are respectively the numerical values of the quantity in the two system of units and $L_1T_1^{-2}$ and LT^{-2} the dimensions of acceleration in the two systems of units.

Let f be the numerical value of the acceleration required.

Then $f[L_1T_1^{-2}] = 32[LT^{-2}]$, or $f = 32\dfrac{L}{L_1}\cdot\left(\dfrac{T_1}{T}\right)^2$

But $\dfrac{L}{L_1} = \dfrac{\text{foot}}{\text{mile}} = \dfrac{1 \text{ ft.}}{1760 \times 3} = \dfrac{1}{5280}$, $\dfrac{T_1}{T} = \dfrac{\text{hour}}{\text{second}} = \dfrac{3600}{1} = 3600$

$$\therefore\ f = 32.\frac{1}{5280}.(3600)^2 = 78545{\cdot}45 \text{ miles per hour}^2.$$

(2) Find the value of a horse-power in watts, one horse-power being equal to 550 ft. lb. per sec., the value of 'g' being $32{\cdot}18$ ft./sec^2.

As the ft.-lb. is the gravitational unit it is to be multiplied by g to reduce it to the absolute unit.

$\therefore$ 550 ft./lb. $= 550 \times 32{\cdot}18$ ft. poundals

Then as in Example 1. We have

$$n_1[M_1L_1^{2}T_1^{-3}] = 550 \times 32{\cdot}18[ML^2T^{-3}]$$

Here $[ML_1^2T_1^{-3}]$ and $[ML^2T^{-3}]$ are the dimensions of power in the two systems of units.

But $\dfrac{M}{M_1} = 453{\cdot}6$ $\qquad \therefore\ n_1 = 550 \times 32{\cdot}18.\dfrac{M}{M_1}\cdot\left(\dfrac{L}{L_1}\right)^2\left(\dfrac{T_1}{T}\right)^3$

$\dfrac{L}{L_1} = 30{\cdot}48$ $\qquad = 550 \times 32{\cdot}18 \times 453{\cdot}6 \times (30{\cdot}48)^2$

T_1, T being the same in both the systems $\Big\}$ $= 745{\cdot}8 \times 10^7$ ergs per second $= 745{\cdot}8$ watts.

QUESTIONS

1. What do you understand by the dimension of physical quantity in terms of mass, length and time? [C. U. 1948, '50, '52]
Illustrate your answer by two examples.
2. Find the dimensions of Young's modulus in terms of the fundamental units of length, mass and time. [C. U. 1927]
3. Find the dimensions of Young's modulus. [C. U. 1953]
4. Define modulus of rigidity and find its dimensions. [C. U. 1954]

EXAMPLES

1. Find the number of dynes in a force which acting on 15 kilograms for one minute produces a velocity of $3{\cdot}6$ kilometres per second. [C. U. 1948, '50, '52]

We know that P (force) $= mf = \dfrac{15 \times 1000 \times 3{\cdot}6 \times 1000 \times 100}{60} = 9 \times 10^7$ dynes

2. Find the number of dynes in the force which, acting on 1 cwt. for one minute produces a velocity of one mile per hour. Given 1 ft. $= 30{\cdot}5$ cm. and 1 lb $= 453$ gms. [C. U 1952]

We have $P=mf$, where P=force ; m=mass and f=acceleration,

$m=1$ cwt. $=1\times4\times28$ lbs $=4\times28\times453$ gms.

$$f=\frac{v}{t}\ ;\ v=1 \text{ mile per hour}=\frac{1760\times3}{60\times60}=\frac{44}{30}\text{ ft./sec.}$$

$$\therefore\ f=\frac{44}{30\times60}\text{ ft./sec.}^2.\ \text{Hence, } P=mf=4\times28\times453\times\frac{44\times30{\cdot}5}{30\times60}\text{ dynes}$$

$$=3{\cdot}78\times10^4\text{ dynes.}$$

3. Assuming that the period of vibration of a tuning fork depends upon the length of its prongs, and on the density and Young's modulus of the material, find by the method of dimensions a formula for the period of vibration. [C. U. 1950]

Let $t=Kl^x\rho^yY^z$ where x, y, z are unknown powers positive or negative, fractional or integral ; and K is a constant.

The dimension of period t is T.

" " " length l is L.

" " " density ρ is ML^{-3}.

" " " Young's modulus Y is $ML^{-1}T^{-2}$

$$\therefore\ T=KL^x(ML^{-3})^y(ML^{-1}T^{-2})^z=KL^{x-3y-z}\cdot M^{y+z}\cdot T^{-2z}$$

$$\text{Hence, }\left.\begin{aligned} x-\quad -z&=0\\ y+\quad\ &=0\\ -2z\quad &=1\end{aligned}\right\}\ \begin{aligned}&\text{Solving these equations,}\\ &z=-\tfrac{1}{2},\ y=\tfrac{1}{2}\text{ and }x=1\end{aligned}$$

$$\therefore\ t=Kl.\rho^{\frac{1}{2}}Y^{-\frac{1}{2}}=Kl\sqrt{\frac{\rho}{Y}}.$$

SOUND

CHAPTER I

INTRODUCTORY

1. Sound and Acoustics : The term sound is used to signify the sensation received through our ears, as also the stimulus or rather the external disturbances which produces this sensation.

When we say, a sound is shrill or grave, we mean the sensation ; when we say, sound moves slower in gases than in solids, we refer to the external processes involving the propagation of sound.

The branch of Physics which deals with the production, propagation and perception of sound is called "**Acoustics**".

2. Production of sound : Sound is due to the vibratory movement of material bodies. The vibrations may be slow or rapid and the ear recognises these vibrations as audible sounds if they lie within certain limits known as the **Limits of audibility,** the lower limit being about 20 vibrations per second and the upper limit about 20000 vibrations per second. These limits differ for different persons. The audible sounds are known as **sonic sounds,** and the sources emitting them are called "Sonic source".

If the frequency of a vibrating body be slower than the lower limit as in the case of vibration of a pendulum, no sound will be heard. The disturbances produced by these slower vibrations are known as **Infra-sonic** disturbances or waves.

If again, the frequencies are very rapid and greater than the upper limit of vibration no sensation of sound is obtained. The disturbances produced by these vibrations are known as **Super-sonic or Ultra-sonic** disturbances or waves. They are produced by **piezo-electric crystal** oscillators.

3. Propagation of Sound : For the propagation of sound a transmitting medium is necessary, and it must be **continuous and elastic.** The medium may be a solid, liquid or a gas. The mode of propagation in the medium is dealt in details in the chapter on wave motion.

4. Perception of Sound : For the reception and perception of sound a healthy organ of hearing, ear, is necessary. The disturbance produced by any vibratory source is transmitted through the medium in the form of waves and when these waves pass into the ear they strike the drum of the ear and cause it to vibrate and the vibrations

of the drum are ultimately conveyed to the brain through a chain of bones and nerves and then the sensation of sound is produced.

5. Characteristic properties of Sound :

(1) *Sound takes time to travel :*

The thunder clap is heard sometime after the flash of lightning is visible although they take place simultaneously. This shows that sound takes much longer time than light to reach the observer. In other words velocity of light is much greater than that of sound.

(2) *Sound travels only through a material medium :*

The ticks of a watch are distinctly heard by placing the watch on one end of a table and the ear against the other end, the sound travelling through the wood of the table.

(3) *Sound may be reflected :*

An echo is an outcome of sound reflection.

(4) *Sound is refracted :*

Sound travelling with the wind is more clearly heard than when travelling against it. This is explained by the phenomenon of refraction.

(5) *Sound exhibits interference :*

The production of beats between two sounds of nearly equal frequencies is caused by the phenomenon of interference.

(6) *Sound exhibits diffraction :*

The phenomenon of sound shadow is an instance of diffraction.

6. Characteristics of sound sensations : Sound sensations differ from one another in three different characteristics, such as **Pitch, Loudness** and **Quality**.

The subject has been treated in a separate chapter.

7. Characteristics of Waves :

(1) Waves take time to travel from one point to another.

(2) For the propagation of the waves a material medium is necessary.

(3) Waves are reflected and also refracted.

(4) Waves exhibit the phenomena of interference and diffraction.

8. Classification of Sound : Sounds which can affect the human ear may be divided into two types :—(1) **Noise** and (2) **Musical sound**. The first type as a rule consists of short-lived sounds which last for a short interval of time only, or if they last for some appreciable time, they continually change their character. There is as a rule no periodicity and harmony in noisy sound. The musical sound is that sound, the main **attribute** of which is that the vibrations by which it is produced are **periodic and regular**.

The clatter of the hoofs of horses, the clapping of the hands, the rolling of the thunder of lightening, the explosion of a bomb are sounds of the first type. The sound emitted by a plucked or bowed string of the violin, or the sound produced by pressing a key of the harmonium, are instances of musical sound.

QUESTIONS

1. What is meant by the term sound? What is the scope of the study of acoustical science? What is meant by supersonic sound?
2. What are the characteristic properties of sound? State the characteristics of sound sensations.
3. How do you distinguish between musical sound and noise?

CHAPTER II

WAVE MOTION

9. Disturbance in a medium: Wave motion: We have already studied the periodic motion of a single particle in connection with simple harmonic motion (*Vide* General Physics). Now we are to consider in detail the resultant motion of various particles in any medium executing periodic motion at different instants of time. In the time known as the **periodic time**, all the particles of the medium situated on a right line will be disturbed successively and the displacements of all these particles at different instants will be different. The simultaneous positions of the successive particles after displacement at any instant determine the form of the wave. Now if the motion of the particles be continued, the curve connecting the particles appears to move steadily forward and the motion of this curve constitutes what is known as a **wave**, and the motion itself is called **Wave motion.**

A wave may be defined as a form of disturbance which travels through a medium and is due to the parts of the medium performing in succession certain periodic motions about their mean positions. In the above case the wave motion has been considered to be due to the periodic vibration of particles and the wave form is known as the periodic wave.

10. Two types of Waves: There are two distinct types of vibration of a material medium—*transverse* and *longitudinal*. The waves arising out of these two types of vibrations are respectively called transverse and longitudinal waves.

11. Transverse Waves ; *The waves in which the particles of the medium through which they advance execute simple harmonic vibrations about their positions of rest at right angles to the direction of propagation of the waves are known as Transverse Waves.*

12. Transverse Waves represented by Displacement curve : When a transverse wave passes through any medium the particles of the medium are disturbed and execute similar simple harmonic vibrations with constant difference of phase and the result is a wave in the form of a sine curve travelling in the direction in which we find a gradual retardation of phase as we pass along the series of particles. The curve is called a **Displacement curve** since it represents the displacements of the different particles at different instants of time.

13. Displacement curve : Let there be a number of particles situated at positions denoted by 1, 2, 3,......9 etc. on the line AE in an elastic medium (*say* water) the particle No. 1 being at A. (Fig. 1).

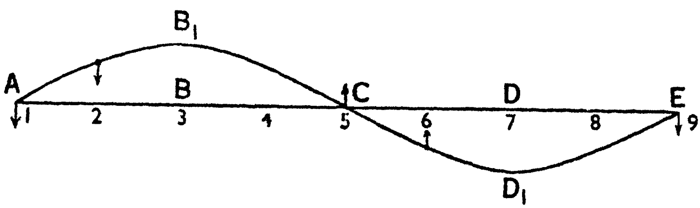


Fig. 1

Suppose transverse waves move in the direction AE in the medium considered. The particles of the medium will perform S. H. M's in directions at right angles to AE. Consider particle No. 1 at A to be just finishing one complete vibration by starting from its undisturbed position at A, going downwards to maximum displacement, then moving through A to the point of maximum displacement in the upward direction and again returning to its normal mean position A just to repeat its movement downwards, due to elastic forces in the medium. Let a length AE of the medium be disturbed during the time the particle No. 1 has taken to make one complete oscillation.

Suppose the particle No. 3 of the medium at B be at a distance $\frac{1}{4}$AE from A. If the period of vibration of the particle No. 1 be T, then the disturbance will reach the particle No. 3. at a time T/4 later than it reached the particle No. 1. Hence, the phase of

vibration of particle No. 3 being T/4 behind that of particle No. 1, it, after finishing its downward motion starting from its undisturbed normal position, must have gone up to B_1 undergoing maximum displacement. Let us now consider the particle No. 5 of the medium at C, at a distance from A equal to $\frac{1}{2}$ AE. Disturbance arrived at C T/2 sec. after it came to A, so that the phase of the particle at C will be behind that of A by T/2 sec. The particle No. 5 has, therefore, finished its downward motion and is just about to pass through the mean position in its upward journey.

The particle No. 7 at D and at a distance from A equal to $\frac{3}{4}$ AE receives the impulse $\frac{3}{4}$T sec. later than the particle No. 1 at A, so that the phase of the particle No. 7 is 3T/4 behind that of A. Hence, the particle No. 7 becomes displaced to the maximum extent in the downward direction to D_1. The particle No. 9 at E is affected at a time T sec. after the particle No. 1 at A received the disturbance, so that the phase of particle No. 9 is T sec. behind that of particle No. 1. The particle No. 9 is, therefore, just about to move downwards from its undisturbed position E.

A continuous smoth line drawn through the displaced positions of the particles at any instant represents the **displacement curve** or the wave curve AB_1CD_1E. The wave curve which resembles a sine or cosine curve, represents a transverse wave and propagates with crests and hollows along AE.

14. Velocity of the displaced particles : At the points A, C, E etc., (Fig. 1) the particles of the medium pass through their mean positions and hence, have maximum velocity. At the points B, D etc. the particles have been displaced to the maximum extents and therefore their velocities are zero. Tangents drawn to the curve at A, C, E will form maximum angle with AE, while tangents at B_1 or D_1 will describe zero angle with AE. Thus the slope of the curve at any point gives the velocity of the particle at that point. It will be noted also that the rate of change of slope at any point on the curve indicates the acceleration of a vibrating particle at that point.

15. Velocity curve : While studying the motion of a particle vibrating harmonically we have also seen that the velocity of the particle at any instant is not proportional to its displacement at that instant but to the displacement which it will have a quarter of a period afterwards. So to represent a transverse wave by a velocity curve we are to draw a curve with the velocity of the particle at any instant as ordinate and the corresponding instant as abscissa and the curve thus drawn is a quarter of a period behind the curve of displacement.

15a. Illustration of Transverse Wave :

(1) The transverse wave may be demonstrated *in a simple way. A solid* India rubber cord (Fig. 2) of fairly good length is suspended vertically from the ceiling or from some suitable support. When the lower free end of the cord is moved laterally to and fro, a disturbance in the form of a wave is found to travel along the cord. In this case, the motion of the different parts of the cord is transverse, *i.e.*, at right angles to the direction of the propagation of the wave and the corresponding waves set up are transverse waves.

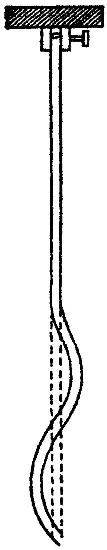


Fig. 2

(2) The familiar waves produced when a stone is thrown into still water are transverse waves. (Fig. 3)

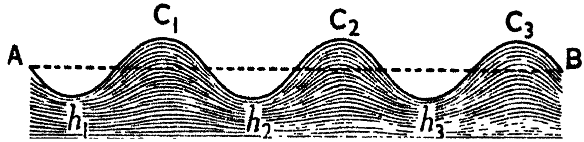


Fig. 3

The particles of water move up and down executing periodic S.H. motion, while the wave travels in a perpendicular direction. The waves consisting of a series of crests (C_1, C_2 etc.) and troughs (h_1, h_2 etc.) travel outwards from the centre of disturbance in gradually-widening circles.

16. Certain terms :

(1) **Wave-length** : *The distance* AE (Fig. 1) *traversed by the wave during one complete vibration of any of the vibrating particles of the medium is called the wave-length of the wave. It is denoted by* λ (Lamda).

It is to be noted that since the particles at A and E are passing through their undisturbed positions in the same direction at the same time they are in the same phase. Any other particle of the medium having the same phase of vibration would be at a distanc AE or its **integral multiple**. Hence, the wave length is the shortest distance between any two layers of particles in the same phase of vibration.

As the particles at A and C are moving through their mean positions in the opposite directions, they have opposite phases of vibration. Again the distance AC is traversed by the wave in half the periodic time of a particle of the medium. Therefore, the shortest distance between two particles having **opposite phases** is equal **to half the wave-length**. Likewise the distance between particles 8 and 7 is equal to half the wave-length, while the distances AB, BC, CD or DE are each one-fourth of the wave-length.

A point B_1 on the wave, at which a particle has been displaced to the maximum value in the positive direction is termed a "crest" whereas, a point D_1 on the wave, at which a particle has suffered maximum displacement in the negative direction is known as a "trough" or "hollow". The two terms are used more frequently in respect of water waves or ripples.

(2) **Amplitude** : *The Amplitude of the wave is the maximum displacement of a particle on the wave from its normal mean position.* The distances BB_1 or DD_1 are each the amplitude of the wave.

(3) **Period** : *Period of a wave is the time which it takes to advance through a distance equal to its wave-length.* It is equal to the time for a complete vibration of the particles of the medium producing the wave. It is denoted by T.

(4) **Frequency** : *Frequency of vibration is the number of vibrations performed in one second by a particle of the medium or by the source sending out the waves.* Now, for each complete vibration of a particle of the medium or of a source, one wave is generated. Hence, frequency of a wave is the number of waves formed in one second. The frequency is denoted by n. If we say that a wave is of frequency n we mean n waves are formed and propagated in one second in the given medium.

(5) **Velocity of propagation** : The space or distance through which a wave moves in one second in a medium is the velocity of propagation of the wave in that medium. It is denoted by V and it depends on the density and the elasticity of the medium.

17. Mathematical relation between T, n, λ and V : If T be the period of a wave and λ its wave-length, then the wave moves through a distance equal to λ in time T ; but the wave also moves through a distance VT in T sec. where V is the velocity of propagation of the wave.

Therefore we have, $VT = \lambda$ ... (i)

Again, if n be the frequency of the wave, *i.e.*, number of vibrations performed in 1 sec., then $nT = 1$ or $T = 1/n$

Hence, relation (i) becomes, $V.\dfrac{1}{n} = \lambda$ or $V = n\lambda$

Thus velocity of propagation of wave = frequency of the wave × its wave-length.

18. Longitudinal Waves : *The waves in which the particles of the medium through which they advance execute simple harmonic vibrations about their mean positions of rest along the direction of propagation of the waves, are called Longitudinal Waves.*

The particles will not be displaced as in the case of a transverse wave, to take the form of a sine curve but will come very close to one another at one

part and separated at the other part and so the longitudinal wave is said to comprise in itself a condensed and a rarefied part of the medium.

Here also the simultaneous displacements of the particles in the time equal to the periodic time of the vibrating particle, may be represented by a curve in which the ordinates at the undisturbed position of the particles measure *displacements to the right-hand, upwards, and to the left-hand, downwards, at different instants of time.*

Thus a longitudinal wave may be represented by a displacement curve; but the curve does not show directly the displacements in the directions in which they actually take place, but merely represent them conventionally.

19. Graphical representation of a Longitudinal Wave: Displacement curve: Let us suppose a row of equispaced particles of a medium occupying their mean undisturbed positions and

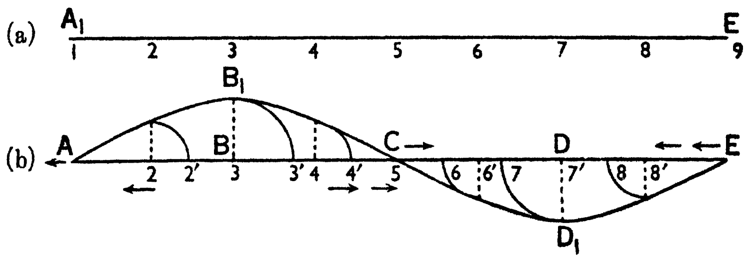


Fig. 4

arranged on a line AE in the direction of the propagation of the disturbance as shown in Fig. 4(*a*). As the longitudinal disturbance moves over the medium, these particles are set into S. H. M, along the line of advance of the disturbance. At a certain instant, suppose the particle No. 1 is at its mean position A and tends to move towards the left [Fig. 4(*b*)]. At that instant, the particle No. 2 is at the position 2′ and moves towards the mean position from the right. Particle No. 3 which is at the extreme right position B_1 is at rest momentarily. Particle No. 4 is at a distance same as that of the particle No. 2 from their respective mean position, but moves from left to right. Particle No. 5 occupies the mean position but tends to move towards the right. Reasoning in the same way, the displacement of other particles can be obtained and the particle No. 9 will be found to occupy the mean position E tending to move to the left as in the case of particle No. 1 at A. It will be observed that during any instant some particles after displacement are more crowded at a region (C) constituting what is called a **compression**, and some particles are rarely distributed at another region (A or E)

constituting what is called a **rarefaction**. The other particles in the line AE produced, in course of their displacements will give no new form but simply repeat the same form as shown by the particles along AE. Thus the longitudinal wave is propagated with a condensed and a rarefied part along AE.

A longitudinal wave, therefore, consists of a sequence of compressions and rarefactions separated by regions [B, D in Fig. 4(b)] of normal pressure.

If the displacements of the various particles be plotted against time along an axis perpendicular to AE, time being taken along AE, we get a curve AB_1CD_1E which is a characteristic sine-curve as in the case of a transverse wave.

Note: The changes of velocity and pressure in the wave may also be represented by a velocity and a pressure or compression curve. If the displacement curve is a sine-curve, the velocity and the compression curves are also represented by sine-curves, which are moved a quarter of wave-length to the right as compared with the displacement curve.

19a. Illustration of Longitudinal Wave: A spiral of copper wire is suspended from a wooden frame [Fig. 5(a)]. The

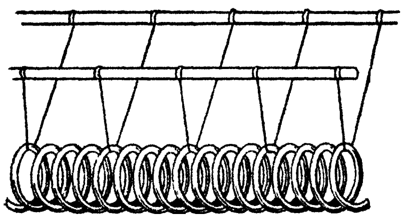


Fig. 5(a)

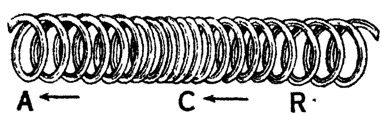


Fig. 5(b)

bifiliar suspension prevents lateral motion of the spiral. When a push is applied to one end of the spiral, a pulse of compression moves along the spiral. Again on giving a mild pull, a pulse of rarefaction moves along it. When the said end is alternately pushed and pulled, a wave of compression (C) and rarefaction (R) [Fig. 5(b)] is found to move along the spiral. As each turn of the spiral moves in the direction of propagation of the wave, the waves are longitudinal.

20. Transverse Waves can not exist in a gas: To explain this, let us consider a number of layers L_1, L_2, L_3 etc. in the medium arranged (Fig. 6) parallel to one another and perpendicular to the direction of propagation AB of the waves which are supposed to be transverse in character. Now if the first layer L_1 be moved upwards perpendicular to the direction of propagation of the waves,

it will tend to drag the second layer L_2 up by exerting a force on it so that a wave may be formed in the medium. But as air or any other fluid does not possess shear elasticity, this force cannot exist and thus no transverse wave can be propagated.

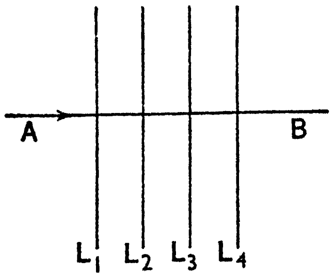


Fig. 6

But when the first layer moves up dragging the second layer the only force that can be exerted between the adjacent layers is the viscous force which is always opposed to the direction of motion and disappears as soon as the motion ceases. Such is the case with other adjacent layers. So the movement of the layers in a direction perpendicular to the direction of propagation cannot be maintained. This process of transmission is associated with a great loss of kinetic energy. Thus a transverse wave although created in the medium cannot be propagated very far. Since sound waves exhibit no evidence of such unusual subsidence, they cannot be of transverse type.

21. Certain terms : The physical quantities *period, frequency wave-length* and *velocity* of propagation with reference to longitudinal wave may be defined in the same way as in the case of the transverse waves, the symbols denoting them being identical for both the waves.

If T be the period, n the frequency, V the velocity of propagation of the wave and λ the wave length, then as in transverse wave

$$VT = \lambda \text{ or } V.\frac{1}{n} = \lambda \text{ or } V = n\lambda.$$

The wave-length in longitudinal wave is the length covered by a compression and a rarefaction.

21a. Wave front : *A wave front is defined as the trace drawn through all the points on a wave at which the particles are in the same phase of vibration.*

In the case of water-wave generated in still water surface by producing a periodic disturbance at a point in it, the wave front consists of concentric circles on the water surface, the point of disturbance being the common centre.

In a homogeneous medium a wave produced at a point travels in all directions around the point with the same velocity. At any instant of time, the wave-motion lies upon the surface of a sphere whose centre is the generating point and radius equal to the product of the velocity and time. On this sphere the particles of the medium are all in the same phase of motion. This equi-phase surface is the wave front at the said instant of time. At a very large distance from the source of disturbance, the spherical surface, over a limited region, may be treated as plane surface. Hence, the wave-front may be taken as plane, when the source is at a very large distance.

21b. Vibration : A particle is said to be vibrating or in a state vibration when it repeats the same movement over and over again.

21c. Undulation : A body is said to be undulating when different points of the same substance perform a similar movement one after the other.

A substance undergoes undulating motion when a sound-wave passes through it.

21d. Pulse : The term *pulse* is used to denote a condensation or a rarefaction.

22. Mechanism of the propagation of sound-wave in air : To explain the mode of propagation, let us consider a column of air (Fig. 7) enclosed in a tube open at both ends. Let this column

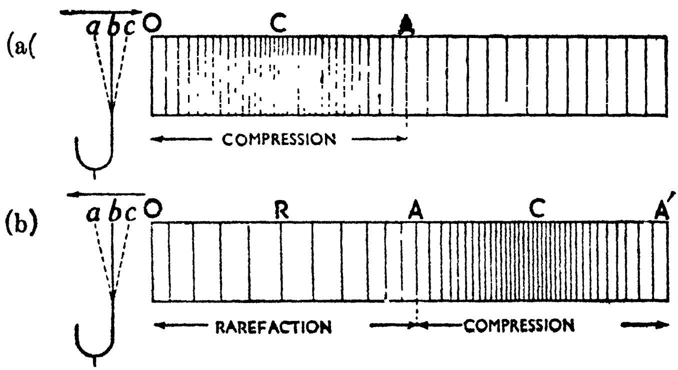


Fig. 7

of air be supposed to be divided by a number of very thin imaginary layers. Let a vibrating tuning fork be placed in front of the end O of the tube.

As the prong *b* [Fig. 7(*a*)] of the fork moves forward from *a* to *c*, the layer of air just in front of it gets compressed. This compressed layer then expands and in so doing compresses the second layer, itself returning to its original condition of pressure and volume by virtue of its elastic property. The second compressed layer again expands, compresses the third layer in front of it and regains its original pressure. In this way the different layers of air become compressed in turn and thus during the forward motion of the prong from *a* to *c*, a pulse of compression travels forward from layer to layer with a definite velocity. Again when the prong moves backward from *c* to *a* [Fig. 7(*b*)], the air in the first layer and that in successive layers further away from the prong inside the tube would become rarefied and move to fill up the gap created

due to the backward motion of the prong, each in turn coming to rest at the moment at which it is restored to its original volume and pressure. Thus when the prong moves forwards and backwards a wave of condensation followed by that of rarefaction is transmitted forward with a definite velocity and forms a complete sound-wave.

Fig. 7(*b*) represents the condition of air confined in a horizontal tube in front of which a tuning fork is vibrating. In consequence of this condition sound-waves with compressed and rarefied parts are produced and maintained. In Figure 7, C represents the compressed part and R represents the rarefied part.

Thus when the prong moves from *a* to *c* and again from *c* to *a i.e.*, makes a complete vibration, the layers of air in the tube are compressed at one part and this compression is followed by a rarefied part. The combined thickness of the compressed and the rarefied parts is the *wave-length* of the disturbance travelling in the tube.

It is important to note that the degree of compression or of rarefaction along the whole length of the compressed or the rarefied part of the wave is not uniform but varies from point to point, for the velocity of vibrating prong varies in its path of vibration. The maximum compression occurs in the layer in front of the prong when it is at its normal position in the forward movement, and maximum rarefaction occurs when it is in the same position in its backward movement. In intermediate positions of the prong, the amount of compression or of rarefaction is proportional to the velocity of the prong and becomes minimum when it is at either of its extreme points of vibration.

When a train of sound waves travels through any medium, the particles over which it passes, move to and fro about their mean positions, taking up the direction of the advance of the waves during compression, and opposite direction during rarefaction.

We can now summarise the property of a longitudinal wave.

(1) The velocity of the particle is maximum if it is passing through its equilibrium position.

(2) The velocity of the particle is zero when it is at either end of the swing and in the immediate neighbourhood has its normal density with compression on one side and rarefaction on the other.

(3) The particle moves along the line of advance of the waves during compression and in the opposite direction during rarefaction.

22a. Alternative Method : Consider a row of equidistant particles (Fig. 8) in an elastic medium denoted by points 0, 1, 2,...20 etc. lying in a straight line. Suppose the particle 0 performs S. H. motion in the same straight line. As the particles are bound by elastic forces, other praticles will be in turn thrown into S. H. motion each lagging in phase behind its predecessor. Let us now

consider four typical stages at the interval of one quarter of a period from the instant the particle 0 just starts from its normal position

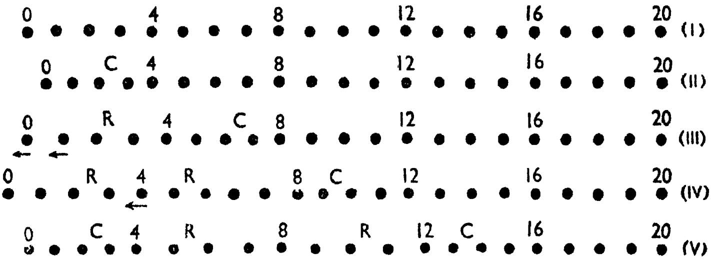


Fig. 8

of rest in the positive direction, as shown in Fig. 8(I). Now, as particle 0 moves, particle 1 is displaced and picks up the S. H. motion of 0 with slight lag of phase behind it. In the same way, particle 2 follows the motion of particle 1, 3 of 2 and so on, and at the end of the first quarter period (T/4) when particle 0 will lie at the positive extremity of its path of motion, particles 1, 2, 3 will have become displaced through decreasingly smaller distances and particle 4 will just be on the point of being displaced from its normal position of rest, all particles beyond 4 still remaining undisturbed. The relative displacements of the particles from 0 to 4 are shown in Fig. 8(II). These particles will appear more densly packed in this condition, and as such the condition is termed *Compression* or condensation.

Now as the particle 0 during the second quarter period of its oscillation goes towards its position of rest, particles 1, 2, 3, do so only after finishing journeys up to their positive extremities of oscillation respectively. Hence, when 0 comes back to its position of rest after the end of its second quarter of vibration (T/2 from start), particles 1, 2, 3 are on their journey towards their respective positions of rest lagging in phase behind particle 0 by increasingly greater amounts. At this instant the particle 4 is displaced to the positive extremity of its vibration and particles from 5 to 8 will lie at the same relative positions as the particles from 1 to 4 at the instant $t=T/4$. The particle 8 will just become disturbed, other particles beyond 8 still remaining unaffected at $t=T/2$. The relative positions of the particles 0 to 8 are shown in Fig 8(III).

It will appear from this figure that the compression C is shifted from the particles 0-4 to the particles 4 to 8, the spacing between 0-4 being more sparse than the normal condition. This condition is said to be rarefaction.

The displacements at the end of 3rd quarter (*i.e.* at $t=3T/4$) and 4th quarter (*i.e.* at $t=T$) are shown by Fig. 8(IV) and 8(V) respectively. It will be evident that at $t=T$ the disturbance has just reached the particle 16. At this instant

16 will just start for its first and particle 0 for its second oscillation in the positive directions. The particles 0 and 16 are, therefore, in the same phase at this instant, and their equal phase will be maintained during all time to come.

22b. Discussious :

(*a*) The distance between two nearest points (from 0 to 16) having same phases is termed the wave-length (λ) of the longitudinal wave. It is to be noted that no two other particles within this length can possess identical phases.

(*b*) Particles 0 and 8, or 8 and 16, which at time $t=T$ indicate identical displacements, are not in same phase, but are exactly in opposite phases, since the directions of velocities are opposite.

Note : It is to be noted that compression *i.e.*, compressional wave is transmitted to further and further particles with advance of time.

(*c*) The rarefaction produced in particles from 0 to 4 during the second quarter of period, persists even during the 3rd quarter. Similarly the compression set up in the 4th quarter continues during the next quarter of the period.

Thus the particles from 0 to 4 alternately undergo a state of compression and rarefaction each of which continues on the average for half the period of oscillation. As the conditions of displacement of particles from 0 to 4 are simply transmitted to the successive particles, it may be said in general that every portion of the row of particles alternately undergoes states of compression and rarefaction, each persisting for half the period on the average.

The above is briefly the mechanism of longitudinal wave propagation. Hence, a longitudinal wave is characterised by the setting up of alternate compressions and rarefactions in the medium.

22c. Diagramatic display of different factors in Longitudinal wave : A longitudinal wave passing along a row of particles may be conveniently shewn in different ways by the different lines of the diagram in Fig. 9, the lines corresponding to the actual positions, velocities, accleration and pressure variations of the particles respectively.

The wave may be supposed to move from left to right. The first line (a) gives the actual positions of the particles in the medium during their vibration. The second line (b) exhibits the conventional displacement curve for the same longitudinal wave. In the third line (c) which represents velocities, the arrows give the direction and a comparative magnitude of the velocities of different particles. In the fourth line (d) which represents accelerations, the arrows give the direction and comparative magnitude of the different particles. The fifth line (e) indicates densities or pressures of different regions. The

R's (capital) denote region of maximum rarefaction while C's (Capital) denote region of maximum compression. The r's (small)

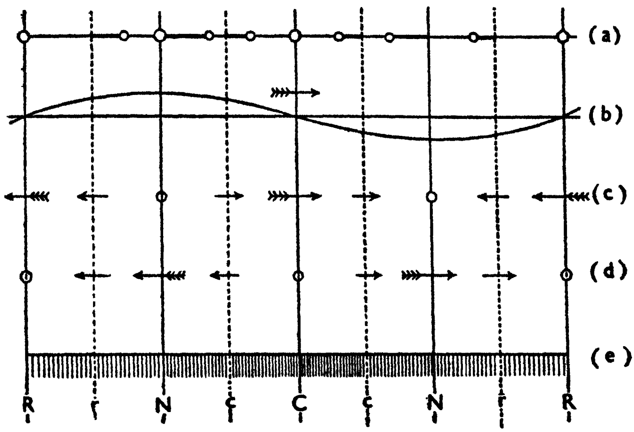


Fig. 9

and c's (small) indicate region of lower rarefaction or compression respectively. The N's represent regions of normal density or pressure.

23. Progressive Wave : A progressive wave is a wave that involves a continuous transfer of a definite state from one region of a medium to another by same series of movements performed by the successive particles of the medium with the passing of time and the wave-form travels onwards with a definite velocity depending on the properties of the medium.

The propagation of light from the sun through ether is an example of transverse progressive wave. The propagation of sound from one place to another through air or any other gas is an example of longitudinal progressive wave.

In a progressive wave whether of transverse or of longitudinal type, the various properties (say, velocity, acceleration, energy) of any particle of the medium go through the same cycle of changes.

24. Equation of Progressive Wave (Analytical treatment) : The displacement of any of the particles the motion of which constitutes a progressive wave of the simple harmonic type is a function of the time. Let a progressive wave proceed in the positive direction along the axis of x, and suppose y denotes the displacement of a particle at a time t. For a transverse progressive wave, y is normal to the direction of x, while for a longitudinal progressive wave y is along the direction of x. The displacement

equation of the vibrating particle can be written for each case, as

$$y=a \sin \omega t \quad \dots (1) \qquad \text{or } y=a \sin 2\pi t/T \quad \dots (1a)$$

where a denotes the amplitude of vibration of the particle (or amplitude of the wave); $\omega=2\pi/T=2\pi n$, where T and n are respectively period and frequency of the wave; and ωt is the phase of the particle in motion.

But the relation (1) refers to only a single point where the displacement is zero when time is zero, *i.e.* at the beginning of any period. A particle vibrating at a distance x from the origin of the waves will perform similar vibration but its phase is retarded. To find the displacement at x we must subtract from t in equation (1a) the time the wave takes to travel the distance x in order to take into account this phase retardation. If V be the velocity of propagation of the wave, then this time is equal to x/V, so that the displacement equation becomes

$$y=a \sin \frac{2\pi}{T}\left(t-\frac{x}{V}\right) \quad \dots \quad \dots \quad \dots \quad (2)$$

$$=a \sin 2\pi\left(\frac{t}{T}-\frac{x}{VT}\right)=a \sin 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right) \quad \dots \quad (3) \quad [\because \ VT=\lambda]$$

where λ is wave-length. Again putting $\frac{1}{T}=\frac{V}{\lambda}$ we can write

relation (2) as $y=a\frac{2\pi V}{\lambda}\left(t-\frac{x}{V}\right)=a \sin \frac{2\pi}{\lambda}(Vt-x) \quad \dots \quad (4)$

Each of the equations above (2, 3 or 4) gives the equation of an harmonic progressive wave (either longitudinal or transverse), travelling in the positive direction.

24a. An important mathematical relation :

From relation (4) we have, $y=a \sin \frac{2\pi}{\lambda}(Vt-x) \quad \dots \quad (4a)$

Differentiating y with respect to x, $\frac{dy}{dx}=-\frac{2\pi a}{\lambda}\cos\frac{2\pi}{\lambda}(Vt-x)$

Differentiating again, $\frac{d^2y}{dx^2}=-\frac{4\pi^2 a}{\lambda^2}\sin\frac{2\pi}{\lambda}(Vt-x) \quad \dots \quad (5)$

Now differentiating y, with respect to t, in (4a)

$$\frac{dy}{dt}=\frac{2\pi a}{\lambda}V\cos\frac{2\pi}{\lambda}(Vt-x)$$

„ again $\frac{d^2y}{dt^2}=-\frac{4\pi^2 aV^2}{\lambda^2}\sin\frac{2\pi}{\lambda}(Vt-x) \quad \dots \quad \dots \quad (6)$

From (5) and (6), we have $\frac{d^2y}{dt^2}=V^2.\frac{d^2y}{dx^2} \quad \dots \quad \dots \quad (7)$

24b Velocity of a particle in the progressive wave :

The velocity of a particle in the wave at a place defined by its co-ordinate x and at a time t can be obtained by differentiating its displacement with time.

The displacement equation is given by

$$y = a \sin \frac{2\pi}{\lambda}(Vt - x) \quad \ldots \quad \ldots \quad (1)$$

Velocity of the particle,

$$\frac{dy}{dt} = \frac{2\pi a V}{\lambda} \cos \frac{2\pi}{\lambda} (Vt - x) \quad (2)$$

Differentiating y in (1) with respect to x, the slope $\frac{dy}{dx}$ of the displacement curve is given by

$$\frac{dy}{dx} = -\frac{2\pi a}{\lambda} \cos \frac{2\pi}{\lambda} (Vt - x) \quad (3)$$

From (2) and (3), $\frac{dy}{dt} = -V.\frac{dy}{dx}$

or **Particle velocity = $-$V $\times$ slope of displacement curve,** where V is the wave velocity.

It is to be noted that the **particle velocity** depends on the time and position but the **wave velocity** depends on the properties of the medium only.

24c. Acceleration of a particle in the progressive wave :

The acceleration of the particle is the rate of change of velocity with time and is expressed by d^2y/dt^2. The rate of change of slope of the displacement curve is given by $\frac{d^2y}{dx^2}$

From relation (7) in Art. 24(a) $\frac{d^2y}{dt^2} = V^2 \frac{d^2y}{dx^2}$

or **Acceleration of the particle**

$= V^2 \times$ rate of change of slope of displacement curve.

When a wave passes over a medium the particles generating the wave move up and down and it is this mode of vibration of the particles which determines the form of the wave.

[It has been found that during the propagation of the waves the phase of the motion of the particle travels along the direction of propagation with the same velocity as that of the waves. So the wave-velocity is sometime called the *phase-velocity*.]

25. Energy of the air waves : We know that when a body vibrates in air, waves consisting of regions of condensation and rarefaction are produced in it. As the body vibrates it communicates its energy of vibration to the surrounding air which

then travels outwards with the waves. The energy of the waves is partly kinetic and partly potential. The kinetic energy is greatest in the most condensed and the most rarefied portions of the wave and at portions in the medium where there is no compression or rarefaction, the air has no energy above what it would have if no waves were passing. So the energy is not uniformly distributed over the waves and it can be easily shown that the condensed and rarefied portions taken together contain more energy than an equal volume of air under normal condition.

It can also be shown that the total energy per unit volume of the medium is proportional to the square of the amplitude of the wave and inversely proportional to the square of the wave-length.

26. Energy of Progressive waves : Let the progressive wave be represented by $y=a \sin \frac{2\pi}{\lambda}(Vt-x)$, where y is the displacement at a distance x and in time t. Then,

$$\text{velocity}=\frac{dy}{dt}=\frac{2\pi a V}{\lambda} \cos \frac{2\pi}{\lambda}(Vt-x)$$

During the vibration of a particle the sum of the kinetic and potential energies of the particle at any instant is constant and when one form of energy is at its maximum value, the other form is zero.

The kinetic energy K. E. at instant t, per unit volume of the medium is given by

$$\text{K. E.}=\tfrac{1}{2}\rho\left(\frac{dy}{dt}\right)^2=\tfrac{1}{2}\rho.\ \frac{4\pi^2 a^2 V^2}{\lambda^2}\cos^2\frac{2\pi}{\lambda}(Vt-x)$$

where ρ is the density of the medium.

The Potential energy P. E. at instant t per unit volume of the medium is given by $\text{P. E.}=\int_0^y \rho\omega^2 y.dy=\tfrac{1}{2}\rho\omega^2 y^2$

Here $\rho\omega^2 ydy$ is the work done in the small displacement dy

$$\therefore\ \text{P. E.}=\tfrac{1}{2}\rho.\ \frac{4\pi^2 V^2}{\lambda^2}a^2.\ \sin^2\frac{2\pi}{\lambda}(Vt-x). \qquad \left[\because\ \omega=\frac{2\pi V}{\lambda}\right]$$

Thus $\text{K.E.}+\text{P.E.}=\tfrac{1}{2}\rho\frac{4\pi^2 a^2 V^2}{\lambda^2}\left(\cos^2\frac{2\pi}{\lambda}(Vt-x)+\sin^2\frac{2\pi}{\lambda}(Vt-x)\right)$

$$=\tfrac{1}{2}\rho\frac{4\pi^2 a^2 V^2}{\lambda^2}=\tfrac{1}{2}\rho.4\pi^2 n^2 a^2 \quad \ldots (1)\ [\because\ V=n\lambda.]$$

Thus from (1), the total energy of the particle varies as the square of the amplitude of vibration of the particle.

This quantity of energy contained in unit length of the wave per unit area of the wave front is called **energy density.**

The flow of energy through the medium per second per unit area of the wave front is regarded as the **intensity** of the sound wave.

This is expressed as $\frac{1}{2}\rho.\ V.4\pi^2a^2n^2$ i.e. $2\pi^2\rho Va^2n^2$, since the wave travels a distance V in one second.

QUESTIONS

1. Clearly explain the meaning of the expression 'velocity of propagation of a longitudinal wave'. Represent graphically the state of disturbance at any time at any place during the propagation of such a wave. Detail the mechanical process involved. [C. U. 1924]

2. A wave is propagating along a row of particles from a source having S. H. M. Find an expression for the displacement of a particle at a distance x from the source at a time t.

3. Show that the intensity of sound emitted by a vibrating body is proportional to the square of the amplitude and the square of the frequency of vibration. [C. U. 1941]

CHAPTER III

VELOCITY OF LONGITUDINAL WAVES

27. Velocity of longitudinal waves along a solid: Let us consider a uniform rod of unit cross-section of a solid (elastic) medium and let A and B be two plane sections at distances x and $x+\delta x$ respectively from some fixed position to the left of A (Fig. 10), where δx is the distance between A and B.

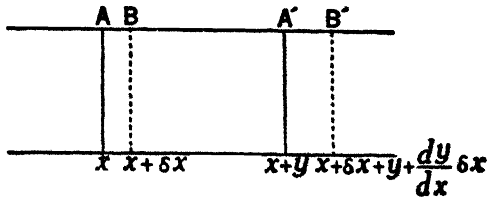


Fig. 10

Let the axis of x be along the length of the rod and let the longitudinal wave propagate along the length of the rod in the direction of x and thereby displace the planes A and B to different extents.

Let y be the displacement of the plane A at certain instant. The new position A′ of A is then at a distance $x+y$ from the fixed

position of reference. Now y is the displacement in the direction of x, for in longitudinal wave, the direction of displacement and propagation of the wave, are along the same line. Then as $\frac{dy}{dx}$ gives the rate of displacement with respect to distance along the direction of x, the displacement of the plane $B = y + \frac{dy}{dx}.\delta x$. The new position B$'$, of the plane B is thus at a distance $= x + \delta x + y + \frac{dy}{dx}.\delta x$. from the fixed position of reference.

The length of the slice A$'$B$'$, in the displaced position of AB

$$= x + \delta x + y + \frac{dy}{dx}.\ \delta x - (x + y) = \delta x + \frac{dy}{dx}.\ \delta x.$$

$\therefore$ Change in the length of the slice

$$= \delta x + \frac{dy}{dx}.\delta x - \delta x = \frac{dy}{dx}\delta x.$$

The fractional change in length, *i.e.*, the longitudinal strain

$$= \frac{\frac{dy}{dx}.\ \delta x}{\delta x} = \frac{dy}{dx}.$$

Let f be the stretching force per unit area of A and if Y be the Young's modulus of the material of the solid, then

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{f}{\frac{dy}{dx}} \text{ or } f = Y.\frac{dy}{dx} \qquad \ldots \quad (1)$$

The stretching force on unit area of B is

$$= f + \frac{df}{dx}.\delta x = Y.\ \frac{dy}{dx} + \frac{d}{dx}\left(Y.\frac{dy}{dx}\right).\ \delta x = Y.\frac{dy}{dx} + Y.\frac{d^2y}{dx^2}.\delta x, \qquad \ldots \quad (2)$$

since Y is independent of x.

The increase of this force in the thickness δx of the slice between A and B is therefore equal to $\frac{df}{dx}.\delta x$ or $Y.\frac{d^2y}{dx^2}.\ \delta x$ and is the moving force on the slice AB of the solid.

$\therefore$ If ρ be the density of the solid, then the mass of the moving slice is $\rho.\delta x$ since the area of the cross-section is unity.

Therefore by Newton's second law of motion ($P = mf$), we have $Y.\frac{d^2y}{dx^2}.\delta x = \rho.\delta x.\frac{d^2y}{dt^2}$, where $\frac{d^2y}{dt^2}$ represents the acceleration of the motion.

$$\frac{d^2y}{dt^2} = \frac{Y}{\rho} \frac{d^2y}{dx^2} \tag{3}$$

But we have the differential equation of a progressive wave travelling along x-axis with the velocity V given by

$$\frac{d^2y}{dt^2} = V^2.\frac{d^2y}{dx^2} \qquad \ldots \quad (4) \qquad \text{[Art. 24(a)]}$$

Thus from (3) and (4), $V^2 = \frac{Y}{\rho}$ or $V = \sqrt{\frac{Y}{\rho}}$, which gives the expression for the velocity.

27a. Alternative method :—Consider a rod of the solid medium (Fig. 11) of unit cross-section. Let the wave travel through it with velocity V in the direction AC. Let us consider two plane sections of the rod AB and CD fixed in space, AB being a region of normal pressure and CD that of rarefaction. Suppose that the rod is moved backwards in the direction CA with velocity V, so that waves in the zone ABDC cannot advance onward. Then evidently, the velocity of the particles of the solid, their states of displacement, the density and pressure of the medium though not same at different parts of the region ABDC, maintain their values all the time. The section AB being a place of normal pressure, the particles of the medium will themselves possess no velocity; owing to the backward motion of the solid medium with velocity V, a volume V will however come out of the region ABDC in one sec. through the section AB. The section CD being a region of rarefaction, the particles of the medium here will have a velocity V′ greater than V, which is the sum of the velocity of the medium and the velocity of the particles from right to left. Evidently a volume V′ will pass into the region ABDC in one sec. through CD.

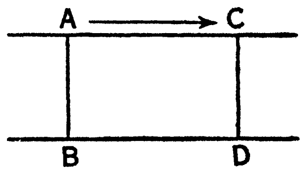


Fig. 11

As the pressure and density at various points inside ABDC remain unchanged, the mass of the medium occupying this region also remains the same. Therefore the mass of the medium entering the region through CD per second is equal to the mass of the medium passing out of the region through AB per second. If ρ and ρ' be the densities of the medium at AB and CD respectively

the equation of continuity, as it is called, is given by $V\rho = V'\rho'$...(1)

or $V' = \frac{V.\rho}{\rho'}$ ... (1*a*)

Now, the momentum gained by this mass of the medium in passing through CD per second is $V'\rho' \times V'$ or $V'^2.\rho'$, and the momentum lost by it while passing through AB per second is equal to $V\rho \times V$ or $V^2.\rho$.

Therefore the change of momentum per second in the said region $= V'^2\rho' - V^2\rho$.

According to Newton's second law of motion this change of momentum per second, is equal to the external force acting on the layer of the medium inside ABDC. As the cross section of the rod is unity, the difference in pressures between the ends AB and CD gives the external force. Denoting the pressures at AB and CD by P and P′ respectively,

$$\text{we have } P - P' = V'^2\rho' - V^2\rho \quad \ldots \quad (2)$$

$$= \left(\frac{V.\rho}{\rho'}\right)^2 .\rho' - V^2.\rho. \quad \text{Putting value of } V' \text{ from (1a)}$$

$$P - P' = \frac{V^2.\rho^2}{\rho'} - V^2.\rho = V^2\rho\left(\frac{\rho}{\rho'} - 1\right) = V^2\rho.\frac{\rho-\rho'}{\rho'}$$

$$V^2 = \frac{P-P'}{\rho.\frac{\rho-\rho'}{\rho'}} = \frac{1}{\rho}.\frac{P-P'}{\frac{\rho-\rho'}{\rho'}} \qquad (3)$$

Suppose l and l' denote the lengths per unit mass at AB and CD respectively. Then since the cross-section is unity,

$$\text{specific volume at } AB = l \times 1 = \frac{1}{\rho} \text{ or } \rho = \frac{1}{l}$$

$$\ldots \quad \ldots \quad \ldots \quad CD = l' \times 1 = \frac{1}{\rho'} \text{ or } \rho' = \frac{1}{l'}$$

Substituting for ρ and ρ' in (3)

$$V^2 = \frac{1}{\rho}.\frac{P-P'}{\frac{\frac{1}{l}-\frac{1}{l'}}{\frac{1}{l'}}} = \frac{1}{\rho}.\frac{P-P'}{\frac{l'-l}{l}}$$

The quantity $\frac{P-P'}{(l'-l)/l}$ being the ratio of the longitudinal stress to the longitudinal strain is the Young's modulus of the material of the solid. Denoting this by Y, we have

$$V^2 = \frac{1}{\rho}.Y \quad \text{or} \quad V = \sqrt{\frac{Y}{\rho}}.$$

28. Velocity of sound in solids and liquids: The velocity of sound in solids and liquids can be calculated from the expression $V = \sqrt{\frac{E}{D}}$, where E is the elasticity, and D the density of the substance. The value for the velocity of sound in any medium does not depend separately on E and D but it depends on the ratio of E and D.

In the case of solids, the density is greater than that in liquids or gases but the elasticity (longitudinal elasticity or Young's Modulus in this case) being in the order of 10^{12} dynes per sq. cm. is much greater, the ratio $\frac{E}{D}$ is much greater and hence, the velocity of sound is greater in solids.

Liquids have greater density than gases and since the elasticity (Bulk Modulus) being in the order 10^{10} dynes per sq. cm. it is more than enough to make good the loss of velocity caused by the effect of greater density. Thus the velocity of sound in liquids depending on the factor $\frac{E}{D}$ is greater than that in gases.

Thus sound travels faster in solids and liquids than in gases and is therefore heard at greater distances in solids and liquids than in gases.

The adiabatic elasticity is practically the same for all gases at normal pressure. So the velocity of sound is greater in gases which have smaller densities.

29. Velocity of longitudinal waves through a gaseous medium: Let us consider a cylindrical tube of the gas (Fig. 12) of unit area of cross-section with its axis in the direction of propagation of the waves.

Let us suppose that the conditions are uniform at each instant across any plane perpendicular to the axis of the cylinder.

Let A and B be two such planes at distances x and $x+\delta x$ respectively from some fixed position to the left of A.

Let the axis of x be along the axis of the tube and let the longitudinal wave propagate along the tube in the direction of x and thereby displace the planes to different extents.

Let A′ and B′ be the displaced positions of the gas originally at A and B respectively.

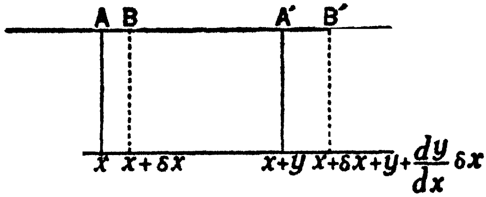


Fig. 12

Suppose y is the displacement of the plane A at the instant t measured from the same origin and along the same line as x, since the waves are longitudinal.

Then the actual position of A after displacement *i.e.*, of A′ $=x+y$.

Since y is the displacement in the direction of x, $\frac{dy}{dx}$ is the space rate of displacement in the direction of x. Hence, the change of displacement between the planes A and B δx apart $=\frac{dy}{dx}.\delta x$, so that the displacement of the plane B $=y+\frac{dy}{dx}.\delta x$.

Therefore the actual position of B after displacement *i.e.*, of B′ $=x+\delta x+y+\frac{dy}{dx}.\delta x$.

The distance apart of the planes A and B in the displaced positions A′ and B′ $=\delta x+\frac{dy}{dx}.\delta x$.

As the cross-section of the tube is unity, the volume of the gas enclosed between A′ and B′ is $\delta x+\frac{dy}{dx}.\delta x$, and volume of the between A and B is δx.

The change in volume of the gas enclosed is therefore

$$=\delta x+\frac{dy}{dx}.\delta x-\delta x=\frac{dy}{dx}.\delta x.$$

The fractional change in volume of the enclosed gas or the *volume strain* $=\dfrac{\frac{dy}{dx}.\delta x}{\delta x}=\dfrac{dy}{dx}$

Let p be the excess of pressure over the normal pressure, over the plane A, and K the bulk modulus of elasticity of the gas, we have $K = -\frac{p}{dy/dx}$ or $p = -K.\frac{dy}{dx}$ (1)

The negative sign is due to the fact that an increase of pressure causes a diminution in volume and *vice-versa*.

Then the excess of pressure (above normal) over the plane section B

$$= p + \frac{dp}{dx}.\delta x = -K\frac{dy}{dx} + \frac{d}{dx}\left(-K.\frac{dy}{dx}\right)\delta x = -K.\frac{dy}{dx} - K.\frac{d^2y}{dx^2}\delta x \quad \ldots \ (2)$$

So from (1) and (2), the difference of pressure over the plane sections A and B $= K.\frac{d^2y}{dx^2}.\delta x$.

This is the moving force on the enclosed gas.

If ρ be the density of the gas, then the mass of the moving gas is $\rho.\delta x$ since the cross-sectional area is unity.

Therefore by Newton's Second Law of motion ($P = mf$) we have

$K\frac{d^2y}{dx^2}\delta x = \rho.\,\delta x\frac{d^2y}{dt^2}$, where $\frac{d^2y}{dt^2}$ is the acceleration of motion.

$$\therefore \frac{d^2y}{dt^2} = \frac{K}{\rho}.\frac{d^2y}{dx^2} \quad \ldots \quad \ldots \quad (3)$$

Let us assume that in time dt the wave advances through the distance dx, then, $V = \frac{dx}{dt}$ or $Vdt = dx$

When y is a given function of x, $\frac{dy}{dx} = \frac{dy}{Vdt}$.

$$\frac{d^2y}{dx^2} = \left(\frac{d}{Vdt}\right)^2 y = \frac{d^2y}{V^2dt^2} \text{ i.e., } \frac{d^2y}{dt^2} = V^2\frac{d^2y}{dx^2} \quad (4)$$

Note: The above exp. can also be obtained from Art. 24 (a).

Thus from (3) and (4) we have $V^2 = \frac{K}{\rho}$ or $V = \sqrt{\frac{K}{\rho}}$

29a. Alternative Method: Suppose that a gaseous medium is contained in a tube (Fig. 13) of unit cross-section, and that the velocity with which the wave travels in the medium is V. Let the advance or the progress of the wave from left to right be opposed by moving the medium from right to left with the same velocity as the wave such that the waves in the medium cease to advance. Each portion of the medium as it is moved backwards will

successively acquire states of compression, normal state and rarefaction and the waves will appear to remain fixed in position on account of the backward motion of the medium. Let the direction of advance of the waves be parallel to the axis of the tube and let us consider two imaginary partitions A and B placed across the length of the tube such that when the waves are one of unchanged type *i.e.* fixed in position, the mass of the medium entering the tube through B in one second is equal to the mass passing out through A in the same time.

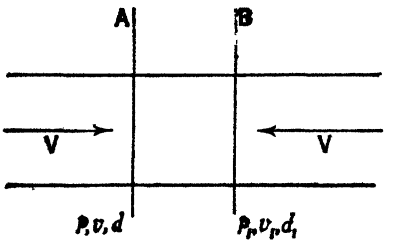


Fig. 13

Suppose for the sake of definiteness that the partition A is situated in a region where the gas has normal density and the partition B in a region of rarefaction where the pressure is less than that at A.

Let the pressure (force per unit area), the velocity of the particles at A and B with reference to the medium, and the density at A and B, be p, v, d, and p_1, v_1, d_1 respectively. Let p and d be respectively greater than p_1 and d_1. Here v and v_1, the velocities of the particles at A and B with reference to the medium, are the respective volumes of the medium which pass through the sections A and B in unit time, for the area of the cross-section of the tube is unity. Again since $d > d_1$, v is less than v_1.

Therefore the mass of the medium entering through B in one sec. is v_1d_1 and that passing out through A in the same time is vd.

Since the pressure and hence the density at any point within the region enclosed between the partitions A and B remain unaltered, the mass of the medium contained in this region also remains the same.

Therefore $vd = v_1d_1$ (1)

Again, since the velocity at B is greater than that at A, the gas must have gained momentum between A and B and by Newton's Second Law of motion, the change of momentum per second of the layer of gas between A and B at any instant is equal to the force *i.e.*, the difference of pressures at the partitions A and B on the same mass of gas at the same instant. We have, therefore, the net gain of momentum per second in the region between A and B $= v_1d_1.v_1 - vd.v = v_1^2d_1 - v^2d$.

$$\therefore \quad p - p_1 = v_1{}^2 d_1 - v^2 d = \frac{v^2 d^2 d_1}{d_1{}^2} - v^2 d$$

$$= \frac{v^2 d^2}{d_1} - v^2 d \text{ [from (1)], or } p - p_1 = v^2 d\left(\frac{d}{d_1} - 1\right) \quad \ldots \quad (2)$$

Here the velocity of the particles v at A with reference to the medium becomes equal to V, the velocity of the medium *i.e.*, of the wave, since the velocity of the particles at A is zero, A being at normal pressure. So the expression (2) becomes

$$p - p_1 = V^2 d\left(\frac{d}{d_1} - 1\right) = V^2 . d\left(\frac{u_1}{u} - 1\right) \quad (3)$$

where u and u_1 are the volumes of unit mass or the reciprocal of the densities, d and d_1.

Therefore $$V^2 = \frac{1}{d} \cdot \frac{p - p_1}{u_1 - u} \quad \ldots \quad \ldots \quad (4)$$

From the definition of the coefficient of volume elasticity *i.e.*, bulk modulus K,

we have $$K = \frac{p - p_1}{\frac{u_1 - u}{u}}$$

Therefore from (4) we have $V^2 = \frac{K}{d}$ and $= \sqrt{\frac{K}{d}}$

30. Newton's Calculation of the Velocity of Sound :

Newton first used the expression $V = \sqrt{\frac{K}{d}}$ to calculate the velocity of sound in air. In calculating the velocity he assumed that during the passage of sound waves in air, the compression and rarefaction were effected under *isothermal conditions* and so from Boyle's Law he deduced that the modulus of elasticity E of air was measured by the pressure P of air. (See General Physics Art. 110a)

Thus according to Newton the equation for the velocity of sound in air becomes $V = \sqrt{\frac{P}{d}}$

If v be the volume of unit mass, *i.e.*, the specific volume, we have $v = \frac{1}{d}$ and therefore the expression for velocity becomes

$$V = \sqrt{Pv}$$

The velocity of sound as calculated by Newton by the above formula has been found to be less by about $\frac{1}{6}$th of the theoretical value and he tried to explain this discrepancy by assuming that linear distance travelled over by sound was occupied by the molecules which were incompressible and that the formula applied only to the inter-spaces, the sound passing instantly through the molecules.

31. Laplace's Correction : Laplace pointed out that condensations and rarefactions produced by the passage of sound waves in air do not take place under isothermal conditions as assumed by Newton, but they really occur so rapidly in the medium that the heat generated during compression or the heat lost during rarefaction is not allowed for.

Under these conditions Boyle's law does not hold and therefore K cannot be put equal to P.

The changes which now take place in the medium are called *adiabatic changes* and the relation between pressure and volume under this condition is given by the relation,

$$PV^{\gamma} = \text{constant}$$

where P is the pressure, V the corresponding volume, and γ the ratio of the sp. heat of gas at constant pressure to that at constant volume, which in the case of air is 1·41.

From this condition Laplace deduced that K is not equal to P but to γP. (See General Physics Art. 111a).

So the velocity of sound is finally given by $V = \sqrt{\frac{\gamma P}{d}}$

The velocity of sound calculated from this formula agrees with the observed value and this is therefore the true expression for the velocity of sound.

Note : Velocity of sound is independent of the wave-length and is therefore the same for all frequencies.

32. Show that $V = \sqrt{gH}$

we know that $V = \sqrt{\frac{P}{d}}$ taking K = P

But $P = gdH$ where H is the height of the homogeneous atmosphere, and g, the acceleration due to gravity. Therefore,

$V = \sqrt{\frac{gdH}{d}} = \sqrt{gH} = \sqrt{\frac{2gH}{2}}$ = velocity acquired by a body falling in vacuo through half the height of the homogeneous atmosphere.

33. Effect of Pressure on the Velocity of sound : If the temperature of the medium be not altered, then according to Boyle's law $PV = P'V'$ or $\frac{P}{d} = \frac{P'}{d'}$ = a constant, for unit mass of any

medium. So the pressure and the density change in the same ratio and therefore the velocity of sound as given by

$V=\sqrt{\frac{\gamma P}{d}}$ remains the same, for a change in the numerator causes a proportional change in the denominator and therefore the *Velocity of sound is independent of any change of pressure in the medium.*

34. Effect of temperature on the Velocity of sound: The change of the temperature of the atmosphere does not alter its pressure much but causes a change in its density. Therefore in the expression

$V=\sqrt{\frac{\gamma P}{d}}$, the numerator is not affected but the denominator is altered with the change of temperature. Thus we have,

$$\left.\begin{array}{l} V_o=\sqrt{\frac{\gamma P}{D_o}} \\ V_t=\sqrt{\frac{\gamma P}{D_t}} \end{array}\right\}$$

where D_o and D_t are the densities and V_o and V_t, the velocities at 0°C and t°C respectively.

$$\therefore\ \frac{V_t}{V_o}=\sqrt{\frac{Do}{Dt}}$$

But since $D_o=Dt(1+\alpha t)$ where α is the coeff. of expansion of air at constant pressure we have $\frac{V_t}{Vo}=\sqrt{(1+\alpha t)}$,

or $\quad V_t=V_o(1+\alpha t)^{\frac{1}{2}}$ or $V_t=Vo\left(1+\frac{1}{2}\alpha t\right)=Vo(1+\cdot 0018t)$

where $\alpha=\cdot 0036$.

$V_t=Vo+60\cdot 5t$ in cms. as obtained by taking $V_o=33000$ cms. per sec.

35. Effect of Humidity: The presence of moisture reduces the density of air and hence the velocity of sound in moist air is greater than that in dry air.

36. Correction in the velocity of sound for the presence of moisture in the atmosphere: We know that,

$$\left.\begin{array}{l} V_{mt}=\sqrt{\frac{\gamma P}{D_{mt}}} \\ V_{dt}=\sqrt{\frac{\gamma P}{D_{dt}}} \end{array}\right\}$$

where V_{mt} and V_{dt} are the velocities of sound in moist and dry air respectively, and Dmt and Ddt are the densities of moist

$\therefore \quad V_{dt} = V_{mt}\sqrt{\dfrac{D_{mt}}{D_{dt}}}$ } and dry air respectively, at a temp. t°C and pressure P.

Consider the air to be saturated with moisture. Then 1 c.c. of moist air at pressure P and temp. t°C contains 1 c.c of dry air at pressure P − p and 1 c.c. of moisture at pressure p, and temp. t°C. Therefore the mass of 1 c.c. of moist air at pressure P and temp. t°C is equal to the sum of the masses of 1 c.c. of dry air at pressure P − p at t°C and 1 c.c. of moisture at pressure p and temp. t°C.

We know that 1 c.c. of dry air at N. T. P. weighs ·001293 gram.

$\therefore$ The mass of 1 c.c. of dry air at P − p at t°C

$$= \frac{\cdot 001293 \times (P-p) \times 273}{(273+t)760} \text{ s}$$

1 c.c of dry air at P − p at t°C occupies $\dfrac{(P-p)273}{760 \times (273+t)}$ c.c. at N. T. P.

Similarly the mass of 1 c.c. moisture at pressure p and temp. t°C is ·622 × mass of dry air at pressure p and temp t°C., i.e.

$$= \frac{\cdot 622 \times 001293 \times p \times 273}{(273+t)760}$$

$\therefore$ The mass of 1 c.c. of moist air at pressure P and temp t°C

$$i.e., \quad D_{mt} = \frac{\cdot 001293 \times (P-p)273}{(273+t)760} + \frac{\cdot 622 \times \cdot 001293 \times p \times 273}{(273+t)760}$$

$$D_{mt} = \frac{\cdot 001293 \times 273}{(273+t)} \{P - p + \cdot 622p\}$$

$$= \frac{\cdot 001293 \times 273}{(273+t)760} \{P - p(1 - \cdot 622)\} = \frac{\cdot 001293 \times 273}{(273+t)760} \{P - \cdot 378\}$$

Again since the density of dry air at pressure P and temp. t°C is equal to

$$D_{dt} = \frac{\cdot 001293 \times P \times 273}{(273+t)760} \quad \therefore \quad D_{mt} = D_{dt}\left\{1 - \cdot 378\frac{p}{P}\right\}$$

$$\therefore \quad \frac{D_{mt}}{D_{dt}} = 1 - \cdot 378\frac{p}{P}$$

$$\therefore \quad V_{dt} = V_{mt}\sqrt{\frac{D_{mt}}{D_{dt}}} = V_{mt}\left(1 - \cdot 378\frac{p}{P}\right)^{\frac{1}{2}}$$

where p is the vapour pressure and P, the pr. of atmosphere at t°C

$$= V_{mt}\left(1 - \cdot 189\frac{p}{P}\right)$$

The velocity of sound in dry air after correcting for moisture only, is given by V_{dt}. But if the temp. be reduced to 0°C then $V_{d0} + 60{\cdot}5t = V_{dt}$ in cms. per sec., where V_{d0} is the velocity of sound in dry air at 0°C $\therefore$ $V_{d0} = V_{dt} - 60{\cdot}5t$

But since $V_{dt} = V_{mt}\left(1 - \cdot189\frac{p}{P}\right)$

$$\therefore \quad V_{do} = V_{mt}\left(1 - \cdot189\frac{p}{P}\right) - 60\cdot5t \text{ in cms. per sec.}$$

37. Determination of Velocity of Sound in Air :

(1) Direct method : Numerous instances can be cited to show that sound takes appreciable time to travel from one place to another. The thunder clap is heard some time after the lightning is visible ; the tremendous sound of discharge when a gun is fired is also heard a few seconds after the flash is seen.

The two events, the lightning and the thunder clap or the sound of discharge and the flash of light take place simultaneously, but the apparent interval between the two events is due to the difference in the velocities of light and sound. Since light travels with a much greater velocity than sound, it affects us more quickly. Hence, to determine the velocity of sound we can neglect the time taken by the light and take the interval between the flash of light and the sound of discharge as the time taken by sound to travel from the source to the observer.

To determine the velocity of sound directly, a gun is fired at a station and the report is heard a few miles apart at another station. The interval T between the flash and the report *i.e.*, the time taken by sound to travel from one station to another is noted. If D be the distance between the two stations, then the velocity $V = \frac{D}{T}$.

This method suffers from several disadvantages and the result is inaccurate due to the following factors :—

(1) *Wind*, (2) *Temperature variation in the medium*, (3) *Humidity of the air*.

The effect of wind in increasing the velocity of sound when travelling in the direction of the wind and in diminishing it when travelling in the opposite direction, is eliminated by the method of *reciprocal firing i.e.*, by firing alternately from each station and the correct time is obtained from the mean of the times taken by sound to travel from one station to another in opposite directions.

To correct for the second, and third factors the temperature and pressure of air are observed at several points between the two stations and corrections made according to the following formulæ. For temperature correction, $V_t = V_o\sqrt{1 + \alpha t}$; for humidity correction, $V_{dt} = V_{mt}\sqrt{1 - \frac{\cdot378p}{P}}$, where p is aqueous tension at t°C.

The error known as the *personal equation* of the observer comes in, for the observer can not record the time at the exact moment when the flash is seen or the report is heard, as his brain is a little late in realising that the flash has occurred and his hand is later in making the record.

This difficulty is avoided by making the flash and the arrival of sound record themselves by some electrical arrangement. The instant of firing is observed on a revolving drum by causing the bullet to a break wire conveying an electric current placed across the muzzle of the gun when the style of the electromagnet ceases to make its usual mark on the drum. The instant of arrival of the sound at the receiving station is recorded on the revolving drum by the style of the electromagnet through which a current is passed as soon as the circuit is completed by a stretched receiving membrane when pushed forward by the arriving sound waves.

If the rate of revolution of the drum is known, the interval between the times when the style ceases to make its mark on the drum and when the mark is made gives the time taken by the sound to travel from the gun to the observer.

(II) Resonance column method : When a vibrating tuning fork is held over the mouth of a long tube containing a certain volume of air no sound is at first audible but if the column of air be adjusted to a suitable length by immersing a certain length of it in a jar containing water, the tube is found to speak and the intensity of the sound at this particular length becomes maximum. Thus intensification of the sound is produced by what is known as Resonance (*For fuller details see Article under Resonance*).

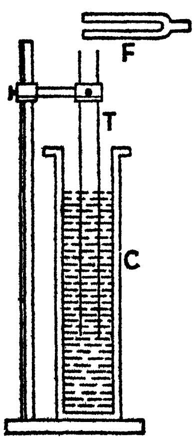


Fig. 14

Let a tuning F be made to vibrate and held at the mouth of a narrow cylindrical tube T (Fig. 14) dipped into water contained in the jar C. As the prong moves in the downward direction the air in front of it becomes compressed and a wave of compression moves down the tube, gets reflected at the surface of the water in the tube as a wave of compression and reaches the open end of the tube and is reflected there as a wave of rarefaction. This occurs just at the instant when the prong after reaching its extreme downward limit moves up and sends a rarefied wave down the tube. Here resonance occurs for the vibration of fork agrees with the vibration of the air inside the tube.

Again the rarefied wave formed at the open end of the tube travels down, gets reflected at the closed end as a wave of rarefaction and reaches the open end of the tube and reflected again as a wave of compression just at the time when the prong has

come back to its original position and is on the point of starting in the downward direction producing a wave of compression.

Thus we see that when the length of the column of air inside the tube is suitably adjusted the motion of the air is helped by the motion of the prong at every instant and that the period of vibration of the fork has become equal to the natural period of the air particles. So resonance takes place and at this stage due to the increase in the amplitude of the vibrating particles the intensity of the sound is maximum.

Thus we see that during the time the fork makes one complete vibration the wave travels over four times the length of the tube. Then if L_1 be length of the column of air and λ be the wave length,

$$\text{we have,} \quad L_1 = \lambda/4$$

The above can be found in another way. In first resonance stationary wave is produced having only a node at water surface and an antinode at the open end. The distance between a node and an antinode is $\lambda/4$. Hence $L_1 = \lambda/4$.

End Correction : The reflection of the waves at the open end of the tube does not cease exactly at the open end but a little distance beyond it. So a correction x, known as the end correction is to be added to the length L_1. The correction x has been found to be equal to $\cdot 6r$ where r is the radius of the tube.

$$\text{Then} \quad L_1 + x = \lambda/4 \qquad \ldots \qquad \ldots \qquad (1)$$

If the length of the air column be different from that required for resonance the fork and the wave would not agree at any instant, sometimes assisting and sometimes opposing each other and therefore there would be no resonance.

Now if the length of the air column be increased gradually a position would be reached at which a second resonance will take place. At this stage the compressed wave sent down the tube and reflected from the water contained in the tube will reach the open end just at the moment when the prong moves in the upward direction from its extreme lower limit after executing one complete and a half vibration.

If T be the periodic time of vibration of the prong or of the air particles when the second resonance takes place, then in time $3T/2$ the wave travels over twice the length of the air column of length L_2. Therefore the wave length λ or the distance travelled in time T by the wave is $4L_2/3$ *i.e.* $L_2 = 3\lambda/4$.

Alternately, in 2nd resonance, there are two antinodes and two nodes in the stationary wave set up in the air column, water surface being a node. But distance between a node and second antinode is $3\lambda/4$. Hence $L_2 = 3\lambda/4$.

Therefore, the true length of the air column considering the end correction $= L_2 + x = 3\frac{\lambda}{4}$ ... ... (2)

Subtracting equation (1) from equation (2)

$$L_2 - L_1 = \frac{\lambda}{2} \text{ or } \lambda = 2(L_2 - L_1)$$

But $V = n\lambda$ where V is the velocity of sound, n the frequency of the sound (Same as that for the fork).

$\therefore$ $V = 2n(L_2 - L_1)$ whence V can be found out taking the known value of n.

The value of the end correction x is obtained from equations (1) and (2).
From the equations (1) and (2) we have $3(L_1 + x) = L_2 + x$
$\therefore$ $2x = L_2 - 3L_1$ or $x = \frac{1}{2}(L_2 - 3L_1)$
The value of x is found to be near about ·6r where r is the radius of the tube.

(III) Determination of Velocity by Kundt's Tube Method: The longitudinal vibration of a rod is utilised to set up stationary vibration in a column of air enclosed in a tube, the frequency of vibration of the air particles being the same as that of the rod. Based on this, the velocity of longitudinal waves in a gas or solid can be found when one of the two is known.

The apparatus consists (Fig. 15) of a wide, long glass tube AB placed horizontally on suitable supports. One end of the tube is closed by a lightly fitted piston P_1. While the other end is provided with a loose piston P_2 attached to one end of a long

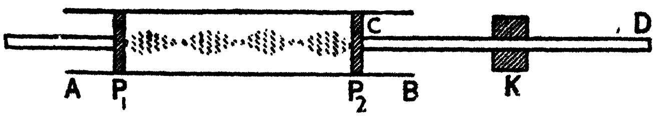


Fig. 15

metal or glass rod CD firmly clamped at the middle point of it, having its axis in the same line with the axis of the tube. The clamped rod is called the *sounding rod*, and the glass tube the *wave tube*.

Before introducing the piston P_2 inside the tube, the tube is throughly dried by passing a current of hot air through it and then some dry lycopodium powder or cork dust is spread in thin layers along the length of the tube.

For performing the experiment, the rod is rubbed with a moist cloth if the rod be of glass or with a leather dusted with resin if

the rod be metal. The length of the air column enclosed in AB is adjusted by pushing in or pulling out the piston P_1 until a distinct loud note is heard. Longitudinal vibration developed in the rod by rubbing it sets the enclosed air into resonant vibration. Longitudinal waves emanating from the piston P_2 which also vibrates with the rod, travel through enclosed air and become reflected by the piston P_1. Then due to superposition of the direct and the reflected waves, **stationary waves** having fixed *nodes* and *antinodes* are produced inside the tube. The particles of lycopodium which become strongly agitated, move from antinodes, the regions of maximum displacement of air particles to the nodes, the regions of minimum displacement, and accumulate in heaps near the nodes. The distances between two successive nodes are measured and their mean is taken. If this distance be l_1, the wave length of sound in air is $2l_1$. Again, the sounding rod is vibrating in its fundamental note with antinode at each end and a node at the middle. Hence, wave-length of sound in the rod is $2l_2$, where l_2 is the length of the rod. Now as the rod and the air column are in resonant vibration, the frequency of the sound wave in the rod is the same as that in the air column. Let the common frequency be n. If V_1 and V_2 be velocity of sound wave in air and in the rod respectively, then by relation $V = n\lambda$, we have

$$V_1 = n.2l_1 \; ; \quad V_2 = n.2l_2 \qquad \therefore \; \frac{V_1}{V_2} = \frac{l_1}{l_2}.$$

Here l_1 and l_2 are obtained by actual measurement and if V_1 the velocity of sound in air be known, then V_2 the velocity of sound in the rod may be found out and vice versa.

The method can be utilised for finding the velocity of sound V_g in a gas by replacing the air by the given gas and determining the distance "l_g" between successive nodes, when resonance occurs, with the rod under same conditions.

For air $V_1 = n.2l_1$

For gas $V_g = n.2l_g$, n being same in both the cases

$\therefore \; \dfrac{V_g}{V_1} = \dfrac{l_g}{l_1}$ knowing l_g, l_1 the ratio V_g/V_1 gives the comparison required. Knowing V_1, V_g can be found out.

For accurate result some precautions are necessary in Kundt's tube experiments, namely (i) the rod should be clamped exactly at its centre; (ii) the piston P_1 should be adjusted carefully to obtain resonance; (iii) to get well defined dust figures at nodes, the tube and the lycopodium powder or cork dust should be perfectly dry, and too much dust must not be taken.

Kundt's expt. is not well suited for determining the absolute velocity of sound in a gas as we require in this case the frequency of the note emitted with accuracy. It is commonly used for comparing the velocities in different gases.

In the earlier form of his apparatus Kundt used only one tube and clamped the rod in the middle part, but in the later form, he used two tubes fitted at their inner ends with caps through which pass the ends of a glass rod of which one-quarter of its length is inside each of the tubes, the outer ends of the tubes being provided with adjustible pistons. The sounding rod was excited by rubbing near the middle. In this way he was able to keep the gases under investigation at the same temperature as was not possible with the earlier form, and moreover in the later form he was certain that the notes were of the same pitch in the gases contained in the tubes for any change in temperature.

(IV) By Methods of Echoes : The velocity of sound in air is roughly determined by using a metronome which beats loudly but slowly. The metronome is placed in front of a reflecting surface and gradually moved away until the ticks coincide with the echoes of the immediately preceding ticks. The interval between successive ticks being known, the velocity of sound V in air is determined by the expression $V = 2d/t$, where d is the distance of the wall from the metronome and t, the interval between two successive ticks or rather the time taken by the sound to move from the metronome to the reflecting surface and back to the metronome.

38. Determination of Young's Modulus or Longitudinal Elasticity of a rod :

From the expression $V_R = \sqrt{\frac{E}{D}} = \sqrt{\frac{Y}{D}}$

where V_R is the velocity of sound in the rod, and D the density of the material of the rod. E and Y are respectively the longitudinal elasticity and the Young's modulus of the rod.

$$\therefore \text{ E or } Y = V_R^2 . D$$

Thus the longitudinal elasticity or Young's modulus of the rod is determined at the given temperature at which the velocity of sound in the rod is obtained by Kundt's tube experiment.

39. Determination of γ : We know that $V = \sqrt{\frac{\gamma P}{D}}$,

where V is the velocity of sound in air, P the pressure of air in the tube, D the density of air, and γ the ratio of the sp. heat of air at constant pressure to the sp. heat of air at constant volume.

If in the above expression V, P and D be known, γ is easily found out. The determination of γ lends itself to a study of the number of atoms per molecule in gases *i.e.*, gives valuable information about the constitution of the molecules of a gas.

40. Velocity of Sound in Solids : Biot's experiment : Biot took a long chain of cast-iron pipes to one end of which a bell was mounted. The bell was struck by a hammer and two sounds, the first through the iron and the second through the air inside the pipes, were heard. The time interval between these two sounds gives a measure of the velocity of sound in iron in terms of the velocity of sound in air.

Let V_a and V_i be respectively the velocities of sound in air and in iron and let l be the length of the cast-iron pipe.

Then the time taken by sound to travel through the air in the pipe $=\dfrac{l}{V_a}$ and that through the iron in the tube $=\dfrac{l}{V_i}$

Therefore the time interval t between the two sounds is given by $t=\dfrac{l}{V_a}-\dfrac{l}{V_i}$, since the velocity of sound in iron is much greater than that in air.

The total pipe length was 950 metres and the velocity (V_i) determined was about 3500 metres per sec.

QUESTIONS

1. Find an expression for the velocity of propagation of longitudinal vibrations in a solid. [C. U. 1623, '44, '54]

2. Find a general expression for the velocity of sound in a gas and discuss the formula due to Newton and Laplace [C. U. 1930, '38, '42, '47, '50, '58]

3. Shew how the velocity of sound in a gas is affected by (*a*) wind, (*b*) temperature, (*c*) pressure and humidity. [C. U. 1925, '26, '27, '38, '47]

Find by theory of dimensions the expression for the velocity of sound in air. [C. U. 1924]

4. Give an account of the classic determination of the velocity of sound in free air and state the results obtained. [C. U. 1949]

Mention the different factors or sources of error that influence the velocity of sound as determined by open air observations and explain how they may be corrected in order to obtain an absolute value. [C. U. 1921, '45]

5. Describe in detail an experimental arrangement for the determination of the velocity of sound in gases at different temperatures and pressures. [C. U. 1917, '36, '55]

6. Explain how the velocity of sound in a gas can be measured with the help of a Kundt's tube. [C. U. 1958]

7. Describe some method of finding the coefficient of longitudinal elasticity by an acoustical experiment. [C. U. 1923, '44]

8. Describe an experimental arrangement by which the result can be utilised to measure the young's modulus of the material of solid rod. [C. U. 1954]

9. How can the ratio of the sp. heat of a gas at constant pressure to that at constant volume be determined from the velocity of sound ? [C. U. 1955]

EXAMPLES

1. Find the velocity of sound at any moment from the following data.

Pressure of the atmosphere = that of 760 mm. of mercury. Ratio of the Sp. heats = 1·41. Temp. of the air = 10°C.

A sound is emitted by a source placed at one end of a long iron tube and two sounds are heard at the other end at an interval of 2·5 secs. If the length of the tube is 951·25 metres, find the velocity of sound in iron. [C. U. 1914]

We know that $V = \sqrt{\frac{\gamma p}{D}}$.

. The velocity of sound at 0°C and 760 mm. pressure

$= \sqrt{\frac{1·41 \times 76 \times 13·6 \times 981}{·001293}}$ cms. per sec. = 332·5 metres per sec.

∴ The velocity of sound at 10°C and 760 m.m. pressure $= V_0(1 + \frac{1}{2}\alpha t)$.

$= 332·5(1 + \frac{1}{2} \times \frac{1}{273} \times 10)$ since $\alpha = \frac{1}{273} = ·0036$

= 338·6 metres per sec.

Let V be the velocity of sound in iron expressed in metres per sec. Then the time taken by the sound travel 951·25 metres along the iron tube is $\frac{951·25}{V}$ sec. Similarly the time taken by the sound to travel through the same distance in air $= \frac{951·25}{338·6}$, where velocity of sound in air = 338·6 metres per sec. (approx.)

Since the velocity of sound in solids is greater than that in air, the time taken by the sound to travel through the iron in the tube is smaller than that when it travels through the air inside the tube.

$$\therefore \ 2·5 = \frac{951·25}{338·6} - \frac{951·25}{V}; \ \therefore \ V = 3076 \text{ metres per sec. (approx).}$$

2. A sound emitted by a source placed at one end of an iron tube one kilometre long, and two sounds are heard at the other end at an interval of 2·8 secs. If the velocity of sound in air in the condition of the experiment is 330 meters per second, find that for iron.

[Ans. 363·6 metres per sec.] [C. U. 1942]

3. A litre of hydrogen at normal temperature and pressure weighs 0·0896 gm. Find the velocity of sound in hydrogen at a temperature of 16°C when the pressure is 750 mm., the ratio of the Sp. heats of Hydrogen being 1·4. [Density of mercury = 13·6, g = 980 cms./sec².] [C. U. 1915]

We know that the velocity of sound in any medium is independent of any change of pressure in the medium but changes with the alteration of temperature.

Therefore the velocity of sound in H at 0°C and 760 mm. = the velocity of sound in H at 0°C and 750 mm. is given by

$V = \sqrt{\frac{1·41 \times 13·6 \times 980 \times 76}{·0000896}}$ cm. per sec. since the density of hydrogen

$= \frac{·0896}{1000}$ or ·0000896 gm. per. c.c. ∴ V = 1269 metres per sec.

$\therefore$ The velocity of sound in H_2 at 16°C and 750 mm.

$= V_0\sqrt{1+\alpha t} = 1263(1+\frac{1}{2}\times \cdot 0036\times 16) = 1299 \cdot 37$ meters per sec.

4. Young's modulus for steel is 210×10^{10} and the sp. gravity is 7·8. Find the velocity of propagation of sound through a steel bar. [C. U. 1916]

In solid, the velocity of sound V is expressed by $V=\sqrt{\frac{E}{D}}$, where E is the Young's modulus for the material of the bar, and D its density.

$\therefore \quad V=\sqrt{\frac{210\times 10^{10}}{7 \cdot 8}}$ cm. per sec. $=5 \cdot 18\times 10^5$ cm. per sec. approx.

5. Determine the velocity of sound in air at N. T. P. and deduce the change n velocity per centigrade degree rise in temperature if $\alpha = \cdot 00367$. [C. U. 1917]

As before $V= \sqrt{\frac{\gamma P}{D}} = \sqrt{\frac{1 \cdot 41\times 76\times 13 \cdot 6\times 981}{\cdot 001293}}$ cm. per sec.

$=332$ meters per sec. Then $V_t = V_0\sqrt{(1+\alpha t)} = V_0(1+\frac{1}{2}\alpha t)$

$=332 \cdot 5(1+\frac{1}{2}\times \cdot 00367t) = (332 \cdot 5+\cdot 61t)$ metres per sec.

$\therefore$ the change of velocity per degree centigrade $= \cdot 61$ metres per sec.

6. Assuming that the velocity of sound in air at N. T. P. is 1090 feet per sec., find its value at 50° and 70 cm. pressure.

[*Ans.* 1185 ft. per. sec.] [C. U. 1938, '58]

7. Calculate the velocity of sound at N. T. P. (The ratio of the sp. heats for air = 1·41) [*Ans.* 332 metres per sec.] [C. U. 1947]

8. The velocity of sound in air at 30°C and saturated with aqueous vapour is found to be 350 meters per second. Calculate the velocity in dry air at N. T. P.

Mass of 1 c.c. of dry air at N. T. P. = ·001293 gms.; Density of aqueous vapour = ·62; Barometric height (corrected) = 760 mm.; Maximum pressure of vapour at 30°C = 31·5 mm. [C. U. 1920]

We know that $V_{Do} = V_{mt}\sqrt{\frac{D_{mt}}{D_o}}$ where V_{Do} is the velocity of sound in dry air at N. T. P. and V_{mt}, the velocity in moist air at 30°C.

D_{mt}, the density of moist air at 30° and 760 mm. and Do, the density of dry air at N. T. P.

D_{mt} = the mass of 1 c.c. of aq. vapour at 30° and 31·5 m.m. + the mass of 1 c. c. of dry air at 30°C and (760 − 31·5) mm. or 728·5 mm.

The mass of 1 c.c. of aq. vapour at 30°C and 31·5 mm.

= ·62 × mass of 1 c.c. of dry air at 30°C and 31·5 m.m.

$$= \cdot 62\times \frac{1\times 31 \cdot 5}{273+30}\times \frac{273}{760}\times \cdot 001293$$

Mass 1 c.c. of dry air at 30° and 728·5 mm. $= \frac{1\times 728 \cdot 5}{273+30}\times \frac{273}{760}\times \cdot 001293$

$$D_{mt} = \frac{\cdot 62\times 31 \cdot 5\times 273\times \cdot 001293}{303\times 760} + \frac{728 \cdot 5\times 273\times \cdot 001293}{303\times 760}$$

$$= \frac{273\times \cdot 001293}{303\times 760}\{\cdot 62\times 31 \cdot 5+728 \cdot 5\}$$

$$V_{Do} = \sqrt{\frac{D_{mt}}{\cdot 001293}} = 350\sqrt{\frac{273}{303\times 760}(\cdot 62\times 31 \cdot 5+728 \cdot 5)}$$

= 329·59 metres per sec.

9. Calculate the velocity of sound in dry air at 0°C and 760 mm. from the following data, supposing the density of moist air inside the tube to be ·00120 gm. per c.c. [C. U. 1922]

Length of the column of air for the 1st max	= 33 cm.
" " " " 2nd...	= 101·6 cms.
Temp. of the air inside the tube	= 30°C
Barometric height (corrected)	= 760 mm.
Frequency of the fork	= 256.

For 1st and 2nd max., $(33+x)=\frac{\lambda}{4}$; $(101·6+x)=\frac{3\lambda}{4}$, $\therefore \lambda=2(101·6-33)$ cm.

$=137·2$ cms ; $\therefore$ $V_{mt}=n\lambda=256\times137·2$ cm. $=351·2$ metres per se

Since $V_{Do}=V_{mt}\sqrt{\frac{D_{mt}}{Do}}$, $V_{Do}=351·2\sqrt{\frac{·0012}{·001293}}=338·38$ metres per sec. (approx)

where V_{Do} is the velocity of sound at N. T. P. and V_{mt}, that in moist air at 30°C.

10. Give a brief account of Kundt's experiment. If the length of the rod is 1 metre, the density of the material 8 grammes per c.c. and its Young's modulus $7·2\times10^8$ gms. per sq. cm., find the distance between the heaps in the Kundt's tube when it is filled with CO_2 at 25°C.

[Velocity of sound in CO_2 at t°C $=260+·478t$ (metres/sec.)] [C. U. 1917]

Velocity of sound in a solid rod $=\sqrt{\frac{E}{D}}$ where E = Young's modulus and D, the density of the material.

Young's modulus of the given material $=7·2\times10^8\times980$ dynes per sq. cm.

$\therefore$ Velocity of sound in the rod $=\sqrt{\frac{7·2\times10^8\times980}{8}}$ cm. per sec.

$=3\times9·9\times10^4$ cm. per sec. nearly $=297000$ cm. per sec.

Velocity of sound in CO_2 at 25°C $=(260+·478\times25)$ metres per sec.

$=27195$ cms. per sec.

Now in Kund't tube experiment, the two ends of the rod are antinodes with a node in the middle.

$\therefore$ The wave length in the rod = twice the length of the rod = 200 cms.

Let l be the length between the two consecutive heaps in the tube. Hence the length of the wave in $CO_2=2l$. Let n be the frequency of the waves.

Then, $297000=n\times200$ and $27195=n\times2l$.

$\therefore$ $\frac{l}{100}=\frac{27195}{297000}$ $\therefore$ $l=\frac{27195\times100}{297000}=9·2$ cm.

11. If the velocity of sound through Hydrogen at 0°C is 4200 feet per sec., what will be the velocity of sound at the same temperature through a mixture of two parts by volume of hydrogen to one of oxygen ? (The density of Oxygen is sixteen times that of Hydrogen). [C. U. 1928]

Density of the mixture $D=\frac{v_1d_1+v_2d_2}{v_1+v_2}$

Here v_1, v_2 are the respective volumes of H_2 and O_2 d_1 and d_1 are the respective densities of H_2 and O_2.

Thus, $D=\frac{2\times1+1\times16}{2+1}$ Then proceed as usual. [*Ans.* V = 1714 ft/sec.]

CHAPTER IV

REFLECTION AND REFRACTION OF SOUND WAVES

41. Reflection of Sound: Like light waves, sound waves are reflected when they meet an obstacle or the surface of separation of two different media. The side of a hill, the wall of a building, thick bushes or rows of trees serve as reflectors of sound.

The laws of reflection of sound are identically same as the laws of reflection of light, namely, the angle of incidence is equal to the angle of reflection and they lie in the same plane. As the wave length of sound is in general large, a large extended surface (not necessarily polished) is required for the reflection of the sound waves. The wave length of light being very small, a small polished surface is capable of producing reflection of light. Huyghen's principles of wave theory of light involving idea of secondary wavelets apply also in respect of sound waves.

41a. Experiments (Illustrative) :—

(1) In Fig. 16, sound wave proceeding along the axis of the tube WS falls on a flat plane surface AB and the reflected sound wave travelling along the axis of the tube SL is heard the loudest. The axis of the two tubes are found to be equally inclined to the reflecting surface, and hence the law of equality of angles of incidence and reflection verified.

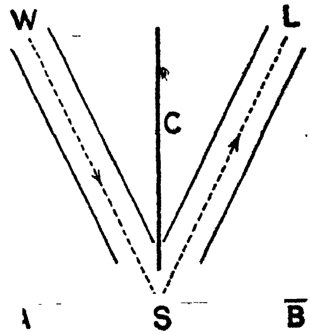


Fig. 16

(2) Two concave (Fig. 17) spherical surfaces A and B of very large aperture are mounted co-axially so that their concave sides face each other. A watch is placed at the principal focus C of one of the surfaces, say B. The sound of the watch being reflected by the concave surface B according to the optical laws of reflection, falls upon the second surface A. The sound is again reflected by the second surface and is brought to its principal focus D. In order to receive the reflected sound a funnel connected to a rubber tube is placed on the common axis near the concave surface A, the free end of the rubber tube being placed near to the ear.

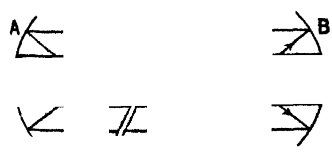


Fig. 17

The sound of the watch is found to be most loudly heard when

the funnel has been adjusted at the principal focus D of the concave surface A.

42. Echoes : Echoes are the repetition of a sound in air caused by its reflection from more or less distant obstacles. Echoes are generally heard when a person speaks aloud near a pond, a wall or huge buildings and mountains and will only be distinct when the distance between the observer and the reflecting surface is sufficiently big to allow the reflected sound to reach him without interfering or overlapping with the direct sound.

It has been noticed that for articulate sounds, the distance of the reflecting surface from the observer should be 112 ft., so as to produce a distinct echo of a monosyllable. A man generally takes $\frac{1}{5}$th of a second to utter a single syllable and during that time the sound moves onwards, gets reflected and travels through 224 ft. which is $\frac{1}{5}$th of 1120 ft., the velocity of sound in air per sec. This distance is again twice the distance between the observer and the obstacle. The reflected sound reaches him as he is about to utter the second syllable.

For inarticulate sounds, the minimum distance of an obstacle from the observer for a distinct echo should be 56 ft. for the sensation of sound remains in our brain for $\frac{1}{10}$th of a second. So to get a distinct echo the sound should travel onward, get reflected from the obstacle and come back to the source after covering a total distance of 120/10 or 112 ft.

Thus the distance between the source and the obstacle should be $=\frac{112}{2}=56$ ft.

Note :—An echo to be audible must be produced by reflection at a surface having dimensions comparable with the wave lengths of the incident sound, since otherwise sound energy becomes dispersed in all directions when intercepted by the reflecting surface. The returned echo therefore becomes inaudible. A cliff or high wall is suitable for the formation of good echo of the sound of a gun, whereas a smaller surface is enough to give a distinct echo of the shrill sound of a whistle.

When a sound is reflected from a number of suitably placed reflecting surfaces a number of echoes may be heard. The rumbling and rolling of thunder is probably the echoing of the peal of thunder caused by reflection from cloud surfaces, rocks and high cliffs.

Successive reflections may also be produced between two reflecting surfaces suitably placed.

The principle of echo has been successfully applied for the measurement of ocean depth, altitude of the aircraft from the ground and for determining the depths of geological strata containing minerals or oil below the earth's surface.

42a. Harmonic Echo : In Building accoustics the reflection of sound plays a very important part. It is a fact that reflection of sounc

depends on an important factor of the reflecting surface, which is its dimensions. The echo of a complex note (which is a mixture of sounds of different wave lengths), returned by a reflecting surface may not be essentially an exact reproduction of the original incident sound, since the reflector can reflect certain wavelengths better than others. The reflector having small dimensions reflects sounds of larger frequencies better than the smaller ones. Cases in which upper frequencies of a complex note predominated in the echo were noticed by Rayleigh who called this echo "*harmonic echo*" since the pitch of the reflected sound was apparently raised.

42b. Musical Echo : When the reflecting surface is made up of a number of obstacles arranged at regular intervals, the echo consists of a number of reproductions of the incident sound received by the ear at regular intervals of time. Wooden fences the palings of which are arranged at equal distances apart as in echelon formation, serve as such a reflector. A clap of hand is reflected from such a reflector as succession of claps at regular intervals and is perceived by the ear as a short musical note, called *musical echo.*

42c. Whispering Gallery : An interesting illustration of the reflection of sound is found in the case of whispering gallery. In the circular gallery inside the hemispherical dome of St. Paul's Cathedral in London, a whisper made near any point of the circular wall is reflected round the wall of the gallery so that the sound can be distinctly heard at any other part of the gallery, if the listener keeps his ear near to the wall. The whisper however will not be heard at any point remote from the wall. Rayleigh gave a theory regarding the phenomenon, which was confirmed by experiments of Raman and Sutherland in 1921. It appears however that a complete and conclusive explanation of the phenomenon is yet to be given.

43. Depth sounding by Echo : The measurement of the depth of the sea is called the **depth sounding.** The production of echo caused by the reflection of sound from the sea-bed is utilised to determine the depth of the sea. For this, a hydrophone (under-water microphone receiver) is placed under water at some depth and an explosion is produced near it. Two sounds are heard, one coming directly through the hydrophone and the other coming a little after by reflection from the sea-bed.

The interval between the two sounds is determined by some special arrangement. Let T secs. be this interval. Then, in this interval the sound has travelled twice the depth of the sea. If D is the depth of the sea-bed and V, the velocity of sound in water $2D = VT$ or $D = \frac{1}{2}VT$, whence D can be found out.

44. Reflection on Rigid and Yielding Surface : When sound waves fall upon a rigid surface and reflected, there occurs no change of phase between the incident and the reflected waves. Thus when the sound waves are incident on the boundary surface of a second medium which is denser than the first, the incident pulse of compression is reflected as a pulse of compression, and similarly the incident pulse of rarefaction is reflected as rarefaction. In resonance column experiment reflection from water (rigid or denser) surface takes place without change of phase.

When again sound waves are reflected from an yielding surface, there is a change of phase so that reflected and incident waves are of opposite phases. In this case a pulse of compression is reflected as a pulse of rarefaction, and a pulse of rarefaction as a pulse of compression. In the resonance column experiment, the sound wave reflected from the surface of water travels up to the open end where it meets outside air of density less than that of inside air and becomes reflected with change of phase, the outside air behaving as an yielding surface.

45. Refraction of Sound : When sound waves travelling in one homogenous medium meets the surface of separation of another medium, it deviates to an appreciable extent from its rectilinear path. Thus sound waves are also refracted. Phenomena based on refraction of sound are less common than those of light since the most usual case of sound propagation, practically met with, occurs in air. But phenomena do occur as a result of refraction of sound from one region of the atmosphere to another of different density.

The refraction of sound at curved surface may be studied by using a bag of rubber or leather shaped into a convex lens and filled with some gaseous medium, say CO_2 gas. Sound travels more slowly in CO_2 than in air so that when each pulse of compression or rarefaction reaches the lens (Fig. 18), it advances more slowly at its centre than at the edges, so that when it emerges from the other side, it is concave instead of convex towards the direction in which it is advancing. The sound produced at S is thus brought to a focus at point M, by the lens.

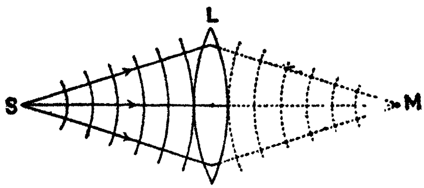


Fig. 18

45a. Effect of wind : It is well known that sound travelling with the wind is more clearly heard than when travelling against it. This is explained by a phenomenon similar to refraction.

The wind travels more slowly near the ground than higher up and consequently the upper portions of the wave-fronts will bend downward and help the observer to hear the sound more distinctly. If the wind blows in the opposite direction, the tilt of the wave-fronts will be in the opposite direction and the observer will not hear the sound as distinctly as he would if there were no wind.

A similar effect is observed when there is a gradual change of temperature of the air from the ground upwards.

We know that the velocity of sound becomes greater, the higher the temperature. So any variation of temperature causes the refraction of sound waves. The phenomenon of the bending of the sound rays is explained in a similar manner as before.

QUESTIONS

1. Explain the formation of an echo. Why are succession of echoes sometimes heard? [C. U. 1922, '28, '29, '49]

2. Explain the formation of an echo. Do you know any use of the phenomena in obtaining scientific data about distant unseen objects? [C. U. 1949]

EXAMPLES

1. You start walking away from a high wall clapping your hands once every second. How far must you go from the wall before you hear the echo of one clap simultaneously with the next clap? [C. U. 1928]
(Speed of sound in air 1120 ft. per sec.) *Ans.* 560 ft.

2. A gun is fired on the sea shore in front of a line of cliffs. A man standing 300 feet away from the gun and equidistant from the cliffs notices that the echo takes twice as long as to reach him as does the direct report. Find by calculation the distance of the gun from the cliffs. (Vel. of Sound in air = 1100 ft. per sec.) *Ans.* 259·8 ft. [C. U. 1929]

CHAPTER V

PITCH, LOUDNESS AND QUALITY

46. Characteristics of a musical sound : Musical sounds differ from one another in three essential particulars *viz.*, *Pitch*, *Loudness* and *Quality*.

46a. Pitch and Frequency : Pitch is a kind of sensation produced in the ear by the harmonic waves falling upon it. If the frequency of the waves be greater, the pitch will be higher, and if it be smaller the pitch will be lower. The strokes of a hammer at regular intervals will produce sensation of separate impulses but if the frequency of the impulses be increased the separate impluses, will blend together into a continuous musical sound and the pitch of the sound will rise. Thus we see that the *pitch of a note depends on the frequency of the impulses* or the rate of vibration of the sounding body.

Pitch is defined as that characteristic which distinguishes a sharp note from a flat one. If the frequencies of the two notes be different, different sensations will be produced in the ear and therefore this difference in the sensations determines the difference of pitch in the notes. The pitch of a sound depends on the frequency of vibration of the source of sound being directly proportional to it. If the frequency be increased the pitch is raised, if the frequency be decreased the pitch is lowered.

47. Determination of the pitch of a note :

(1) By Resonance Column Method. (2) By Seebeck's Siren (2a) By Cagniard de la Tour's Siren. (3) By Sensitive Flame Experiment. (4) By Melde's Experiment. (5) By Sonometer Method.

(1) Resonance Column Method : In Art. 37 (II) we have seen that $V=2n(L_2-L_1)$, where V is the velocity of sound in the air at the time of the experiment, n, the frequency of the tuning fork and (L_2-L_1), the difference between the lengths of the air column corresponding to the second and first resonances respectively. Knowing L_1, L_2 and V, n can be found out.

(2) Seebeck's Siren : It consists of a metal or cardboard disc D (Fig. 19) perforated with a number of equidistant holes along the circumference of the disc. This is mounted on a whirling table which can be rotated at any desired speed.

A current of air is blown through a jet T whose mouth is kept very close to the disc exactly opposite to one of the holes.

As the disc rotates, the current of air blown through the jet is alternately stopped, and allowed to pass through the holes producing a series of puffs at regular intervals. When the disc is quickly rotated, the puffs result in a musical note.

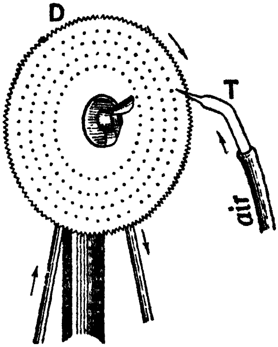


Fig. 19

The siren as well as the tuning fork of which the frequency is to be determined are sounded together. The rotation of the disc of the siren is so adjusted that the musical note produced by the siren is exactly in unison with that of the tuning fork. Thus if we know the number of puffs per second this number will be the frequency of the tuning fork.

Let m be the number of holes in the siren disc rotating at a speed of n revolutions per second.

Then the frequency of the fork = Frequency of sound emitted by the siren = $m \times n$.

The number of revolution of the siren disc is determined by a speed counter attached to the spindle of the whirling table.

Note : This method may be utilised to determine the **frequency of a note of high pitch.**

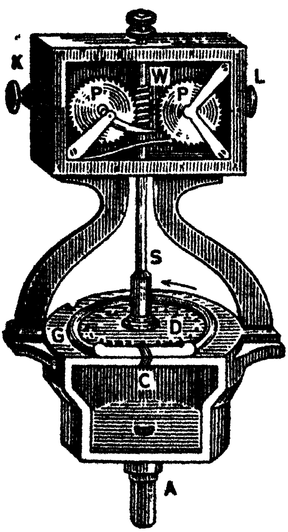


Fig. 20

(2*a*) **Cagniard de la Tour's Siren** : It is an improved form of siren which can produce sounds of high pitch. It consists (Fig. 20) of a cylindrical metal chamber C called the wind-chest the lid G of which has a few rows of equidistant concentric holes. A metal disc D with exactly equal number of holes is supported just above the lid over a vertical spindle S about which the disc can rotate freely. The two systems of holes which are drilled obliquely make an angle with each other, so that air blown within the wind-chest through the leading pipe A with which the wind-chest is provided, rushes out and strikes against the sides of the holes of the upper disc D. This causes the disc D to revolve. There is a speed counter PP attached to the vertical shaft W of the disc.

Whenever the two rows of holes in the fixed and the rotating discs are opposite to each other during the rotation of the disc, a puff of air escapes through each hole of the upper disc. If the upper

disc have m holes and if it makes n revolutions per second indicated by the speed counter, then the frequency of the sound emitted is equal to $m \times n$. For a given value of m, sounds of different frequency can be obtained by altering the value of n. The value of n can be changed by regulating the pressure of air delivered from the foot-bellows to which the wind-chest is connected through A.

(3) Sensitive Flame Experiment : A bird-call or a shrill whistle giving out a note of high pitch is placed several feet in front of a vertical board. A sensitive flame is also placed between the whistle and the board. Starting with the flame close up to the board it is moved along the normal from the whistle to the board. The flame flares until it occupies a *node* when it shoots up straight. The position of the flame is noted. As it is moved further a series of equidistant points (nodes) is found at which it is undisturbed. The mean distance between the successive nodes is found out. We know that the distance between any two consecutive nodes is half the wave-length.

Then the frequency of the note of high pitch is calculated from the expression $V = n\lambda$, where V is the velocity of sound, n the frequency, and λ the wave-lengh,

(4) Melde's Experiment : One end of a string is fixed to one of the prongs of a vibrated tuning fork and the other end passes over a pulley and supports a scale-pan. There are two ways of fixing the string to the fork. In the **first mode** (Fig. 21) *the string*

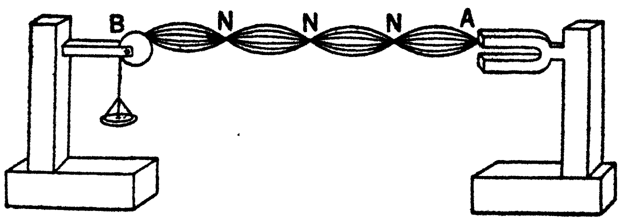


Fig. 21

is attached to the prong in such a way that the prong virbrates in a direction at right angles to the length of the string which is seen to be divided into a number of segments of equal length. The vibration of the string, in this mode, is the same, as the vibration of the fork. In the **second mode** (Fig. 22), *the fork and the string are fixed in such a way that the prong vibrates along the length of the string dividing it into a number of segments of equal length.*

In the first mode, resonant vibration may be set up in the string by adjusting the load on the pan supported from the free end of the

string until the frequency of the stationary vibration set up along the string is *nearly of the same frequency as the fork.*

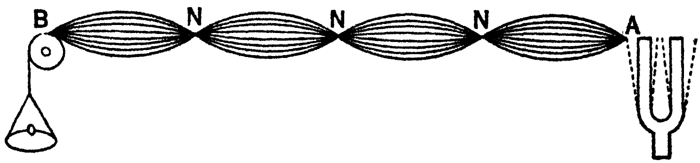


Fig. 22

We know that the frequency n of the fundamental sound emitted by the string is expressed by $n=\frac{1}{2l}\sqrt{\frac{T}{m}}$, where T is the tension in dynes applied to the string, m the mass per unit length of the string, and l the distance between the consecutive nodes.

If N is the frequency of the fork, $N=n$.

When T, m and l are known, the frequency of the string and hence of the fork can be determined.

In the second mode, when the fork and the string AB (Fig. 23) are fixed in such a way that the prong vibrates along the length of the string, resonant vibration may also be set up when the load on the pan P is such that the frequency of the stationary vibration set up is half that of the fork. For, when the end of the vibrating prong is at the extreme limit of one of its swings *i.e.* when the prong B is furthest from A, the string is tightened and when it moves back and reaches its extreme outward limit towards A, the string sags down and becomes loosest. Hence, when the prong moves from its extreme inward to its extreme outward limit or in other words when it completes half a vibration, the string moves from its horizontal position to its downward limit, it executing ¼th vibration.

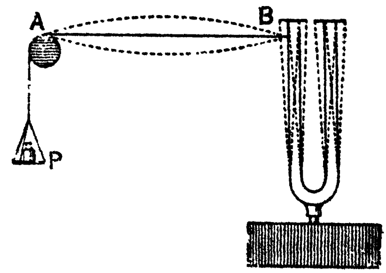


Fig. 23

In this case the string must vibrate with a period which is twice the period of the fork.

Thus we see that the *frequency of the fork is double that of the string.*

The frequency n of the string is obtained by the formula $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$. Therefore the frequency n' of the fork is equal to $2n$.

Note: In this case, the vibration of the prong causes a periodic variation of tension in the string. So long as the fork continues to vibrate, the up and down motion of the string in a direction transverse to that of the prong is maintained.

(5) Sonometer Method: The pitch of a note can be determined by an instrument known as sonometer in which a metal string is stretched on a hollow wooden-box AA (Fig. 24). One end of the string is fixed to a peg on the box and the other passes over two bridges B_1 and B and a pulley and is kept stretched by a number of weights W. A movable bridge C is placed at any part under the string and a portion of it is selected which vibrates in unison with a given note.

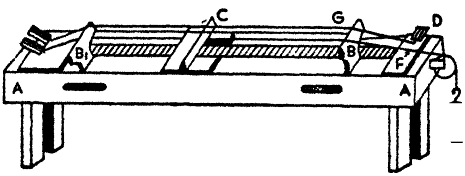


Fig. 24

The unison between the given note and that emitted by the string is attained when the rider (paper strip or short length of metal string bent in the form of Λ) is seen to jump off from the vibrating string, when it is placed at the middle of the portion of the string producing the sound. The frequency n of the note is obtained by the expression

$$\frac{1}{2l}\sqrt{\frac{T}{m}}$$

(The mass per unit length of wire is obtained by dividing the mass of a certain length of the wire by its length.)

Sources of Errors: The rigidity of the string makes its frequency greater than that given by the formula. This is avoided by making the string as flexible as possible for the formula is applicable only to a perfectly flexible string.

Owing to the small mass of the string, it is difficult to determine when the actual unison between the notes takes place *i.e.*, when the resonance is maximum. The error is avoided by adjusting the length of the string until no beats are heard when the two notes are sounded together. The method of adjustment for unison by using riders is not accurate and should be avoided.

The friction of the pulley is another source of error as it alters the tension of the string to some extent. This error is minimised by suspending the sonometer wire in the vertical position.

48. Intensity and Loudness: The term *loudness* is applicable to all kinds of sounds whether musical sound or noise, but pitch as we have said before, applies only to musical sounds. *Pitch relates to the character of the sensation produced in the ear while loudness relates only to the quantity of the sensasion.*

The two terms *intensity* and *loudness* are not exactly identical as to their meaning. The intensity of any sound is a purely physical quantity independent of the ear and is proportional to the wave energy passing per unit time through unit area, whereas loudness of the sound is not wholly physical but depends on the ear and the listener and relates to the degree of sensation produced by the energy of the waves transmitted to the ear. So loudness may be said to be depending upon the intensity of the waves producing it.

The intensity and therefore the loudness of any sound depends on the energy of the vibrating source which is again influenced by several factors.

Thus the *intensity* or *loudness* of any sound which depends on the energy of vibrating body producing the sound, is proportional to the *square of the amplitude* of vibration. That is, the greater the amplitude of vibration of the sounding body, the louder is the sound emitted by the body. The intensity also varies inversely as the *square of the distance of the listener from the sounding body.*

The *size of the vibrating body*, the *density of the medium* in which sound is produced, *the direction of the wind* and several other factors very greatly influence the intensity of sound.

The intensity of any note may also be increased by holding a sounding body in contact with or near a sounding box containing a large volume of air. The causes of this intensification of sound may be due either to *resonance* or to *forced vibration.*

49. Quality : It is the peculiarity in the impression produced on the ear by two sounds alike in pitch and intensity but coming out from two different instruments, say a flute and a harmonium. Quality may be defined as that which enables us to distinguish between notes of the same pitch and intensity produced by different sources. The difference in the *quality*, *timber* or *colour* between the notes is due to the complexity of the vibration producing the note. The note produced by any sounding body is generally a compound sound in which several harmonics or overtones are present in combination with the fundamental tone which masks the effect of all the harmonics and becomes predominant. The quality of a musical note depends on the *number*, *order* and *relative strength* of its harmonic constituents and *not in their difference of phase.*

50. Representation of Loudness, Pitch and Quality by Diagrams : We know that the character of any musical sound is determined by its intensity, pitch and quality and we know also that the intensity of the musical sound depends on the *amplitude* of the wave at the point in the medium at which the sound is heard and its pitch which is expressed by the rate of vibration depends on the *length* of the wave or displacement curve of motion.

We may have two simple sounds differing in amplitude and pitch and these two sounds may be represented by two simple harmonic displacement curves as A and B, in each of which the maximum amplitude and also the wave-length

are different (Fig. 25) If these two sounds are sounded together and if the resulting sound or wave motion in the medium be represented by a displacement curve C, it will not be a simple harmonic curve and the form of it will greatly differ from that of the simple sound.

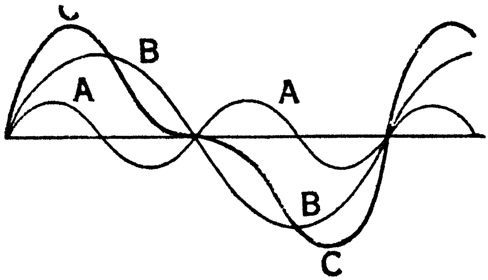


Fig. 25

Again if we have two sounds, one simple and the other complex having the same intensity and pitch but differing only in quality, the amplitudes and the wave-lengths of the two sounds will agree but their *forms* will differ, for the simple sound will be represented by a harmonic curve like A or B and the complex sound which consists of a number of simple sounds will be represented by a curve of the type C obtained by adding or subtracting the ordinates of the constituent harmonic curves when in the same or in the opposite directions respectively.

QUESTIONS

1. Define pitch, loudness and quality (timbre) of a musical note. Explain how they are represented in the wave curve. [C. U. 1922, '28]

2. To what physical characteristics do the loudness, pitch and quality of a musical note correspond ? [C. U. 1925]

3. Describe Melde's experiment for determination of the frequency of a tuning fork. [C. U. 1956]

CHAPTER VI

DOPPLER'S PRINCIPLE

51. Doppler's Principle: It has been observed that the pitch of any sound rises or falls according as the distance between the source and the observer or listener diminishes or increases, due to the relative motion of either or both. This change in pitch is noted when a locomotive engine blowing its whistle approaches and then passes the listener. *This apparent* change in the pitch of a note due to the relative movement of the source and the listener is known as *Doppler's Effect* and the principle by which this change in pitch has been explained is known as *Doppler's Principle.*

To study Doppler's effect four cases are to be considered.

1. The pitch of a note when both the source and the listener are at rest. 2 The apparent change in the pitch when the source is only in motion. 3. The apparent change in the pitch when the listener only is in motion. 4. The apparent change in the pitch when the source and the listener are both in motion, and (5) that due to the presence of wind.

Case I. Source and listener at rest: Let a source be at rest and give out n waves per sec. These waves will travel through a distance V, the velocity of sound waves in air, in one sec. The listener in this case will hear a sound of pitch n.

Case II. Source in motion and Listener at rest: Let S be the position of the source and let L be the position of the listener at rest (Fig. 26)

Fig. 26

The source emits n waves per sec. and let these waves travel towards the listener with a velocity V and let the source follow them with a velocity V_s.

At the end of one second, the first wave from the source has reached a position A where $SA = V$, and at the same instant the source has moved up to the position S_1 where $SS_1 = V_s$.

If the source be at rest n waves emitted by the source in one second would have occupied the length SA but now, since the

source has moved towards the listener through a distance SS_1, these n waves occupy the length S_1A.

Thus the effect of the motion of the source is to *shorten* the wave length and the length of each wave now becomes equal to λ_1 (say) given by

$$\lambda_1 = \frac{V - Vs}{n}.$$

Since the velocity of the waves is independent of the wave-length the listener receives a larger number of waves per sec. and so he hears a sound of pitch n_1.

Thus we have $V = n_1 \lambda_1 = n_1 \cdot \frac{V - Vs}{n}$

Therefore $$n_1 = \frac{nV}{V - Vs} \qquad \ldots (1)$$

Thus n_1 is **greater than** n and so the pitch is raised.

Case III. Listener in motion and source at rest : Let the source S be at rest and let the listener L be moving away from the source with velocity V_0 (Fig. 27)

Fig. 27

As before, the source emits n waves per sec. and these waves reach the listener L and at the end of one second let the particular wave that is now at L reach a position B so that $LB = V$, and at that instant let the listener have moved up to the position C so that $LC = V_0$.

In this case the *wave length is not affected* by the motion of the listener and the listener only receives waves lying in between C and B *i.e.*, in the length $V - V_0$ and hears a sound of pitch n_1.

We know that n waves occupy a length V, so for a length $V - V_0$ we will have $n\frac{(V - V_0)}{V}$ waves.

That is $$n_1 = n\frac{(V - V_0)}{V} \qquad \ldots (2)$$

Thus n_1 is **less than** n and the pitch is lowered.

Case IV. Source and Listener both in motion: The expressions (1) and (2) can be combined to give an expression for the pitch of the sound when both the source and the listener are in motion in the same direction.

When the source is in motion and the listener is at rest

$$\text{we have } n' = \frac{V}{V - Vs} . n \quad ...(1)$$

If now, the listener is also in motion

$$n_1 = \frac{V - V_o}{V} n' \qquad ...(2)$$

Substituting in exp. (2) of the above equations the value of n_1 given by exp. (1), we have

$$n_1 = \frac{V - V_o}{V - Vs} n \qquad ...(3)$$

If again, wind blows in the direction in which the waves travel with a velocity w the resultant pitch is obtained by substituting $V + w$ for V in (3) and so the expression for the resultant pitch becomes

$$n_1 = n \frac{V + w - V_o}{V + w - Vs}.$$

51a. General Expression for Doppler Effect: Let a sounding body be situated at S and give out n waves per sec. Let a listener standing at D receive n waves per sec. He, therefore, hears a sound of pitch n (Fig. 28)

Let V be the velocity of sound waves *i. e.*, the distance travelled by waves in one sec. be represented by SB. If w the velocity of wind in the same direction as the waves be denoted by BC then SB + BC *i.e.*, SC is the distance travelled by the wave in one sec.

Let the velocity V_s of the source be represented in magnitude and direction by SA. Now if we consider the source at rest, the waves

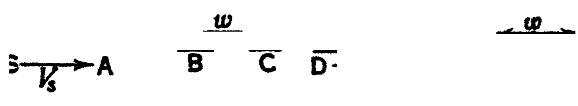


Fig. 28

started from the source would pass the observer at D and would travel through DG in one sec. and if there be wind moving with a velocity w in the same direction as the waves, the distance travelled by the waves in one sec. is DG + GF or DF and is the same as SC.

We will now consider a case when the source and the listener are moving in the same direction *i.e.*, towards right with velocities v_s and v_0 respectively.

At any instant of motion, let n be the number of waves emitted by the source per sec. and let n_1 be the number of waves received by the listener per sec. Let S be the position of the source at the commencement of the time when the first wave starts from it.

Now A is the position of the source just after one second and at this instant $(n+1)$th wave is about to start from it and by this time the first wave has reached the point C. Therefore there are n waves in the length AC in the first second.

Let the first wave reach the listener at certain instant when he is at D and after 1 sec. the first wave will reach F and during this interval the listener has moved to E to receive the last wave of the second.

Therefore there are n_1 waves in the length EF in one second.

Thus we have, $\dfrac{n_1}{EF}=\dfrac{n}{AC}$ or $\dfrac{n_1}{n}=\dfrac{EF}{AC}=\dfrac{DF-DE}{SC-SA}=\dfrac{V+w-v_0}{V+w-v_s}$

i.e. $n_1=n\,\dfrac{V+w-v_0}{V+w-v_s}$ = the pitch of the sound as perceived by the listener.

Proper signs of w, v_s and v_0 are to be taken if the motion of the wind, source and the listener be not in the direction from the source to the listener.

If $v_s=v_0$, then $n_1=n$ *i.e.*, the pitch is not altered if the source and the listener move in the same direction with the same velocity.

Again if $v_s>v_0$, $n_1>n$ *i.e.*, the pitch is raised. But is $v_s<v_0$, $n_1<n$ *i.e.*, the pitch is lowered.

It can be shown very easily that when v_s and v_0 are very small compared with $V+w$, a motion of the source has the same effect as an equal and opposite motion of the listener but this is not the case when v_s and v_0 are not small compared with V.

52. Shew that in the absence of wind a given speed of approach of the source raises the apparent pitch more than that for the same speed of approach of the listener.

The exp. for the apparent pitch is $n_1=n\,\dfrac{V+w-v_0}{V+w-v_s}$, when the source, wind and listener are all moving in the same direction.

Case I: Here $w=o$ and $v_0=0$, then $n_1=n\,\dfrac{}{V-v_s}=nV(V-v_s)^{-1}$

$$=nV.V^{-1}\left(1-\frac{v_s}{V}\right)^{-1} \text{ or } n_1=n\left(1+\frac{vs}{V}+\frac{vs^2}{V^2}+\right)=n\left(1+\frac{v}{V}+\frac{v^2}{V^2}\right) \quad ...(1)$$

where $v_s=v$ the velocity of approach of the source

Case II : Here $w=o$ and $vs=o$ we have $n_2=n\left(\frac{V+v_o}{V}\right)$, since the sign of v_o, the speed of listener, is opposite to that of vs $\quad n_2=n\left(1+\frac{v_o}{V}\right)$

Now since $vo=vs=v$ ...(2)

The apparent pitch n_2 is given by

$$n_2=n\left(1+\frac{v_s}{V}\right)=n\left(1+\frac{v}{V}\right) \quad ...(3)$$

From (1) and (3) we see that $n_1>n_2$

53. Demonstration of Doppler's Principle A whistle of high frequency is fitted into a long rubber tubing. Taking the free end of the tubing into his mouth an experimenter blows through it steadily and at the same time whirls the whistle around his head. The other experimenter standing at a distance notes the gradual rise and fall in the pitch of the sound.

Doppler's Principle has been applied in many astronomical investigations especially in determining the velocity of a star in the line of sight.

54. Measurement of a small interval by a fork of known frequency : This is done by means of an instrument known as chronograph. It consists of a hollow metal cylinder fitted on to an axle which when rotated makes it revolve and at the same time causes it to move forward. (Fig. 29).

A piece of smoked paper is gummed round the cylinder A, and a tuning fork B of known frequency provided with a style resting against the smoked paper is made to vibrate and the cylinder is rotated. Due to the combined motion of the fork and the cylinder a wavy line is traced on the paper which for each complete vibration of the fork assumes the appearance of a zig-zag line.

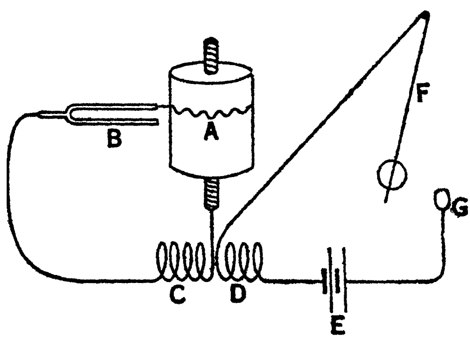


Fig. 29

From the knowledge of the frequency of the fork *i.e.*, the number of zig-zag lines traced in one second we can determine any unknown interval of time by counting the number of such similar lines in that unknown period.

The interval of time is recorded automatically on the smoked paper by connecting the vibrating tuning fork with one end of the secondary C of an

induction coil, the other end of which being connected to the revolving drum. The current in the primary circuit D of the induction coil is made and broken by connecting it to an oscillating pendulum F through a drop of mercury placed in a metal cup G at which contact is made when the bob of the pendulum is at its lowest position, and broken when it moves away.

At each break of the current in the primary, a spark passes between the point of the style attached to the fork and the smoked paper on the cylinder and produces a dark spot on it.

By counting the number of the lines between two consecutive dark spots and knowing the frequency of the fork, the small interval between two consecutive passages of the pendulum bob through the mercury drop can be calculated.

QUESTIONS

1. Explain Doppler's principle and describe how it can be demonstrated. Deduce the formula connecting the pitch of the note heard with velocities of the wind, source and the observer. [C. U. 1926, '27, '29, '37, '40, '45, '47, '50, '52]

2. What is Doppler effect? Deduce and explain the formula for apparent frequency due to it. [C. U. 1957, '59]

3. Explain clearly Doppler effect in sound. Show that the Doppler effect is greater when the source approaches the observer than when the observer approaches the source with same speed. [C. U. 1954]

EXAMPLES

1. A man standing on a train moves at the rate of 108 kilometres per hour, blows a whistle the pitch corresponds to 1200 vibrations per second. What is the apparent pitch of the whistle to a person standing at a point in the direction towards which the train moves? Temperature of the atmosphere at the time of observation was 30°C; Velocity of sound in air at 0°C is 33060 cms. per sec. [C. U. 1913]

The exp. for the apparent pitch of a note is given by $n_1 = n \dfrac{V+w-vo}{V+w-vs}$

In this case $w=o$, $vo=o$, therefore $n_1 = n \dfrac{V}{V-vs}$

But V = the velocity of sound at the time observation at 30°C.

$$= V_o\left(1+\frac{\alpha t}{2}\right) = 33060\left(1+\frac{\cdot 0036\times 30}{2}\right) \text{ cms/sec.}$$

$$= (33060+1785\cdot 24) \text{ cms/sec.} \quad = 34845\cdot 24 \text{ (approx.)}$$

$$n_1 = 1200\times\frac{34845}{34845-3000}, \quad \text{since } \frac{10800000}{60\times 60} = 3000 \text{ per sec.}$$

$$= \frac{1200\times 34845}{31845} \text{ per sec.} = 1314 \text{ vibrations per sec. nearly.}$$

2. Two trains are approaching from opposite directions with the same speed of 100 ft/sec. The whistle of the first train is of frequency 1028. Find the variation of the apparent pitch calculated by an observer in the second train as the trains pass, supposing there is no wind and that the velocity of sound in air to be 1100 ft./sec. [C. U. 1918]

The expression for the apparent pitch when two trains approach each other is $n_1 = n\,\dfrac{V+vo}{V-vs}$,

since $w = o$ and the motions of the source and the listener are in opposite directions *i.e.*, vo is negative
$$n_1 = 1028 \times \frac{1100+100}{1100-100} = 1028 \times \frac{1200}{1000}$$
$$= 1233{\cdot}6 \text{ vibrations per sec.}$$

Again when the two trains have passed each other the velocity of the sound is in opposite direction to that of the source and in the same direction as the listener.

Therefore the exp. for the apparent pitch is $n_1 = n\,\dfrac{V-vo}{V+vs}$
$$= 1028 \times \frac{1100-100}{1100+100} = 856{\cdot}6 \text{ vibrations per sec.}$$

3. Shew that if the source moves away with the velocity of sound from an observer who is at rest, the frequency of vibration is halved. [C. U. 1937, '57]

When the source is in motion and the observer is at rest, the frequency or the pitch is given by $n_1 = \dfrac{nV}{V-vs}$

As the source moves away from the observer we have $vs = -V$
$$\therefore \quad n_1 = n.\frac{V}{V+V} = \tfrac{1}{2}n.$$

4. A locomotive whisto emitting 2000 waves per second is moving towards you at the rate of 60 miles an hour on a day when the thermometer stands at 24°C. Calculate the apparent pitch of the whistle [Velocity of sound in air at 0°C = 1093 ft. sec.] C. U. 1940 Ans. 2167·3 per sec.

5. A train is passing a railway station with a speed of 40 m. p. h. and blowing continuously a whistle of frequency 256 per sec. What will be the frequencies apparent to a person waiting on the platform when the train is (*a*) approaching (*b*) departing ? What is the interval between these two notes ? [Velocity of sound = 1120 ft. per sec.] [C. U. 1945]

(*a*) Use the formula $n_1 = n\,\dfrac{\quad}{V-V_s}$

Here V = 1120 ft. per sec. $V_s = \dfrac{40 \times 1760 \times 3}{60 \times 60} = \dfrac{176}{3}$ ft. per sec., and $n = 256$

Ans. $n_1 = 270{\cdot}15$ per sec.

(*b*) Use the formula $n_2 = n\,\dfrac{V}{V+V_s}$. Ans. $n_2 = 243{\cdot}26$ per sec.

Interval $= \dfrac{n_1}{n_2} = \dfrac{270{\cdot}15}{248{\cdot}26} = 1{\cdot}1105.$

6. A spectroscopic examination of light from a certain star shows that the apparent wave-length of a certain spectral line is 5001 A. U., whereas the observed wave-length of the same line produced by a terrestrial source is 5000 A. U. In what direction and at what speed do these figures suggest that the star is moving relative to the earth ? [C. U. 1947]

From the formula : $n_1 = n\frac{V - v_0}{V - vs}$, we have $\frac{n_1}{n} \quad \frac{V - v_0}{V - vs}$

Now $n_1 = \frac{V}{\lambda_1}$; $n = \frac{\quad}{\lambda}$, $\frac{n_1}{n} = \frac{\lambda}{\lambda_1}$ $\because$ $\frac{\lambda}{\lambda_1} = \frac{V}{V - vs}$ $\therefore$ $_1 = 0$

Or, $\frac{\lambda_1}{\lambda} = \frac{V - vs}{V} = 1 - \frac{vs}{V}$ Or, $1 - \frac{\lambda_1}{\lambda} = \quad V$

Since $\lambda_1 = 5001$ A. U. ; λ 5000 A. U.

We have $1 - \frac{5001}{5000} - \frac{vs}{V}$ Or, $-\frac{1}{5000} = \frac{vs}{3 \times 10^{10}}$

Since V, velocity of light $= 3 \times 10^{10}$ cm. sec.

$$Vs = \frac{3 \times 10^{10}}{5000} \cdot -6 \times 10^6 \text{ cm/sec.}$$

Thus the star is moving *away* from the earth with a speed of 6×10^6 cm/sec.

7. A train approaches a stationary observer at a speed of 75 kilometers per hour sounding a whistle of frequency 1000. What will be the apparent frequency of the whistle to the observer ? Ans. 1067 (approx.) [C. U. 1948]

8. Calculate the velocity at which a source of frequency 10 thousands per second should approach the observer at rest in order to produce a Doppler shift of 200 per second. [C. U. 1954]

We know that $n_1 = \frac{nV}{V - v^s}$ where n_1 = apparent pitch of the sound

n = original pitch of the sound,
V = vel. of the sound
v_s = vel. of the source

Here $n_1 > n$

$$10200 = \frac{10000V}{V - v_s}, \text{ or } 10200(V - v_s) = 10000V \text{ or } v_s \quad 51$$

If V=1120 ft. per sec., $v_s = \frac{1120}{51} = 21{\cdot}9$ ft/sec.

9. Calculate the percentage difference between the frequency of a note emitted by the whistle of a train approaching an observer with a velocity of 100ft./sec and that heard by the observer, the velocity sound in air being 1100ft./sec. [C. U. 1957]

Let n be the frequency of the note emitted by the whistle, and n' the frequency of the note heard by the observer when the train approaches the observer. Then we have $n' = n.\frac{V}{V - v}$,

where V is the velocity of the waves emitted by the whistle and v the velocity of the train.

Or, $n' = \frac{n \times 1100}{1100 - 100} = \frac{11}{10}$, Then the difference $= n' - n = n\left(\frac{11}{10} - 1\right) = \frac{n}{10}$

$\therefore$ Percentage difference is 10.

10. An engine sounding a whistle of frequency 500 vibrations per sec., is approaching an observer at a velocity of 60 miles per hour. What is the frequency of the note heard by the observer ? (Velocity of = 1120 ft./sec..

[C. U. 1959] *Ans.* $n = 542{\cdot}6$

CHAPTER VII

FORCED VIBRATION AND RESONANCE

55. Resonance and forced vibration: These terms are better understood by the aid of a simple pendulum. If the pendulum be displaced from its position of equilibrium and left free to vibrate under the force called into play by the displacement, the vibrations thus executed by the pendulum are called **free vibrations** and the pendulum is then said to vibrate with its **natural period** or frequency depending on its **length.**

Now if the pendulum be struck a number of blows at regular intervals and at such times that the blows help to increase the velocity of the pendulum *i.e.*, act in the direction the pendulum is moving, the amplitude of oscillations will increase at each blow and finally an infinitely great amplitude of the swing in the pendulum will be set up, which in practice is never attained owing to the damping effect of the friction at the point of support.

This happens when the interval between the successive blows is equal to the natural period of oscillation of the pendulum. This process of increasing the amplitude of oscillation of any vibrating body by the application of a periodic force having its period equal to the natural period of vibration of the body is what is known as **Resonance** and the vibrating body is said to resound to the periodic force. [In this illustration it has been assumed that the blows have no effect on the period of the pendulum]. Resonant vibration can also be set up if the period of the applied non-harmonic force or impulse is an exact submultiple of the natural period of the vibrating body. It has also been found that a periodic force which increases and decreases harmonically does not cause resonant vibration of periods which are submultiplies of its own, though the force which varies non-harmonically may do so.

Again if the interval between two successive blows be either greater or less than the natural period of the vibrating body the vibrations will at first be irregular and after some time the body, will be constrained to move or vibrate with a very small amplitude and in a period equal to that of the applied force *i.e.*, equal to the interval between two successive blows. Such vibrations which do not agree in period with the natural period of the body but are produced by the action of a periodic force are called **forced vibrations.**

In the above illustration it has been assumed that the applied force is intermittent and that it has no action in the period of the pendulum. But if the applied force be periodic in character, resonance will take place when the period of the applied force is the same as that of the vibrating body. Again if

the period of the force does not agree, or very nearly agrees with that of the vibrating body, the body will execute a forced vibration with a very small amplitude of oscillation of the vibrating body when acted on by forces of *nearly* the same period as the natural period of the body depends on the mass of the vibrating body and also on the amount of friction resisting its motion. If the mass be large and the friction small as in the case of a pendulum or a tuning fork, resonant vibration is very small compared with that produced by forces of the same period as the natural period of the body *i.e.*, the fork.

Again if the mass be small and the friction large as in the case of air vibrating in a tube resonant vibration will be *nearly as strong* as that produced by forces of the same period as that of the vibrating air column.

55a. Mechanical Illustrations of Resonance and Forced vibration : Resonance and Forced vibration may be beautifully illustrated by suspending four pendulums A, B, C and D (Fig. 30) from a light horizontal elastic support, the plane of vibration being perpendicular to the plane of the paper. The pendulums B and C are of the same length and hence of the same period. The pendulum D being slightly longer have greater period, and the pendulum A, slightly shorter have shorter period. The pendulum B with a heavy bob is set into vibration which again sets the horizontal support into forced vibration of the same period as B and also causes the points of suspension of C, A and D to vibrate with the same period.

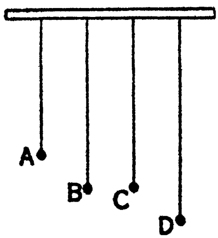


Fig. 30

Since the period of C is the same as the period of B it is readily thrown into resonant vibration and thus resonance is produced.

The pendulums A and D on the other hand move with a small swing, come to rest and then continue the process until they make a compromise with one another and finally vibrate with the period equal to that of B.

It will be seen that the amplitudes of oscillation in A and D are less than that in B or C, and that B, C and A are in the same phase but D is in the opposite phase.

55b. Other illustrations :

(A). Resonance : A vibrating tuning fork is held at the mouth of a vessel contaning a certain volume of air and the vessel is found to speak *i.e.*, to produce a maximum sound. This is the case of resonance and is possible only when the period of the air particles is equal to the period of the tuning fork.

The regular tramp of soldiers crossing a bridge breaks it down if its period of vibration agrees with the interval between the successive steps.

In sonometer experiments, resonance is produced when the note yielded by the vibrating string is in unison with the note emitted by the tuning fork.

For analysis of compound musical sound by Koening's resonators, the principle of resonance is utilised.

The detection of wireless signals by a receiving circuit depends on the principle of electrical resonance.

(B). **Forced vibration** : If the tuning fork or a string be put into vibration no sound is practically heard. For, since the surface area of the vibrating fork or string is not very large, the fork or the string cannot set a large quantity of air into vibration and moreover the compressed part of the medium slips round the prongs and fills up the rarefied part of the medium behind the fork or the string. Thus the interference between two sets of waves, one rarefaction and the other compression started out from the vibrating prong in opposite phases reduces the intensity of the sound.

But when the fork or the string is held on a table or a sounding box containing air, the vibration of the fork or of the string sets the table or the box into forced vibration and the table on account of its large surface causes a large quantity of air to vibrate and consequently energy of the air set in motion is much greater than if there were no table or box. Hence the sound is intensified. This is not a case of resonance but of forced vibration.

56. **Mathematical Treatment** : To treat the subject mathematically let us consider a particle of mass m executing a S. H. M. under the action of a controlling force (such as gravity in the case of a pendulum and elastic force in the case of a mass attached to a spring). Let the period of the motion be T.

Now when the displacement of m is x, the acceleration of m towards the origin is $-\left(\frac{2\pi}{T}\right)^2 x$ and the outward acting force on m to decrease x is $-m\left(\frac{2\pi}{T}\right)^2 x$.

Suppose a harmonically changing force $F \sin \frac{2\pi}{T_1}t$ of period T_1 acts on m.

The effect of this force on m is to produce a steady vibration of period T_1, the period of the applied force.

Thus, when the displacement of m is x, the acceleration of m is $-\left(\frac{2\pi}{T_1}\right)^2 x$ and the outward acting force on m is $-m\left(\frac{2\pi}{T_1}\right)^2 x$

This force is the resultant of the natural force and the applied force.

Thus we have, $-m\left(\frac{2\pi}{T_1}\right)^2 x = -m\left(\frac{2\pi}{T}\right)^2 x + F\sin\frac{2\pi}{T_1}t$

$$\text{on } 4\,m\pi^2 x\left\{\frac{1}{T^2}-\frac{1}{T_1{}^2}\right\} = F\sin\frac{2\pi}{T_1}t$$

$$\therefore\ x = \frac{F\sin\frac{2\pi}{T_1}t}{4\pi^2 m\left\{\frac{1}{T^2}-\frac{1}{T_1{}^2}\right\}}$$

(1) If the period of the applied force T_1 be greater than the natural period T, x is positive and is of the same sign as the force $F\sin\frac{2\pi}{T_1}t$ *i.e.*, the forced vibration is in phase with applied force.

(2) If $T_1 < T$ *i.e.*, if the period of the applied force be less than the natural period, x is negative *i.e.*, the vibration is in opposite phase to that of applied force.

(3) If $T_1 = T$ *i.e.*, if the applied and the natural periods are equal, $\left(\frac{1}{T^2}-\frac{1}{T_1{}^2}\right)=0$, and x is infinite.

It means that the displacement becomes larger and larger until the loss of energy due to air resistance and various other causes is balanced by the energy given to the vibrating body by the applied force. This is the case of **Resonance**.

QUESTIONS

1. Distinguish between Resonance and Forced vibration with illustrations.

Write a short essay on resonance, pointing out how the subject may be experimentally illustrated. [C. U. 1938, '58]

2. Write a short note on forced and free vibrations. [C. U. 1955, '56]

CHAPTER VIII

SUPERPOSITION OF WAVES

57. Principle of Superposition: The principle states that when a particle of a medium is struck simultaneously by two or more waves, the resultant displacement of the particle (if the displacement of the particle is small) is the algebraic sum of the component displacement which the particle would have if each acted alone.

Three cases are to be considered.

(1) **Interference** of the two waves of the same frequency moving in the same direction.

(2) **Beats** which are due to the superposition of two waves of *nearly* equal frequencies moving in the *same* direction.

(3) **Stationary waves** which are due to the superposition of two identical waves moving in *opposite* directions.

58. Interference: When two systems of waves are sent out in a particular direction in a medium, the medium will be greatly agitated and the actual disturbance at any part of it is the resultant of the component disturbances produced by the two waves separately. If two similar waves travel in any direction so that the hollows of one fall on the crests of the other, the result of this superposition will be the absence of any disturbance in the medium. Again if the hollows and crests of one fall respectively on the hollows and crests of the other, the medium will be doubly disturbed and produce an increased effect due to this superposition. This principle is known as the **principle of interference**. If the waves differ in period, amplitude and phase and travel in any direction the effect of interference will be complicated.

We know that in air, only longitudinal waves can be propagated and that a wave consists of a condensed and a rarefied part. Then, in order to determine the resultant disturbance at any part of the medium in which longitudinal waves travel in the same direction, condensation and rarefaction are to be taken as of opposite signs and then to be added algebraically.

We know that a longitudinal wave can be represented graphically by displacement curve, similar to that in the case of a transverse wave, in which the ordinates at the undisturbed positions of the particles measure displacements to the right-hand, upwards, and to the left-hand, downwards, at different instants of time. If two or more waves are superposed one upon another and if they travel in the same direction the resultant disturbance at any part of the medium is obtained by adding the ordinates of the component curves when in the same direction and subtracting them when in the opposite directions.

58a. Study of two particular cases: (1) If two waves travel in the same direction and are of equal periods and start with the same phase *i.e.*, the crest of one falls on the crest of the other

the resultant disturbance and the actual wave form is obtained by the algebraic sum of the ordinates at any instant in the region under consideration.

(2) If the waves are of equal period (of the same wave-length), of equal amplitudes and in opposite phase *i.e.*, the crests of one falling on the hollows of the other) the resultant disturbance will be nil and the wave form is a straight line *i.e.*, there is no wave in the region and there will be silence.

If in the above case the amplitudes are not equal, the resultant disturbance will be very much diminished in intensity.

59. Conditions of interference : 1. The two notes must have the same frequency, phase and quality. 2. The displacement due to individual waves must be in the same line. 3. The waves must continue to arrive at the given point in opposite phases.

60. Illustrations : (a) Interference near a tuning fork : The phenomenon of interference may be illustrated by twisting the stem of a vibrating tuning fork round between the fingers. The sound will be heard to swell out and die away four times in each rotation of the fork.

The explanation is simple. The prongs of the tuning fork move inwards and outwards together. When they move outwards they condense the air outside them and rarefy the air between them. The condensation starts off in all directions from the outer surfaces with maximum amplitude in a direction joining the prongs and with minimum amplitude in a direction at right angles to it.

At the same instant, rarefaction also starts off from the inner surfaces in all directions with maximum amplitude in a direction at right angles to the line joining the prongs and with minimum amplitude along the line joining the prongs.

Thus there will be four directions such as N. E, N. W, S. W, and S. E where condensation and rarefaction have the same amplitude and interfere with one another and produce silence.

(b) Interference by Quinke's tube : Quinke's apparatus consists of a bent tube CBBC (Fig. 31) which is open at the sides A, E, and D is a sliding tube which slides over the arms of the tube CBBC. By drawing D in or out the length of the path ACDE can be suitably altered.

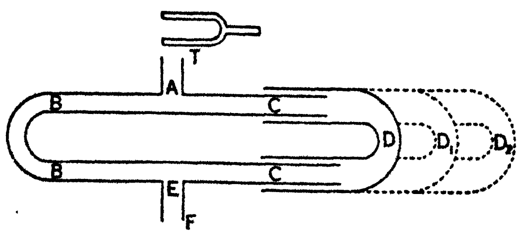


Fig. 31

A vibrating tuning fork T is held at A and the sound is heard by connecting a stethoscope at F. If the two paths ABE and ACDE are equal, the two waves travel over equal lengths and reach the ear at same phase at F. They reinforce each other and a maximum sound is produced.

If the tube D be gradually drawn out till a position D_1 is reached, the sound reaching the ear will be reduced to a mimimum and the difference in the path between ACDF and ACD_1F is half the wave-length. The sound waves in this case reach the ear in opposite phases and interfere with one another.

The tube D is further drawn to the position D_2 when the two paths ACDE and ACD_2F differ by one wave length. In this case a maximum sound is again obtained.

The apparatus can be used to determine the velocity of sound in a tube, if the frequency of the fork is known and *vise versa.*

(c) Sound Shadow : We know that when an obstacle is interposed between a source of light and a screen, a shadow is formed on the screen. Similarly when an obstacle is placed between a source of sound and the ear, sound shadow may be formed under certain conditions.

If the wave-length of the sound waves be much less than the dimension of the obstacle, a sound shadow is obtained but if the wave-length be greater or even not much smaller than the dimension of the obstacle, the waves will close in round the edge of the obstacle and the sound is heard at all points.

To explain the formation of sound shadow let us consider a sound wave, either spherical or plane to be divided into a number of half-period zones round a point in the wave-front nearest to a point at which the resultant effect is to be considered. Let an obstacle intercept the first few half-period zones and the unintercepted zones in the immediate neighbourhood of the edges of the obstacle being of a lower order will produce some effect at the point inside the geometrical shadow and consequently sound is heard at all points. [*Consult Article on Interference of Light.*]

Thus, to form a shadow, the obstacle should be large in comparison with the wave-length whether the waves be those of light or of sound.

The wave-length of light depends upon its colour and varies between $\frac{1}{30000}$ in. and $\frac{1}{60000}$ in. So an ordinary obstacle is very large compared with the wave-length and therefore shadows are easily formed.

Again since the wave-length of sound as uttered by a man ranges from 8 to 10 feet and that of woman from 4 to 5 feet, the obstacle must be very large in dimensions to cast shadow *i.e.*, to weaken the sound of the human voice.

If a large screen (such as a pile of open newspapers) be placed between the ear and a watch, the sound is not cut off, but if a packet of post-cards be interposed, the sound is cut off.

61. Beats : When two notes of **nearly equal frequency** and of the same quality are sounded together, a fluctuation in loudness occurs due to the mutual action of two notes. The waves started by those two notes differ slightly in length and consequently when superposed they reinforce each other at some regions and destroy each other at other places causing a throbbing effect in the sound. This phenemenon is known as **Beats** and is detected by the alternate loud and soft pulses in the sound.

In Fig. 32 the displacement curves represented by two dotted lines for the two waves differing slightly in length are drawn. It is seen that in the neighbourhood of A waves are in the same phase and assist each other and consequently the resultant disturbance is increased and at B the waves are in the opposite phases and neutralise each other and so the disturbance is diminished.

At C, the waves are again in the same phase and assist each other and consequently the resultant disturbance is increased.

So the resultant disturbance represented by the thick line does not produce a continuous sound but shews clearly the alternations in loudness and softness which are called *Beats*.

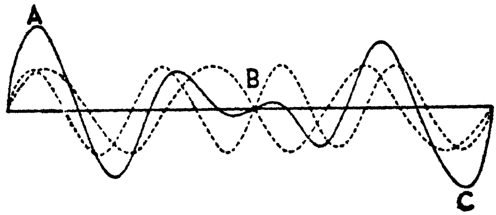


Fig. 32

The existence of beats can be easily demonstrated by sounding two forks or two organ pipes of slightly different frequencies.

If two notes having frequencies m and n respectively are sounded together the number of beats per sec. is $\mathbf{m-n}$, where m is slightly greater than n.

This is better understood by considering two notes of frequencies 101 and 100 respectively. If we start from an instant, say, the beginning of a second when the maximum condensation of one coincides with the maximum condensation of the other, next coincidence will take place exactly a second later. During the interval the maximum condensation of the wave of higher frequency will gradually gain over those of the other till at the end of a second the gain amounts to one wave-length. Therefore at the end of half a second the gain in wave-length is only half wave-length and the two waves destroy each other since at that instant the condensation of one wave will coincide with the rarefaction of the other. Thus in one second the number of beats is only one and therefore if m and n are the frequencies of the two notes which are sounded together to produce beats, the number of beats per second is $\mathbf{m-n}$.

61a. Mathematical Treatment: Let two wave trains of frequencies m and n respectively start with the same phase. The individual displacements at a point at some instant t due to two wave trains of equal amplitudes a, are given by

$$y_1 = a \sin \omega t = a \sin 2\pi mt \qquad \text{where } \omega = 2\pi m$$
$$y_2 = a \sin \omega' t = a \sin 2\pi nt \qquad \text{where } \omega' = 2\pi n$$

The resultant displacement at a given instant is represented by
$y = y_1 + y_2 = a(\sin\ 2\pi mt + \sin\ 2\pi nt)$

$$= 2a\ \cos \frac{2\pi(m-n)t}{2}.\ \sin 2\pi \frac{(m+n)t}{2}. \quad ...(1)$$

$$= \mathrm{A}\ \sin\ 2\pi\ \mathrm{N}\ t......(1a),$$

where $\mathrm{A} = 2a\ \cos\ \pi(m-n)t$ and $\mathrm{N} = \frac{m+n}{2}$

The equation ($1a$) represents a note or rather a periodic vibration of frequency $\frac{m+n}{2}$ midway between the frequencies of the two notes and of amplitude $2a\ \cos\pi(m-n)t$. If $t=0$, then $\mathrm{A} = 2a$; if $t = \frac{1}{m-n}$, then $\mathrm{A} = -2a$. Thus amplitude varies from positive maximum $2a$ through 0 to negative maximum $-2a$ as the time increases from 0 to $\frac{1}{m-n}$ and therefore the intensity goes through its range of values in time $\frac{1}{m-n}$ secs. In other words there is one maximum and one minimum value of intensity *i.e.*, one beat in $1/(m-n)$ secs, Hence number of beats per second is $m-n$.

61b. *When the amplitudes are unequal.*

Let m and $m+n$ be the frequencies of the sources. Let the two wave systems start with the same phase. The individual displacements at a point at some instant t due to the two wave systems are given by $y_1 = a\ \sin 2\pi mt$; $y_2 = b\ \sin 2\pi\ (m+n)t$,

where a and b are the amplitudes of the waves. Then, by principle of superposition, the resultant displacement is given by
$y = a\ \sin\ 2\pi mt + b \sin\ 2\pi(m+n)t = \sin\ 2\pi mt(b\ \cos\ 2\pi nt + a) + b\ \cos\ 2\pi mt.\ \sin\ 2\pi nt$ ...(1)

The equation (1) may be expressed in the form $y = \mathrm{C} \sin (2\pi mt + \Phi)$ ...(2)

where C is the amplitude and Φ, the epoch. Comparing the two equations (1) and (2) and equating the coefficients of sin $2\pi mt$ and cos $2\pi nt$ we have.

$\mathrm{C}\ \cos\ \Phi = b\ \cos\ 2\pi nt + a$; $\mathrm{C} \sin\ \Phi = b\ \sin\ 2\pi nt$

$$\therefore\quad \mathrm{C}^2 = a^2 + b^2 + 2ab\ \cos\ 2\pi nt \quad ...(3)\quad \tan\ \Phi = \frac{b\ \sin\ 2\pi nt}{a + b\ \cos\ 2\pi nt}$$

The equation (3) shows that the amplitude of the resultant wave varies with time and fluctuates between $a+b$ (maximum) and $a-b$ (minimum).

$$\text{Thus when } t=o, \quad \cos 2\pi nt=1, \quad C=a+b \text{ (max.)}$$

$$t=\frac{1}{2n}, \cos 2\pi nt=-1, \quad C=a-b \text{ (min.)}$$

$$t=\frac{1}{n}, \cos 2\pi nt=1, \quad C=a+b \text{ (max.)}$$

Thus in an interval $\frac{1}{n}$ sec. between two maximum amplitudes one minimum is present. Similarly between two minimum amplitudes, one maximum is present. So the number of beats per second $=(m+n)-m=n=$ the difference of the two frequencies.

62. Combination Tones. Besides the production of beats or throbbing sensation when two tones are sounded simultaneously, in addition to the two primary tones, other tones, known as *combination tones* are produced under certain conditions due to the combined effects of the two primary tones. The combination tones consist of three different classes (1) **Summation tones** (2) **Difference tones** and (3) **Beat tones.**

They are heard when a double whistle is blown hard or when two gongs are struck near together close to the observer.

According to the theory of the combination tones the two primary tones cause the same body or the same portion of air to vibrate violently.

While studying interference and beats, it has been assumed that the resultant displacement at any point is obtained by adding algebraically the displacements produced at the point by the component waves. This is only true when the displacement is proportional to the force that causes it.

But when the sounds are loud, the resultant displacement will not be proportional to the force of restitution of the medium and this defect of proportionality may be great enough to introduce new features and produce tones known as combination tones.

62a. Summation Tone: When two pure tones are sounded together a series of other tones will be produced. One of them is a summation tone and has frequency equal to the sum of the frequencies of the separate tones.

Summation tones of a higher order than the first can also be obtained by the combination of the first order summation tone with one of the primary tones.

The first summation tone is very weak and can sometimes be heard when two notes are sounded very strongly on the harmonium.

62b. Difference Tone: Like summation tones it is a combination tone of frequency equal to the difference in the frequencies of the two primary tones.

Difference tones of higher order can also be obtained by the combination of the first difference tone with one of the primary tones.

The first difference tone is the only one of the series that is easily audible. Combination tones are heard when two organ pipes are blown hard.

62c. Self-Combination Tones. They are combination tones having twice the frequency of the primary tones.

If m and n are the frequencies of the primary tones, the different combination tones are given below :

m, n	..	primary tones.
$m+n$	.	first summation tone.
$2m+n$	.	second summation tone.
$m-n$	.	first difference tone.
$2m, 2n$	.	self-combination tones.

62d. Beat Tone : When the beats produced by the two notes are sufficiently rapid, they combine to form a combination tone, called a Beat tone, whose frequency is the difference in the frequencies of the two primary tones.

It is seen that the Beat tone corresponds in frequency to Difference tone. So Koening considers that the difference tone is really a beat tone produced by the beats having been so rapid as to form a tone. But Rucker and Edser proved conclusively by experiments the objective existence of the difference tone by getting a fork of the right frequency to respond to it and the difference tone is not a beat tone.

The combination tones especially the first summation tone can be detected by blowing two sirens or organ pipes strongly, the ear being placed close to the instrument.

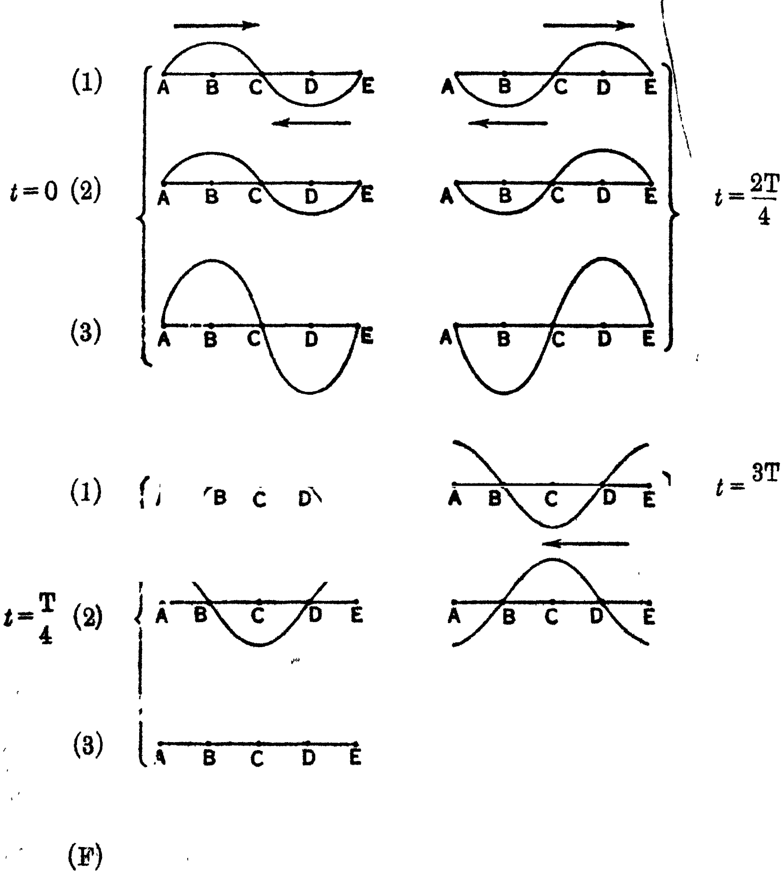
63. Analysis of tones : We know that a note is a compound sound consisting of the fundamental tone together with a certain number of overtones having different frequencies of vibration. Since it is very difficult to detect the presence of the overtones in the compound sound Helmholtz devised a simple instrument known as **Resonator** consisting of a brass shell of nearly spherical form with openings of different dimensions at the ends of a diameter. The larger one is turned towards the source of sound, while the smaller opening is connected to the ear by means of an India-rubber tubing. The resonator is so constructed that its size and consequently the volume of air inside it responds by resonance to a tone of its own natural pitch and thereby intensify the sound. Such resonators are made in sets, the pitch of each one being known and so these resonators can be conveniently used to detect the components in a compound sound which might be too feeble to be detected by the ear alone.

The manometric flame can also be used to analyse the components of a compound sound.

64. Stationary Waves : When two exactly similar wave trains travel in opposite directions with equal velocities along a straight line, due to the superposition of the two sets of waves, interference will take place and the medium will be thrown out into a form of disturbance or waves known as **stationary waves** in which the displacement at each point is fixed or stationary in character.

The nature of the stationary waves can be better understood from the adjoining diagrams.

64a. For transverse waves : Curves (1) and (2) represent the positions Fig. 33 of two similar **transverse waves** at some instant



Fig, 33

and the curve (3) represents the nature and character of the resultant waves produced by the superposition of the waves (1) and (2) travelling in opposite directions in the straight line A B C D E. The curves (1), (2) and (3) represent the displacements of particles at some particular instant as the waves pass over them.

Let us start our investigation from the instant $t=0$. At this instant the isplacement of any particle in the path of the waves is equal in magnitude and n the same direction due to both sets of waves and consequently the resultant isplacement of the same particle as shewn by (3) is double the value of either.

It is to be noted here that in the curve (3) the particles at C and E remain at rest, but those at B and D have maximum isplacements upwards and downwards respectively and have values ouble those at B and D of the elementary waves while the other ntermediate particles have also double displacements in directions etermined by the positions of the particles.

At the instant $t=\frac{T}{4}$ both sets of wave have advanced through $\frac{\lambda}{4}$ in the pposite directions and the displacement of particles due to both sets of waves are xactly equal and opposite in direction and consequently the resultant displacement or any particle is nil and therefore the resultant curve will have the appearance f a straight line.

At the instant $t=\frac{2T}{4}$ and $t=\frac{3T}{4}$, the elementary waves have moved through and $\frac{3\lambda}{4}$ respectively in opposite directions and the resultant displacement urves at the above instants of time are shewn against (3) for each instant.

Owing to persistence of vision, we cannot distinguish the different tages of motion of the medium through which the waves pass, but it appears to be of the form shown at (F).

If we carefully study the resultant curves or rather the resultant waves, we see that there are certain points such as A, C, E situated at regular intervals of $\frac{\lambda}{2}$ which are permanently at rest and are called **nodes** and points B and D half way between two consecutive nodes and at which the medium swings up and down a maximum extent are known as **antinodes.**

The portion of the medium between two consecutive nodes is nown as a loop or a **ventral segment.**

If we further examine the resultant disturbance, we notice that at any instant series of alternate crests and troughs appear at the loops, then the medium attens out and at the next instant a series of alternate troughs and crests ppear at the loops and then again the medium flattens out. It is also to be noticed that the portion of the medium between two consecutive nodes moves up down, the amplitude of the movement gradually decreasing from the antinode the node.

Since at any instant the character of the displacement at any oint of the medium is fixed or stationary and since the disturbance es not move forward the resultant disturbance or wave is known a **stationary wave.**

64b. For longitudinal waves: Stationary waves are als formed when the two **elementary waves are longitudinal.** I this case, instead of considering the displacement curves for tl formation of the stationary waves, the compression and rarefactic and the velocities of the particles at different parts of the mediu at different instants of time are to be considered.

Let us consider a particular instant at which the maximum compressions ar rarefactions of one system coincide with the maximum compressions ar rarefactions of the other, both systems of waves being exactly similar ar travelling in opposite directions. The result of this superposition is to produ double compression and rarefaction at every part of the medium. And sin during compression the velocity of the particles is in the direction of propagatic of the waves and opposite during rarefaction, the result of the superposition regards motion of the particles in the medum will be to produce rest all over.

At the instant $\frac{T}{4}$, both the waves will be displaced in opposite directic through $\frac{\lambda}{4}$ and consequently the maximum compression of one will coincide wi the maximum rarefaction of tbe other and the result of this superpositic will be to produce uniform density and double velocity all over.

At other instants, say $\frac{2T}{4}$ and $\frac{3T}{4}$ the effect for $t=0$ and $t=\frac{T}{4}$ will repeated except in the fact that positions for maximum compression an rarefaction for $t=0$ and $t=\frac{T}{4}$ will be interchanged in $t=\frac{2T}{4}$ and $\frac{3T}{4}$.

If we study the resultant disturbance at different instants o time we find certain points at regular intervals of $\frac{\lambda}{2}$ where maximum compression of one wave system coincides with th maximum compression of the other. These points are **nodes** and a these nodes the displacement and the velocities of the particles always zero.

Half-way between two consecutive nodes where the maximum compression of one system coincides with the maximum refrac tion of the other, there are points known as **antinodes** and a these places the displacement and the velocity of the particles have maximum values.

65. Interference by Reflection—Stationary Vibration When a wave train moves forward and gets reflected at a surface the reflected wave will travel in the opposite direction and interfere with the direct or incident wave. The result of interference is produce stationary undulation in the medium situated between tl source and the reflecting surface with nodes and antindodes distances equal to one-quarter of wave length apart. It is to be no here that when reflection takes place from a medium of great

nsity, a reversal of displacement occurs at reflection and nsequently the point of incidence at which the displacement is ro is a node. Thus the reflected wave will travel in the opposite rection with a **change of phase equal to** π and therefore the pe of the wave will remain unchanged *i.e.* a wave of compression ll be reflected as a wave of compression. Again when the wave reflected in the medium of less density no change of phase curs and the type of the reflected wave will change and thus the ave of compression will be reflected as a wave of rarefaction.

Stationary vibration with nodes and antinodes can be observed the longitudinal vibration of air column in organ pipes.

66. Formation of Stationary waves in organ pipes:

Case 1. Closed pipe: A closed pipe is one in which one nd is closed and the other open. When air is blown into the pipe, pulse of compression started from the open end travels down the ipe and when it reaches the closed end it is reflected as a wave f compression, for when the compression reaches the closed end ne compressed layers of air in contact with the end cannot expand owards this end but react on the adjacent layer of air behind it hich then gets compressed and reacts on the next layer and so n. Thus a pulse of compression is reflected from the closed end nd returns as a compression and similarly a pulse of rarefaction s reflected from the closed end as rarefaction.

So a continuous train of waves arriving at the closed end will be reflected and turn as a similar train only with the velocity reversed. At the instant the rst compressed part of the wave reaches the closed end, the second-compressed is just one wave-length behind it and after reflection, the reflected compressed with reversed velocity moves up the tube and meets the advancing second mpressed part of the incident wave at a distance equal to one-half wave-length m the reflecting surface. Now since the velocities of the particles of the dium due to the incident and the reflected waves are exactly equal and posite, they destroy one another and consequently the medium at that distance m the reflecting surface is at rest and becomes a node. It is to be noted here at the closed end of the pipe at which reflection takes place is a node due to reversal of velocity.

In this way if we follow the incident and the reflected waves e will get a number of nodes at a distance half wave-length apart nd between two consecutive nodes the medium rises up and down nd forms loops.

Case II. Open pipe: An open pipe is one in which the two ds are open. When air is blown through one end, a pulse, say of mpression started from one end travels up to the other end. ere the compression is reflected as rarefaction which again moves the other end to be reflected as a compression. The mechanism f formation of nodes is the same as in case I.

67. Formation of nodes and antinodes in a stationar wave train : Mathematical Treatment :

Let the two systems of exactly similar waves travelling : opposite directions be represented by

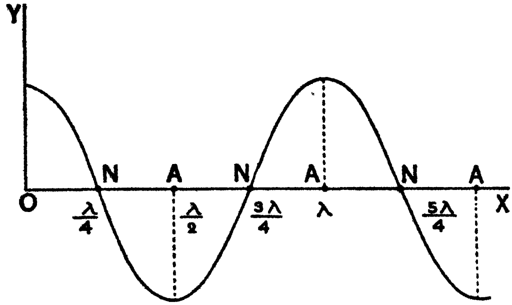


Fig. 34

$$y_1 = a \sin\frac{2\pi}{\lambda}(Vt - x) \text{ ; ar}$$

$$y_2 = a \sin\frac{2\pi}{\lambda}(Vt + x)$$

Let the resultant di placement y be equal t $y_1 + y_2$, then we hav $y = y_1 + y_2 = 2a \cos$

$$\frac{2\pi x}{\lambda} \sin \frac{2\pi}{\lambda} Vt.$$

This is the equatio to stationary wave.

Any equation in which the time factor t and the space factor : appear separately represents a stationary wave.

Amplitude of the resultant $= 2a \cos \frac{2\pi x}{\lambda}$; Phase factor $= \sin \frac{2\pi}{\lambda} Vt$.

At any instant all the particles have the same phase for th phase factor does not depend on x.

The amplitudes of the particles diminish continuously from a maximum at one place A (called an **antinode**) down to zero at the other place N (called a **node**).

(i) Position of Antinodes : For amplitude to be maximum positive or negative,

$$\cos\frac{2\pi x}{\lambda} = 1 \qquad \frac{2\pi x}{\lambda} = S\pi \quad \text{or} \quad x = S\frac{\lambda}{2}$$

where S may assume any value 0, 1, 3, 4, etc.

$$\text{when } S = 0,\ x_0 = 0$$
$$S = 1,\ x_1 = \frac{\lambda}{2}$$
$$S = 2,\ x_2 = \lambda \text{ etc.}$$

Thus the points in the medium at distance equal to 0, $\frac{\lambda}{2}$, λ, etc. from the origin have maximum displacements of value $\pm\ 2a \sin \frac{2\pi}{\lambda} Vt$. These points are called antinodes.

Again the shortest distance between two points having maximum displacements$=x_1-x_0=x_2-x_1=\frac{\lambda}{2}$.

(ii) Position of Nodes : Nodes correspond to zero amplitude for which $\cos\frac{2\pi x}{\lambda}=0 \quad \therefore \quad \frac{2\pi x}{\lambda}=(2S+1)\frac{\pi}{2}$ or $x=(2S+1)\frac{\lambda}{4}$

where S may assume any value such as 0, 1, 2, 3, 4, etc.

$$\text{when } S=0,\ x_0=\frac{\lambda}{4}$$

$$S=1,\ x_1=\frac{3\lambda}{}$$

$$S=2,\ x_2=\frac{5\lambda}{4},\ \text{etc.}$$

The shortest distance between two points which have no displacement

$$=x_1-x_0=x_2-x_1=\frac{\lambda}{2}.$$

Thus there will be alternately **nodes** and **antinodes** which will be *equidistant* and the portion of the medium between two successive nodes is called a **loop** or **ventral segment.**

67a. Velocity of a particle in a stationary wave :

We have $y=2a\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi}{\lambda}Vt$

$$\text{Velocity}=\frac{dy}{dt}=\frac{4\pi aV}{\lambda}\cos\frac{2\pi x}{\lambda}\cos\frac{2\pi}{\lambda}Vt$$

$\frac{dy}{dt}$ is **maximum**, when $\cos\frac{2\pi x}{\lambda}=\pm1$ or $\frac{2\pi x}{\lambda}=S\pi$

$\frac{dy}{dt}$ „ when $x=0,\frac{\lambda}{2},\ \lambda$, etc.

Again $\frac{dy}{dt}$ is **zero**, when $\cos\frac{2\pi x}{\lambda}=0$ or $\frac{2\pi x}{\lambda}=(2S+1)\frac{\pi}{2}$

i. e. $\frac{dy}{dt}$ is when $x=\frac{\lambda}{4},\ \frac{3\lambda}{4},\ \frac{5\lambda}{4}$, etc.

67b. Change of density at a place traversed by a stationary ave :

We have $y=2a\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi}{\lambda}Vt$

Change of density $= \frac{dy}{dx} = \frac{4\pi a}{\lambda} \sin \frac{2\pi x}{\lambda} . \sin \frac{2\pi}{\lambda} Vt$

$\frac{dy}{dx}$ is **maximum** when $\sin \frac{2\pi x}{\lambda} = \pm 1$

or $\frac{2\pi x}{\lambda} = (2S+1) \frac{\pi}{2}$

$\frac{dy}{dx}$ is when $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$, etc.

$\frac{dy}{dx}$ is **zero** when $\sin \frac{2\pi x}{\lambda} = 0$ or $\frac{2\pi x}{\lambda} = S\pi$

$\frac{dy}{dx}$ is when $x = 0, \frac{\lambda}{2}, \lambda$, etc.

Considering the displacement, velocity and change of density we find that at the nodes displacement of the particle is zero, velocity is zero, and the change of density is maximum.

That is, **at the nodes** $y=0$, $\frac{dy}{dt}=0$ and $\frac{dy}{dx}$ is maximum.

At the Antinodes : The displacement is maximum, velocity is maximum and the change of density is zero.

That is, $y = \text{max.}$, $\frac{dy}{dt} = \text{max.}$, and $\frac{dy}{dx} = 0$.

68. Progressive waves : The waves in which the character of disturbance is not fixed but always changing and at the same time moving onward are known as Progressive Waves.

In these waves the maximum displacement is the same for each particle and is reached at different instants and also the particles of the medium are disturbed in turn as the waves pass over them.

69. Distinction between Progressive and Stationary Waves :

Progressive Waves : (1) The motion of every particle in the path of the wave is exactly similar to the motion of every other particle as regards period and amplitude, though at any particular instant the motion of two different particles will generally differ is phase, or the vibration of all these particles in the wave agree in period and amplitude but differ in phase.

(2) No particle in the path of the wave is permanently at rest. At any instant the particles at the top of a crest and those at the bottom of a trough are momentarily at rest.

(3) Every region in the path of the wave becomes successively a region of compression, normal pressure and rarefaction.

Stationary Waves: (1) The vibration of the particles agree in period and phase but differ in amplitude.

(2) In the path of the wave, the particles situated at the nodes, are permanently at rest and have no displacements and no velocity and at any moment the displacement and the velocity of the particles are greater at an antinode than that at any other point.

(3) Condensation and rarefaction do not move along as in progressive waves; they simply appear and disappear again to be succeeded by the opposite condition in the same place.

(4) The period of the stationary waves is the same as the period of either of the elementary waves.

(5) The distance between two consecutive nodes or antinodes is half the wave-length of one of the wave-systems.

(6) Nodes are not places of greater average density than the rest of the air but of greatest variation of density. Each node is a point of maximum and minimum density in turn and as we pass along a series of nodes we find them as points of maximum and minimum density alternately.

Antinodes are always centres of region at normal pressure.

QUESTIONS

1. Discuss the phenomena of interference in case of sound [C. U. 1930, '50]
2. Give some examples of shadows in sound.

Explain clearly why sound shadows are not generally so well marked as those of light.

3. Show graphically the superposition of two vibrations of equal amplitude and almost equal frequency. Give a physical interpretation of it.
4. Explain the formation of beats when two notes are sounded together. [C. U. 1929, '36, '34, 46, '50, '53, '57]
5. Write a short note on combination tones. [C. U. 1955]
6. Give the theory of the formation of stationary waves in air and in other elastic media and illustrate your answer considering the different types of stationary waves with which you are familiar in acoustics. [C. U. 1930, '38, '39, '56]
7. Explain how stationary waves are produced in a closed pipe. How do they differ from progressive waves? [C. U. 1934, '37, '38, '46]

CHAPTER IX

VIBRATION OF STRING

70. Introductory : We have noticed that vibration may be maintained in a string stretched between two points with a certain amount of tension, or in a solid rod either supported at its ends or fixed at one of its ends. In a string which is considered to be very thin and perfectly uniform and flexible, it is the tension which supplies the restoring force when any part of it is displaced and in the solid rod which possesses rigidity, the elasticity of the material, not the tension, restores the successive portions of the rod to their orginal relative positions.

The string or the rod may vibrate transversely and also longitudinally. It has been found that the pitch of a note emitted when the string is rubbed along its length is higher than when it struck transversely.

71. Transverse vibration of strings : Let any point in a string stretched between two points with a certain amount of tension be struck so as to move harmonically and to execute transverse vibration. The waves produced will travel to both the points between which the string is stretched and will be reflected to the opposite ends and again reflected and return to the original point of disturbance in the same phase with the disturbance which is about to start. Thus a stationary undulation will be set up in the string between the points due to the superposition of the similar wave-systems travelling in the opposite directions along the same path and resonance will be produced.

The string thus vibrating gives out a note which continues to be of the same pitch though its intensity is gradually decreasing with its amplitude.

72. Different modes of vibration : The string stretched between two points may vibrate as a whole and the points between which it is stretched are called *nodes* or *places of no motion* and the middle portion of the string which has the greatest amount of vibration is called an *antinode* (Fig 35).

The string vibrating with a single loop or segment gives out a note which is called the **Fundamental tone** *i.e.*, a tone of the lowest pitch.

The string may also be put into different modes of vibration in which it will vibrate in 2, 3, 4 or more loops of equal lengths of segments and the notes in all these different cases will be different and higher in pitch than that of the fundamental tone.

The notes having higher frequencies are generally known as **overtones**, of which those whose frequencies are exact multiples of that of the fundamental are known as *Harmonic overtones* or **Harmonics**. The harmonic having a frequency *twice* that of the fundamental is known as **octave** or the **first harmonic.** The harmonic having frequency equal to three times that of the fundamental, is known as the **second harmonic** (Fig. 35).

Thus generally, if the harmonic has a frequency p times that of the fundamental it is known as the $(p-1)$th harmonic.

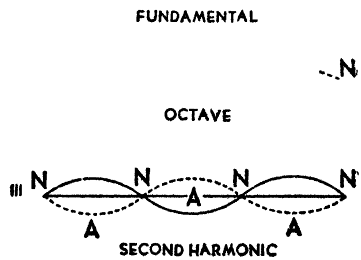


Fig. 35

73. Overtones and Harmonics : Overtones are musical sounds having frequencies which denote higher tones, whether or not they are exact multiples of the fundamental. Harmonics are the musical sounds whose frequencies are multiples of that of the fundamental.

These tones are known as **Overtones or upper partial tones.**

In vibrating strings as well as in organ pipes both closed and open, the musical sounds produced in different modes of vibration are known as **Harmonic Overtones.** Overtones which are not exact multiples of the fundamental in frequencies are known as **Inharmonic overtones.** Sounds produced from the violin, flute and bell are rich in inharmonic overtones.

Of the tones which go to build the Diatonic Scale, the tone C′ the octave which has the frequency 512 is harmonic of the tone C having frequency 256 and the tones between D to C′ are overtones which have frequencies not multiples of the lowest tone C (256). [*Consult Chapter on Musical Scale*].

The tone which has the lowest frequency is called the fundamental tone (Fig. 35. I.).

The presence of harmonics in a musical sound can be detected by a trained ear and by analysing the sound by a resonator and a manometric flame.

Any actual string offers some resistance to bending and consequently the overtones do not fall exactly in the harmonic series—each overtone is a little too high in pitch. (Fig. 35. II.)

73a. Production of Harmonics : To produce all these harmonics the string is to be struck and damped at known positions. Thus to produce, the first harmonic the string is to be struck at a point one-fourth of the length of the wire from one of the points between which the string is stretched and damped lightly at the middle point of the string. The string will then vibrate with two

segments and yield a note having a frequency double that of the fundamental tone which is produced by simply striking the string at its middle point.

The second harmonic can also be produced by striking the string at $\frac{1}{6}$th of the length of the string from one of those points and damping it at $\frac{1}{3}$rd of its length. (Fig. 33. III).

Thus the nature and the number of harmonics are determined by the mode of striking and damping the string.

Note. If the string be struck at and arbitrary position, not according to the different modes of vibration described before, it will vibrate at random and a compound sound known as **note** will be produced which is generally a mixture of the fundamental tone together with several harmonics. The presence of these harmonics determines the quality of any musical note.

73b. Suppression of Harmonics: Some of the harmonics are suppressed depending on the condition of exciting the string.

If the string is struck at the middle part of the string, the middle part can no longer be a node but an antinode. So the tones which have their nodes at the middle point will not be produced.

Thus if n be the frequency of the fundamental tone, the tones having frequencies $2n$, $4n$, $6n$, etc. *i.e.*, **even harmonics** will disappear.

Similarly if the string is struck at one-third of its length from either end, the tones having frequencies $3n$, $5n$, $7n$, etc. (*i.e.*, **odd harmonics**) will disappear.

Note: Thus if we divide the string into p equal parts and strike at one of the points of division, the pth note is absent and this is the tone whose nodes fall on the points of division. But a node for pth tone is also a node for the $2p$th, etc. tones. Hence all these higher tones are absent.

Thus when any point of the string is **Plucked**, **Struck** or **Bowed** all the harmonics having a node at the point of excitement will be absent from the resultant tone.

The *bowed string* differs from both the *plucked string* and the *struck string* in the fact that in the case of a bowed string the production of the sound is sustained and the note is continuously under the control of the performer.

74. Production of audible sound: We have noticed that when a stretched string is not fastened to a solid body the vibration of the string will, no doubt produce waves in air but the sound emitted from the vibrating string would hardly be audible from a distance. But if the same string be fixed to a wooden board by means of pegs or bridges, the vibration of the string will be communicated to the board which will then vibrate and produce air waves. The movement of the board corresponds to a movement

of the air but may not exactly correspond to that of the string but may be said to be made up of the harmonic components of the string. Thus we may say that the movement of the board has a harmonic component corresponding in frequency to each harmonic component of the movement of the string. Over and above these, movement of the board has other components which may not be present in the movement of the string.

Now if a tuning fork be sounded and placed on a wooden board on which a string is stretched the board will begin to vibrate with the same frequency as the fork and will set the string into resonant vibration if the frequency of one of its modes agrees nearly with that of the fork. This may be used to determine the frequency of in the way described before.

75. Harmonics present in the vibration when the string is struck at any distance from one end :

Let the string be of length l and extend from $x=0$ to $x=l$ and let the string be struck at a distance h from the end.

Then the displacement y at any point x of the string is by the application of **Fourier's theorem**

$$n=\sum^{n=\infty} \frac{C}{n\pi l N}\left(\sin\frac{n\pi h}{l}\sin\frac{n\pi x}{l}\sin npt\right)$$

when C is a constant, N, the frequency of the fundamental tone of the string, p the period of vibration and n, any integer from 1 to ∞.

In the particular case, when h has a value equal to $\frac{l}{2}$,

$$\text{we have } y=\sum_{n=1}^{n=\infty} \frac{C}{n\pi l N}\sin\frac{n\pi}{2}\sin\frac{n\pi x}{l}\sin npt$$

$$=\frac{C}{\pi l N}\sin\frac{\pi}{2}\sin\frac{\pi x}{l}\sin pt+\frac{C}{2\pi l N}\sin\frac{2\pi}{2}\sin\frac{2\pi x}{l}\sin 2pt$$

$$+\frac{C}{3\pi l N}\sin\frac{3\pi}{2}\sin\frac{3\pi x}{l}\sin 3pt$$

$$+\frac{C}{4\pi l N}\sin\frac{4\pi}{2}\sin\frac{4\pi x}{l}\sin 4pt+\ldots\ldots$$

Since we know that for all even values of n, $\sin\frac{n\pi}{2}=0$

$$\text{Therefore, } y' = \frac{C}{\pi l N} \sin \frac{\pi}{2} \sin {\pi x} \sin pt$$

$$+ \frac{C}{3\pi l N} \sin \frac{3\pi}{2} \sin \frac{3\pi x}{l} \sin 3pt + \ldots$$

From the above expression we see that the resultant vibration contains only the odd harmonics of frequencies 3N, 5N, 7N etc.

76. Demonstration of the mode of vibration of a bowed string : A screen with a narrow vertical slot S is placed opposite a portion of a vibrating string AB and illuminated by a strong source of light. A lens L is placed behind the slot S (Fig. 36)

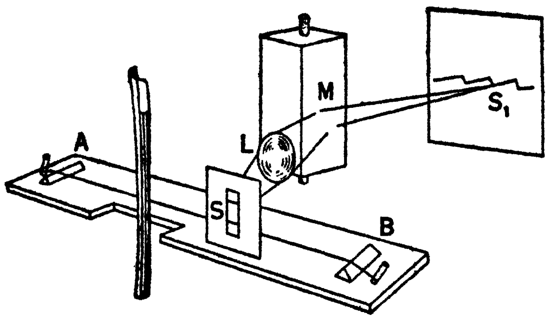


Fig. 36

and the image of it and that of the string are formed on the screen S_1, after being reflected from the rotating mirror M. If the string is not bowed, the image is a straight line, but if it is bowed vertically, the straight line becomes a zig-zag line with straight parts shewing that the vibration of the string is not simple harmonic. If the vibration be simple harmonic the image would have the appearance of a sine curve.

77. Melde's Experiments : Resonant vibration may also be set up in a string attached to one of the prongs of a vibrating tuning fork in such a way that it vibrates in a direction at right angles to the length of the string. Then by adjusting the load on the pan supported from the free end of the string, the frequency in one of the modes of stationary vibration set up along the string is nearly of the **same frequency** as that of the fork.

Again if the length of the string be kept the same and if the load on the pan be altered gradually the string will vibrate in different segments and the number of segments corresponding to each load will be different. From the experiment, Melde found that

the loads required to make the string vibrate in 1, 2, 3 *etc. loops or segments are inversely proportional to the squares of the numbers of loops.*

In Melde's experiment if N be the frequency of the fork and if the string be divided into p segments when the load m is placed on the pan, then $N = \frac{p}{2l}\sqrt{\frac{T}{m}}$

Since N, l and m are constants, we have

$p\sqrt{T} =$ a constant ; $p^2 T =$ a constant.

The experiment is repeated by arranging the fork and the string in such a way that the fork vibrates in the direction of the string. Resonant vibration is set up along the length of the string when the load of the pan is so adjusted that the **frequency** of one of the modes of vibration in the string is half that of the fork.

78. Laws of transverse vibrations of a stretched string : If a string of length l is stretched between two points with a tension T and if m be the mass per unit length of the string, the vibrations produced in the string are found to obey certain laws which are included in the formula

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2lr}\sqrt{\frac{T}{\pi\rho}} \qquad m = \pi r^2 \rho$$

where n is the frequency of vibration of the string r, the radius and ρ, the density of the material of the string.

1. **Law of length** : $n \propto \frac{1}{l}$, when T and m are constants.

The frequency *i.e.* the number of vibrations per second, varies inversely as the length when the tension T and mass per unit length of the wire are constant.

2. **Law of tension** : $n \propto \sqrt{T}$, when l and m are constants.

The frequency varies directly as the square root of the tension when the length and the mass per unit length of the wire are constant.

3. **Law of mass** : $n \propto \frac{1}{\sqrt{m}}$, when l and T are constants. The frequency varies inversely as the square root of the mass per unit length of the string when the length and the tension are constant.

4. **Laws of Radius** : $n \propto \frac{1}{r}$, when l and ρ remain constant.

The frequency varies inversely as the radius of the wire when the length, the tension and the density of the material of wire remain constant.

5. **Law of Density** : $n \propto \frac{1}{\sqrt{\rho}}$, when l, T and r remain constant.

The frequency varies inversely as the square root of the density of the material when the length, the tension and the radius of the wire remain constant.

79. **Verification of the Laws** : (a) **By Sonometer** :

1. **Law of Length** : A string is fitted on the board of a sonometer and stretched by a tension T. Two tuning forks of known frequencies *viz* n_1 and n_2 are successively sounded and the corresponding lengths of the wire in unison with the two forks are selected from the stretched wire with the help of riders. Let these lengths be l_1 and l_2. Then by the law we have $\frac{n_1}{n_2} = \frac{l_2}{l_1}$

The law is verified as the above relation is found to be true from the knowledge of n_1, n_2, l_1 and l_2.

2. **Law of Tension** : A string is stretched by a tension T_1 dynes and a length L is found out so as to be in unison with a fork of frequency n_1. The same length of the string when struck yields a note which is found to be in unison with a fork of frequency n_2 when the tension is increased to T_2 dynes. It will be seen that $\frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}}$

3. **Law of Mass** : Two strings of different material or of different diameters and having therefore different masses m_1 and m_2 per unit length are stretched by the same tension, by the side of an auxiliary wire fitted on the sonometer.

The lenghs l_1 and l_2 are found out on the auxiliary wire in unison with the sound emitted from equal lengths of the two strings. Let n_1 and n_2 be the pitch of the two notes yielded by the above lengths of the two strings. Then according to the law of mass we have to prove $\frac{n_1}{n_2} = \sqrt{\frac{m_2}{m_1}}$

Now, by the first law we have $\frac{n_1}{n_2} = \frac{l_2}{l_1}$. By actual calculation we can show that

$$\frac{l_2}{l_1} = \sqrt{\frac{m_2}{m_1}} \quad \therefore \quad \frac{n_1}{n_2} = \sqrt{\frac{m_2}{m_1}}$$

Thus the law can be verified. The law of tension can be also verified with the help of auxiliary wire, without using tuning forks.

(b) By Melde's Experiment :

(1) Law of length : $n \propto \frac{1}{l}$, when T and m are constants.

One end of a string is attached to one of the prongs of a tuning fork and the other end passes over a pulley and supports a scale pan. The prong is attached to a string in such a way that it vibrates at right angles to the string and the frequency of the string is the same as that of the fork since it moves in the same way as the fork.

The fork is then made to vibrate and loads are placed on the pan until well-defined loops are formed in the string. Let l_1 be the length between two consecutive nodes and n_1 be the frequency of the string and hence of the fork.

The fork is then turned through 90° so that the prongs now vibrate along the length of the string and for the same weight on the pan, let l_2 be the distance between two consecutive nodes in the string of which the frequency is n_2 which is half that of the fork.

Then if we can shew that

$$\frac{n_1}{n_2}=\frac{l_2}{l_1} \text{ or } \frac{n_1}{\frac{n_1}{2}}=\frac{l_2}{l_1}=2, \text{ the law is verified.}$$

(2) Law of Tension : $n \propto \sqrt{T}$, when l and m are constants.

Since the frequency of the fork is constant, the frequency of the string in any one of the above arrangements is constant. So the law of tension is verified if we can shew that $\frac{l}{\sqrt{T}}$ = a constant quantity.

In any of the above arrangments, the length of the string between two consecutive nodes for different loads on the pan are determined. Let l_1, l_2, l_3 etc. be such lengths for loads w_1, w_2, w_3 etc, on the pan. Using different tensions *i.e.* w_1g, w_2g, w_3g etc, and the corresponding lengths l_1, l_2, l_3, etc. in the expression $\frac{l}{\sqrt{T}}$ a constant value is found for all cases. Thus the law is verified. This result shows that the length for a constant frequency varies directly as $\sqrt{T}$. But the frequency for constant T varies inversely as l. Therefore, if the original length were maintained the frequency n would vary $\sqrt{T}$.

(3) Law of Mass : $n \propto \frac{1}{\sqrt{m}}$, when l and T are constants.

Since the frequency of the fork *i.e.* of the string is constant the law of mass is verified if we can shew that $l\sqrt{m}$ is a constant quantity.

When the string vibrates with well-defined loops, measure the length of a loop l for a particular load. Repeat the experiment with strings of different diameters or material but with the same load and measure l in each case. Using the values of l and m for each string the product $l\sqrt{m}$ is found to be

constant and we, therefore, conclude as in the last case that the frequency n of the string varies inversely as $\sqrt{m}$.

The mass of the string per unit length is obtained by dividing the mass of a certain length of the string by its length.

80. Nodes and loops in a vibrating string :

No. of vibrating segments	Length of each segment	Frequency of the note emitted
1	l	n
2	$\frac{1}{2}l$	$2n$
3	$\frac{1}{3}l$	$3n$
4	$\frac{1}{4}l$	$4n$
&	&	&

Thus the frequencies possible for vibration in a string, ar[e] proportional to the numbers 1, 2, 3, 4 etc.

81. Velocity of propagation of a Transverse Wave alon[g] a stretched string : If a string stretched between two points b[e] struck at any point in a direction at right angles to its length, [a] transverse wave will be produced and will move with a velocit[y] depending on the tension with which the string is stretched an[d] on the mass per unit length of the string.

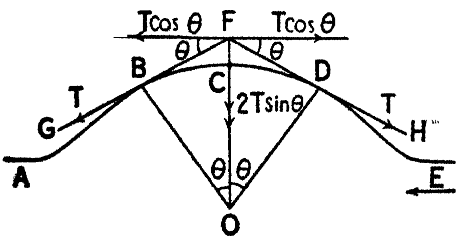


Fig. 37

Let ABCDE represen[t] a portion of the strin[g] (Fig. 37) along which th[e] transverse pulse BCD i[s] travelling from left t[o] right with a velocity V. The condition of the wav[e] is better studied if it i[s] supposed to remain in on[e] position by considerin[g] the string to be movin[g] from right to left with a velocity equal to V.

Since the velocity of the waves does not depend on its form we may consider the portion BCD of the wave as the arc of th[e]

circle of which O is the centre and of which the length is δs. If m be the mass per unit length of the string, the mass of the portion BCD *i.e.*, δs is $m\delta s$ and the centripetal force for its motion in a circle with the velocity V is

$$= \frac{m\delta s.V^2}{r} \quad ...(1)$$ where r is the radius of the circular path.

Let the radii of the two points B and D at equal distance from C on the arc, meet at O. Let the angle BOD be 2θ and since CO bisects this angle, each of the angles BOC and DOC is equal to θ.

The two tensions each of value T at the points B and D act along the tangents BC and DH respectively. These tangents when produced backwards meet at the point F on OC produced. The components of the tensions at B and D. in directions perpendicular to CO are each T cos θ, and being oppositely directed cancel each other. The components of the tensions along CO are each equal to T sin θ and hence the resultant of the tensions at B and L acts along CO and its magnitude is$=2T\sin\theta=2T\theta$ since θ is small.

$$\text{But } 2T\theta=2T.\frac{BC}{r}=2T.\frac{BCD}{2r}=2T.\frac{\delta s}{2r}=T.\frac{\delta s}{r} \quad ... (2)$$

$$\text{Equating (1) and (2)} \quad \frac{m\delta s.V^2}{r}=T.\frac{\delta s}{r} \quad \text{or} \quad V^2=\frac{T}{m}$$

$$\therefore \quad V=\sqrt{\frac{T}{m}}.$$

81(a). Alternative Method (By Calculus): Let ABC be the undisturbed position (Fig. 38) of a string in the positive direction of x fixed at the ends A and C and stretched with a tension T. Due to transverse vibration, suppose the string is laterally displaced in the positive direction of y, into the position indicated by APQC.

Let δs be a small length of the string between two very close points P and Q, so that δs in sensibly straight. The tension T acts at P and Q along the tangents drawn at the points P and Q as indicated in the figure. Let PQ make an angle θ with AX which is small for small amplitude of the wave.

The resolved part of T at P along the negative direction of x is T cos θ or T since for small value of θ, cos $\theta=1$... (1)

The resolved part of T at P along the negative direction of y is

$$T\sin\theta=T\tan\theta \text{ (since } \theta \text{ is small)}=T.\frac{dy}{dx} \quad ...(2)$$

It is found from (1) that the resolved part of T in the direction of x does not depend on x. Obviously, the component of tension

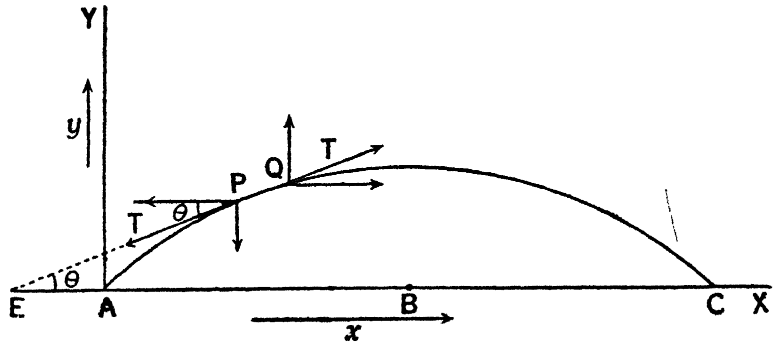


Fig. 38

at Q in the direction of x is T but it acts in the positive direction of x. The other resolved part which acts in the positive direction of y may be found as follows.

From (2) the y-component of tension at $P = T\frac{dy}{dx}$.

Then, space rate of change of y-component of T along x-axis

$$=\frac{d}{dx}\left(T\frac{dy}{dx}\right).$$

$\therefore$ The change in the y-component of tensions betwen the points P and $Q=\frac{d}{dx}\left(T\frac{dy}{dx}\right).\ \delta x$, where δx is the projection of the element PQ on the axis of x. As θ is small we many take $\delta x = PQ = \delta s$.

Hence the y-component of tension at Q

$$=T\frac{dy}{dx}+\frac{d}{dx}\left(T\frac{dy}{dx}\right)\delta s = T\frac{dy}{dx}+T\frac{d^2y}{dx^2}.\delta s.$$

($\because$ T is constant)... (3)

Now, the component forces along x-direction at P and Q neutralise each other being equal and opposite. The component force along y-direction at P and Q are equivalent to

$$T\frac{dy}{dx}+T\frac{d^2y}{dx^2}.\ \delta s - T\frac{dy}{dx} = T\frac{d^2y}{dx^2}.\delta s = \text{Resultant transverse force on}$$

length δs of the string.

If m be mass of unit length of the string, then mass of the length $\delta s = m.\delta s$. Then we have $m.\delta s.\frac{d^2y}{dt^2} = \mathrm{T}.\frac{d^2y}{dx^2}.\delta s$, where $\frac{d^2y}{dt^2} =$ acceleration of motion of the string

$$\text{or} \quad \frac{d^2y}{dt^2} = \frac{\mathrm{T}}{m} \frac{d^2y}{dx^2}$$

Comparing this with the differential equation of a wave given y $\frac{d^2y}{dt^2} = \mathrm{V}^2 \frac{d^2y}{dx^2}$, where V = velocity of propagation of the wave, Art. 24a)

$$\text{we have} \quad \mathrm{V}^2 = \frac{\mathrm{T}}{m} \quad \text{or} \quad \mathrm{V} = \sqrt{\frac{\mathrm{T}}{m}}.$$

Note 1. If we consider the motion of the element δs *i.e.* δx of string, the resultant of the two equal tensions is directed towards the centre of curvature of the element.

[For *Dimensional Proof Consult General Physics*, *page* 167(4).]

Note 2. In the expression $\mathrm{V} = \sqrt{\frac{\mathrm{T}}{m}}$, it is noticed that the velocity depends on the tension. But its variation with tension is given by the following relation.

$$\mathrm{V}^2 = \text{a constant} \times \mathrm{T}$$

Comparing this with the equation of a parabola $y^2 = 4ax$, where $4a$ is a constant, we see that the variation of V with T will be indicated by a parabolic curve.

Note 3. Again we know that $\mathrm{V} = n\lambda$, where V is the velocity of the transverse wave having λ as the wave-length and n, the frequency of vibration.

$$\therefore \quad n\lambda = \sqrt{\frac{\mathrm{T}}{m}} \quad \text{or} \quad n = \frac{1}{\lambda}\sqrt{\frac{\mathrm{T}}{m}}$$

For the **fundamental tone** when the string vibrates in one segment $\lambda = 2l$, $\therefore \quad n = \frac{1}{2l}\sqrt{\frac{\mathrm{T}}{m}}$

For the **first harmonic** when the string is divided into two equal segments $\lambda = l \;\therefore\; n = \frac{2}{2l}\sqrt{\frac{\mathrm{T}}{m}}$

So generally, if the string vibrates in p *segments* the frequency of the $(p-1)$th harmonic $= \frac{p}{2l}\sqrt{\frac{\mathrm{T}}{m}}$, where T is the applied tension and m, **the mass per unit length of the string.**

82. Longitudinal vibration of strings: In the case of longitudinal vibration of strings the particles of the string move backwards and forwards parallel to the length of the string and this vibration does not depend on the tension with which the string is stretched but on the elastic property of the string.

The velocity with which a longitudinal wave travels along a string is expressed by $V_L = \sqrt{\frac{Y}{D}}$

where Y is the Young's modulus of the string and D, its density.

For the fundamental tone to be produced in the string clamped at both ends there should be a node at each end with an antinode in the middle. In this case the wave length of the disturbance is equal to twice the length of the string. That is $\lambda = 2l$, where l is the length of string.

Again, $V_L = n\lambda$; $n = \frac{V_L}{\lambda} = \frac{V_L}{2l} = \frac{1}{2l}\sqrt{\frac{Y}{D}}$

We know that the velocity of transverse waves in a string is expressed as $V_T = \sqrt{\frac{T}{m}} = \sqrt{\frac{T}{S.D}}$

where T is the tension, S the area of cross-section and D, the density of the material of the string of unit length.

Again $\frac{T}{S} = \text{stress} = Y \times \text{strain} = Y\frac{l'}{l}$, where Y is the Young's modulus, l the original length of the string and l' the extension produced in the string by the tension T.

Therefore $V_T = \sqrt{\frac{Y}{D} \cdot \frac{l'}{l}} = V_L\sqrt{\frac{l'}{l}}$

V_T becomes equal to V_L when the string is stretched to double its length *i.e.*, when $l' = l$ which is not possible.

Hence $V_L > V_T$ ...(1)

Thus the velocity of longitudinal waves through a string is greater than that of the transverse waves travelling through the string stretched under a certain tension.

Again, if n_T and n_L are respectively the frequencies of transverse and longitudinal vibrations, we have

$$V_T = 2n_T l \; ; \quad V_L = 2n_L l.$$

Therefore from (1) $n_L > n_T$.

[T]hus the frequency of the longitudinal vibration is greater [than] that of the transverse vibration.

[2]33. Velocity of sound in a stretched string : The velocity [of s]ound produced either by the transverse vibration or the [long]itudinal vibration of a stretched string can be determined with [the] help of a sonometer.

A metal wire is stretched over two bridges on a sonometer and [st]ruck transversely or rubbed along its length so as to produce [a n]ote. The frequency of the note emitted in either case is [dete]rmined by comparison with a tuning fork of known frequency. [The] length of the wire between the bridges is then measured and [the] velocity calculated by the relation $V=2nl$.

It is found that the velocity of longitudinal waves in a stretched [strin]g is much greater than that of transverse waves in the same [strin]g

[N]ote. From the knowledge of the velocity of longitudinal waves *i.e.*, of the [value]s of n and l in the stretched string the value of Y, the Young's modulus [can be ea]sily calculated provided the density D of the material of the string is known.

For we know that $V=\sqrt{\frac{Y}{D}}$ or $V^2=\frac{Y}{D}$ or $Y=V^2D=4n^2l^2D$ since $V=2nl$.

QUESTIONS

[1]. Enumerate the laws of vibrations of a stretched string. [C. U. 1943, '48]

[2]. Explain the production of nodes and loops in the case of strings. [C. U. 1930]

3. Obtain the expression for the velocity of transverse waves in a string. [C. U. 1931, '41, '52, '59]

[4]. Prove that the frequency of vibrations of a stretched string is equal to

$$\frac{1}{2l}\sqrt{\frac{T}{m}}.$$

Explain the symbols used.

Indicate what harmonics will be present and what absent when the string is [stru]ck at the middle point.

[5]. Describe Melde's experiments and indicate how the laws of vibrations of [a st]retched string can be verified with Melde's apparatus. [C. U. 1923 '43]

6. Distinguish clearly between overtones and harmonics. Give a few [ins]tances of overtones that are not harmonics. [C. U. 1944]

Explain how in the case of a bowed string, the higher harmonics may not [be] obtained.

7. Describe an accurate method of measuring the frequency of a note of [(a)] medium pitch, (*b*) high pitch.

8. Describe Melde's experiment for determination of frequency of a tuning [fork]. Compare the vibrations excited in the string in the longitudinal arrange[men]t with those in the transverse arrangement. [C. U. 1948, '56]

EXAMPLES

1. Describe that experiment of Melde's which has for its purpose its illustra[tion] of the laws governing the vibration of strings.

The tuning fork makes 128 vibrations per second ; a mass of one kilogramme [is] placed on the scale-pan, the wire is of platinum of sp. gravity 21, the [dia]meter is ·5 mm.

What must be the effective length of the wire for a node to be formed : the middle? [C. U. 191

Let l be the length of the wire. Then if a node be formed at the middl λ, the wave-length would be equal to l.

Tension of the wire = wt. of 1 kilogramme = 1000×980 dynes

Mass per unit length of the wire = volume $\times$ density = $\frac{22}{7} \times (\cdot 025)^2 \times 21$ gms.

$$= \frac{1}{\lambda}\sqrt{\frac{T}{m}} = \frac{1}{l}\sqrt{\frac{98 \times 10^4}{\frac{22}{7} \times (\cdot 025)^2 \times 21}}$$

Now if the fork vibrates along the length of the wire, frequency of the wi will be half that of the fork *i.e.* $n = 64$ vibrations per sec.

$$\therefore \quad 64 = \frac{1}{l}\sqrt{\frac{98 \times 10^4 \times 7}{22 \times (\cdot 025)^2 \times 21}}$$

$$\text{or} \quad l = \frac{1}{64}\sqrt{\frac{98 \times 10^4}{22 \times (\cdot 025)^2 \times 3}} = \frac{1}{64}\sqrt{\frac{49 \times 10^4}{11 \times (\cdot 025)^2 \times 3}} = \frac{7 \times 100}{64 \times \cdot 025 \times \sqrt{33}}$$

$$= 76 \text{ cms. (approx.)}$$

If the fork vibrates at right angles to the length of wire, frequency of the wi is equal to that of the fork *i.e.*, $n = 128$ vibrations per second. In this case,

$$l = \frac{76}{2} \text{ cm.} = 38 \text{ cms.}$$

2. A fork vibrates along the length of a string that is attached to one pro and is stretched by a load of 41·8 grammes. If the length of the string be : cm. and its mass per unit length be ·325 gms., and if the string vibrate in segments, calculate the frequency of the fork. [C. U. 191

Length of each vibrating segments = 5 cm.; Tension of the string $= 41 \cdot 8 \times 980$ dynes $\therefore$ n the frequency of the string

$$= \frac{1}{2l}\sqrt{\frac{T}{m}} = \frac{1}{2 \times 5}\sqrt{\frac{41 \cdot 8 \times 980}{\cdot 025}}$$

$$\text{Frequency of the fork } 2n = \sqrt{\frac{41 \cdot 8 \times 980}{\cdot 025}} = \frac{1280}{5} = 256 \text{ vibrations per sec.}$$

3. Wires of equal length of brass and steel are stretched on a sonometer an adjusted to emit the same fundamental note. If the tensions in the two cas are 5 and 3 kilogram-weights respectively and the diameter of the steel wire 0·8 mm. find that of the brass wire, the densities for brass and steel being 8 and 7·8 respectively. *Ans.* $2r = d = \cdot 099$ cm. [C. U. 194

4. In a transverse arrangement of Melde's experiment the string vibrated i 3 loops when the tension is 200 gms. Calculate the tension required to make th string vibrate in 2 loops in the longitudinal arrangement. [C. U. 195

$$\text{For the transverse arrangement } N = \frac{3}{2l}\sqrt{\frac{T}{m}} = \frac{3}{2l}\sqrt{\frac{200g}{m}}$$

$$\text{For the longitudinal arrangement } \frac{N}{2} = \frac{2}{2l}\sqrt{\frac{T}{m}} = \frac{2}{2l}\sqrt{\frac{Mg}{m}}$$

Here N is the frequency of the fork, l the mean distance between consecutive nodes, T the tension and m the mass per unit length of the string.

From (1) and (2) we have

$$\frac{3}{2l}\sqrt{\frac{200g}{m}} = \frac{2}{2l}\sqrt{\frac{Mg}{m}} \quad \text{or} \quad \frac{9}{4} \times 200 = 4M$$

$$M = \frac{2 \times 200}{16} = \frac{1800}{16} = 112 \cdot 5 \text{ gms.}$$

CHAPTER X

VIBRATION OF RODS, PLATES AND BELLS

84. Introductory : While considering the transverse vibration of a string we have noticed that the tension with which the string is stretched is the only restoring force which maintains the vibration when any displacement occurs in the string. In strings, we have neglected the rigidity of the material.

In rods which are not stretched by any tension, the vibration either transverse or longitudinal is maintained by the restoring force caused by the rigidity of the material which restores the successive portions of the rod to their original positions.

85. Transverse Vibration of rods : A rod may vibrate in various ways according to whether it is unclamped at both ends or clamped at one end only.

When the rod is clamped at one end and made to vibrate transversely, the clamped end is always a node and the free end an antinode.

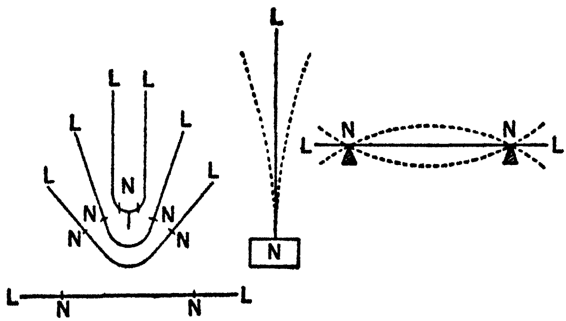


Fig. 39

There are other modes of vibrations in which 1, 2, 3 nodes may be formed between the free end and the clamped end and consequently corresponding to each mode of vibration a note is produced, and if the frequency of the fundamental tone be taken as unity, the frequencies of the overtones are 5·22, 8·87, 10·21 each. The higher notes are not therefore harmonics but overtones, being not exact multiples of that of the fundamental.

A harmoniun reed vibrates in this way.

The rod may vibrate transversely with both ends free but supported at two points near the ends. In this arrangement the free ends are always antinodes and the portions of the rod between

the two points of support which are always nodes may vibrate with one loop, two loops etc. in different modes of vibration. In this case also the higher notes are overtones but not harmonics.

85(a). Tuning Fork : The rod supported at two points near its free ends and vibrating transversely with the free ends as antinodes L, L, and points of support as nodes N, N gives out the fundamental tone (Fig. 39). If the rod vibrating in this way be bent so that the free ends approach one another, the nodes of the fundamental will be seen to approach more and more closely and to lower the frequency of vibration. The amplitude of vibration at the antinode is small compared with that at the ends of the prongs.

The addition of a stem at the bend between the two nodes has the double effect of adding mass at the antinode and of increasing the stiffness of the part of the fork between the two nodes. It will be found that the fundamental tone will come near and near and practically coincide when the two limbs become parallel. So when a handle or a stem is fitted at the place where the nodes coincide without interfering with the vibration of the rod, **the system becomes a tuning fork**. Thus when a tuning fork vibrates, its ends alternately approach and recede from one another and give out a pure tone, the frequency of which depending on the temperature of the fork. (Fig. 39)

The overtones of a fork do not belong to the harmonic series. Their pitch depends on the shape of the fork but they are always much higher than the fundamental. They die away more quickly than the fundamental and are not audible.

Note : (1) The tuning fork when sounded yields a note which is a pure tone and practically free from any overtone. Hence its importance.

(2) The fork can be maintained electrically so as to give out a tone lasting for some time.

(3) The pitch of the fork is raised if metal be removed from the ends of the prongs so as to decrease the moment of inertia of the prongs without altering its stiffness.

The pitch is lowered if metal is removed from the base where the prongs unite. This diminishes the elastic stiffness without altering its moment of inertia.

(4) The effect of rise of temperature is to increase its size and to lower the elasticity of the metal and thereby to diminish its frequency.

(5) The frequency of vibration of a tuning fork varies directly as the thickness of the prong and inversely as the square of the length of the prongs.

86. Period of vibration of the tuning fork by method of dimensions : Let us assume that the period of a tuning fork depends upon the length of the prongs and on the density and Young's modulus of the material.

Let $t \propto l^x \rho^y Y^z$ where x, y and z are unknown powers, and where t is the period, l, the length ρ, the density and Y, the Young's modulus.

The dimensions of period t is T, that of length l is L, and that of density ρ is ML^{-3} (mass per unit volume).

Also the dimension of Young's modulus Y is $ML^{-1}T^{-2}$

$$\therefore \quad T = L^x(ML^{-3})^y . (ML^{-1}T^{-2})^z = L^{x-3y-z} . M^{y+z} . T^{-2z}$$

Hence, $x - 3y - z = 0$; $y + z = 0$; $-2z = 1$

Solving these, $x = 1$; $y = \frac{1}{2}$; $z = -\frac{1}{2}$ $\quad \therefore \quad t \propto l\rho^{\frac{1}{2}} Y^{-\frac{1}{2}}$

Or $t = kl\sqrt{\dfrac{\rho}{Y}}$, K is a constant to be determined by experiment.

87. Electrical Maintenance of Vibration of Tuning Forks: The principle adopted in this method is the conversion of electrical energy into mechanical energy necessary to overcome the frictional resistance and into the energy radiated as sound.

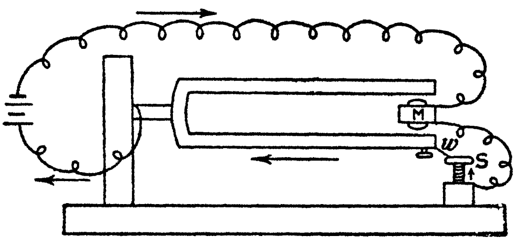


Fig. 40

The fork is clamped on a firm support which also supports an electromagent M (Fig. 40) between the prongs of the fork and a metal collar supporting a screw S. To the lower prong of the fork is attached a thin piece of platinum wire w, just touching the upper end of S when the prong is at rest.

Electric connections are as shewn in the figure.

One of the terminals of the battery is connected to the platinum wire w through the support and the body of the fork and the other terminal is connected to S through the electromagnet M.

As the electromagnet is excited, the prongs of the fork are attracted inwards, thus breaking the contact at S. The prongs then move outwards due to elastic forces and the electric connection is established between the platinum wire w and the screw S. The operation is repeated over and over again and the vibration of the fork is maintained.

During the inward motion of the prongs, the magnet is doing work on the fork and during the outward motion the fork is doing work on the magent.

If these quantities of work are equal, no energy would be drawn from the battery and the vibration would die away.

But owing to self-induction of the coil of the electromagnet and irregular contact, the rise of the current is delayed at *make* (*i.e.*, in the outward journey of the prongs) and prolonged at break (*i.e.*, on the inward journey of the prongs) so that more work is done on the fork than by the fork and the balance represents the energy available for maintenance of vibrations of the fork.

In another form, due to Helmholtz the spring contact w is replaced by a wire attached to one of the prongs and dipping into a cup of mercury. The mode of operation is practically the same as described in the former case.

In this case the balance of energy required for maintenance of the vibration is due to surface tension of mercury and the self-induction of the coil of the electromagent.

88. Longitudinal vibration of rods: As in the case of a column of air in a tube, longitudinal waves with alternate compression and rarefaction travel along the length of the rod and get reflected at the ends and thus stationary waves are set up in the rod.

When the rod is clamped at the middle point and vibrates longitudinally it gives out its fundamental tone when the middle point is a node and the free ends are antinodes.

The overtones in this case are not sufficiently important to be taken into account.

89. Vibration of Plates: If a plate, either square or circular, be clamped at one point and free everywhere else, it can be made to vibrate transversely and give out a note when bowed against its edge. By damping and bowing the plate in different ways it can be put into various modes of vibration giving out different notes. Nodal lines can be easily detected by scattering sand particles over the upper surface of the plate which are thrown away from the vibrating parts and settle down along the nodal lines.

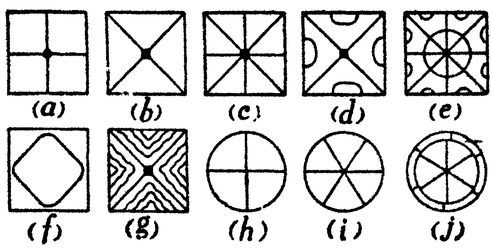


Fig. 41

The figures so produced are known as **Chladni's figures**.

(A). Square plate clamped at the centre. If the plate be bowed at one corner and clamped at the middle of one side, the

nodal lines consist of two straight lines parallel to the sides of the plate and passing through the centre dividing the plate into four segments Fig 41(*a*).

It is found that the segments divided from each other by any nodal line always move in opposite directions.

If the plate be clamped at one corner and bowed at the middle of a side, nodal lines will be formed passing through the corners and intersecting each other at the centre. Fig. 41(*b*).

(B). Circular plate fixed at the centre : In this case two classes of nodal lines are formed. The first class consists of radial lines dividing the plate into even number of sectors and the second class consists of circles concentric with the plate. Fig 41 (*h*), (*i*), (*j*)].

The frequency of a plate is proportional to its thickness and inversely as the area of the plates.

90. Vibration of Membranes : Circular stretched membranes such as the head of a drum, *tabla* etc., vibrate in various modes somewhat similar to those of a plate though the vibrations depend on the tension with which they are stretched.

Vibrations do not form a harmonic series and for this reason the notes emitted are not musical and are only used to accentuate the rhythm of the music.

A circular stretched membrane when loaded at the centre as an *Indian Tabla* produces partially agreeable notes which form a harmonic series.

91. Vibration of Bells : The mode of vibration of a bell which is simply a concave circular plate is very complex. When it is put into vibration by bowing across its edge it divides itself into an even number of vibrating sectors separated by nodal lines N_1, N_2, N_3, N_4 (Fig. 42). The sectors on the opposite sides of a nodal line vibrate radially and in opposite phases and at the nodal lines there is no radial motion ; but there is a tangential motion, for the arc of the vibrating sector on one side of a node while moving outside the normal position is longer than that of the sector on the other side of the node and moving inside the normal position.

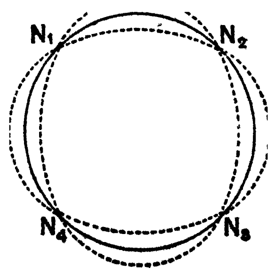


Fig. 42

The existence of the nodal lines may be tested by holding a suspended pith ball against different parts of the edge of the bell, or by filling it with water and bowing the edge, ripples will be seen to proceed from all points except the nodes.

The overtones of the bell are not exact multiples of the frequency of the lowest tone and so they are not harmonics. A number of in-harmonic overtones are produced in a bell and the bell never gives a pure tone.

QUESTIONS

1. Explain the nature of vibrations of a tuning fork. For which specia features, it is a valuable instrument for study of sound. [C. U. 1931 '47
2. Assuming that the period of vibration of a tuning fork depends upon th length of the prongs and on the density and Young's modulus of the materia find by method of dimensions a formula for the period of vibration. [C. U. 1950
3. Describe experiments illustrating the maintenance of vibrations by electricity Explain in a general way the mode of maintenance. [C. U. 1934
4. What are the Chladni's figures ? Explain how they are formed in plates.

CHAPTER XI

VIBRATION IN ORGAN PIPES

92. Organ Pipe : The simplest form of an organ pipe is a tube closed at one end or open at both ends. The ordinary organ pipe consists of a cylindrical or a rectangular hollow tube provided with a short pipe A (Fig. 43) through which air is blown into the tube BD. There is a narrow slit to the left of B through which air rushes out and strikes the sharp edge C called lip and sets up vibrations which are reinforced by the sympathetic vibration of the air column in the tube.

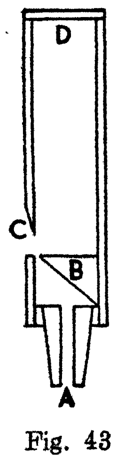


Fig. 43

The organ pipe is closed or open according as it is closed at one end or open at both ends.

In organ pipes, either closed at one or open at both ends, the vibrating medium is air and the vibration of a certain length of the medium determines the pitch of the note emitted. We know that transverse vibration cannot be set up in air and therefore the particles of air vibrating longitudinally send out waves of compression and rarefaction along the whole length of the air column, which are reflected at the opposite ends of the tube and the reflected wave thus produced will interfere with the direct wave and will produce stationary undulation in which nodes and antinodes will occur at different places.

93 Closed pipe : In a pipe closed at one end, the closed end is always a node since the motion of the vibrating particles is stopped there, and at the nodes the change of pressure *i.e.*, of density is maximum, for the particles of air at these places are alternately crowded together and separated. The open end is always an antinode since the motion of the particles of air at the open end is maximum and the change of density and of pressure is minimum.

When a disturbance is produced at the mouth of a pipe closed at one end, it strikes the edge C and sets up a compressed wave which travels up the whole length of the pipe, gets reflected as a compressed wave at the closed end and returns to the mouth which is a free and open end.

The compressed air after reaching the open end expands to the outer air through the mouth and a wave of rarefaction starts inside and moves up the pipe to be reflected again at the closed end as a wave of rarefaction. This wave of rarefaction reaching the mouth again is reflected as a wave of compression. The result is that the reflected wave interfere with the incident wave and produce longitudinal stationary waves with a node at the closed end and an antinode at the open end.

Different modes of vibration may be set up with additional nodes and antinodes between the two extreme ones by causing a greater disturbance at the mouth of the tube.

Again if a constant disturbance is produced by holding a vibrating tuning fork at the mouth of the tube, the same phenomenon will occur and resonance will be produced if the length of the air column in the tube be equal to $\frac{\lambda}{4}$ *i.e.*, one-fourth the wave-length of the disturbance *i.e.*, of the sound wave produced. By gradually increasing the length of the air column, different modes of vibration may be set up with 2nd, 3rd, 4th, etc. resonance when the column of air is divided into segments by the formation of intermediate nodes and antinodes.

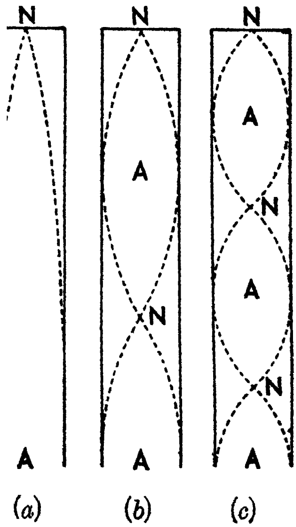


Fig. 44

93. Different modes of vibration.

(1) When the tube sounds its fundamental tone, the closed end is a node and the open end an antinode and the length of the tube is equal to $\frac{1}{4}$th the wave-length of the note emitted. [Fig. 44 (*a*)].

If n be the frequency of the fundamental tone, then

$$n=\frac{V}{\lambda}=\frac{V}{4L}.$$

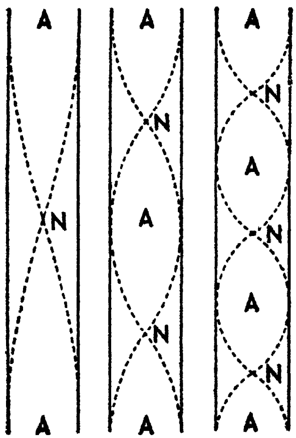
(2) When the tube is sounding its first harmonic, besides the node at the closed end and an antinode at the open end, another additional node is formed dividing the air column into three equal half-segments. Fig. 44 (*b*).

In this case $L=\frac{3}{4}\lambda_1$ *i.e.* $\lambda_1=\frac{4L}{3}$ $n_1=\frac{V}{\lambda_1}=\frac{3V}{4L}=3n.$

Other modes of vibrations may also be produced in which the ai column will be divided into 5, 7, 9 etc. equal half-yielding harmonic of frequencies equal to $5n$ (Fig. 44*c*), $7n$, $9n$, etc.

Thus in a pipe closed at one end, only the odd harmonics ar produced.

93(a). Organ pipe—Open at both ends : When a disturbanc is produced at the open end of an organ pipe it travels up and i reflected at the other end with a change of type *i.e.* a compresse part of the wave is reflected as a rarefied part and *vice versa*. Thu due to the interference between the direc and the reflected waves a stationar undulation is set up and the air colum vibrates with two antinodes at the tw open ends with a node between them.



(*a*) (*b*) (*c*)
Fig. 45

93(b). Different modes of vibra tions : (1) The tube in this mode c vibration will yield the fundamenta tone. The wave-length of the note i twice the length of tube. [Fig. 45 (*a*)].

(2) In the second mode of vibration if the air current be suitably forced o if the length of the tube be increased the column of air inside the tube will b divided into four equal half-segment with the formation of two nodes beside the two antinodes at the open ends anc the wave-length to the first harmonic is equal to L as in Fig. 45 (*b*).

Therefore $n_1 = \frac{V}{\lambda} = \frac{V}{L} = \frac{2V}{2L} = 2n.$

In this way by forcing the air current, other modes of vibration may be set up in which the air column may be divided into 6, (Fig. 45*c*) 8, 10. etc. equal half-segments by alternate nodes and antinodes.

The frequencies of the higher harmonics will $3n$, $4n$, $5n$ etc.

Thus the harmonics that are produced in an open orgen pipe, include all the harmonics, both odd and even.

Note. We know that the pitch of the fundamental note of an open pipe is octave higher than that of a closed pipe and consequently by partially closing one of the ends of open pipe the pitch is considerably diminished.

The results for both closed and open pipes are tabulated below :

KIND OF TUBE	CLOSED			OPEN		
Mode of Vibration	Fundamental	2nd	3rd	Fundamental	2nd	3rd
No. of half-segments ...	1	3	5	2	4	6
Length of half-segments	l	$\frac{1}{3}l$	$\frac{1}{5}l$	$\frac{1}{2}l$	$\frac{1}{4}l$	$\frac{1}{6}l$
Wave-length in air	$4l$	$\frac{4}{3}l$	$\frac{4}{5}l$	$2l$	l	$\frac{2}{3}l$
Ratio of frequency to the fundamental	1	3	5	1	2	3

94. Demonstration of nodes and loops in organ pipes : (1) The existence of nodes and loops in an organ pipe can be demonstrated by suspending a very thin membrane stretched on a circular ring by a string inside an open organ pipe in vibration. The membrane having some fine sand sprinkled on it is gradually lowered in the tube and when it reaches a node, grains of sand will not move at all but when it reaches an antinode where the displacement of the particles of the medium is maximun, the grains of sand will be violently agitated and jump up and down. In this way the positions of nodes and antinodes in an organ pipe can be located.

(2) The positions of nodes and antinodes in organ pipes can also be demonstrated by *Koeing's* manometric flame.

The apparatus used for the production of a manometric flame consists of a chamber (Fig. 46) divided into two compartments by a thin elastic partition, the posterior compartment is fitted with a tube for communication with the external air and the anterior chamber is fitted with two other tubes, one for the passage of the coal gas and the other for leading the gas to the gas jet where it will burn.

If the pressure of the gas inside the compartment be steady the flame is also steady and assumes a tapering form but if the pressure of the gas changes due to a variation of pressure of air in the posterior chamber, the flame will not be steady but will jump up and down and this flickering movement of the flame will not be noticed with a naked eye.

This jumping motion of the flame is beautifully observed if the image of the flame is noticed in plane mirrors attached to the four faces of a cube rotating about a vertical axis. When the flame is steady the image of flame in the revolving mirror is seen drawn out into a continuous band of light but if the

flame jumps up and down the images of the flame at successive intervals will appear like a row of teeth regularly arranged.

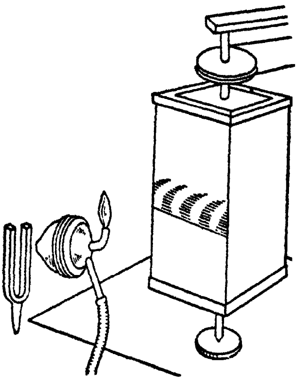


Fig. 46

Now if the tube leading to the posterior compartment of the chamber be connected to a tube fitted into a wooden board sliding over a vertical slit cut on one of the sides of the organ pipe, the flame will remain steady and the image in the revolving mirror will be drawn out into a continuous band of light when the tube fitted into the wooden board reaches an antinode, since at the antinode the change of pressure of air is minimum. Again when the same tube reaches a node where the change of pressure is maximum the flame will jump up and down, and the movement is shewn in the revolving mirror by a row of equally spaced teeth.

This experiment determines not only the position of nodes and antinodes but also the changes of pressure taking place at different parts of the vibrating medium.

From the positions of the two consecutive nodes the wave-length and hence the frequency of the note emitted is easily determined.

Koening's manometric flame can be conveniently used to demonstrate the existence of overtones in the sound emitted either from an organ pipe or from a stretched string vibrating transversely with the help of resonators.

The sonorous body emitting the sound is passed in succession in front of a series of resonators fixed on a stand and having frequencies proportional to 1, 2, 3, 4 etc. The smaller ends of the resonators are connected each with a separate manometric flame situated in front of the rotating mirrors.

Some of the resonators which agree in pitch with the overtones that are present in the sound will respond and the corresponding flames will be affected and the effect can be observed by looking at the flames in the rotating mirror.

QUESTIONS

1. Explain with the help of neat diagrams the modes of vibrations of an open and a closed organ pipe. [C. U. 1927, '28]
2. State the relations which exist between the length of the closed and open organ pipes and the pitch of the notes emitted by them. [C. U. 1928]
3. How would you experimentally demonstrate the existence of overtones in a closed organ pipe? [C. U. 1926]
4. Explain how stationary waves are produced in a closed pipe. [C. U. 1946]

EXAMPLES

1. An open organ pipe emits its fundamental note. Find its length, if the velocity of sound in air = 33000 cm. per sec., and if it vibrates in unison with the violin string of length 33 cms., under a stretching force of $7{\cdot}89\times10^6$ dynes, the mass of the string per centimetre being ·0042 gramme. [C. U. 1916]

Let l be the length of the tube. Then the length of the wave *i.e.*, $\lambda=2l$.

$$n=\frac{1}{2\times33}\sqrt{\frac{7{\cdot}89\times10^6}{{\cdot}0042}}=\frac{4{\cdot}33\times10}{2\times33}=656 \text{ vibration per sec. (approx.)}$$

Since the pipe vibrates in unison with the string its frequency of vibration is also equal to 656, Now, $V=n\lambda$. $\therefore$ $33000=656\times2l$

$$\therefore\quad l=\frac{33000}{1312}=25{\cdot}18 \text{ cm. nearly.}$$

2. The velocity of sound in air at 0°C is 332 metres per second. Find the shortest length of a tube, open at both ends, that will be thrown into resonant vibration by a fork whose frequency is 256, when the temperature of the air is 51°C. [C. U. 1915]

Let l be the length of the tube required.

Since, in an open pipe thrown into resonant vibration two antinodes are formed at the two ends and a node is formed in the middle of the tube, the wave length $\lambda=2l$.

The velocity of the sound in air at 51°C $=332\sqrt{(1+\frac{2}{273}\times51)}$ metres per sec.

$\therefore =332(1\times\frac{1}{2}\cdot\frac{1}{273}\cdot51)$ metres per sec. nearly = 363 metres per sec. nearly.

Now, $V=n\lambda$ or $36300=256\times2l$. $\therefore$ $l=\frac{36300}{512}$ cms. = 70·9 cms. nearly.

3. A closed pipe 4 ft. long full of a certain gas resounds to a given tuning fork. If an open pipe resounding to the same fork and containing air be 5 ft. long, what would be the velocity in the gas taking the velocity in air to be 1120 ft./sec.? [C. U. 1934, '46]

For a closed pipe, we have $V_g=4nl_g$ where V_g is the velocity of the sound in the gas and l_g, the length of the pipe.

For an open pipe we have $V_a=2nl_a$ where V_a is the velocity of sound in air and l_a, the length of the pipe.

n = the frequency of the fork.

Therefore $V_g=\frac{2l_g}{l_a}\cdot V_a=\frac{8}{5}\cdot1120=1792$ ft. per sec.

4. Sixty-four forks are arranged in order of increasing frequency, any one of them giving four beats per second with the one next in order. If the last fork gives the octave of the first, calculate the frequency of first. [C. U. 1934, '46]

Let N be the frequency of the 64th fork and n, the frequency of the first fork and since the common difference between the frequencies of a fork and its next is 4, we have. $N=n+(64-1)4=n+63\times4=n+252$.

But since $N=2n$; $2n=n+252$, $\therefore$ $n=252$.

5. What is the velocity of sound in a gas in which two waves of lengths 1 and 1·01 metres respectively produce 10 beats in three seconds? [C. U. 1936]

Let n_1, n_2 = frequencies of the first and second wave respectively,

V = velocity of the wave in the gas, λ_1, λ_2 = wave length of the 1st and 2nd wave respectively.

$\therefore$ We have $V = n_1\lambda_1$ or $n_1 = \frac{V}{\lambda_1}$; $V = n_2\lambda_2$ or $n_2 = \frac{V}{\lambda_2}$

But no. of beats per sec. $= n_1 - n_2 = V\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = V\left(\frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2}\right) = \frac{10}{3}$

$\therefore$ $V = \frac{10.\lambda_1\lambda_2}{3.(\lambda_2 - \lambda_1)} = \frac{10 \times 1 \times 1{\cdot}01}{3 \times {\cdot}01} = 336{\cdot}66$ metres per sec.

6. An open organ pipe produces 8 beats per second when sounded with a tuning fork of 256 vibrations per second, the fork giving the lower tone. How much must the length of the pipe be altered to bring it into accord with the fork? (Velocity of sound in dry air = 286 metres per second.)

Ans. 1·69 cm. [C. U. 1939

7. Two tuning forks A and B are sounded together and 8 beats per second are heard. A is in resonance with a column of air 32 cm. long in a pipe closed at one end and B is similarly in resonance when the length of the column is increased by one centimetre. Calculate the frequency of the forks. [C. U. 1950

Let the frequency of A and B be n_1 and n_2 respectively.

Then for a closed pipe $n = \frac{V}{4L}$

where V is the velocity of sound and L the length of the pipe

Then $n_1 = \frac{V}{4 \times 32}$, $n_2 = \frac{V}{4 \times 33}$ $\therefore$ $\frac{n_1}{n_2} = \frac{33}{32}$ Or $\frac{n_1 - n_2}{n_2} = \frac{33}{32} - 1 = \frac{1}{32}$

But since $n_1 - n_2 = 8$, $\frac{8}{n_2} = \frac{1}{32}$ $n_2 = 256$, $n_1 = 256 + 8 = 264$

8. An open organ pipe produces 8 beats per second, when sounded with fork of 256 vibrations per sec., the fork giving the lower note. How much must the length of the pipe be altered to bring it in unison with the fork? Proceed as above. [C. U. 1950

Ans. [Length of the pipe to be increased by $\frac{1}{32}$ of the initial length]

CHAPTER XII

EAR AND VOCAL ORGAN

. **Ear**: The organ of hearing *i.e.*, the ear may be divided hree distinct parts *viz.*, the external ear or *pinna* T by which waves are collected, the middle ear or *tympanum* and internal · *labyrinth* (Fig. 47)

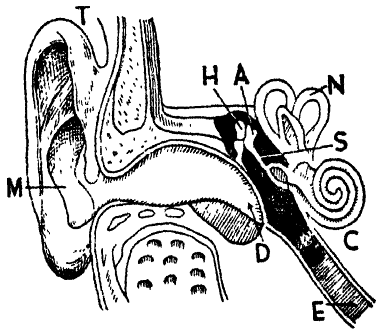


Fig. 47

m pinna T a tube leads directly s and is closed at the inner end by ched membrane or skin D which es the outer ear from the middle 'he membrane is connected by a of three small bones known as the 'r or *malleus* H, the *anvil or incus* . *stirrup or stapes* S to another ane which separates the middle ear the labyrinth. Besides these there o small pieces of bones known as *a ovalis* and *fenestra rotunda* for the ission of motion. The labyrinth is plicated structure filled with a liquid onsists mainly of three semi-circular N and a spiral-shaped *cochlea* or hell C which is separated throughout tire length by a division partly of bony and partly of membraneous re. This membrane known as *basilar membrane* is lined with extremely fibres which are the terminations of the auditory nerves.

'hen sound waves reach the tympanum it vibrates and its ments are transmitted through the linkage of bones to the l which fills the cochlea and then through the fibres of the ir membrane to the auditory nerves by which the sensation of l is produced.

5 **Helmholtz's theory of hearing**: The basilar membrane the cochlea of the human ear is lined with extremely minute which are the terminals of the auditory nerves. The brane is highly stretched across its width but unstretched tudinally.

lmholtz assumed that these fibres act as resonators, each fibre being to a particular definite frequency, so that the series of fibres would respond rations of all the frequencies throughout the range of audition.

us when the disturbance produced by a simple tone reaches the ear it throws ympathetic vibration just those fibres whose natural period agrees with that simple tone and produces the sensation of sound.

7. **Vocal Organ in man—The Human Voice**: The true 1 of vocal sound is known as the **larynx** which consists of cartilaginous plates, the vocal chords, placed at the top of a

windpipe, the *trackea*, edge to edge with a narrow slit betwee them, the other end of the pipe being connected to the lungs. Th vocal chords are connected with muscles which can vary th frequency of vibration and thereby the pitch of the sound. Th vocal chords act as a double reed and as soon as air is force from the lungs, the chords vibrate and give rise to sound. Th cavities of the mouth and the nose act as resonators. The voca chords in men are thicker than those in boys and women an consequently sound produced by men are deeper.

98. Speech—Vowel Theories : During speech two kinds o sound are produced one, the consonants and other vowel Consonants are only passing sounds and not continuous but ar simply noises and are required only to begin or end a vowel sound We distinguish between the different vowels *a*, *u* etc. in the sam way as we distinguish the sound of a violin from that of a flut Vowels are musical sounds which can be maintained indefinitel and contain a long series of harmonics, some of which ar strengthened by the resonance of the mouth.

To recognise a vowel we need not hear the beginning or the end of the soun but we are to recognise the quality of the vowel.

In order to distinguish between the vowel sounds sung on the same scale w have at present two theories, one is known as the **Fixed Pitch Theory** and th other, the **Relative Pitch Theory**.

98(a). Fixed Pitch Theory: In this theory of whic Helmholtz is one of the prominent supporters, the mouth is set t a definite size and shape for each vowel and retains that shap unchanged when a vowel is sung on any scale.

It is to be noted that some overtones of fixed pitch are characteristics for particular vowel.

It is to be noted that the pitch of one or more notes of the constituent of the vowel remains fixed when the pitch of the fundamental is raised.

98(b). Relative Pitch Theory : It states that for a give vowel the harmonic which is strengthened by the mouth is certain definite member of the harmonic series and moves up an down with the pitch of the note on which the vowel is sung always remaining at the same interval above the fundamental.

According to this theory the quality of the vowel is the same for all pitche of the fundamental as the relative strengths of the harmonic constituents remai the same.

Phonograph provides a means of deciding between the two **rival theories**.

According to **relative pitch theory** a phonograph record will give out the same vowel as was sung to it at whatever rate the cylinder or the disc rotates. For, in this case all the harmonics will rise in pitch at the same rate and the interval between them will remain constant. Thus the character of the vowel will remain unchanged.

According to **fixd pitch theory** the pitch of one or more onstituents of the vowel will remain fixed in pitch if the speed rotation of cylinder of the phonograph be the same as at the time hen it was recorded. But if the speed changes the character of he vowel will alter.

The experiment is inconclusive and no decision has yet been rrived at.

Thus to distinguish between the vowels we are to consider the llowing facts.

(1) The change of the shape of the cavity of mouth by the rch of the tongue, thereby strengthening a particular harmonic of e vowel.

(2) The change of the tension of the vocal chords thereby tering the frequency of the harmonics of the vowel.

QUESTIONS

1. Describe the mechanism of human ear with regard to the production of nsation of sound. [C. U. 1945, '47, '49,]

Explain what you know about Helmholtz's resonance theory of hearing. [C. U. 1949]

2. Explain the characteristics of the vocal organ in man. [C. U. 1924, '48]

3. What is that enables us to distinguish sounds of the different vowels hen we hear them and how do their differences arises? [C. U. 1931]

CHAPTER XIII

MUSICAL SOUND

99. Musical intervals : Scales and Temperament : To nderstand a musical scale we are to consider the relationship tween the sounds of various pitch with regard to the pleasing or ther effects, they produce in the ear. The absolute frequencies the notes are immaterial. It is the ratio of the frequencies hich the ear takes cognisance of. We have noticed that two unds when sounded consecutively or simultaneously form a easing combination when their ratio is simple.

The ratio of the frequencies of the two notes is called the **terval** between the notes.

If there are three frequencies n_1 n_2 and n_3, the interval between the first and the second is $\frac{n_1}{n_2}$ and that between the second and the third is $\frac{n_2}{n_3}$.

The interval between the first and the third note is obtained by multiplying the intervals *i.e.*, $\frac{n_1}{n_2} \times \frac{n_2}{n_3} = \frac{n_1}{n_3}$.

A combination of two or more notes when sounded together is called *a chord*. When the cord is agreeable it is called *consonance or concord and when disagreeable it is called dissonance or discord.*

Now let us consider the interval which are expressed by simple integers (small whole numbers) and convey a pleasing sensation to the ear.

The simplest frequency ratios are 1 : 1, 2 : 1, 3 : 2, 4 : 3 etc. These intervals are classed as consonant intervals and have special names assigned to them.

Unison	...	1 : 1	Fourth	...	4 : 3
Octave	...	2 : 1	Major third	...	5 : 4
Fifth	...	3 : 1	Major Sixth	...	5 : 3

Three notes having frequency—ratios as 4 : 5 : 6 when sounded together form a consonant combination and are called a *Harmonic triad* or a *Major chord.*

Three notes whose frequency ratios are as 10 : 12 : 15 when sounded together are not altogether disagreeable and form a *Minor chord.*

Any one of the notes in a major chord may be replaced by its octave without altering the consonant character of the combination.

100. Musical Scale : A musical scale or a series of notes used in a musical scale must be so chosen that the intervals between the notes are expressed by simple integers and that concords can be formed from it.

Between a note and its octave the ear recognises a number of notes of definite frequencies. Including the tonic (the lowest note) and its octave the scale consists of eight notes and is called *Gamut* or **Diatonic scale.**

The notes in the scale are represented by letters

C D E F G A B c

and are called by names

do re mi fa sol la si do

Their vibration frequencies are proportional to the ratios

$1 \quad \frac{9}{8} \quad \frac{5}{4} \quad \frac{4}{3} \quad \frac{3}{2} \quad \frac{5}{3} \quad \frac{15}{8} \quad 2$

or in whole numbers

24 27 30 32 36 40 45 48

The interval between each two consecutive notes is shown below.

C	D	E	F	G	A	B	c
1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2

$\frac{9}{8}$ $\quad \frac{10}{9}$ $\quad \frac{16}{15}$ $\quad \frac{9}{8}$ $\quad \frac{10}{9}$ $\quad \frac{9}{8}$ $\quad \frac{16}{15}$

The frequencies of the notes are given below

C	D	E	F	G	A	B	c
256	288	320	341·4	384	426·7	480	512

100a. Intervals of Diatonic Scale: The note C in the Diatonic scale is called the *key note* or the *tonic* of the scale and this may have any frequency.

The intervals between consecutive pair of notes in the Diatonic scale are of three different kinds.

The interval $\frac{9}{8}$ is called a major tone.

$\frac{10}{9}$,, a minor tone

$\frac{16}{15}$,, a semitone (*a limma*)

The difference between the major tone and the minor tone $\frac{9}{8} \div \frac{10}{9} = \frac{81}{80}$ and is called a *Comma.*

The difference between the minor tone and semitone or limma $\frac{10}{9} \div \frac{16}{15}$ or $\frac{25}{24}$ is called a *diesis*.

When a note is raised by a diesis it is said to be sharpened and when it is lowered by a diesis it is said to be flattened.

101. Musical Temperament: The Diatonic scale shews its advantages from the point of view of harmony as it combines in itself the most consonant intervals but its defect lies in the fact that the scale with a definite set of notes can only suit a music with C as the tonic.

In modern music it is necessary to use scales having different tonics. A music may begin with C as tonic but if it changes into the key of G, it will require a scale with G as tonic and the notes of the new scale for the interval between G and A in the old scale is $\frac{10}{9}$ but the interval between the next note above ought to be $\frac{9}{8}$ in the new scale.

So to use the same series of the notes for music written in different keys, some of the intervals are altered from the true diatonic intervals in such a way that the notes belonging to the scale may be used for the scale with any other key.

This process of adjusting the notes of the scale is called **temperament.**

There are various methods of temperament. In the method of equal temperament the octave is divided into 12 exactly equal intervals called Equal Temperament semitones. It is true that the

tempered scale is not a truly diatonic scale for any key, but it is equally good or bad for all the keys.

This equally tempered semitone is an interval, say $\frac{a}{b}$ which when repeated 12 times doubles the frequency of the note *i.e.* raises it to the upper octave.

That is $\left(\frac{a}{b}\right)^{12}=2$ or $\frac{a}{b}=2^{\frac{1}{12}}=1{\cdot}0595$

Thus the semitone ($\frac{1}{2}$ of a tone *i.e.* $\frac{1}{12}$ of the octave) 1·0595 in the tempered scale is slightly smaller than 1·067 in the diatonic scale.

Thus using the interval 1·059 and applying it 12 times the notes in the equally tempered scale are as follows.

C	C*	D	D*	E	F	F*	G	G*
1	1·059	1·122	1·189	1·260	1·335	1·414	1·498	1·587
				A		B*	B	c
				1·682		1·782	1·888	2

The relative frequencies of the notes in a Diatonic and the equally tempered scale are shewn below.

	C	D	F	F	G	A	B	c
Diatonic scale	1·000	1·125	1·250	1·333	1·500	1·667	1·875	2·000
Tempered scale	1·000	1·122	1·260	1·335	1·498	1·682	1·888	2·000

Thus we see that the frequencies of the notes in the two scales do not differ by as much as 1 per cent. and so the tempered scale may be used for a music beginning with any note as tonic without departing greatly from the diatonic scale.

In a harmonium or a piano the white keys give a scale not much different from the diatonic scale and the five black keys which are added give notes which differ a little from the true diatonic semitones.

102. Consonance and Dissonance : Two tones are said to be *consonant* when they produce an agreeable sensation on the ear on being sounded together. They are *dissonant* when their combination produces an unpleasant sensation on the ear.

When dealing with the musical scale we have noticed that some intervals which are expressed by simple integers are consonant, while others are dissonant.

According to **Helmholtz's theory** dissonance is a phenomenon of beats and is due to the roughness caused by them and according to the same theory the combination formed by two or more notes when sounded together is more consonant, the more it is free from beats of such a rapidity as to cause roughness.

If we start with two tones of the same frequency and gradually increase the frequency of one of them the number of beats per sec. will increase. As long as the number of beats per sec. is 6 the sensation produced on the ear begins to be unpleasant but as the frequency on the beats increases the sound becomes more unpleasant and dissonance is produced which becomes maximum when the number of beats per sec. is about 33.

If the frequency of the beats be still further increased and becomes so rapid at the ear no longer hears the alteration in intensity, no roughness is felt and e two tones again produce consonance when the number of beats per sec. is 80.

When considering the consonance or dissonance produced by ompound notes such as ordinarily occur in music, the presence of armonics of the notes must be taken into consideration, for though fundamentals may produce consonance, the harmonics of ie notes may produce dissonance.

The subject is very complicated and a complete investigation of , is not possible here.

QUESTIONS

1. Explain what you understand by the musical interval between two notes. [C. U. 1932, '35, '52]

What intervals are used in the diatonic scale? [C. U. 1932, '35, '52]

What is temperament and why is the tempered scale used in the keyed istrument? [C. U. 1932]

Write a short note on Diatonic scale [C. U. 1956]

2. What is concord and discord? Discuss the relationship between the equencies of two notes which cause them to be discordant when sounded multaneously. [C. U. 1940, '51, '53]

CHAPTER XIV

TECHNICAL APPLICATIONS

103. Demonstration of Lissajous' figures : A strong beam of light (Fig. 48) produced by a convergent lens L is allowed to fall on a mirror M_1 attached to one of the prongs of a tuning fork F_1 in

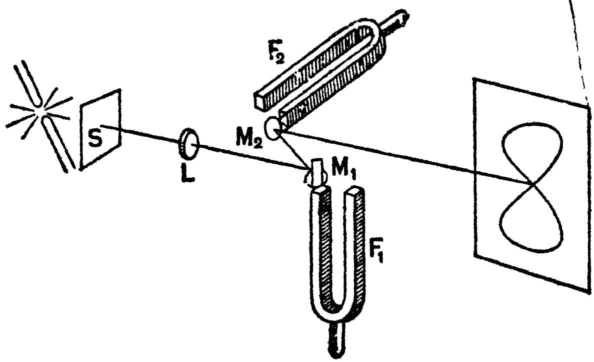


Fig. 48

such a way that after reflection from the mirror it is again reflected at a mirror M_2 attached to a second fork F_2 vibrating in a direction at right angles to that of the first fork.

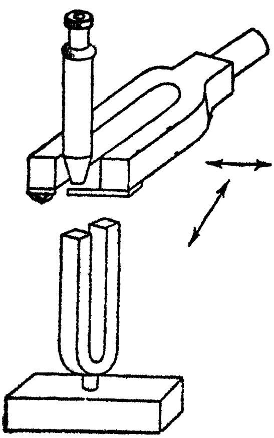


Fig. 49

When both the forks vibrate together in directions at right angles to one another the resulting motion of the beam of light will trace out figures on a screen whose shapes depend upon the frequencies and the phases of the forks. These figures are called Lissajous' figures.

104. Helmholtz's Vibration Microscope : Helmholtz devised a very convenient method of demonstrating the Lissajous' figures with the help of two tuning forks, one fixed vertically and the other horizontally (Fig. 49). At the end of one of the prongs of the fork which is fixed horizontally and whose plane of vibration is vertical an object-glass is fitted which in combination with the tube fitted with an eye-piece placed suitably above it forms a compound microscope.

A small bright dot is marked on the end of the other fork which is fixed vertically and whose plane of vibration is horizontal and viewed through the

microscope and by a careful adjustment of the amplitude and the phase of the motions of the forks, the resulting curves representing the Lissajous' figures are observed in the field of view of the microscope.

105. Lissajous' figures ; How utilised in acoustical experiments : We know that when two simple harmonic motions of the same frequency but vibrating at right angles to one another are compounded together the resultant motion is represented by a curve which remains constant so long as the phase difference between the vibrations is constant.

The form of the curve for the resultant motion will change and will assume different shapes as the phase difference changes from 0^0 through $\pi/2$, π, $3\pi/2$ to 2π. So when the ratio of the frequency of component vibrations is 1 : 1, a set of curves for the resultant motion will be obtained for different phase differences ranging from 0° to 2π.

Phase Difference 0 $\frac{\pi}{2}$ π $3\frac{\pi}{2}$

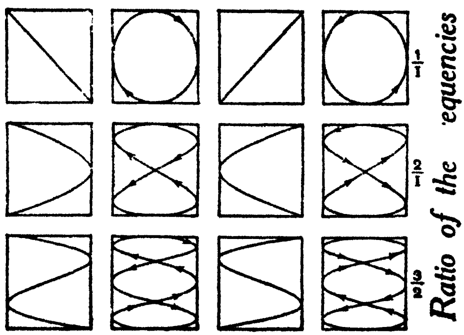


Fig. 50

Different set of curves will be obtained for the same range of phase difference *i.e.*, from 0^0 to 2π when the ratios of the frequencies of the component vibrations are 2 : 1 and 3 : 2 etc.

But if the frequencies of the component vibrations are not exactly commensurable *i.e.*, are not expressed by simple numbers and therefore the ratio of them does not correspond exactly to either $\frac{1}{1}$, $\frac{2}{1}$ or $\frac{3}{2}$ but differs by a very small amount, the figures for one set will gradually change, go through the series and return to its original shape when one fork has executed exactly one complete vibration more than the other. This gives a very accurate method of determining the difference between the frequencies of two vibrations.

If m and $n+\delta$ are the frequencies of the two forks where δ is a small quantity and if these two forks vibrate together in directions at right angles to one

another, the two motions will combine and form figures corresponding to tl ratio $m : n$ and the whole series of figures will be gone through in time Thus one fork makes one complete vibration in time t more than the other, the frequency of one is $\frac{1}{t}$th of vibration greater than the other. That is, $t\delta =$

Hence by noting the time t we can calculate δ and thus the true ratio betwe frequencies m and $n+\delta$ of the forks is determined.

106. Edision's Phonograph. This is an instrument whic not only records the sound it receives but also reproduc them. A wax cylinder C (Fig. 51) is driven by means of som clock-work arrangement in such a way that as the cylinder rotate it also moves endwise. The source, whose sound is to be recorde is placed before the horn F which i closed at its lower end by a thin glas or mica diaphragm D with a shar metallic style S rigidly fixed to th centre. The diaphragm with the shar cutting edge cuts a trace in the wax i the form of a helix running from or end of the cylinder to the other. Th depth of the cut in the cylinder is nc uniform but varies from part to par of the cylinder and this variation in th depth corresponds to the motion of th diaphragm caused by the sound wave falling upon it. This wax cylinder thu records the vibration of the diaphragr and is called a phonograph record.

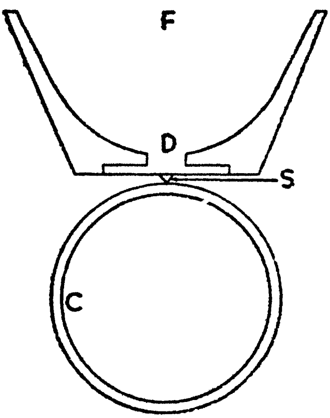


Fig. 51

The diaphragm with the cutting styl is then replaced by the *reproducer*, know as the **sound box** in which the diaphragm is provided with a blun needle point. The blunt point is caused to travel along the cut i the *cylinder* as it is rotated and the diaphragm is made to repea the movements by which the 'cut' was originally made and in doin so the original sounds are reproduced.

Defects :

(1) The wax cylinder on which the impressions are made by the cutting sty is made up of a very soft material and so it does not last long.

(2) The grooves in the cylinder are of varying depth and so the reproducin style with the diaphragm during the motion presses heavily on the wax an consequently the reproductions of the vibrations are not strictly faithful.

107. Gramophone : The improved form of the phonograph i the Gramophone in which the defects of the older type have bee overcome to a large extent.

In the new model the records are in the form of flat discs i which the grooves run from the circumference to a place near th

centre. These grooves instead of varying in depth contain transverse *zig-zag* curves with varying width.

In this case the needle or style instead of moving up and down moves from side to side in the grooves of the records.

Note. The records which are sold in the market are copies of the record which is an *electro* on a metal plate of the original record which is composed of a mixture of different varieties of wax and some other substances.

The modern method of recording sounds is electrical and requires fuller treatment.

At the time of recording, sound is produced in front of a microphone and the style is attached to the diaphragm of a telephone receiver.

During reproduction of sound the sound box is provided with a pick-up which modulates a feeble current as the style moves through the grooves and amplifies the sound with the help of radio-valves.

108. Telephone : The complete arrangement for transmitting sound from one place and receiving the same at a distant place consists of two instruments, the *transmitter* and the *receiver* connected together by a pair of wires.

In the transmitter the sound-waves cause vibration in a thin soft iron diaphragm D placed in front of a steel magnet M (Fig. 52) round one end of which is fixed a thin flat coil C of fine insulated copper wire so that it can vibrate freely.

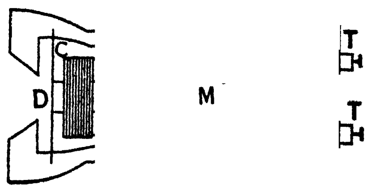


Fig. 52

The movement of the disc backward and forward alters the magnetic field and causes currents to flow through the coil and through the line wire to the coil of the distant telephone *i.e.*, the receiver in which the diaphragm is set to vibrations similar to those of the diaphragm in the sending station and similar sounds are produced.

Originally the Bell Telephone was used both as a transmitter and as a receiver but now improved forms of transmitters have superseded it for long distance transmission. As a *receiver* the Bell Telephone is universally used with some alterations in form.

109. Microphone as transmitter : Hughes' Microphone : It consists of a carbon rod with pointed ends which rest loosely and vertically in cups hollowed out in two blocks of carbon supported horizontally and parallely on a vertical stand.

These blocks are connected to the terminals of a battery and in the circuit a Bell's telephone is included.

When a sound is produced before the microphone, the rod vibrates between the two cups and causes a variation of resistance in the circuit by varying the pressure at the loose contacts and consequently due to a change of current, produces a variation

in the magnetisation of the magnet in the telephone and the original sound is reproduced.

In improved forms of microphones single carbon contacts are not used as were used in Hughes Microphone but granulated carbon is placed between two thin carbon plates, one C_1 at the front and the other C_2 at the back. The centre of the front carbon plate C_1 is rigidly fixed to the middle of the steel diaphragm D in front of which there is a mouth piece E. T, T are the terminals connecting the diaphragm and the carbon plate C_2. The movement of the diaphragm due to sound-waves produces changes in the resistance of the carbon contacts and causes a corresponding variation of electric current passing into the receiver.

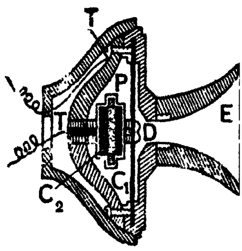


Fig. 53

110. Receiver—The Bell Telephone : The single bar magnet in the original Bell Telephone has been replaced by a horse-shoe magnet each prong of which nearly but do not quite touch the soft iron diaphragm placed in front of it. A flat coil of many turns of fine wire is wound round each pole-piece and the coils are connected in series with the line connecting the transmitter.

111. Action of Phonograph and Telephone : The action of the phonograph is to record permanently the sound or rather the vibration in especially prepared cakes or cylinders and to reproduce it when desired but the action of the telephone is to transmit sound to a distant place by means of wires connected to the transmitting and the receiving sets.

In telephone sound is reproduced immediately after the original sound is produced.

In both these instruments the receiving diaphragms are first put into vibration and then the exact reproduction of the sound is obtained by causing the reproducing diaphragm to vibrate in exactly the same manner as when the original sound was produced.

Both these instruments, the phonograph and the telephone work on different principles. The action of the phonograph depends entirely on some mechanical action whereas the action of the telephone depends not only on the mechanical action but also on the induced current due to the mechanical action.

112. Loud-Speaker : There are various types of loud-speakers such as Moving Iron type, Moving Coil type, Induction type etc. Of these the Moving Coil type is usually used in radio receiving sets and in a large gathering.

The description is given below. In fig. 54, a coil of wire CC having a large number of turns known as the *speech or voice coil* is attached to the conical diaphragm D and is free to move in the gap of the pot magnet NS. The magnet may be either an electro-magnet or a permanent magnet and the diaphragm may be made of stiff paper or of aluminium of radius 10 to 15 cms.

ttached to a flexible ring-type support E. At its periphery the diaphragm is hed to flexible support F which in turn is connected to a frame G.

A ction. The speech current from the microphone after amplification by suitable amplifiers is passed through the voice coil CC of the loudspeaker and the varying currents in the coil cause it to oscillate at the same frequency in the magnetic field and the diaphragm to which it is attached vibrates in the corresponding manner and generates sound waves.

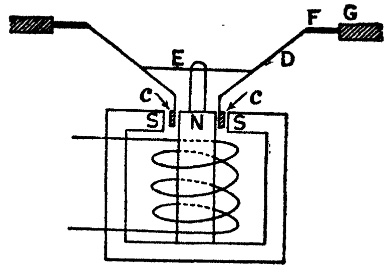


Fig. 54

113. Vibration maintained by heat: It has been found that a periodic supply of heat to a body causes it to vibrate and the

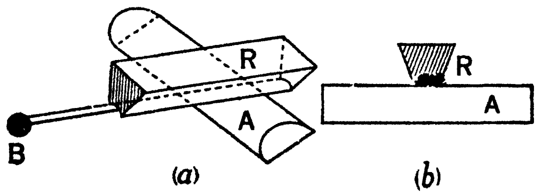


Fig. 55

body is found to emit a musical note of definite frequency.

Trevelyan Rocker. The Trevelyan Rocker R (Fig. 55) consists of prism of brass almost triangular in secion but one edge is taken away and a groove is made in the small flat thus produced.

When working, the prism rests with its groove downwards on a block of lead A with a rounded top. The end of the rocker is carried by a rod terminating in a ball B which rests on a table.

When heated, one of the edges of the groove comes in contact with lead and this expansion will tilt the rocker to the other edge. The portion of lead under the second edge expands and causes the rocker to tilt back to the first edge. This process is repeated and the rocker is in vibration and a musical note is produced.

114. Singing Flame: If a jet Hydrogen gas be placed within a vertical tube open at both ends a musical sound is produced which will continue so long as the gas remains alight.

These observations indicate that stationary waves are set up in the supply tube as well as in the surrounding tube.

This note corresponds to the resonant frequency of the pipe. This effect may be produced with other gases but to a much less degree.

It has been experimentally determined that the flame is in vibration and the emission of gas from the supply-tube is not uniform but intermittent and consequently the supply of heat to the air contained in the surrounding tube is also intermittent.

In has also been found that the length of the gas supply-tube determines the notes which can be obtained with a given flame.

115. Sound Ranging : This is primarily a 'war method' and has been successfully applied to determine the exact position of the enemy gun from the sound of discharge.

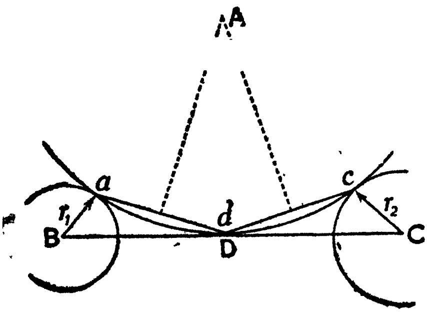


Fig. 56

For this purpose, three or more receivers, each provided with a hot-wire microphone are mounted at known positions on surveyed base lines.

As the gun is fired, explosion wave passes over the receivers placed at the stations B, D and C at the instants t_1 0, and t_2 respectively.

Suppose D is the nearest of the three receivers to the gun. So the sound will reach B later than it reaches D by an interval t_1. Similarly the sound will reach C later than it reaches D by intervals t_2.

If V be the velocity of sound in the medium and if circles of radii r_1 or Vt_1 and r_2 or Vt_2 with centres at B, C are drawn, the gun must therefore be at the centre A of a circle of which the circumference passes through D and touches the small circles at B and C.

116. Supersonic Sounds : Certain crystals *e. g. Tourmaline, Quartz,* Rochelle salt etc. exhibit electric charges on the opposite faces when they are subjected to stresses at suitable points. The electricity thus developed is called **Piezo-electricity** and was discovered by J and P Curie.

Conversely, mechanical strain appears in the crystal when it is placed in an electric field *i.e.*, when its opposite faces are placed in contact with two metal plates connected to the terminals of a battery, and in consequence elastic deformations (linear extension and contraction) are developed in it.

If the field be an alternating one, the quartz wil be subject to oscillations and will thereby undergo linear extensions and contractions at the frequency equal to that of the applied field. The crystal is then said to undergo mechanical vibration having a frequency much above 30,000 and hence beyond the range of audible frequencies. Sound produced by such a high frequency is known as **Supersonic Sound.**

If one face of the crystal be kept fixed the other will move to and fro owing to the changes in thickness. If this face is in contact with air, water, oil or

ny other fluid, waves will be generated in the fluid and these waves are known s **Supersonic waves**.

116(a). Applications of Supersonic Waves :

Supersonic waves can be used for a large variety of purposes.

(1) They are used for depth sounding and for determining the positions of sunken obstacles.

(2) They have been used to produce a directed sound beam.

(3) Telephony under water along a supersonic beam has become possible by using supersonic waves as carriers.

117. Talking Picture : The talking picture involves two distinct processes *viz recording on film* and *reproduction from the film.*

Recording of Sound : During recording of sound on film two methods are usually employed of which (1) *Variable density method* and (2) *Variable width method* need special mention.

(1) *Variable density method.*

In variable density method the illumination from a lamp fills the whole width of the sound track but its intensity is varied so that the record varies in density. The slit whose image is projected on the film is illuminated with light of variable intensity.

The intensity may be modified by modifying the source of light or by modifying the beam after it has left the source.

The mercury arc may be used for modifying the source of light. But a variable slit or light valve is essentially an electromagnetic shutter and consists of a narrow slit between two strips of duralumin ribbon, the strips carry the amplified microphone current and are placed in a strong transverse magnetic field.

The strips are subjected to electrodynamic forces tending to open or close the slit and depending on the microphone currents.

(2) *Variable-width method.*

In this system, a constant source of light illuminates a variable width of the slit whose image is projected on the moving film.

A narrow beam of light of constant intensity is reflected from an oscillograph mirror on to the slit. The oscillograph may be a wire loop having a mirror attached to it and it is suspended with the plane of the loop parallel to the lines of force of a strong magnetic field. Amplified speech currents flow through the oscillograph and cause the mirror to vibrate according to the current variations.

Variable width recording gives negatives of uniform density but having the appearance of a serrated edge.

In this way recording of sound on film is produced.

Note : The sounds from the speaker are received by a sensitive microphone amplified, and current variations are used to fluctuate the light emitted from a special form of lamp. The light is focussed on a fine slit placed close to the cinema film and occupying a width of ·1 inch of the film.

Reproduction from film: In reproduction from sound film which runs along one side of the picture passes through the sound gate and light from an electric lamp is focussed on the film by

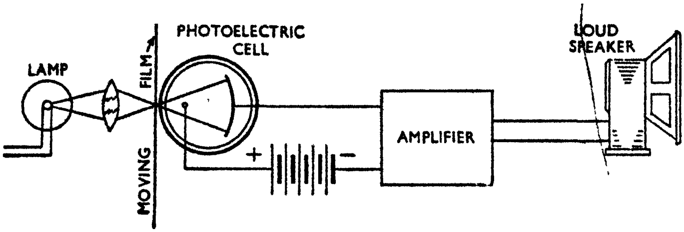


Fig. 57

lens. Having passed through the film, the light falls on a photo-electric cell, the illumination of which depends on the density or the width of sound film.

The varying current produced from the cell is amplified by an amplifier and passed on to the loudspeaker which is mounted behind the screen on which the moving pictures are projected, for reproduction of speech or music.

QUESTIONS

1. What are Lissajous' Figures? How would you demonstrate them by Helmholtz's vibration microscope?

Show how they may be practically utilised in acoustical determination. [C. U. 1921

2. Write notes on Trevelyan rocker and explain its action.

3. Describe an apparatus by which vibration of a body can be maintained by heat.

4. Describe the construction and operation of a gramophone. [C. U. 1940, '50

5. Compare the actions of the phonograph and telephone. [C. U. 1924

6. Describe the construction of a Loud-speaker and its use. [C. U. 1946

7. Write a short note on the talking pictures. [C. U. 1944

LIGHT

CHAPTER I

INTRODUCTORY

1. **Optics:** The science of Optics deals with the nature and properties of light and vision. Light, which is a form of radiant energy is the external physical cause of sensation of vision or light. The vision of an object presented one of the most intricate problems to the earlier investigators. The earliest investigations regarding the nature and behaviour of light were most probably made in ancient Egypt. The work of the Egyptians was then taken up by the ancient Greeks. A rapid survey of the work on optics extending over a few centuries is given below.

2. **Development of Optics:** Pythagorus in 540 B.C. postulated the theory that vision was caused by particles projected from the object as emanations, into the pupil of the eye. Plato (430 B.C.) maintained the view that vision was effected by means of something emitted from the eye itself, which after combining with something else emanating from the objects caused the sensation of sight. It is believed that Platonic school formulated two of the fundamental facts of light namely, rectilinear propagation and laws of reflection of light. Aristotle and his followers contended that light was not a material emission from any source but simply a quality of a medium. Archimedes in 287 B.C. constructed huge mirrors with which he employed the sun's rays to burn the Roman fleet, by reflection. The Egyptian astronomer Ptolemy (100 A.D.) first mentioned about refraction which is the bending of light when it travels from one transparent medium into another.

No material progress in the science of light was made for over a period of one thousand years. Roger Bacon (1214 A.D.) published a book in which properties of mirrors and lenses were explained. During the period from 1270 A.D. to 1611 A.D., *Vitellio*, *Copernicus*, *Galileo* and *Kepler* made important contributions to the science of optics including the construction and use of telescopes for astronomical work. Snell (1621 A.D.) discovered the laws of refraction, but the discovery was published by Descarte's in his own name in 1626 A.D. after Snell's death. Romer in 1676 A.D. first measured the finite velocity of light from astronomical observations. At this period, Newton made many spectacular discoveries in the

domain of optics. He was an exponent of corpuscular theory of ligh and he first showed the composite nature of white light from su or other sources. Bradley in 1728 A.D. discovered the phenomenon c aberration of light. Huygens in 1678 A.D. first propounded the wav theory of light. The double refraction of light in calcite crysta or ice-land spar was observed by Bertholinus in 1670 A.D. The wav theory of Huygen was established on a sound basis by th contributions of Young and Fresnel in the first part of the 19th century.

Clerk Maxwell in 1856 A.D. purely from theoretical investigation discovered the electromagnetic nature of light waves, which wa confirmed experimentally by Hertz in 1888 A.D.

Planck in 1901 A.D. postulated his Quantum theory according to which light is considered to be emitted in bundles or quanta in a discontinuous way, to interpret many complex problems about light and other radiations, and this led to the revival of the abandoned corpuscular theory in a new form. Eienstein in 1905 A.D. in his *Relativity theory*, stated that the velocity of light is the maximum limit of velocity of any wave or a particle. Compton in 1925 A.D. showed that the *photon* which is a quantum of light can collide like elastic balls with matter. De Broglie in 1924 A.D. announced that beam of particles like atoms or electrons under necessary experimental conditions should act like a stream of light waves, the wave-length depending on the velocity and mass of the particles. Thus, so far as the present knowledge of light and radiations goes, we can assert that light has a dual nature, behaving both as waves and particles.

3. The Cause of Light; Certain Fundamental Concepts: Light owes its origin to the vibrations of molecules and their fundamentals, *i.e.* electrons, of a luminous body. It is transmitted by means of waves in the hypothetical medium called the luminiferous ether. Unlike the sound waves light waves can travel in vacuum. The wave length of light waves is extremely short being of the order from $\cdot 4 \times 10^{-4}$ to $\cdot 8 \times 10^{-4}$ cm.

Bodies which themselves produce light are called self-luminous. Many bodies can reflect light coming from self-luminous objects and thus become themselves luminous. The sun is self-luminous, but the moon is made luminous by sunlight falling upon it.

Li ht itself is invisible, since light simply consists of undulations in an invisible medium. When sunlight enters through a small window into an otherwise dark room, the flash of light makes us think that we are seeing light. What is actually seen is not the light itself but a large number of dust particles in air illuminated by sunlight.

A medium is any substance which may be in the way of light. Bodies like air, water and glass which permit light to pass through more or less completely are transparent media. Bodies like wood, metal, etc. which permit little or no light to pass through them are opaque media. Bodies such as wax, ground, glass, etc. which transmit light partially but do not enable one to see clearly through them are translucent media. A medium is said to be *homogeneous* when its every part has same uniform structure, properties and composition. A medium which is not uniform throughout is called *heterogeneous*.

The term *ray of light* is applied to the straight path along which light travels in any direction from a point of a luminous object. As light consists of waves, a ray has really no physical existence but the idea of ray is helpful since it enables us to interpret many phenomena in light. A *beam* of light is an assemblage of adjacent rays of light, and may be parallel, divergent or convergent, *i.e.*, the constituent rays may run parallel, diverge from a point or converge to a point. A parallel beam is obtained from a luminous source at a great distance ; the sun's rays received on earth are parallel. Any ordinary source of light such as a candle flame produces a divergent beam of light. With the help of mirrors and lenses divergent rays coming from a luminous source can be rendered *convergent* or parallel.

CHAPTER II

RECTILINEAR PROPAGATION OF LIGHT AND REFLECTION

4. Rectilinear Propagation of Light : In a physically homogeneous medium a ray of light travels in a straight line. The formation of shadows, occurrence of the solar and lunar eclipses, formation of images by pin-hole camera, etc., are all due to this rectilinear mode of propagation of light. It will be seen later on that the rectilinear propagation of light is only approximately true.

5. Reflection of Light ; Laws of Reflection : When a ray of light passes from one homogenous medium into another, the path of the ray is bent on incidence at the interface between the media, a portion of light turns back and is said to be reflected into the medium in which the light was originally moving. The reflected ray obeys two laws called laws of reflection.

Note : The remaining portion of light goes into second medium and is said to be refracted.

5(a). Laws of Reflection :

(*i*) *The incident ray, the normal to the surface at the point of incidence, and the reflected ray lie in one plane.*

(*ii*) *The angle of incidence is equal to the angle of reflection.*

Note : The laws of reflection do not depend on the wave-length or colour of the incident light. The amount of light reflected from a surface depends on (a) Condition of the surface, (b) angle of incidence and the wave-length of incident light.

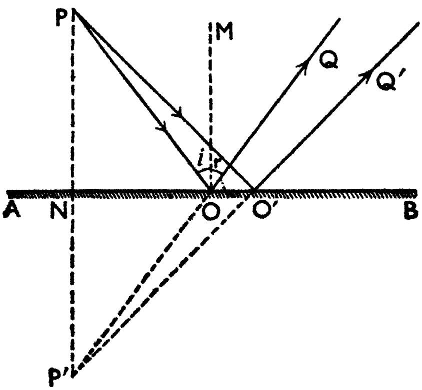


Fig. 1

6. Deviation by Reflection : As the angle of incidence and angle of reflection have same value i, then deviation which is the angle between the incident and the reflected ray is given by $D = \pi - 2i$ (Fig. 1).

6(a). Fermat's Principle of least Path or least Time : The principle states that a ray of light in passing from one point to another by way of either a reflecting or a refracting surface traverses a minimum path or takes a minimum time. This principle is true for plane surfaces only.

6(b). Case of Reflection Let a ray PO (Fig. 2) start from the point P, get reflected at O and reach the point E. The path PO + OE would be less than any other adjacent path such as PQ + QE for which the laws of reflection are not obeyed.

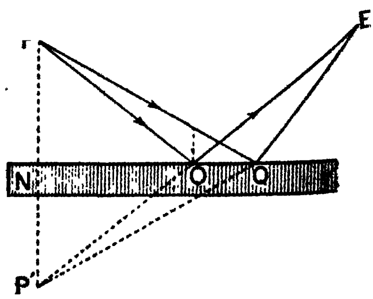


Fig. 2

In the figure 2, P′ is the image of P and lies on EO produced. Join PP′ cutting the mirror at N and join QP′.

Then $OP = OP'$; $QP = QP'$

$\therefore \; PO + OE = P'O + OE = P'E$;

and $PQ + QE = P'Q + QE$

Then in the triangle P′QE,

$$P'E < P'Q + QE, \text{ i.e. } PO + OE < PQ + QE$$

That is, the actual path traversed by the ray during reflection is less than any other adjacent path.

This proves the principle of least path and also the principle of least time since the velocity of light in a homogeneous medium is constant.

7. **Path of ray is reversible** : If a ray of light starting from a point P reaches a second point Q after one or more reflections or refractions along a certain path, then the source being placed at Q, the ray will travel back to P by the same path in the reversed direction, all other conditions remaining unaltered. Thus path of ray is reversible.

8. **Image of an object** : An image is an optical counterpart of an object formed by reflection or refraction of rays coming from the object. When a pencil of rays coming from a point source of light suffers a change of direction due to reflection (or refraction) such that the reflected (or refracted) pencil actually converges to, or appears to diverge from a second point, then the second point is called the image of the first point. We usually distinguish between two types of images :—(*a*) *Real image* and (*b*) *Virtual image.*

8(a). The **real image** : A definite point Q is said to be the real image of another point P when a bundle of rays diverging from P actually passes through the point Q after reflection or refraction.

A real image can be received on a screen, and can be seen by an eye placed in the path of the beam diverging from the image. A real image is always inverted and it may be magnified or reduced. A concave mirror or a convex lens can form real image.

8(b). The **virtual image** : A point Q is called the virtual image of another point P when a bundle of rays diverging from P appears after reflection or refraction to diverge from Q.

A virtual image can not be received on a screen, but can be seen by an eye placed in the path of the beam of rays appearing to diverge from the image. A virtual image is always erect and its size is sometimes greater and sometimes smaller than that of the object. A virtual image can be formed by mirrors and lenses of all types.

8(c). **Real and Virtual object** : If the rays of light forming an image originally come diverging from a point P actually, the point P is called real object. If the rays appear to come diverging from P it is then called a virtual object.

9. **Moving Mirror** : An object situated in front of a plane mirror forms an image at the same distance behind the mirror. But if the mirror moves parallel to itself through a distance d (the object remaining fixed) the image moves through a distance $2d$.

10. Parallel Mirrors : A luminous object P situated between two plane mirrors KL and MN (Fig. 3) and having their reflecting faces turned towards each other will form infinite trains of images behind both the mirrors by successive reflections.

These images are divided into K-series and M-series according as the first reflection takes place at the mirror K or at the mirror M.

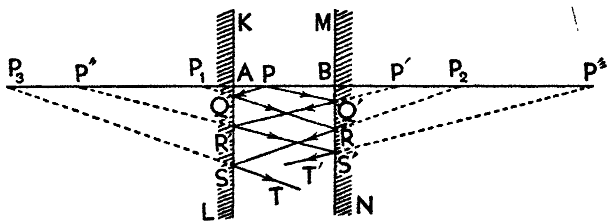


Fig. 3

Let the distance of the object P from A and B of the mirrors KL and MN be a and b respectively and c, *i.e.*, $a+b$ be the distance between the mirrors.

In the K-series all the odd numbers lie to the left of the mirror KL and the even numbers to the right of the mirror MN.

K-Series : Let P_1, P_2, P_3......P_{2n}, P_{2n+1} be the positions of images behind the mirrors.

Then $PP_1 = 2a$; $PP_2 = 2a + 2b = 2c$; $PP_3 = 2c + 2a$;

... ...

$PP_{2n} = 2nc$: $PP_{2n+1} = 2nc + 2a$.

M-Series : Let P', P'', P'''..........P'_{2n}, P'_{2n+1} be the positions of the images behind the mirrors of which the odd numbers lie to right of MN and the even numbers to the left of KL.

Similarly we have $PP'_{2n} = 2nc$; $PP'_{2n+1} = 2nc + 2b$.

N. B. The number of images is limited for light is reduced at each successive reflection.

11. Inclined Mirrors : If a luminous point be situated between two mirrors A and B inclined to each other (Fig. 4) at an angle θ, a number of images is formed by successive reflections and the images are divided into **A-Series** and **B-Series** according as the first reflection takes place in A or B.

In the **A-Series** the images are formed at P_1, P_2, P_3 etc. and in the **B-Series** the images are formed at P', P'', P''', etc., by successive reflections from the mirrors A and B.

All the images lie on the circumference of the circle having the centre at O the point of intersection of the mirrors and radius equal to the distance of the source from the point of intersection.

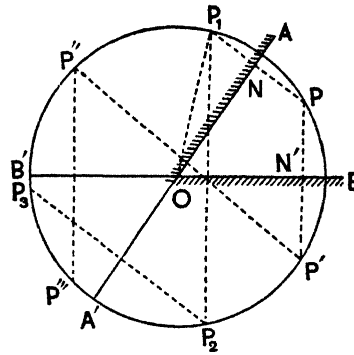


Fig. 4

It is to be noted that there is a limit to the number of images formed. The last numbers of either series is that image which is formed behind the reflecting surfaces of both the mirrors.

It can be proved that if the angle between the mirrors be θ or a submultiple of π the total number of images formed by successive reflections at the two mirrors is given by $\left(\frac{2\pi}{\theta}-1\right)$.

But if θ is not a submultiple of π the total number of images is an integer next greater than $\left(\frac{2\pi}{\theta}-1\right)$.

12. Rotating Mirror : It can be easily proved from the laws of reflection that when a plane mirror is rotated through any angle θ, the reflected ray is rotated through twice the angle, *i.e.*, through 2θ.

This principle is utilised in measuring a small deflection of a suspended system capable of rotation about a vertical axis.

A small mirror is attached to the suspended system and the image of a horizontal scale reflected in this mirror is viewed by a telescope.

When the mirror is rotated through an angle θ; the image moves through 2θ which is observed by the shift of the scale division in the telescope.

If α be the displacement or rather the shift of the scale division and D the distance between the mirror and the scale, then $\tan 2\theta = d/D$, where θ is the angle through which the mirror is rotated.

Again, when θ is small $\tan 2\theta = 2\theta = d/D \quad \therefore \quad \theta = d/2D$.

The **optical lever** used to measure small displacements, rotates a plane mirror and causes a reflected beam to rotate through twice the angle through which the mirror is turned.

13. Spherical mirrors : When the mirror is a portion of a sphere it is called a spherical mirror. If reflection occurs from the inside hollow portion, the mirror is called **concave** [Fig. 5(a)] ; if it occurs from the outer bulging portion, the mirror is called **convex** [Fig. 5(b)].

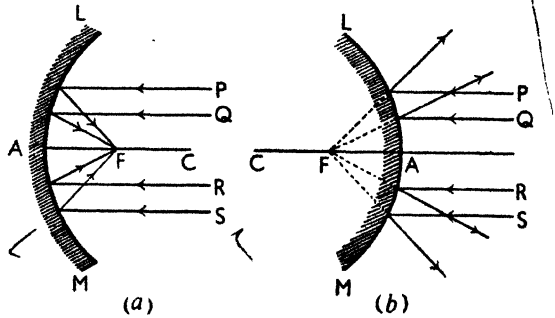


Fig. 5

The centre of the sphere, of which the mirror is a part is called the **centre of curvature** and the centre of the face of the mirror the **pole**. The line joining the centre of curvature and the pole is called the **principal axis**. The surface of the mirror from which reflection can take place is the **aperture** and the width of the aperture is the diameter of the mirror. **Radius of curvature** of the mirror is the radius of the sphere of which the mirror is a part. The secondary axis is any radial line other than the principal axis, which passes through the centre of curvature.

The laws of reflection at a plane surface hold good for reflection at a spherical mirror. Any line joining the centre of curvature of the mirror and any point on the mirror will be **normal** to the mirror at that point. Hence, for a given incident ray the reflected ray can be easily drawn, according to the laws of reflection.

If a paraxial beam of ray (*i.e.* beam of parallel rays also parallel to the axis) falls on a spherical mirror, then the rays, for small aperture, after reflection actually converge to a point (in case of concave mirror) and appear to diverge from a point (in case of convex mirror), on the principal axis of the mirror. This point is called the **principal focus** of the mirror. The focus which is real for concave and virtual for convex is midway between the pole and the centre of curvature of the mirror. The distance between the pole and the focus is called the **focal length**.

Two points are said to be conjugate points or **conjugate foci** if when an object being placed at one point the real image will be formed at the other point. The centre of curvature at which object coincides with its real image is called **self-conjugate point.**

13(A). Relation between u (object distance), v (image distance) and f (focal length), taking pole as origin :

(a). Concave mirror :

In the figure 6, LM is the principal section of a concave mirror of which A and C are respectively the pole and centre of curvature. A point object is placed at P on the principal axis of the mirror. The incident ray PO is reflected along OQ and the axial ray PCA falling normally on the mirror retraces its path. Thus two reflected rays actually meet at point Q which is the real image of P.

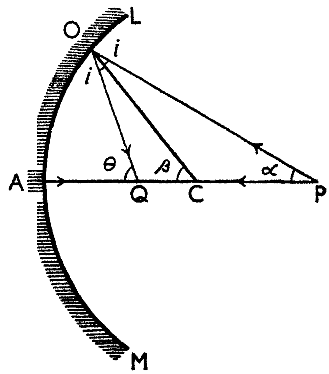


Fig. 6

Taking the pole as origin AP $= u$, AQ $= v$ and AC $= r$, the radius of curvature.

Let $\angle POC$ (angle of incidence) $= i$; $\angle QOC$ (angle of reflection) $= i$; $\angle OPA = \alpha$; $\angle OCA = \beta$; and $\angle OQA = \theta$. Then from the figure,

$$\theta = i + \beta \text{ ; and } \beta = i + \alpha \text{ or } i = \beta - \alpha$$

$$\therefore \quad \theta = \beta - \alpha + \beta = 2\beta - \alpha$$

$$\text{Hence,} \quad \alpha + \theta = 2\beta \qquad \ldots \quad \ldots \quad (1)$$

From small aperture α, β and θ are small, so that we can write

$$OA = AP \times \alpha \text{ or } \alpha = \frac{OA}{AP} = \frac{OA}{u}$$

$$OA = AC \times \beta \text{ or } \beta = \frac{OA}{AC} = \frac{OA}{r}$$

$$OA = AQ \times \theta \text{ or } \theta = \frac{OA}{AQ} = \frac{OA}{v}$$

Putting these in (1)

$$\frac{OA}{u} + \frac{OA}{v} = 2. \frac{OA}{}$$

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r} \qquad \ldots \quad \ldots \quad (2)$$

When $u = \infty$, *i.e.*, when incident rays are paraxial, the corresponding image distance v becomes equal to focal length f,

$$\text{Hence} \quad \frac{1}{\infty} + \frac{1}{f} = \frac{2}{r}$$

Thus relation (2) becomes,

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r} = \frac{1}{f} \qquad (3)$$

This relation is also known as conjugate foci relation.

Note : In deducing the above equation the following conventions are followed : (1) All distances are measured from the pole ; (2) All distances measured in the direction of the incident ray are taken negative and those in opposite direction as positive. In other words all distances on the incident side are positive and those behind the mirror are negative. In the above case u, v, r and f are all $+ve$

(b). Conjugate foci relation for concave mirror taking focus as origin :

Let x and x' be the distances of object and image respectively from the focus. Then $x = u - f$ and $x' = v - f$

$$\text{Now we have} \quad \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$
$$\text{or} \quad uf + vf = uv$$
$$\text{or} \quad uv - uf - vf = o$$
$$\text{or} \quad uv - uf - vf + f^2 = f^2$$
$$\text{or} \quad u(v-f) - f(v-f) = f^2$$
$$\text{or} \quad (u-f)(v-f) = f^2$$

$\therefore$ $xx' = f^2$ which is positive and a constant quantity. This relation is known as **Newton's formula.** Since f^2 is a positive quantity, x and x' must have same sign *i.e.* object and image must always lie on the same side of the focus.

13(B). Convex mirror : The $u - v$ relation for convex mirror can be shown to be given by the same formula. In deducing it, v and r are taken negative while u as positive according to conventions followed.

Note : Image of an object produced by a concave mirror may be (i) real, magnified, inverted, (ii) real, reduced, inverted, (iii) real, equal to object, inverted, (iv) virtual, magnified and erect, depending on the distance of the object from the pole. But image of an object produced by a convex mirror is always virtual, reduced and erect for all positions of the object.

14. Magnification of image by spherical mirrors :

Case I. Linear magnification : For a thin long object placed normally on the axis, a thin long image is formed normally on the

axis, and the ratio of the length of the image to that of the object is called the linear magnification. Mathematically, magnification m is given by $m=\frac{\text{I}}{\text{O}}=\frac{v}{u}$, where I and O are sizes (lengths) of the image and object, and v and u are image and object distances from the pole respectively. When image is erect, magnification is positive and when it is inverted it is taken negative. For plane mirror, the relation $\frac{1}{v}+\frac{1}{u}=\frac{2}{r}$ becomes $\frac{1}{v}+\frac{1}{u}=\frac{2}{\infty}=0$ so that $v=-u$ and $m=1$. Thus in a plane mirror the image of a real object is virtual and of same size.

Case II. Longitudinal magnification : It is defined as the ratio of the amount of displacement of the image to the corresponding displacement of the object along the principal axis.

Let the displacement of image be dv when displacement of object is du, then longitudinal magnification $=\frac{dv}{du}$

$$\text{Now we have} \quad \frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\text{Differentiating} \quad -\frac{1}{v^2}\cdot\frac{dv}{du}-\frac{1}{u^2}=0$$

$$\text{or} \quad \frac{1}{v^2}\cdot\frac{dv}{du}=-\frac{1}{u^2} \quad \text{or} \quad \frac{dv}{du}=-\frac{v^2}{u^2}=-m^2$$

Thus the longitudinal magnification is the square of the linear magnification.

Case III. Angular Magnification :

It is the ratio of the angles θ' and θ which the reflected and the incident rays respectively make with the principal axis.

The angular magnification $=\frac{\theta'}{\theta}$. It may be proved that the longitudinal magnification is the reciprocal of the angular magnification.

15. Caustics formed by Reflection.

We have considered reflection at spherical surfaces of such small apertures that all rays from a luminous point on the axis are reflected so as to pass through a single point on the axis. If the aperture of the mirror is large, the reflected rays will no longer all pass through a single point on the axis.

When the incident rays are parallel to the axis of the mirror, those rays alone which are reflected near the pole will pass through a single point on the axis, known as the **Principal focus** Q (Fig. 7), while others which are reflected from the periphery of the mirror will cut the axis at points which are situated nearer to the mirror than the principal focus.

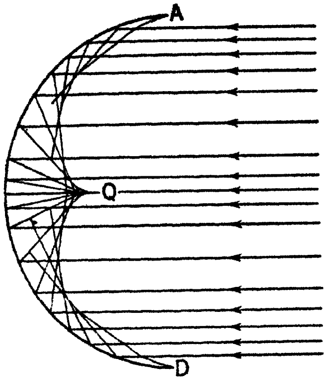


Fig. 7

This phenomenon is known as **Spherical Aberration** and the distance of the point on the axis at which a ray reflected from the periphery of the mirror cuts the axis from the principal focus of the mirror gives a measure of the *spherical aberration.*

It is also noticed that any two rays reflected from two neighbouring points on the mirror situated at a distance from the pole intersect each other before reaching the axis and the point of intersection is a sort of focus and very bright. Each pair of such neighbouring reflected rays produce a focus and a curve AQD drawn through all these foci and touching all the reflected rays is known as **Caustic curve.** It is a curve of great luminosity and has its brightest point or **cusp** at the principal focus of the mirror.

The caustic is very clearly seen on the surface of tea in a cup illuminated by a distant lamp.

16. Oblique Pencils: When the axial ray of an incident pencil makes an angle with the principal axis of a reflecting or refracting surface, the pencil is called an *oblique pencil.* The pencil may be *centric* or *eccentric.*

When the axis of the pencil is incident at the centre of the reflecting surface it is called the oblique centric pencil. In all other cases the oblique pencil is eccentric.

17. The image of an extended object: The image of a small object on the axis formed by a narrow pencil incident near the pole of a reflecting or refracting surface is bright and distinct but when the object is large the image is *curved and distorted.*

In Art. 15. we have noticed, that the rays parallel to the axis of the mirror and incident near the pole will pass through a single point on the axis known as the principal focus while others reflected from the periphery of the mirror cut the axis at points which are situated nearer to the mirror than the principal focus.

In Fig. 8 a narrow cone of rays is drawn from a luminous point O on the axis PO and incident at the surface DQ of the spherical mirror APB.

The two extreme rays OQ, OD in the plane of the paper converge after reflection at F_1 and cuts the axis at two points, as shown at F_2. All the other rays striking the arc DQ will pass through F_1 and cut the axis on the line at

, If the diagram be now rotated about the axis PO through a small angle, F_1 icribes a small arc but F_2 remains a line.

From F_1 to F_2 there is a gradual twist of the pencil from a line at right gles to the plane of the paper called the **First Focal** line F_1 to a line in the ne of the paper, the **Second Focal line** F_2. At some point between F_1 and the cone passes through a circle which is called the **circle of least confusion**.

This circle is the nearest approach to a point focus.

The pencil striking the small area described by DQ is after flection brought not to one int but to two lines. This ncil is called an **astigmatic ncil** and the effect is called stigmatism.

The term astigmatism is rived from Greek words *a*, eaning without and *stigma*, a int. It thus means the operty of not allowing the ys to pass through a point.

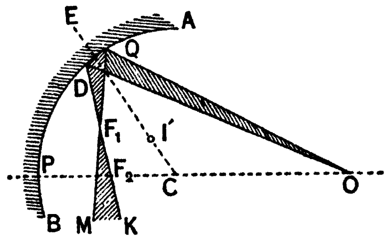


Fig. 8

Note: Spherical aberration is due to wide angle pencil and astigmatism is e to oblique incidence.

It can be proved that for oblique centric pencils the mirror acts if its focal length were numerically less than for direct reflection d that the greater the obliquity of the incident centric pencil, the orter is the effective focal length of the mirror.

Removal of spherical aberration in a spherical mirror: Spherical berration can not be completely removed, but it can be considerbly minimised (i) by limiting the number of rays by using a op, (ii) by using a parabolic or ellipsoidal mirror.

18. Aplanatic Surface and Aplantaic foci: It is a surface f a mirror of large aperture which brings rays, derived from a articular point on the axis to a point focus on the axis. Such a urface or a mirror is said to be *aplanatic* and the conjugate foci re called its *aplanatic foci.*

A lens of wide aperture may also be constructed so as to bring ll of the rays falling on it from any point on the axis to a point cus on the axis. Such a lens is called an aplanatic lens.

19. Uses of spherical mirrors: Convex spherical mirrors are sed as driving mirrors to enable a view of the rear. Concave pherical mirrors are used as shaving mirrors and by dentists to iew the posterior surfaces of patient's teeth. They are also used concentrate light on microscopic slides. Concave parabolic irrors are used as reflectors in motor headlights and search lights.

QUESTIONS

1. What is Fermat's principle of least path ? How would you prove th principle in the case of reflection ?
2. A candle is placed in any position between two parallel plane mirror Shew by tracing rays of light how an infinitely large number of images are form behind both the mirrors. Determine their positions.
3. What are aplanatic surface and aplanatic foci ?
4. Explain the formation of caustic curves by reflection at a spheric surface.
5. Explain the terms—the first and second focal lines of a reflected penc and the circle of least confusion. Explain how they are formed.

EXAMPLES

1. In front of a concave mirror, having a radius of 3 ft., an object 6 inche high is placed, first at 6 ft., then at 3 ft., then at 2 ft. distance from the mirro Find the position and height of the image in each case. [C. U. 1910
[*Ans.* Position of the images. 2 ft., 3 ft, and 6ft. Heights ...2 in., 6 in. an 18 inches.]
2. The radius of a concave mirror is 10 inches. If a rod of length 3 inche is placed at a distance of 4 in. from the mirror find the size of the imag [*Ans.* 15 inches.] [C. U. 1911
3. The radius of curvature of a convex mirror is 4 ft. and an object 1 inc in length is held 3 ft. in front of the mirror. Find the nature, position an magnitude of the image. [*Ans.* $v = 1.2$ ft. $1 = \frac{2}{5}$th inch.

CHAPTER III

REFRACTION

20. Refraction of Light : A ray of light travels in a straight line so long as its course lies in the same homogenous medium (transparent), but when it passes from one transparent medium into another, it becomes bent and undergoes a change of direction at the surface of separation of the two media. This *bending of light* is called refraction. There is however no bending when the ray meets the surface of separation of the two media normally.

21. Laws of Refraction : In 1621 Snell enunciated the two following laws of refraction :

(1) The incident ray, the refracted ray and the normal at the point of incidence lie in the same plane ; (2) The ratio of the sine of the angle of incidence to the sine of the angle of refraction for any two given transparent media is constant for a light of definite colour (or wave length).

The constant is called *index of refraction* from one medium into the other for the given light, and it is denoted by μ which is equal

to $\sin i/\sin r$, where i and r are respectively angles of incidence and refraction. If light passes from air into another medium we speak of **refractive index** of the second medium. If light passes from vacuum into second medium the index of refraction is called its *absolute refractive index.* The absolute refractive index of a medium is represented also as the ratio of the velocity of light in vacuum to its velocity in the medium. The refractive index of all transparent media changes as a rule with density and temperature of the medium. In figure 9, a glass block ACDB is surrounded by same medium.

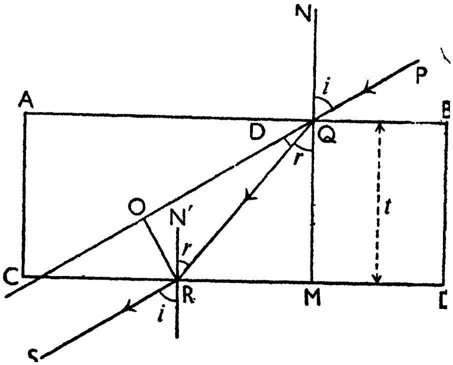


Fig. 9

If a ray PQ passes from one medium (1) say air, into another medium (2), say glass, along QR, the index of refraction of the second medium with respect to the first is given by

$$_1\mu_2 = \frac{\sin i}{\sin r} \qquad \ldots(a)$$

where i and r are angles of incidence and refraction respectively.

A ray of light RQ in (2) incident at Q will by principle of reversibility of light be refracted along QP in medium (1) so that

$$_2\mu_1 = \frac{\sin r}{\sin i} \qquad \ldots(b)$$

From (a) and (b) $_1\mu_2 \times {}_2\mu_1 = \frac{\sin i}{\sin r} \times \frac{\sin r}{\sin i} = 1 \qquad {}_2\mu_1 = \frac{1}{_1\mu_2}$

In Figure 9, the ray QR suffers second refraction at R and emerges out along RS, so that angle of emergence is i'. The ray enters the block from the medium (1) and emerges in to the same medium (1).

Then $\quad {}_2\mu_1 = \frac{\sin r}{\sin i'}$ and $_1\mu_2 = \frac{\sin i}{\sin r}$

$$\frac{\sin i}{\sin r} \times \frac{\sin r}{\sin i'} = {}_1\mu_2 \times {}_2\mu_1 = 1 \text{ ; or } \frac{\sin i}{\sin i'},$$

Hence $i = i'$; *i.e.* angle of incidence is equal to the angle of emergence and the incident and the emergent rays are parallel, the first and the last medium being same [*i.e.* (1)].

21(a). Relation between the refractive indices of different media taking part in refraction :

It can be shown similarly that when a ray of light traverses n number of layers of different transparent substances represented by $_1$, $_2$, $_3 \ldots n$, $_1$ etc, and are bounded by parallel faces the emergent ray is parallel to the incident ray and that

$$_1\mu_2 \times {}_2\mu_3 \times {}_3\mu_4 \times \ldots\ldots \times {}_n\mu_1 = 1 \qquad .. (1)$$

the first and the last media being the same.

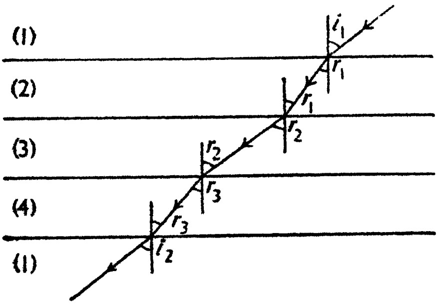


Fig. 10

If a compound plate consisting of two parallel plates having absolute indices of refraction $_v\mu_1$ and $_v\mu_2$ respectively be bounded by vacuum above and below, then, as before

$$_v\mu_a \times {}_a\mu_b \times {}_b\mu_v = 1$$

$$\text{or } {}_a\mu_b = \frac{1}{_v\mu_a \times {}_b\mu_v} = \frac{_v\mu_b}{_v\mu_a} = \frac{\mu_2}{\mu_1}$$

Here the letters v, a and b denote respectively the vacuum, the first and the second medium.

Again if μ_1 and μ_2 are the absolute indices of refraction of the first and the second medium respectively, the relative refractive index is given by

$$\frac{\sin i}{\sin r} = {}_1\mu_2 = \frac{\mu_2}{\mu_1} \text{ or } \mu_1 \sin i = \mu_2 \sin r.$$

That is, the product of the refractive index and the sine of the angle made with the normal at the point of incidence is *constant* for a given ray in both media.

Note : Referring to Figure 10, relation (1) in Art. 21(a), becomes

$$= \frac{\sin i_1}{\sin r_1} \times \frac{\sin r_1}{\sin r_2} \times \frac{\sin r_2}{\sin r_3} \times \frac{\sin r_3}{\sin i_2} = \frac{\sin i_1}{\sin i_2} = 1 \because i_1 = i_2.$$

22. The Optical Path or Equivalent Air-path :

The Optical Path or Equivalent Air-path between two points P and Q is the *equivalent air-path in vacuo* or air that will be traversed by light in the same time in which light would travel from P to Q through the medium connecting them,

Let t be the time of travel and l the equivalent air-path, then

$$t = \frac{l}{v_0} = \frac{PQ}{v}$$

ıere v_0 and v are respectively the velocities of light in vacuo and the medium.

$$l=\frac{v_0}{v}\,.\,PQ=\mu PQ$$

ıere μ is the absolute refractive index of the medium.

Thus if l be the equivalent air-path, then $l=\mu PQ$.

23. Principle of Least Time or the Law of Fermat in case Refraction : If a ray of light passes from any point M in a ədium to any point M′ in another medium undergoing refraction A, the time taken by the light to traverse the path MAM′ is a inimum (Fig. 11).

To prove this, suppose that the time ong the path MAM′ is a minimum. If , the time over this path must also be ual to that occupied in traversing the ɪry near path MA′M′.

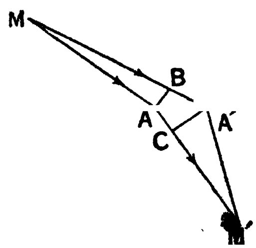


Fig. 11

Hence if AB and A′C be drawn perpendicular MA′ and M′A respectively, it follows that the nes taken by light to traverse the distances ; and A′B are equal. Since MA=MB and ′A′=M′C (nearly),

Therefore $\frac{A'B}{v}=\frac{AC}{v'}$ -- $\frac{AA' \sin i}{v}=\frac{AA' \sin r}{v'}$

$\frac{}{\sin r}=\frac{v}{v'}=\frac{\mu}{\mu}$

Here i is the angle of incidence and r, the angle of refraction, , μ', and v and v' are respectively the refractive indices and elocities of light in the upper and the lower medium.

Thus the ordinary law of refraction is obeyed if the time ccupied in traversing the path is a minimum. The converse is lso true.

If the rectilinear paths in the two media be denoted by l and l' the law of ermat is expressed as

$$\frac{l}{v}+\frac{l}{v'}=\text{a minimum or } \mu l+\mu' l'=\text{a minimum}$$

Fermat's principle is analytically expressed as $\int\frac{ds}{v}$ or $\int\mu ds$=minimum

where ds is a small element of the curved path traversed by light in a ıedium of variable refractive index.

24. Deviation by refraction at a single plane surface : In Figure 9, PQ is the ray incident from medium (1) on the surface ⅠB of a second medium (2), so that refracted ray is QR. If the econd medium were absent the ray would have gone along PQC.

Hence deviation of the refracted ray is equal to angle $OQR = i - r$ where i and r are angles of incidence and refraction respectively.

24(a). Refraction from denser to lighter medium ; Refracted image.

Case 1. Rectangular glass slab (plane surface).

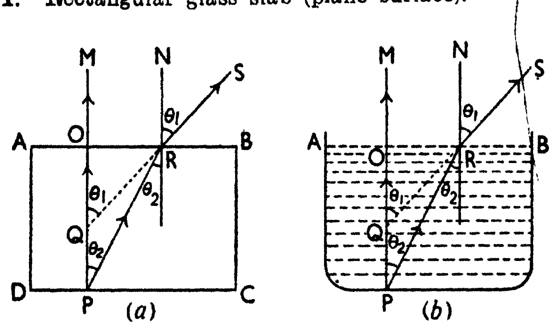


Fig 12

Let PO be the path [Fig. 12(a)] of a normal ray from the point P in the denser medium, the point O being on the surface of separation AB of two media of refractive indices μ_2 (denser) and μ_1 $(\mu_2 > \mu_1)$. Let PR be the path of an oblique ray which is refracted along RS in the rarer medium. The refracted rays OM and RS when produced backwards meet at Q which is the image of the point P. Denoting angle of incidence in the denser medium by θ_2 and angle of refraction in the lighter medium by θ_1 we have from geometry of the figure,

$$\frac{\mu_2}{\mu_1} = \frac{\sin \theta_1}{\sin \theta_2} = \frac{OR/QR}{OR/PR} = \frac{PR}{QR}$$

when the pencil of light from P is narrow, then in the limit $PR = OP$ and $QR = OQ$.

$$\text{Thus} \quad \frac{\mu_2}{\mu_1} = \frac{OP}{OQ} = \frac{u}{v}$$

where u = object distance OP and v = image distance OQ.

If $\mu_1 = 1$ (air) and $\mu_2 = \mu$ refractive index of the denser medium

$$\mu = \frac{\text{Real thickness}}{\text{Apparent thickess}} \quad [\text{ for glass slab, Fig. 12}(a)\ ]$$

$$= \frac{\text{Real depth}}{\text{Apparent depth}} \quad [\text{ for liquid, Fig. 12}(b)\ ]$$

Note : The point Q is the refracted image of the point P. The image is virtual since the rays after refraction appear to diverge from Q.

The shift in the direction normal to the surface is AB *i.e.*, PO − QO which equals

$$PO\left(1-\frac{QO}{PO}\right)=PO\left(1-\frac{\mu_1}{\mu_2}\right)=t(1-\tfrac{2}{3})=$$

where PO is thickness t of the medium b, $\mu_1=1$ and $\mu_2=\frac{3}{2}$. (for glass)

Note 1. If a beam of light passes from the first 'a' to the second medium, 'b' and undergoes refraction v is greater than u when $\mu_1<\mu_2$ and v is less than u when $\mu_1>\mu_2$.

Note 2. Again when the first medium has refractive index equal to unity and the second medium has refractive index μ, $v=\mu u$ when the refraction takes place from the first to the second medium and $u=\mu v$ when refraction takes place from the second to the first.

24(b). Apparent thickness of a glass plate : For a glass plate silvered on the back as ordinary mirrors are, no transmitted rays arise, but reflection will take place at the surface which is at a distance behind the front surface equal to $\frac{2}{3}$rd the thickness of the glass plate.

That is the apparent thickness of the glass plate

$= t/\mu = 2t/3$ (since μ for glass $=3/2$

In the same way an object within water appears to be at $\frac{3}{4}$th its actual distance from the surface, for μ from water to air $=\frac{3}{4}$.

24(c). Determination of μ by travelling microscope : Case I. Glass slab :—A fine scratch is made on a thin glass plate and placed on the horizontal platform of a travelling microscope. The scratch mark is viewed and focussed by the microscope EO whose position x is noted on a graduated upright rod having a vernier scale V attached. The microscope with the vernier scale V_1 can slide up and down. The glass slab whose refractive index is to be determined is placed over the scratch and the scratch is focussed by the miscroscope and second reading y is obtained. A small quantity of lycopoduim powder is then sprinkled on the upper surface of the slab and a very thin layer of it is then focussed for the 3rd reading z. The difference between the third and the first reading *i.e.* $z-x$ gives the value of u and that between the 3rd and the second reading *i.e.* $z-y$ gives v.

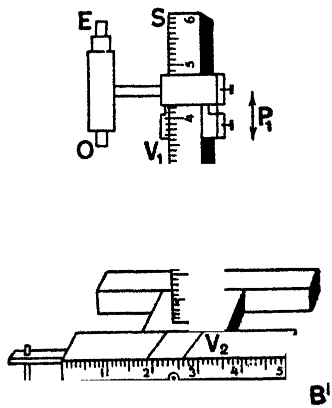


Fig. 13

Then from formula $\mu = \dfrac{z-x}{z-y} = \dfrac{u}{v} = \dfrac{\text{Real thickness}}{\text{App. thickness}}$,

the refractive index can be found out.

Case 2, Liquid : The refractive index of a liquid may be determined by using a cylindrical glass vessel with a scratch mark at its bottom in a similar manner. In this case $\mu = (z-x)/(z-y) =$ Real depth/Apparent depth, whence μ can be found out.

Note : For theory and formula consider Art. 24(a) and take point P in Figure 12, to correspond to the scratch mark both for glass slab and the liquid vessel.

25. Apparent depth of a point in case of refractions through different liquids of different thickness.

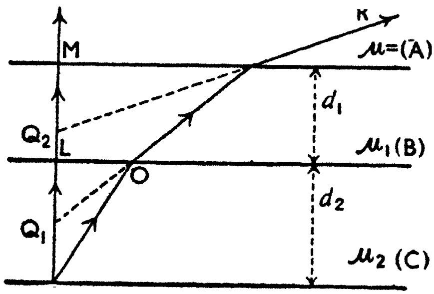


Fig. 14

In figure 14, C is the denser liquid of refractive index μ_2 and B the lighter liquid of refractive index μ_1 and the medium A above the liquids is air. Let the thickness of C and B be d_2 and d_1 respectively.

In Figure 14 refering to normal ray PLMN, PL $= d_2$ and LM $= d_1$

For refraction at O, LP is the object distance and LQ_1, is the image distance.

Hence. $\dfrac{\mu_2}{\mu_1} = \dfrac{\text{Obj. dist.}}{\text{Imag.dist.}} = \dfrac{LP}{LQ_1}$. $LQ_1 = \dfrac{\mu_1}{\mu_2}$ $LP = \dfrac{\mu_1}{\mu_2} \cdot d_2$

Again for refraction from B to A (air), MQ_1, is the object distance and MQ_2 is the image distance.

Hence we have $\dfrac{\mu_1}{1} = \dfrac{\text{Obj. dist.}}{\text{Image. dist.}} = \dfrac{MQ_1}{MQ_2}$ ∴ $MQ_2 = MQ_1 \cdot \dfrac{1}{\mu_1}$

or $MQ_2 = \dfrac{1}{\mu_1}(ML + LQ_1) = \dfrac{1}{\mu_1}\left(d_1 + \dfrac{\mu_1}{\mu_2} d_2\right) = \dfrac{d_1}{\mu_1} + \dfrac{d_2}{\mu_2}$.

Thus the final apparent depth becomes MQ_2 given by above relation, when viewed normally from above.

For n different liquids of refractive indices $\mu_1, \mu_2, \mu_3 \ldots \mu_n$ having thicknesses $d_1, d_2, d_3 \ldots d_n$ respectively the apparent depth of a point lying at the bottom of the densest liquid is evidently given by $x_n = \dfrac{d_1}{\mu_1} + \dfrac{d_2}{\mu_2} + \ldots\ldots + \dfrac{d_n}{\mu_n}$.

when the medium above the topmost liquid is air.

25(a). Lateral displacement of a ray on refraction through transparent plate or Block.

In Figure 9, PQRS is the path of ray through the slab ABDC laced in air. The perpendicular RO drawn from R on the incident ay PQ produced is the lateral displacement of the ray. Let it be and let t be thickness of the plate. From the figure deviation

$=i-r$; $\quad d=\text{RO}=\text{QR}\sin\theta=\dfrac{t\sin\theta}{\cos r} \qquad \cos r=t/\text{QR}.$

Thus $d \propto t$ for a given angle of incidence.
If μ be the refractive index of glass.

$$\mu=\frac{\sin i}{\sin r} \text{ or } \sin r=\frac{\sin i}{\mu}, \text{ and } \cos r=\sqrt{1-\sin^2 r}$$

$$\text{Then} \quad d=\frac{t\sin\theta}{\cos r}=\frac{t\sin(i-r)}{\cos r}=\frac{t(\sin i\cos r-\cos i\sin r)}{\cos r}$$

$$=t\left(\sin i-\cos i.\frac{\sin r}{\cos r}\right)$$

$$=t\left(\sin i-\frac{\cos i.\sin i}{\mu}.\frac{1}{\sqrt{1-\sin^2 r}}\right)$$

$$=t\left(\sin i-\frac{\cos i.\sin i}{\sqrt{\mu^2-\mu^2-\sin^2 r}}\right)$$

$$=t\left(\sin i-\frac{\cos i.\sin i}{\sqrt{\mu^2-\sin^2 i}}\right)$$

$$=t\left(1-\frac{\cos i}{\sqrt{\mu^2-\sin^2 i}}\right).\sin i$$

Thus lateral displacement depends on t, i and μ of the medium.

26. Object viewed through a transparent plate : Let an object placed at a distance u from a transparent plate having two parallel refracting surfaces be viewed normally through it.

Let μ be the refractive index of the plate and let the medium surrounding it have refractive index equal to unity.

The position of the image formed by refraction at the first surface of the plate opposite to the object is given by μu. This image serves as the virtual object with respect to refraction at the second surface and is at distance $\mu u+t$ where t is the thickness of the plate.

The position of the final image is therefore equal to

$$\frac{\mu u+t}{\mu}=u+\frac{t}{\mu}$$

since refraction takes place during second refraction from the plate to the medium having μ equal to unity.

That is, when an object is viewed normally through a transparent plate, the object appears to be nearer than it really is by a distance equal to

$u+t-\frac{\mu u+t}{\mu}=t\left(1-\frac{1}{\mu}\right)$. If $\mu=\frac{3}{2}$, (for glass) the displacement in the line of sight is $t\left(1-\frac{2}{3}\right)$ or $\frac{t}{3}$

27. Determination of μ of a liquid by a concave mirror: The centre of curvature A of the concave mirror CD is determined by observing no-parrallax between a pin B and its image. A certain quantity of liquid is then poured into the mirror and the new position of the pin B is found for which there is no parallax between the pin and its image. (Fig. 15).

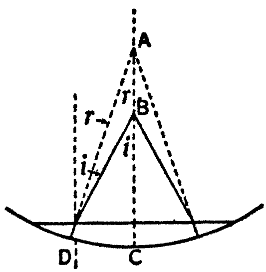


Fig. 15

In this case, the rays from the pin at B after refraction fall normally on the mirror and pass through the centre of curvature A when produced backward.

From fig. 15, $\sin i=\frac{DC}{DB}$, $\sin r=\frac{DC}{AD}$

$$\frac{\mu}{\mu'}=\frac{\sin i}{\sin r}=\frac{DC}{DB}\times\frac{AD}{DC}=\frac{AD}{DB}$$

where μ and μ' are the refractive indices of the liquid and air respectively.

If the image is viewed normally $\frac{AD}{DB}=\frac{AC}{BC}$ That is. $\frac{\mu}{\mu'}=\frac{AC}{BC}$

The above expression is valid if the layer of liquid is thin.

28. Total Internal reflection : When a ray of light passes from an optically denser medium of refractive index μ_2 into an optically rarer medium of refractive index μ_1, the angle of refraction in the rarer medium goes on increasing with the increase of the angle of the incidence in the denser medium. For a particular value of the angle of incidence (i_c), the angle of refraction becomes equal to $90°$, so that the refracted ray grazes the surface of separation of the two media. This limiting angle of incidence is called the *critical angle* for the two given media. If the angle of incidence exceeds

the critical angle there will be no refraction and hence no refracted ray, and the incident ray becomes internally reflected. This phenomenon is known as **total reflection of light.**

Consider a ray of light PO_1 which travelling from glass comes out through surface AB in air making certaing angle of refraction r_1 corresponding to angle of incidence i_1. As the angle of incidence increases the angle of refraction also increases, but when the angle of incidence (i_c) becomes such that the angle of refraction is equal to $90°$, the refracted ray O_cB grazes the surface of the glass. If again the angle of incidence be increased further, the angle of refraction becomes greater than $90°$, *i.e.*, the refracted ray instead

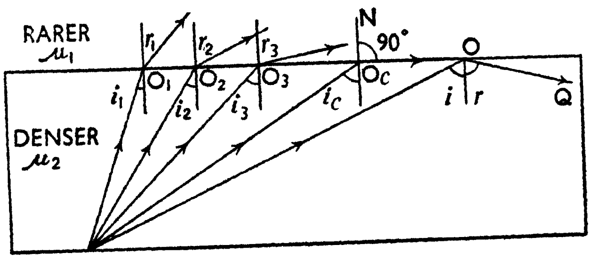


Fig. 16

of emerging out into air is internally reflected in the direction OQ so that incident angle becomes equal to the reflected angle $(i=r)$ Fig. 16.)

28(a). Critical Angle : It is that particular angle of incidence of ray of light for two given media, in passing from denser one to the rarer one for which the corresponding angle of refraction is equal to $90°$.

If $w\mu a$ be the refractive index from water (denser) to air (lighter) and if i_c be the critical angle and r the angle of refraction, then

$$w\mu_a = \frac{\sin i_c}{\sin r} = \frac{\sin i_c}{\sin 90} = \sin i_c$$

$$\text{or } \sin i_c = \frac{1}{a\mu w} \quad \text{or } i_c = \sin^{-1}\left(\frac{1}{a\mu w}\right)$$

where $a\mu w$ is the refractive index of water relative to air

Note : The phenomenon of total internal reflection has many practical uses, nce it allows us to get a totally reflecting surface. Critical angle for water and ir is 48·5°, for crown glass and air 40·5° and for flint glass and air 36·5°. The

fact that the critical angle for glass is less than 45° makes it possible to use 45° prisms as good reflectors of light.

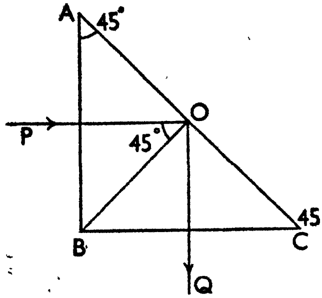


Fig. 17

In Figure 17, ray of light PO normally incident on the face AB of a right-angled isosceles prism meets the face AC at O at angle of 45° which is grater than the critical angle for glass (41°50′) and is totally reflected along OQ, the face AC serving as the reflector of light.

29. Determination of μ for a liquid by total reflection method · Air film method : A thin film of air is enclosed between two plane-parallel glass A, B and mounted in a rectangular frame so that it can be turned about a vertical axis to which a pointer P is attached (Fig. 18). The rotation is measured on a graduated circular disc placed above the frame.

At first the air-film is placed vertically in the liquid contained in the vessel V so that its sides are parallel to the sides of the vessel. Rays of light from the slit S illuminated by the burner at F, pass through the lens L and fall normally on the film and the image of the slit is observed by a telescope T focussed for parallel rays.

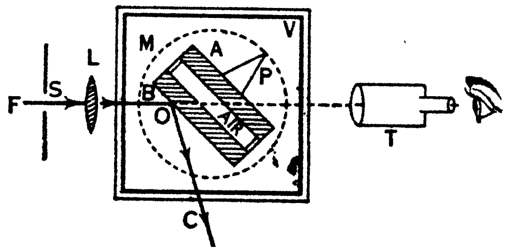


Fig. 18

The air-film is then turned until the image just disappears. The position of the film is noted. The film is now turned in the opposite direction until the image just disappears again. The position of the film is noted again. Half the angle between the two positions of the film is the critical angle for the liquid and air.

Let i be the angle of incidence at the first face of glass plate A, and r the angle of refraction. The glass wall being a parallel plate, the angle of refraction r at the first face is equal to the angle of incidence at the second face.

When total reflection occurs, the angle of refraction at the second face is 90°.

Then $_l\mu_g = \dfrac{\sin i}{\sin r}$, $_g\mu_a = \dfrac{\sin r}{\sin 90°}$.

Here $_l\mu_g$ and $_g\mu_a$ stand respectively for the indices of refraction from liquid to glass and glass to air.

But we know that $_l\mu_g \times {_g\mu_a} \times {_a\mu_l} = 1$ or $\dfrac{\sin i}{\sin r} \times \dfrac{\sin r}{\sin 90°} \times {_a\mu_l} = 1$

$$_a\mu_l \text{ (air to liquid)} = \frac{1}{\operatorname{Sin} i}$$

Thus from the knowledge of i already determined, the refractive index of the liquid is obtained.

30. Prism ; Principal section of a prism : A prism is a portion of transparent medium bounded by three plane faces meeting in three sharp lines called the *edges* of the prism. The angle at which any two plane faces meet is the *angle* of the prism. A section of the prism in a plane at right angles to the edges of the prism is called its **principal section** and the plane itself the principal plane of the prism.

Note : Two faces of a trianglular prism are often kept optically plane and polished while the remaining face is ground to make it rather opaque. In that case the angle contained by the polished faces called refracting faces, is the angle of the prism, while the ground face is what is called the **base** of the prism.

31. Deviation produced by a prism : When a ray of light passes through a prism of glass or of some optically dense medium, it is deviated through a certain angle after emergence. Let ABC be the trace of a prism and let OP, PR and RS, (Fig. 19) be the incident ray, refracted ray and the emergent ray respectively. Let i, i' and D be respectively the angles of incidence, emergence and deviation. Let r and r' denote the angles LPR and PRL respectively and A the angle of prism.

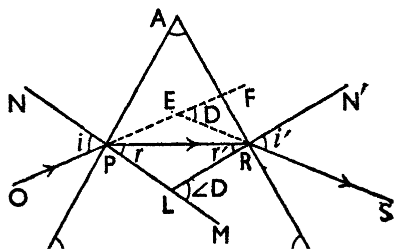


Fig. 19

Then $D = \angle EPR + \angle ERP = i - r + i' - r' = i + i' - (r + r')$ (1)

In the triangle APR, $\angle APR + \angle ARP + A = 180°$

and $\angle APL + \angle ARL = 180°$ for each of them is a right angle.

$\therefore$ $\angle LPR + \angle LRP = A = r + r'$.

$\therefore$ Deviation $D = i + i' - A$.

32. Deviation produced by a thin prism, angle of incidence being small : Let us consider a ray making a small angle of incidence i with one face of a thin prism (small angle prism) and passing in a principal plane. Let μ be the refractive index of the material of the prism.

From relation (1) in Art. 31 $D = (i - r) + (i' - r')$

We know that

$\mu = \dfrac{\sin i}{\sin r} = \dfrac{\sin i'}{\sin r'}$ Since i and r are small, i' and r' are also small, we have therefore $\mu = \dfrac{i}{r} = \dfrac{i'}{r'}$

Hence $D=(\mu r - r)+(\mu r' - r')$; $D=(\mu-1)(r+r')$
Now $A=(r+r')$ $\therefore$ $D=(\mu-1)A$.

This shows that the deviation is independent of the angle of inciden(

Thus, when the angle of the prism is small and the inciden(is nearly normal, the deviation is directly proportional to the ang of the prism and it is so small that the question of maximum an minimum deviation does not arise.

33. Case of normal Incidence : Let the ray PQ (Fig. 20) incider normally on the face AB of th prism ABC traverse the prism alon the path PQRS. Considering refra(tion at the point R of the fac AC, let i be the angle of incidenc and ψ the angle of emergence.

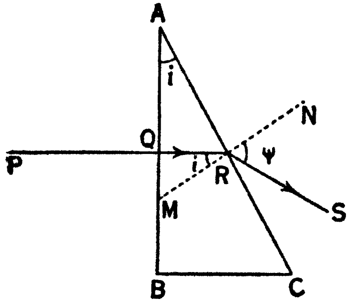


Fig. 20

Then the angle of deviation] is given by $D=\psi-i$

But from geometry of the figur $i=A$, where A is the angle of th prism.

$\therefore$ $D=\psi-A$ or $D+A=\psi$

Again from the law of refraction, we have $\sin\psi=\mu\sin i$

Hence $\sin(D+A)=\mu\sin A$ $\therefore$ $\mu=\dfrac{\sin(D+A)}{\sin A}$

If, in addition, the prism is thin, it behaves like a parallel plat(with small deviation so that $\mu=\dfrac{D+A}{A}$ or $D=(\mu-1)A$

34. Angle of minimum Deviation, in a prism : The angl(of deviation of a ray of light passing through a prism depends on the angle of incidence. It is the angle between the incident and the emergent ray. If a curve is plotted with angles of incidence as abscissae and angles of deviation as ordinates, it will be found that the deviation at first decreases with the increase in the angle of incidence and then attains a *minimum value* AA$'$ for a particular value of the angle of incidence OA (Fig. 21). Then with a further increase in the angle of incidence the deviation increases again.

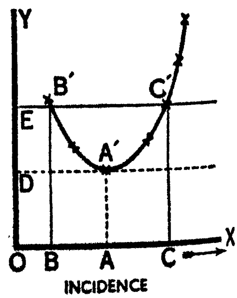


Fig. 21

It is seen that when minimum deviation occurs, the ray passes **through the prism symmetrically** and $i=i'$ and $r=r'$.

35. Condition of Minimum Deviation, in a Prism : We know that for a particular value of the angle of incidence the angle of deviation in the prism is minimum. It happens only when the incident and the emergent rays cut the prism symmetrically.

If this is not the case, suppose the ray QPRS (Fig. 19) cuts the prism ABC unsymmetrically, the angle of incidence being i. Then according to the principle of reversibility of rays, the ray SRPQ would undergo the same deviation although its incident angle i' is not equal to i. The deviation produced in this ray while passing through the prism can not be minimum. since the same value for the minimum deviation occurs for the two different values of the angles of incidence. Therefore, for the deviation to be minimum there should be one value for the angle of incidence, *i.e.*, $i=i'$ and the ray should pass symmetrically through the prism.

35(a). Mathematical treatment : Referring to Fig. 19 in Art. 31 we have

$$D=i+i'-A \quad ...(1) \quad ... \quad A=r+r' \quad ...(2)$$

for D to be maximum or minimum $dD=0$

Differentiating (1). we have $0=di+di'$ (A being constant) ...(1*a*)

Differentiating (2), we have $0=dr+dr'$ ... (2*a*)

From (1*a*) and (2*a*) $\frac{di}{di'}=\frac{dr}{dr'}$

Again $\sin i=\mu \sin r$ ...(3)

$\sin i'=\mu \sin r'$ ...(4)

Differentiating (3) and (4)

$\cos i\,di=\mu \cos r\,dr$ (3*a*) and $\cos i'di'=\mu \cos r'd'r$ ... (4*a*)

From (3*a*) and (4*a*) $\frac{\cos i}{\cos i'} \quad \frac{\cos r}{\cos r}$ Squaring, $\frac{\cos^{\cdot}}{\cos^{\cdot} i} \; \frac{\cos^2 r}{\cos^2 r'}$

$$\text{or} \quad \frac{1-\mu^2 \sin^2 r}{1-\mu^2 \sin^2 r'}=\frac{1-\sin^2 r}{1-\sin^2 r'}=\frac{\sin^2 r\ (1-\mu^2)}{\sin^2 r'\ (1-\mu^2)}=\frac{\sin^2 r}{\sin^2 r'}$$

$\frac{1-\sin^2 r}{1-\sin^2 r'} \quad \frac{\sin^2 r}{\sin^2 r'}$ $\qquad r=r'$ and consequently $i=i'$

from (3) and (4).

Hence, deviation is minimum, when the angle of incidence i is equal to the angle of emergence, i' and $r=r'$, that is when the ray passes symmetrically through the prism.

35(b). Prism Formula : The deviation D is given by $D=i+i'-A$; again $A=r+r'$. When deviation is minimum (D*m*) $i=i'$ and $r=r'$

$$\therefore \quad Dm=2i-A \qquad i=\frac{Dm+A}{2} \text{ and } A=r+r' \quad \therefore \quad r=\frac{A}{2}$$

Now μ (refractive index of glass) $= \frac{\text{Sin}\, i}{\text{Sin}\, r} = \frac{\text{Sin}\, \frac{Dm + A}{2}}{\text{Sin}\, \frac{A}{2}}.$

This is the required prism farmula.

36. Image formed by a Prism : If from a point P a very narrow pencil of rays be incident on a prism such that the axial ray undergoes minimum deviation, then all the rays will suffer almost the same deviation and the emergent rays will appear to diverge from a point Q which for all practical purposes can be taken as the image, of P (Fig. 22).

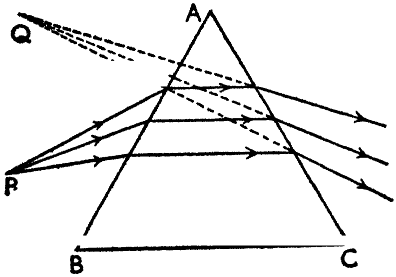


Fig. 22

It can be shown that when the prism is placed in the minimum deviation the image distance is equa to the object distance.

If the prism is not in the minimum deviation position, the incident rays will suffer unequa deviation and so the emergent rays will not diverge from a single point and the image will not be bright and well defined. This accounts for the fact that the prism should be placed in the position o minimum deviation for mean ray and the incident pencil is made narrow to produce a pure solar spectrum.

For narrow pencil of rays the object distance u and image distance v are related by

$$\frac{}{u}\,\frac{\cos^2 i'}{\cos^2 r'}\Big/\frac{\cos^2 i}{\cos^2} = y/x. \quad \text{(say)}$$

when deriation is minimum $i = i'$ and $r = r'$, $\therefore\ v = u$, *i.e.* obj distance = Image distance, *when* $i > i'$, $y > x$

$\therefore\ v > u$, i.e., object distance is less than image distance.

when $i' < i$, $v < u$, *i.e.* object distance is greater than image distance.

37. Determination of the Index of refraction of a solid in the form of a prism : The prism formula is given by

$$\frac{\text{Sin}\, \frac{Dm + A}{2}}{\text{Sin}\, \frac{A}{2}}$$

where μ is the refractive index, Dm the angle o

minimum deviation and A the angle of the prism. The value of μ can be found if Dm and A are experimentally determined.

Experiment : The determination of the index of refraction is performed with the help of an instrument known as Spectrometer (Fig. 23) which mainly consists of a graduated circle mounted on a

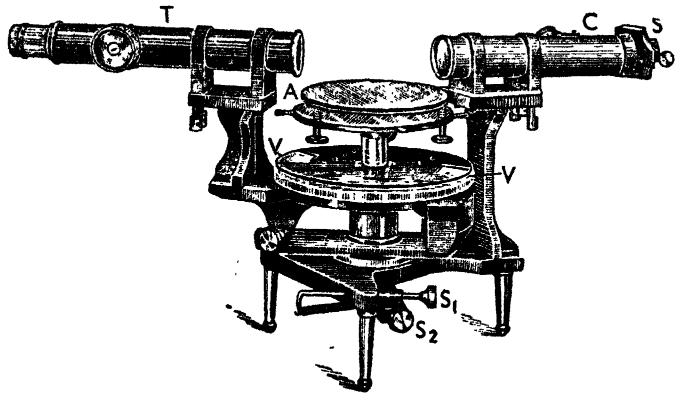


Fig. 23

vertical support. Round the circle there is a movable arm carrying a telescope T whose position on the circle can be read off by means of two verniers V attached to it. There is also a tube known as a collimator (C) fixed round the circle, which consists of an adjustable slit at one end and a convex lens at the other end of the tube. By turning the screw of the collimator, the slit S can be placed at the focus of the lens and the rays of light emerge from the lens as a parallel beam.

There is also a small circular table A known as the prism table which can be raised above the larger table and can be clamped at any desired position. The prism table is provided with three levelling screws and can be rotated about the vertical axis of the instrument.

Before commencing the experiment with the spectrometer the following adjustments are to be made.

(1) To adjust the eye-piece of the telescope, the telescope is turned towards the sky or a bright wall and the eye-piece tube is made to slide in and out of the telescope tube until the cross wires are distinctly seen.

(2) The telescope is then focussed on a distant object till there is no parallax between the image of the distant object and the cross wires, *i.e.*, the image is formed in the focal plane of the telescope.

(3) The telescope adjusted for parallel rays is then turned so that its axis coincides in direction with the axis of the collimator. The slit of the collimator is then brightly illuminated by a sodium flame and the image of it is observed through the telescope and distinctly focussed by moving the slit in and out of the collimator tube. The adjustment is correct when there is no parallax between the edge of the image of the slit and the cross wires. The collimator is now focussed for parallel rays,

Adjustment for parallel rays may also be performed by Schuster's method. (Consult any Text Book of Practical Physics).

(4) Having performed all the above adjustments, the slit is illuminated by a sodium flame and a glass prism is placed on the prism table over its centre so that one of the faces which includes the angle to be measured is placed perpendicular to the line joining two of its levelling screws and the table is adjusted by altering the positions of the screws until the faces of the prism which include the angle to be measured are parallel to the axis about which the telescope and the table turn.

Having done this, the prism is placed by rotating the table in such a position that the parallel rays from the collimator fall on both the faces of the prism containing the angle to be measured and are reflected in directions the angle between which is found to be twice the angle of the prism.

The telescope is then turned alternately to view the reflected image from the two faces of the prism and its position determined in each case. The difference between the readings for these two positions of the telescope divided by 2 gives the angle of the prism

The table is then rotated and the prism is placed so that the rays from the collimator pass through the prism and are thus refracted. The telescope is turned to view the refracted image and the angle which the direction of the telescope makes with that of the collimator is the angle of deviation. To determine the angle of minimum deviation the table with the prism is rotated so that the image moves towards the axis of the collimator and a position, is found when the image is as near as possible to the axis and is coincident with the cross-wire in the telescope. At this position the telescope is clamped and its position noted on the graduated circle. The prism is now removed from the table and the image of the slit is observed directly through the telescope whose position is also noted again. The difference of the readings for these two positions of the telescope gives the *Angle of minimum deviation.*

Having determined A, the angle of the prism, and D_m the angle of minimum deviation μ for glass is determined by the relation.

$$= \sin \frac{(D_m + A)}{2} \Big/ \sin \frac{\quad}{2}$$

The refractive index of a liquid may also be found by using a hollow prism filled with the liquid.

38. Limiting Angle of a prism In the adjoining Figure 23, POQR is the path of a ray through the principal section ABC of a prism placed in air. The emergent ray grazes the second face AC of the prism, PO and OQ being respectively the incident ray on the first face AB, and the refracted ray inside the prism. Let the angles of incidence and refraction at the face AB be i and r, and corresponding angles at the face AC be r_1 and 90° (Fig. 24)

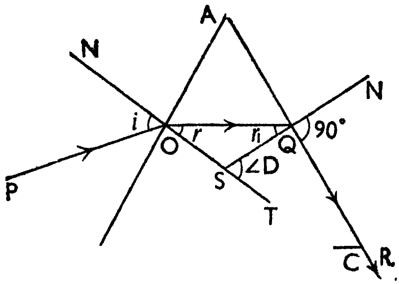


Fig. 24

Evidently $r_1 = \theta$, the critical angle between glass and air. We have from the figure, angle of the prism $A = r + r_1$ ... (1)

For refraction at O, $\sin i = \mu \sin r$, or $r = \sin^{-1}\left(\frac{\sin i}{\mu}\right)$

For refraction at Q, $\frac{\sin 90}{\sin r_1} = \mu$, or $\sin r_1 = \frac{1}{\mu}$

$$\therefore \quad r_1 = \sin^{-1}\left(\frac{1}{\mu}\right) = \theta$$

Putting these values for r and r_1 in (1), we have

$$A = \sin^{-1}\left(\frac{\sin i}{\mu}\right) + \sin^{-1}\left(\frac{1}{\mu}\right) \qquad (2)$$

Cor. I. For normal incidence on face AB, $i = 0$, then relation (2) becomes

$$A = \sin^{-1}\left(\frac{1}{\mu}\right) = \theta$$

Hence the normal ray will emerge out when $\angle A$ is equal to the cirtical angle θ of prism in air and it will not emerge out if be greater than θ.

Cor. II. For Grazing incidence on face AB, $i = 90$; then relation (1) becomes

$$A = \sin^{-1}\left(\frac{1}{\mu}\right) + \sin^{-1}\left(\frac{1}{\mu}\right) = \theta + \theta = 2\theta$$

Hence the angle of the prism which permits just emergence of the ray at grazing incidence is equal to 2θ. If the angle of the prism be greater than 2θ, even the grazing incident ray will not emerge out of the prism.

Thus from above, we find that no incident ray can emerge ou of the prism if the angle of the prism be greater than twice th critical angle of the material of the prism with respect to th surrounding medium.

QUESTIONS

1. Prove Fermat's principle of least time in the case of refraction.

2. Establish a relation between the real depth and the apparent depth of liquid.

Describe an experiment for finding the refractive index of a liquid by measu ing its apparent depth. [C. U. 1951

3. State Snell's law. Find an expression for the displacement of the imag when an object is seen normally through a transparent plate with plane ar parallal faces.

4. Define critical angle and total internal reflection. Determine the refra tive index of a liquid by total internal reflection.

5. What is meant by the principal section of a prism ? Show that the devi tion of a ray which goes symmetrically through a prism in a principal sectic is less than that of any other ray. [C.U. 194

6. If δ be the minimum deviation produced by a prism of refracting angle when a ray of light proceeds through its principal section,

prove that the refractive index. $\mu = \dfrac{\operatorname{Sin} \dfrac{\delta + a}{2}}{\operatorname{Sin} \dfrac{a}{2}}$.

If the critical angle for total reflection of the material of the prism be show that no ray can pass through the prism if the angle a be great than 2θ. [C. U. 1937

7. Explain what you mean by the minimum deviation of light in a pris of glass.

Show that the deviation produced by a ray of light by a thin acute-angl prism is $(\mu - 1)a$ where a is the angle of the prism and μ the refractive ind of the material. [C. U. 1948

8. Explain the principle of finding the refractive index of a thin prism wi the spectrometer by the method of normal incidence, giving the adjustments detail. [C.U. 1924, 25, 3

9. Deduce an expression for the limiting angle of a prism for which the will be no emergent ray. [C. U. 195

EXAMPLES

1. Explain why the horizontal bottom of a pool of water appears to raised to an eye placed above it. Find how deep the pool will appear to be, the real depth is 15 ft. and the refractive index of water is 1·3 [C.U. 19

we know that $\dfrac{u}{v} = \mu$, *i.e.*, $\dfrac{\text{Real depth}}{\text{Apparent depth}} = \mu$

or $\dfrac{15}{v} = 1·3$ $\qquad v = \dfrac{15}{1·3} = 11·54$ ft.

2. A glass plate of thickness 3 *cms.* is in contact with the surface of water Find how much the image of a point on the water surface will be elevated. μ of glass=3/2].

3. A ray of light passes from oil to water. The refractive index of water is ·33 and that of oil is 1·45. Find the critical angle between oil and water.

Let $a\mu_\omega$ be refractive index of water relative to air, $a\mu_o$ the refractive index of oil relative to air and $o\mu_\omega$ the refractive index of water relative to oil.

Now $a\mu_o \times o\mu_\omega \times \omega\mu_a = 1$; or $1{\cdot}45 \times o\mu_\omega = \frac{1}{\omega\mu} = 1{\cdot}33$ $\qquad a\mu_\omega = 1{\cdot}33$

$$o\mu_\omega = \frac{1{\cdot}33}{1{\cdot}45} = {\cdot}917. \text{ But } \frac{\text{Sin C}}{\text{Sin } 90} = o\mu_\omega \text{ where C is the critical angle}$$

Hence Sin C$=$·917.$=$Sin 66°30′ $\therefore$ C$=$66°30′.

4. A vessel of depth d is half-filled with a liquid of refractive index μ_1 and the other half is filled with liquid of refractive index μ_2. Show that the apparent depth of the vessel when viewed normally is $\frac{d}{2}\left(\frac{1}{\mu_1}+\frac{1}{\mu_2}\right)$.

[C.U. 1951]

For refraction from lower liquid (A) to upper liquid (B). From figure

$$A\mu B = \frac{\text{Apparent depth}}{\text{Real depth}} = \frac{QR}{PR} = \frac{\mu_1}{\mu_2}$$

$$QR = \frac{\mu_1}{\mu_2} PR = \frac{\mu_1}{\mu_2} \cdot \frac{d}{2}.$$

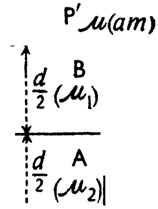


Fig. 25

For refraction from upper liquid to air above

$$B\mu\ air = \frac{\text{Apparent depth}}{\text{Real depth}} = \frac{ST}{QT} = \frac{ST}{QR+RT} = \frac{ST}{\frac{\mu_1}{\mu_2}\cdot\frac{d}{2}+\frac{d}{2}} = \frac{\mu}{\mu_1} = \frac{1}{\mu_1}$$

$$\therefore \text{ Apparent depth } ST = \frac{1}{\mu_1}\left(\frac{\mu_1}{\mu_2}\cdot\frac{d}{2}+\frac{d}{2}\right) = \frac{d}{2}\left(\frac{1}{\mu_2}+\frac{1}{\mu_1}\right).$$

5. The refracting angle of a prism is 60° and the index of refraction is $\sqrt{\frac{7}{4}}$. What is the limiting angle of incidence of a ray that will be transmitted through the prism ?

Let A be the angle of the prism ABC and let the incident ray on the face AB of the prism make such an angle i (called the limiting angle) with the normal that the refracted ray strikes the second face AC at the critical angle c and the emergent ray grazes the surface, *i.e.*, makes angle 90° with the normal.

Let i and r be the angle of incidence and refraction at the first face. Then we have at the first face $\frac{\text{Sin } i}{\text{Sin } r} = \mu$ (1) ; at the second face Sin $c = \frac{1}{\mu}$ (2) ; and $r+c = A$ (3)

Therefore from (1) Sin $i = \mu$ Sin r.

From (3) Sin $i = \mu$ Sin $(A-c) = \mu$ Sin A cos $c - \mu$ Cos A Sin c.

From (2) $\text{Sin } i = \mu \text{ Sin A}\sqrt{\frac{\mu^2-1}{\mu^2}} - \text{Cos A} = \text{Sin A} \quad \sqrt{\mu^2-1} - \text{Cos A}$

$= \text{Sin } 60^\circ \sqrt{\frac{7}{3}-1} - \text{Cos } 60^\circ = \frac{\sqrt{3}}{2}.\frac{2}{\sqrt{3}} - \frac{1}{2} = \frac{1}{2}$ $\quad i$ (limiting angle of incidence) $= 30^\circ$. This is the lower limit. The upper limit is 90°.

6. Find the range of the angles of incidence for which light incident on glass prism ($\mu = 1{\cdot}5$) of refracting angle 60° will be transmitted without an internal reflection.

Proceeding as in problem 5 above

$$\text{Sin } i = \mu \text{ Sin } r = \mu \text{ Sin } (\text{A} - c)$$

$$= \mu \{\text{Sin A Cos } c - \text{Cos A Sin } c\}$$

$$= \mu \left\{\text{Sin } 60\sqrt{1-\frac{1}{\mu^2}} - \text{Cos } 60 \cdot \frac{1}{\mu}\right\}$$

$$= \text{Sin } 60\sqrt{\mu^2-1} - \tfrac{1}{2}$$

$$= \frac{\sqrt{3}}{2}\sqrt{\left(\frac{3}{2}\right)^2 - 1} - \tfrac{1}{2} \qquad \mu = 1{\cdot}5 = \frac{3}{2}$$

$$= \frac{\sqrt{3}\cdot\sqrt{5}}{4} - \tfrac{1}{2} = \frac{3{\cdot}87}{4} - \tfrac{1}{2} = \frac{1{\cdot}87}{4} = {\cdot}4675 \qquad i = \text{Sin}^{-1}\,{\cdot}4674 = 27^\circ$$

CHAPTER IV

REFRACTIONS AT SPHERICAL SURFACES : LENSES

39. Refraction at a Concave Spherical Surface :

Pole as orgin : Let RP be the section of a concave spherical surface separating two media of refractive incides μ_1 and μ_2 ($\mu_2 > \mu_1$), μ_2 being the refractive index of the medium on the left and μ_1, that on the right respectively. Let OP be the axis for the concave surface and let P (the middle point) and C be the pole and the centre of curvature of the surface respectively. Let a ray OR from an object O (Fig. 26) on the axis, be incident on the surface at R and let the refracted ray when produced backward meet the axis forming the virtual image at I. Let $\angle$ORC and $\angle$IRC denote respectively the angle of incidence i and angle of refraction r and let the angle RCO be θ.

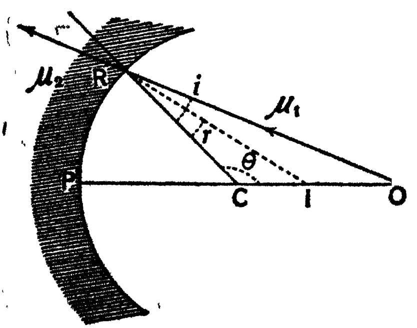


Fig. 26

Then we have

$$\frac{\sin i}{\sin r}=\frac{\mu_2}{\mu_1} \text{ or } \mu_1 \sin i=\mu_2 \sin r.$$

Dividing by $\sin \theta$ we have $\mu_1\dfrac{\sin i}{\sin \theta}=\mu_2\dfrac{\sin r}{\sin \theta}$

From the Δs ORC and IRC, $\dfrac{\sin i}{\sin \theta}=\dfrac{CO}{OR}$ and $\dfrac{\sin r}{\sin \theta}=\dfrac{CI}{RI}$

$$\therefore \quad \mu_1\frac{CO}{OR}=\mu_2\frac{CI}{RI}$$

When the point R is very close to P, *i.e.*, for small aperture, we have OR = OP and RI = PI

$$\therefore \quad \mu_1\frac{CO}{PO}=\mu_2\frac{CI}{PI}$$

Let $OP=u$, $IP=v$, $CP=r$;

then $\mu_1 \dfrac{(u-r)}{u}=\mu_2\dfrac{(v-r)}{v}$, since u, v and r are all $+ve$.

$$\text{or } \mu_1 \left(1-\frac{r}{u}\right)=\mu_2\left(1-\frac{r}{v}\right) \text{ or } \frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{r} \qquad \ldots \qquad (1)$$

40. Refraction at Convex Surface :

(a) Pole as Origin : Let O, I, P and C denote the same things as before and let the $\angle$ECO be again denoted by θ (Fig. 27).

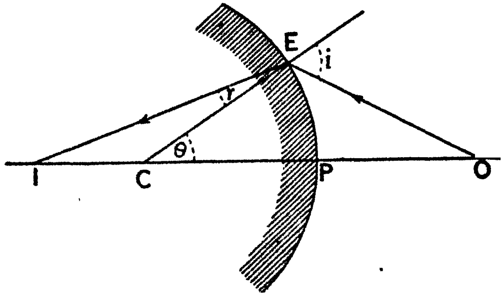


Fig. 27

Then, we have $\mu_1 \sin i=\mu_2 \sin r$

Dividing this by $\sin \theta$

$$\mu_1\frac{\sin i}{\sin \theta}=\mu_2\frac{\sin r}{\sin \theta} \quad \mu_2\frac{\sin (180-i)}{\sin \theta}=\mu_2\frac{\sin r}{\sin (180-\theta)}$$

From Δs OEC and IEC, $\dfrac{\sin(180-i)}{\sin\theta}=\dfrac{CO}{EO}$ and $\dfrac{\sin r}{\sin(180-\theta)}=\dfrac{CI}{EI}$

$$\therefore\quad \mu_1\frac{CO}{EO}=\mu_2\frac{CI}{EI}$$

When E is very close to P, *i.e.*, for small aperture,

$$EO=PO \text{ and } EI=PI$$

$$\mu_1\frac{CO}{PO}=\mu_2\frac{CI}{PI}$$

Now $PO=u$, $PI=v$, $CP=r$ $\quad\therefore\quad \mu_1\dfrac{u-r}{u}=\mu_2\dfrac{[-v-(-r)]}{-v}$ Using convention in sings, v and r are negative.

$$\mu_1\frac{u-r}{u}=\mu_2\frac{v-r}{v} \text{ or } \mu_1\left(1-\frac{r}{u}\right)=\mu_2\left(1-\frac{r}{v}\right)$$

$$\therefore\quad \frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{r} \qquad \ldots \qquad \ldots \qquad (2)$$

The equations (1) and (2) are identically similar.

When $\mu_2=\mu$ and $\mu_1=1$, *i.e.*, when light passes from vacuum or air to a medium of refractive index μ the expressions (1) and (2) become $\dfrac{\mu}{v}-\dfrac{1}{u}=\dfrac{\mu-1}{r}$.

(b) Centre as Origin: If the **distances** of the object and the image are **measured from the centre** of curvature of the refracting surface, and denoted by x and y respectively, then,

$$\mu_1\frac{CO}{PO}=\mu_2\frac{CI}{PI} \text{ or } \mu_1\frac{x}{x-r}=\mu_2\frac{y}{y-r}$$

$$\mu_2\left(\frac{x-r}{x}\right)=\mu_1\left(\frac{y-r}{y}\right) \text{ or } \mu_2\left(1-\frac{r}{x}\right)=\mu_1\left(1-\frac{r}{y}\right)$$

$$\text{or}\quad \frac{\mu_1}{y}-\frac{\mu_2}{x}=\frac{\mu_1-\mu_2}{r} \qquad (3)$$

Note: Distances measured towards the incident rays are considered positive and against negative. The above relation (3) is also true for a concave refracting surface.

(c) Conjugate Foci: Two points situated on the same axis of a curved surface are said to be conjugate when an object at ... of these points forms after reflection or refraction an image at the second point. In the above case I and O are conjugate

(d) Principal Foci : In the equation $\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{}$

If v [illegible] u [illegible] $-1 = f_1$

This indicates, a point situated at a distance u from the pole on the axis is called the **First Principal Focus.** The focal length is denoted by f_1.

Thus a point situated on the axis such that an incident ray diverging from or tending to converge to the point becomes parallel to the axis after refraction, is called the *First Principal Focus.*

Hence the first principal focal length is the object distance corresponding to the image distance equal to infinity.

Again if $u = \infty$, $v = \frac{\mu r}{\mu - 1} = f_2$

This indicates a point on the axis such that an incident ray parallel to the axis will after refraction either converge to or appear to diverge from the point is called the *Second Principal Focus.*

The socond principal focal length is the image distance corresponding to the object distance equal to infinity.

The point on the axis at a distance v from the pole is called the **Second Principal Focus** and the focal length is denoted by f_2.

$$\therefore \quad \mu f_1 + f_2 = 0$$

If refraction takes place from a medium of refractive index μ_1 to a medium of refractive index μ_2, then by substituting $\frac{\mu_2}{\mu_1}$ for μ, we have $\frac{f_1}{\mu_1} + \frac{f_2}{\mu_2} = 0.$

Here $f_1 = -\frac{\mu_1 r}{\mu_2 - \mu_1}$ and $f_2 = \frac{\mu_2 r}{\mu_2 - \mu_1}$

[f_1 and f_2 are of opposite signs. For a **concave surface** f_1 is negative and f_2 is positive and for a **convex surface** f_2 is negative f_1 is positive.]

41. Certain Other Relations :

We know that $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{r}$

$$\text{or} \qquad \frac{\mu_2 r}{(\mu_2 - \mu_1)} \cdot \frac{1}{v} - \frac{\mu_1 r}{(\mu_2 - \mu_1)} \cdot \frac{1}{u} = 1 \qquad (1)$$

Again since we know that $f_1 = -\frac{\mu_1 r}{\mu_2 - \mu_1}$ and $f_2 = \frac{\mu_2 r}{\mu_2 - \mu_1}$

therefore from (1) we have $\frac{f_2}{v} + \frac{f_1}{u} = 1 \qquad (2)$

41(a). Focal points at Origin. Now from (2) $f_2u+f_1v=uv$ or $f_2u+f_1v-f_1f_2-uv=-f_1f_2$ or $(u-f_1)(v-f_2)=f_1f_2$ or $U.V.=f_1f_2$ $=$ constant
where U is the object-distance from the first principal focus and V, the image-distance from the second principal focus.

42. Relative position of the image and the object in curved refracting surfaces : (a) Concave Surface : The image due to refraction is always virtual and is nearer to the centre of curvature than the object.

From the formula, $\frac{u}{v}-\frac{1}{u}=\frac{\mu}{f_2}$, where $f_2=\frac{\mu r}{\mu-1}$

or $\frac{\mu}{v}=\frac{1}{u}+\frac{\mu}{f_2}=\frac{1}{u}+\frac{\mu-1}{r}$

If $u>r$, v is also $>r$, but if $u=r$, $v=r$; and if u is $\angle r$, v is also $\angle r$.

(b) **Convex Surface :** In this case r is negative and so f_1 is positive.

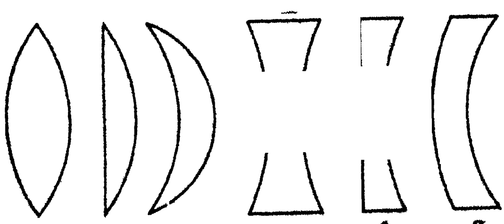
From the formula, $\frac{\mu}{v}-\frac{1}{u}=\frac{\mu-1}{r}=-\frac{1}{f_1}$ or $\frac{u}{v}=\frac{1}{u}-\frac{1}{f_1}$

If u is $>f_1$, v is negative, *i.e.*, a real image is formed inside the refracting surface ; but if u is $<f_1$, v is positive and the image is formed on the same side as the object and virtual.

The magnification $m=\frac{I}{O}=\frac{u}{\mu u}=-\frac{V}{f_2}=-\frac{f_1}{U}$.

43. Spherical Lenses : Lens is a portion of a transparent medium bounded by one or more spherical surfaces.

The various forms of spherical lenses (Fig. 28) are (*a*) double-convex ; (*b*) plano-convex ; (*c*) concavo-convex ; and (*e*) double-concave ; (*f*) plano-concave ; (*g*) convexo-concave.



a *b* *c* *f* *g*

The line passing through the centres of curvatures of the two bounding surfaces is called the *principal axis* of the lens. A section

of the lens through the principal axis is called the principal section. The *optical centre* of a lens is a point on the principal axis so that all rays whose paths within the lens pass through this point must have, therefore, corresponding incident and emergent rays parallel to each other. The *principal focus* is the point on the principal

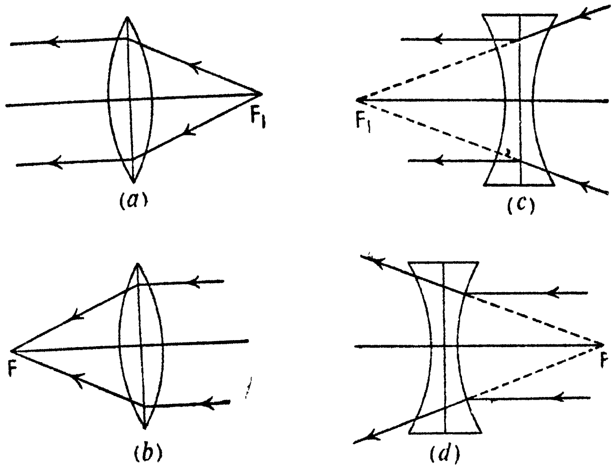


Fig. 29

axis to which a paraxial beam incident on the lens will after refraction converge (in case of convex) [Fig. 29(*b*)] or from which the paraxial beam after refraction will appear to diverge (in case of concave) [Fig. 29(*d*)]. This focus is some time called the second principal focus of a lens. The first principal focus of a lens is a point on the principal axis such that rays diverging from the point (in case of convex) [Fig. 29(*a*)] or directed towards the point (in case of concave) [Fig. 29(*c*)] when incident on the lens will emerge out parallel after refraction.

(a) Spherical Lens: Let x be the distance of the object, y the distance of the image formed by the refraction at the first surface and z, the distance of the image formed by second refraction.

Let all these distances be measured from the centre of the spherical lens and considered positive when measured towards the incident light.

Let r be the radius of the sphere and let μ be the refractive index of the material of the lens.

Then according to formula (3) of Art. 40(b) we have for the first refraction at the convex surface from air to glass,

$$\frac{1}{y} - \frac{\mu}{x} = \frac{1-\mu}{r} \quad \ldots \quad \ldots \quad (1)$$

since in this case, $\mu_2 = \mu$ and $\mu_1 = 1$

For the second refraction at the concave surface from glass to air.

$$\frac{1}{z} - \frac{\frac{1}{\mu}}{y} = \frac{1-\frac{1}{\mu}}{-r}$$ Since refraction takes place from glass to air. Here r is negative.

$$\text{or } \frac{1}{z} - \frac{1}{\mu y} = -\frac{\mu-1}{\mu r} \text{ or } \frac{\mu}{x} - \frac{1}{y} = \frac{1-\mu}{r} \qquad (2)$$

Adding (1) and (2) we get

$$\frac{\mu}{z} - \frac{\mu}{x} = \frac{2(1-\mu)}{r} \text{ or } \frac{1}{z} - \frac{1}{x} = -\frac{\mu-1}{\mu} . \frac{2}{r}$$

If $x = \propto$, $z = f = -\frac{\mu}{\mu-1} . \frac{r}{2}$ *i.e.*, the principal focus f is at distance equal to $\frac{\mu}{\mu-1} . \frac{r}{2}$, from the centre on the side remote from the incident light.

(b) Lens of any shape : Let an object be placed at P on the axis of a lens at distance u from the nearer surface of the lens of radius r_1 (Fig. 30). Due to refraction at the first surface an image will be formed at Q′ at a distance v_1 from the pole B of the first refracting surface, given by the expression

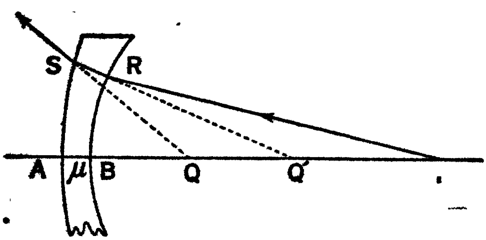


Fig. 30

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{r_1} \ldots (1)$$

where μ_2 and μ_1 $(\mu_2 > \mu_1)$ are the refractive indices of the substance of the lens and of the medium surrounding it.

This image is situated at a distance $v_1 + t$ from the pole A of the second refracting surface where t is the thickness of the lens (*i.e.*, the distance between the poles of the refracting surface) and will give rise to a second image at Q at a distance v from the pole A of the second refracting surface of radius r_2.

Since the refraction at the second surface takes place in passing from a denser to a rarer medium the position of the final image is given by $\frac{\mu_1}{v} - \frac{\mu_2}{v_1 + t} = \frac{\mu_1 - \mu_2}{r_2}$ ... ... (2)

Neglecting t, the thickness of the lens, we have by adding (1) and (2) $\frac{1}{v} - \frac{1}{u} = \frac{(\mu_2 - \mu_1)}{\mu_1}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$ ... ... (3)

If $\mu_2 = \mu$ and $\mu_1 = 1$, we have $\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$ ... (4)

Principal Foci : If $v = \infty$, we have $\frac{1}{u} = -(\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

This value of u is known as the first Principal Focal distance and is denoted by f_1. Then $\frac{1}{f_1} = -(\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

Again if $u = \infty$, we have $\frac{1}{v} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

This value of v is known as the Second Principal Focal distance and is denoted by f_2. Then $\frac{1}{f_2} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

When we speak of the focal length of a lens we generally mean the second principal focal distance of the lens.

The expression (4) may be written as

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}, \text{ where } \frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

44. Focal length of different lenses :

(a). **Double—Convex Lens** : The general expression for focal length is $\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

According to usual convention in signs, r_1 is negative and r_2 is positive. Therefore, $\frac{1}{f} = -(\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

Again if the surface of the radius r_2 faces the incident ray, r_2 is $-ve$ and r_1 is $+ve$ and we get the same expression. Thus the focal length of a double convex lens is negative.

(b). Double—Concave or Bi-concave Lens :

In this case r_1 is + and r_2 is −. Therefore $\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$

The focal length of a double concave lens is positive.

(c). Plano—Convex Lens :

If the plane surface faces the incident ray $r_1=\propto$ and r_2 is +.

$$\therefore \quad \frac{1}{f}=(\mu-1)\left(\frac{1}{\propto}-\frac{1}{r_2}\right)=-(\mu-1)\frac{1}{r_2}$$

The focal length is not altered if the convex surface faces the incident ray. It is negative.

(d). Plano—Concave Lens :

If the plane surface faces the incident rays $r_1=\propto$ and r_2 is −.

$$\therefore \quad \frac{1}{f}=(\mu-1)\left(\frac{1}{\propto}+\frac{1}{r_2}\right)=(\mu-1)\frac{1}{r_2}.$$

The focal length is positive in this case.

45. Conjugate foci for a lens by Fermat's Principle :

Let us consider the case of a double-convex lens (Fig. 31) having surfaces AM and AN of radii of curvature r_1 and r_2 respectively.

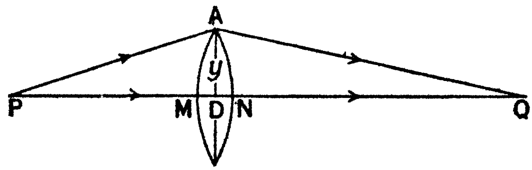


Fig. 31

Let AD $=y$, PD $=u$ and QD $=v$, Q being the image of the point P.

Since P and Q are conjugate, the time taken by the ray to travel over the path PAQ must be the same as that taken by the ray to travel over the path PMNQ. That is, optical paths joining P and Q through the lens must be the

Therefore PA + QA = PM + μMN + NQ

$$= PQ - MN + \mu MN = PQ + (\mu-1)MN \qquad (1)$$

$$y^2 = 2r_1 MD - MD^2 = 2r_2 ND - ND^2$$

$$\therefore \quad MD = \frac{y^2}{2r_1} \text{ and } ND = \frac{y^2}{2r_2} \text{ (approx.)}, \; MN = \frac{y^2}{2}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

Again $PA^2 = PD^2 + DA^2 = u^2 + y^2$, or $PA = \sqrt{u^2+y^2} = (u^2+y^2)^{\frac{1}{2}}$

$$= u\left(1+\tfrac{1}{2}\frac{y^2}{u^2}\right) \text{ (approx.). Similarly } QA = v\left(1+\tfrac{1}{2}\frac{y^2}{v^2}\right)$$

Therefore, substituting in (1) we have

$$(u+v)+\frac{y^2}{2}\left(\frac{1}{u}+\frac{1}{v}\right)=(u+v)+(\mu-1)\frac{y^2}{2}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

or $$\frac{1}{u}+\frac{1}{v}=(\mu-1)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

46. The Prismatic effect of a thin lens : A lens is upposed to be made up of a succession of truncated prisms with heir refracting edges placed perpendicularly to the axis of the lens.

Let a ray from the object O (Fig. 32) on the axis of the convex ens be incident on it at a height h from the axis and after efraction through the ens form the image I on the same axis. Then O and I are conjuate oci.

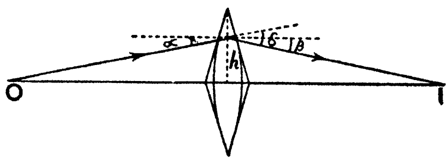


Fig. 32

Let α and β be respectively the inclinations of the object and image rays to the axis of the lens.

If δ be the deviation experienced by the ray in passing through the lens, we have $\delta=\alpha+\beta$

Again, if u and v are the distances of O and I from the centre of the lens, $\alpha=\frac{h}{u}$ and $\beta=\frac{h}{v}$ $\therefore$ $\delta=\frac{h}{u}+\frac{h}{v}=h\left(\frac{1}{u}+\frac{1}{v}\right)$

For the convex lens, we have $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ where f is the focal length. $\frac{h}{v}+\frac{h}{v}=\frac{h}{f}$ $\therefore$ $\delta=\frac{h}{f}$

That is, the deviation of any ray through any lens depends only on its height of incidence and the focal length.

The same result may also be obtained with a concave lens.

47. Two or more lenses in contact : Let two lenses A and B, one concave and the other convex having focal lengths f_1 and f_2 respectively, be placed in contact so as to have a common axis. (Fig. 33) An object O situated on the axis at a distance u from the lens A gives rise to an image P′ due to refraction in the first lens A at a distance v_1 from it.

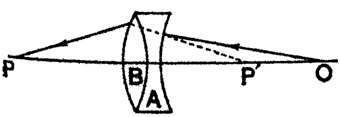


Fig. 33

Then $$\frac{1}{v_1}-\frac{1}{u}=\frac{1}{f_1} \qquad ...(1)$$

The image P′ as formed by the first lens and situated at a distanc nearly equal to v_1 from it, serves as an object with respect to tl second lens B and gives rise to another image P at a distance v from the second lens B. Then $\frac{1}{v}-\frac{1}{v_1}=\frac{1}{f_2}$ ... ...(2)

Adding (1) and (2) we have $\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2}$. ...(3)

When $u=\infty$; $v=F$. the focal length of the lens combination.

If a single lens of focal length F be placed in the position occupied by the lens combination so that it produces an image of the object in the same position and of the same size as that produced by the combination, such a lens is called an **equivalent lens** of the combination.

Then $\frac{1}{v}\quad\frac{1}{u}\quad\frac{1}{F}$ $\qquad \frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$

In this case the image produced by the equivalent lens is not only in the same position but also of the same size as that produced by the combination.

It can be easily proved that $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\cdots\cdots+\frac{1}{f_n}$

where f_1, f_2, f_3......f_n are the focal lengths of a number of thin lenses.

The equation suggests that for purpose of calculation it is convenient to deal with the reciprocals of the focal lengths rather than the focal lengths themselves.

48. Powers of a lens: Power of a lens is the degree of divergence (in case of concave lens) or convergence (in case of convex lens). Since the degree of divergence or convergence varies inversely as the focal length of the lens, the reciprocal of the focal length of a lens expressed in metres is termed the power or dioptric strength of the lens.

The unit of power is called the *dioptre*. This is the power of a lens of one metre focal length.

The power of a combination of lenses is equal to the algebraic sum of the powers of the constituent lenses.

If D, D_1 and D_2 are respectively the powers of the combination and the individual lenses, we may write. $D=D_1+D_2$

49. Equivalent lens: When two lenses are separated by a distance, it is impossible to find a single thin lens which when placed in any fixed position shall produce an image of the same size as that produced by the combination.

But a single thin lens can be found which when placed in a suitable position produces an image of the same size but not generally in the same position as that produced by the combination. This lens is said to be equivalent in this restrictive sense, to the combination.

50. Two thin Lenses separated by a distance :

Let two **concave lenses** AH_1 and BH_2 (Fig. 34) of focal lengths f_1 and f_2 respectively, be situated at a distance d from each other on a common axis AB. Let a ray SH_1 parallel to the axis cut

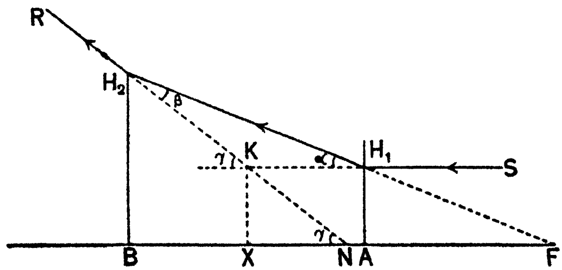


Fig. 34

the first lens at H_1 and refracted in the direction H_1H_2 and appear to come from the second principal focus F of the first lens.

On reaching the second lens, a further refraction takes place so that the ray now takes the direction H_2R and appears to proceed from the point N.

For the second lens, FB is the distance of the virtual object and NB the distance of the image so that we have,

$$\frac{1}{NB}-\frac{1}{FB}=\frac{1}{f_2} \text{ or } \frac{1}{NB}=\frac{1}{f_2}+\frac{1}{FB}=\frac{1}{f_2}+\frac{1}{f_1+d}, \text{ since } FB=f_1+d.$$

$$\therefore \quad \frac{1}{NB}=\frac{f_1+f_2+d}{f_2(f_1+d)} \qquad (1)$$

Produce SH_1 to meet NH_2 at K and from K draw KX perpendicular to the axis.

A lens is said to be equivalent to a combination when it produces the same deviation in a ray parallel to the axis as that produced by the combination.

Thus a lens at X of focal distance NX would be equivalent to the combination considered, for the path of the light refracted by this lens would be the same as the final direction of the light refracted by the combined system.

We have from similar triangles KXN and H_2BN

$$\frac{NX}{NB}=\frac{KX}{H_2B}=\frac{H_1A}{H_2B}=\frac{FA}{FB}=\frac{f_1}{f_1+d}$$

$$\therefore \quad NX=\frac{f_1}{f_1+d}.NB$$

But we have from (1) $NB=\dfrac{f_2(f_1+d)}{f_1+f_2+d}$

$$\therefore \quad NX=\frac{f_1}{f_1+d}.\frac{f_2(f_1+d)}{f_1+f_2+d}=\frac{f_1f_2}{f_1+f_2+d}$$

$$\therefore \quad \frac{1}{NX}=\frac{1}{F}=\frac{f_1+f_2+d}{f_1f_2}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{d}{f_1f_2}$$

where $NX=F$, the focal length of the equivalent lens.

Thus we have, $\dfrac{1}{F}=\dfrac{1}{f_1}+\dfrac{1}{f_2}+\dfrac{d}{f_1f_2}$

To determine the position of the lens we have,

$$\frac{NB}{NX}=\frac{H_2B}{KX}=\frac{H_2B}{H_2A}=\frac{FB}{FA} \quad \therefore \quad \frac{NX+XB}{NX}=\frac{FA+AB}{FA}$$

Since $NB=NX+XB$, $FB=FA+AB$

$$\text{or} \quad 1+\frac{XB}{NX}=1+\frac{AB}{FA} \quad \text{or} \quad \frac{XB}{NX}=\frac{AB}{FA}$$

$$\therefore \quad XB=NX.\frac{AB}{FA}=d\,.\frac{F}{f_1}$$

Thus the equivalent lens should be placed at distance XB or $d.\dfrac{F}{f_1}$ in front of the second lens or $d\left(1-\dfrac{F}{f_1}\right)$ behind the first lens of the combination.

But if the beam be divergent or convergent, then the position of the equivalent lens is not immaterial.

Note : If the incident beam consists of parallel rays, in whatever position the equivalent lens be placed, the image will be of the same size.

It can be shown that if the equivalent lens be placed at the distance $d.\dfrac{F}{f_2}$ in front of the first lens, the size of the image produced by the equivalent lens may be the same as that produced by the combination. But their position are not the same *i.e.* the two images, one by the equivalent lens and the other by the combination are not formed in the same position.

50(a). Alternative method :

(*a*) *When the lenses are divergent* (Concave) :

In Fig. 34, let the incident ray SH_1 parallel to the axis cuts ie first lens at H_1 at a distance x from the axis. After refraction rough the first lens the emergent ray H_1H_2 cuts the second ns at H_2 at a distance y from the axis.

Let α ($\angle H_2H_1K$) be the deviation by the first lens, β $\angle H_1H_2K$) the deviation by the second lens and γ ($\angle H_2NB$) the otal deviation produced by the combination or equivalent lens.

Therefore, $\gamma = \alpha + \beta$ ... (1)

Then we heve, $\alpha = \frac{x}{f_1}$, $\beta = \frac{y}{f_2}$ $\gamma = \frac{x}{F}$

Here f_1, f_2 and F are respectively the focal lengths of the first, econd and the equivalent lens.

Hence from (1)

$$\frac{x}{F} = \frac{x}{f_1} + \frac{y}{f_2} = \frac{x}{f_1} + \frac{x+a}{f_2} = \frac{x}{f_1} + \frac{x}{f_2}\left(1+\frac{d}{f_1}\right)$$

Here $a = y - x = \frac{d.x}{f_1}$ d being the distance between the lenses.

$$\therefore \quad \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}\left(1+\frac{d}{f_1}\right) = \frac{1}{f_1} + \frac{1}{f_2} + \frac{d}{f_1 f_2}.$$

Note : Let the ray SH_1 be produced to meet the second lens L. Then H_2L is equal to a *i.e.* $a = y - x$.

Then $\tan \alpha = \frac{H_2L}{H_1L} = \frac{H_2L}{BA} = \frac{a}{d}$. Since α is small $\alpha = \frac{x}{f_1}$ $\quad a = \frac{xd}{f_1}$.

(*b*) *When the lenses are convergent* (Convex) :

Let A and B are two convergent lenses A and B of focal lengths f_1 and f_2 respectively and separated by a distance d. (Fig. 35)

Let a ray PQ parallel to the axis passes after refraction through the lens A in the direction QR, strikes the second lens B at R and then after refraction travels along RF intersecting the axis at F.

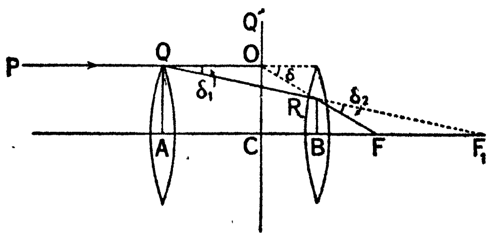


Fig. 35

Let $AQ = h_1$, $RB = h_2$, δ_1 = the deviation by the first lens, δ_2 = the deviation by the second lens, and δ the total deviation by the combination or the equivalent lens.

Then according to Art. 46, $\delta_1 = \dfrac{h_1}{f_1}$; $\delta_2 = \dfrac{h_2}{f_2}$ and $\delta = \dfrac{h_1}{F}$,

where F is the focal length of the equivalent lens.

The total deviation $\delta = \delta_1 + \delta_2 = \dfrac{h_1}{f_1} + \dfrac{h_2}{f_2}$

But $h_2 = h_1 - d.\delta_1 = h_1 - \dfrac{h_1 d}{f_1} = h_1\left(1 - \dfrac{d}{f_1}\right)$

$\therefore \quad \delta = \dfrac{h_1}{f_1} + \dfrac{h_1}{f_2}\left(1 - \dfrac{d}{f_1}\right) = h_1\left[\dfrac{1}{f_1} + \dfrac{1}{f_2} - \dfrac{d}{f_1 f_2}\right]$. But $\delta = \dfrac{h_1}{F}$

$\dfrac{h_1}{F} = h_1\left(\dfrac{1}{f_1} \quad f_2 \quad \dfrac{d}{f_1 f_2}\right)$ or $\dfrac{1}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2} - \dfrac{d}{f_1 f_2}$.

51. Determination of μ for a convex Lens :

The focal length of a convex lens is given by the expression

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

So in the above expresion if f, r_1 and r_2 are known, the refractive index μ for the material of the lens can be obtained.

The focal length of the convex lens is determined by noting the positions of the object and its image with the help of the formula $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$ or by any other known method.

The radii r_1 and r_2 of the two surfaces of the lens are determined by a spherometer.

Then, by substituting the values of f, r_1 and r_2 in the above formula, the value of μ is easily determined.

52. Radius of curvature of the surface of a convex lens:

The lens is made to float on mercury with the surface whose radius is to be determined, in contact with mercury.

A pin is moved along the axis of the lens until it coincides with its image formed by reflection in the lower surface of the lens in contact with mercury.

Rays from the object after refraction strike the lower surface normally, as if they come from the centre of curvature of the lower surface. Here the position of the object and the centre of curvature are conjugate foci.

In the lens, $\dfrac{1}{v} - \dfrac{1}{u} = -\dfrac{1}{f}$ or $\dfrac{1}{r} - \dfrac{1}{u} = -\dfrac{1}{f}$

Since here $v=r$, the radius of curvature of the lower surface of he lens in contact with mercury.

$$\therefore \quad \frac{1}{r}=\frac{1}{u}-\frac{1}{f} \quad \text{or} \quad r=\frac{fu}{f-u}$$

53. Determination of μ for a very small quantity of a iquid : A convex lens of moderate focal length is placed on a lane horizontal mirror and its focal length f_1 is determined by iding the position of a pin on the axis at which there is no rallax between the pin and its real image. The distance of the in from the centre of the lens is equal to the focal length of the lens. A very small quantity of a liquid is then introduced into he space between the lower surface of the lens and the plane irror. The liquid forms a plano-concave lens, the radius of urvature r of the upper surface of the liquid being the same as hat of the lower surface of the glass lens.

The focal length F of the compound lens of glass and liquid is determined by pin method in the same way as before.

Then, $\left.\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}\right\}$ where f_2 is the focal length of the liquid lens.

$$\text{or } \frac{-}{f_2}=\frac{-}{-}-\frac{1}{f_2}$$

Thus f_2, the focal length of the liquid lens is determined by using proper signs for the numerical values of F and f each of which should be taken negative in the above relation.

Since $\frac{1}{f_2}=(\mu-1)\frac{1}{r}$...(1) when μ is the refractive index of the liquid, the value of μ for the liquid is obtained by substituting the values of f_2 and r in the expression (1) the radius r being determined by a spherometer.

Note : For a plano-concave or a plano-convex lens. If $\mu=1{\cdot}5$ nearly, $f_2=2r$ *i.e.* focal length $=2\times$radius of curvature.

Hence for an equi-convex or equi-concave lens $f_2=r$ (nearly).

54. Optical Centre of a Lens : If a ray of light passes through a lens in such a way that the emergent ray is parallel to the direction of the incident ray, the path of the ray within the lens intersects the axis at a point which is called the optical centre of the lens.

Let C_1 and C_2 be the centres of curvature of the spherical surfaces AR and A_2Q of the lens (Fig. 36) of axial thickness t. Draw two parallel radii to cut the surfaces at R and Q. These two radii

being normal to the respective surfaces, the two surface at R and Q are parallel to each other. So the ray SR incident at the point

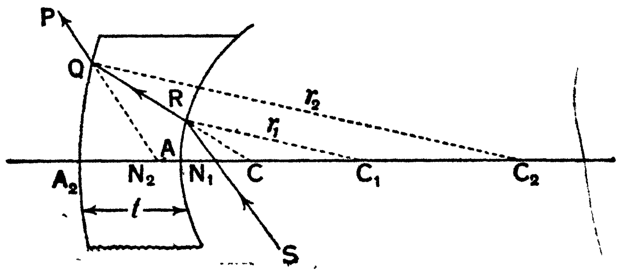


Fig. 36

R emerges out from the point Q along QP in a direction parallel to the incident ray SR. Thus the portion of RQ within the lens when produced backwards cuts the axis at the point C. The point C is called the **optical centre** of the lens.

In the case of double convex lens and a double concave lens, the optical centre lies within the lens but in the case of plano-convex and plano-concave lenses the optical centre lies on the curved surface.

In the Figure 36, the rays SR and QP cut the axis at the points N_1 and N_2 respectively. These points N_1 and N_2 are called the nodal points.

55. The cardinal points of a thick lens : In the case of a thin lens distances of the object and the image are measured from the surfaces of the lens. If however, the lens is a thick one, very different values of the object and the image distances are obtained according as they are measured from the front or back surface of the lens.

The formula for the conjugate distances are wonderfully simplified by measuring conjugate distances from certain points in the object and image spaces. These points are called **cardinal points** and also **Gauss points** after their investigator.

There are **six** such points for an optical system. They are known as **two focal points, two principal points** and **two nodal points.**

55(a). Focal Points—Focal Planes : A parallel bundle of rays after refraction at spherical surfaces converge to a point which is called the **focal point**. Since any of the two faces of the lens can be used for the first incidence, a lens must possess two focal points.

Planes drawn through these focal points perpendicular to the axis of the system are called the **focal planes.**

55(b). Principal Points—Principal Planes : In the case of a number of spherical refracting surfaces, two transverse planes with

reference to the axis can always be drawn such that a small object on the axis in one of them has an erect image of equal size formed in the other and for which the *magnification* is *unity* and of positive sign. Such planes are called the **principal planes.**

55(c). Nodal Points—Nodal planes :

The points of intersection of the the principal planes with the principal axis are called **principal points** or **Gauss points.**

Nodal points have the property that a ray directed to one of these points emerge on the other side of the lens in a direction parallel to the incident ray and radiating from the other nodal point.

Planes passing through the nodal points and perpendicular to the axis are called **nodal planes.**

56. Least distance between an object and its image formed by a convex lens is equal to four times the focal length of the lens :

For a lens we have, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

For a convex lens and for real images, using proper signs we get,

$$\frac{1}{v}-\frac{1}{u}\quad\frac{1}{f} \text{ or } \frac{1}{v}+\frac{1}{u}=\frac{1}{f} \;\ldots (1) \quad \text{or } \frac{u+v}{uv}=\frac{1}{f} \qquad (2)$$

Since f is constant, $u+v \propto uv$ ... (3)

From (1) we have, $\left(\frac{1}{v}+\frac{1}{u}\right)^2=\frac{1}{f^2}=\left(\frac{1}{v}-\frac{1}{u}\right)^2+\frac{4}{uv}$

Since f is constant, we have, $\frac{4}{uv}=\text{const.}-\left(\frac{1}{v}-\frac{1}{u}\right)^2$

$\frac{4}{uv}$ will be maximum, when $\left(\frac{1}{v}-\frac{1}{u}\right)^2$

That is, when $v=u$, $\frac{4}{uv}$ is maximum, *i.e.*, uv is minimum.

Hence from (3) we have $u+v$ is minimum, when uv is minimum, *i.e.*, when $u=v$. Thus from (2) $\frac{2v}{v^2}=\frac{1}{f}$ or $v=2f$

∴ The least distance between the object and the image is $2f+2f=4f$ or 4 times the focal length of the lens

56(a). Alternative Proof : Let u and v be the distances of the object and the image from the lens and let l be the distance between the object and the image. Then $l = u + v$

$$\text{Differentiating } \frac{dl}{du} = 1 + \frac{dv}{du}$$

For minimum value of l, $\frac{dl}{du} = 0$, $\therefore \frac{dv}{du} = -1$

But we know that $\frac{dv}{du} = -\frac{v^2}{u^2}$ (longitudinal magnification)

$\therefore \frac{v^2}{u^2} = 1$, (for minimum value of l) or $v = \pm u$ gives minima.

When $v = +u$, the eqn. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ gives $u = 2f$ and $v = 2f$.

Therefore $l = 4f$

Note : We have, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Differentiating we have, $\frac{dv}{v^2} + \frac{du}{u^2} = 0$, $\because$ f is constant,

or $\frac{dv}{du} = \frac{v^2}{u^2} = \mathrm{M}$, where M is the *longitudinal magnification.*

57. If I_1 and I_2 are the sizes of the images of the object for the two positions of the lens, prove that :

$O^2 = I_1 I_2$, where O is the size of the object.

Let D be the distance between the object and the screen and let u_1 and u_2 be respectively the distances of the object from the first and second positions of the lens. Then, for the first position,

$$\frac{I_1}{O} = \frac{D - u_1}{u_2} \qquad (1)$$

for the second position $\frac{I_2}{O} = \frac{D - u_2}{u_2}$ (2)

$$\frac{I_1 I_2}{O_2} = \frac{(D - u_1)(D - u_2)}{u_1 u_2}$$

$$= \frac{D^2 - D(u_1 + u_2) + u_1 u_2}{u_1 u_2} = \frac{D^2 - D^2 + u_1 u_2}{u_1 u_2}, \text{ since } u_1 + u_2 = D$$

$\therefore \frac{u_1 u_2}{u_1 u_2} = 1$. That is $O^2 = I_1 I_2$ or $O = \sqrt{I_1 I_2}$

58. If m_1 and m_2 be the magnifications in these two positions and if d be the distance between them, prove that the focal length $f=\dfrac{d}{m_1-m_2}$

Let u_1 and v_1 denote respectively the distance of the object and the image from the first position of the lens and let u_2 and v_2 denote the corresponding values for the second position of the lens. Let m_1, m_2 be magnifications for 1st and 2nd positions respectively.

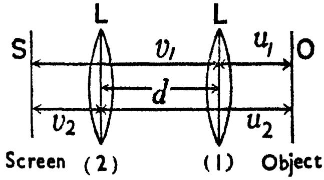


Fig. 37

For the first position, we have

$\dfrac{1}{v_1}+\dfrac{1}{u_1}=\dfrac{1}{f}$ ... (For real image by convex lens).

then $\dfrac{v_1}{v_1}+\dfrac{v_1}{u_1}=\dfrac{v_1}{f}$

or $1+m_1=\dfrac{v_1}{f}$ (1) since $\dfrac{v_1}{u_1}=m_1$

Similarly for the 2nd position $\dfrac{1}{v_2}+\dfrac{1}{u_2}=\dfrac{1}{f}$ (For real image)

then $\dfrac{v_2}{v_2}+\dfrac{v_2}{u_2}=\dfrac{v_2}{f}$ or $1+m_2=\dfrac{v_2}{f}$ ... (2)

Subtracting (2) from (1) $m_1-m_2=\dfrac{v_1}{f}-\dfrac{v_2}{f}=\dfrac{v_1-v_2}{f}$

But $v_1-v_2=d$, the distance between two positions of the lens

$\therefore\ m_1-m_2=\dfrac{d}{f}$ or $f=\dfrac{d}{m_1-m_2}$

Hence the relation is proved.

QUESTIONS

1. Deduce the relation $\dfrac{\mu}{v}-\dfrac{1}{u}=\dfrac{\mu-1}{r}$, when an image is formed by refraction through a single spherical surface.

Apply this relation to obtain the formula $\dfrac{1}{v}-\dfrac{1}{u}=\dfrac{1}{f}$, for refraction through spherical lenses. [C. U. 1928, '40]

Find the limiting distance between a fixed object and screen, so that for two positions of a thin convex lens placed between them, sharp images are formed. [C. U. 1944. '50]

2. Prove that for a combination in air of two thin lenses of focal lengths f_1 and f_2, $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$. [C. U. 1942, '58]

where f is the focal length of the combination and d the distance between the lenses.

Deduce the position of the equivalent lens and shew that it is equivalent to the combination in a restricted sense. [C. U. 1958]

3. Deduce the relation between the conjugate foci for lenses by Fermat's Principle.

EXAMPLES

1. An object 10 mm. long and 5 mm. broad is placed 30 cms. in front of a double convex lens of focal length 15 cms., behind which at a distance of 20 cms., a concave lens of focal length 30 cms. is placed. Find the position and size of the image produced by refraction through the two lenses. [C. U. 1915]

For a lens we have, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

Considering the convex lens, let v_1 be the distance of the first image from the convex lens. Then $\frac{1}{}-\frac{1}{30}=-\frac{1}{15}$, *i.e.*, $v_1=-30$ cms.

The image due to the convex lens is formed at a distance of 30 cms. on the other side of the lens, *i.e.*, 10 cms. from the concave lens on the side away from the object.

Considering the concave lens, let v_2 be the distance of the final image from the concave lens. Then $\frac{1}{v_2}-\frac{1}{(-10)}=\frac{1}{30}$, *i.e.*, $v_2=-15$ cms.

That is, the final image is formed at a distance of 15 cms. behind the concave lens. Again $\frac{I}{O}=\frac{v}{u}$ where I and O are the sizes of the image and the object respectively.

Therefore, the length of the first image $=\frac{v_1}{u}\times 10=\frac{30}{30}\times 10=10$ mm.

,, the breadth of the ... ... $=\frac{30}{30}\times 5=5$ mm.

Again the length of the *final image* $=\frac{v_2}{10}\times 10=\frac{15}{10}\times 10=15$ mm.

,, breadth ... ... $=\frac{15}{10}\times 5=7{\cdot}5$ mm.

2. Explain, why the horizontal bottom of a pool of water appears to be raised to an eye placed above it. Find how deep the pool will appear to be, if the real depth is 15 ft. and refractive index of water is 1·3. [C. U. 1927]

We know that $\frac{u}{v}=\mu$, *i.e.*, $\frac{\text{Real depth}}{\text{Apparent depth}}=\mu$

$\frac{15}{}=1{\cdot}3$ $\qquad \frac{15}{1{\cdot}3}=11{\cdot}54$

3. The rays of a vertical sun are brought to a focus by a lens at a distance of 1 ft. from the lens. If the lens is held just over a smooth and deep pool of water, at what depth in the water will the rays come to a focus? The index of refraction of water is $\frac{4}{3}$. [C. U. 1923]

Let the depth below the free surface of water at which the rays come to a focus be v, then $\frac{v}{u} = \frac{4}{3}$. But here $u = 1$ ft. $\therefore$ $v = \frac{4}{3}$ ft. $= 1{\cdot}3$ ft.

4. Rays which from a real image on a screen are intercepted by a concave lens of 12 in. focal length at a distance of 8 in. from the screen. How far must the screen be moved that it may receive the new image?

[C. U, 1924]. (Ans. 16 inches)

5. On an optical bench an object is clamped at 10 cm., a convex lens ($f = 15$ cm.) at 25 cm. and a concave lens ($f = 15$ cm.) at 46 cm.

Calculate the position of the image formed and the magnification produced. [C. U. 1933]

[The image is real and is formed at distance 30 cm. from the concave lens. Magnification $= 6$]

6. A convex and a concave lens, each 10 inch focal length are held co-axially at a distance 3 in. apart. Find the position of the image if the object is at a distance of 15 in. beyond (a) the convex lens (b) the concave lens. [C. U. 1934]

[Ans. (a) The image is virtual and is formed at a distance of $15\frac{1}{7}\frac{?}{}$ in. in the same side as the object.

(b) The image is virtual and is formed at a distance of 90 inches in front of the concave lens.]

7. When a convex lens is placed above an empty tank an image of a mark on the bottom of the tank, which is 45 cm. from the lens is formed 36 cm. above the lens. When a liquid is poured into the tank to a depth of 40 cm., the distance of the image of the mark above the lens is 43 *cm.* Find the refractive index of the liquid. [C. U. 1944] (Ans. $\mu = 1{\cdot}36$)

CHAPTER V

DISPERSION

59. Dispersion: When a ray of white light is refracted through a prism it is dispersed into its constituent colours. The angle included by the emergent rays of any two given colours is termed the **dispersion between these two rays.**

Let the angle included by the red and violet emergent rays, *i. e.*, the dispersion, between the red and the violet rays be equal to the angle VOR. (Fig. 38)

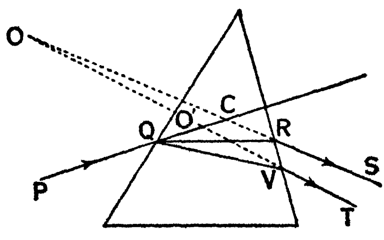


Fig. 38

If D_v and D_r stand respectively for the deviations of the violet and the red rays then the dispersion between the violet and the red rays is $D_v - D_r$.

The deviation of the violet ray is given by $\angle VO'C = D_v = (\mu_v - 1)$ A, where A is the refracting angle of the prism.
The deviation of the red ray is givin by $\angle O'CO = D_r = (\mu_r - 1)A$.

$\therefore$ The dispersion between the violet and red rays is
$\angle VOR = \angle VO'C - \angle O'CO = D_v - D_r = (\mu_v - \mu_r)A$.

60. Dispersive Power: It has been found by experiments that the refractive index of a transparent medium depends on the nature of the light used and it is greater for violet than for red rays while for rays corresponding to the intermediate portion of the spectrum the refractive index will have inter-mediate values. Let μ_v and μ_r be the refractive indices for violet and red rays respectivily and let μ be that for the mean ray such that $\frac{\mu_v + \mu_r}{2} = \mu$.
The mean ray between any two rays is that ray for which the refractive index of a given medium is the avarage of the refractive indices of the same medium for those two rays.

We know that the deviation D of the mean ray of light through a prism is given by $D = (\mu - 1)A$, where A is the angle of the prism.

Then, for violet rays $D_v = (\mu_v - 1)A = \frac{\mu_v - 1}{\mu - 1}.(\mu - 1)A$

$$= \frac{\mu_v - 1}{\mu - 1}. D$$

and for red rays $D_r = (\mu_r - 1)A = \frac{\mu_r - 1}{\mu - 1}.(\mu - 1)A = \frac{\mu_r - 1}{\mu - 1}. D$

$\therefore$ Dispersion between violet and red rays

$$= D_v - D_r = (\mu_v - \mu_r)A = \frac{\mu_v - \mu_r}{\mu - 1}. D$$

The factor $\frac{\mu_v - \mu_r}{\mu - 1}$ is called the **Dispersive Power** of the medium with respect to the violet and red rays and is measured by the ratio of deviation between these two rays after refraction through a prism of the same medium having a small refracting angle to the deviation of the mean ray produced by the same prism.

Using the notation of Differential Calculus $\omega = \frac{d\mu}{\mu - 1}$

From (1) We have, $\omega = \frac{D_v - D_r}{}$ for red and violet rays.

61. Deviation without Dispersion : It is possible by using two prisms of different substances to produce deviation without any great amount of dispersion ; such a combination of prisms is called an *achromatic combination.*

62. Condition of achromatism in prisms : When a beam of white light is allowed to fall on a prism, it is dispersed as well as deviated. To do away with dispersion, only a second prism of different index of refraction is to be used in combination with the first prism with its vertex in the opposite direction.

If μ_v and μ_r be the refractive indices for violet and red rays respectively in the first prism, the dispersion through it is given by

$$D_v - D_r = (\mu_v - \mu_r)\ A,$$

where D_v and D_r are the respective deviations for the violet and red rays, μ_v, μ_r are the refractive indices of the prism for violet and red rays, and A the angle of the prism.

Similarly the dispersion for the violet and red rays in the second prism of refractive index μ' (for the mean ray) and angle A' is given by $D'_v - D'_r = (\mu'_v - \mu'_r)A'$,

where D'_v D'_r and D' are deviations produced by rays for which the refractive indices are equal to μ'_v, μ'_r and μ'.

(A). To do away with the dispersion of the rays produced by the first prism, the second prism is placed in contact with the first prism but with its vertex in the opposite direction. The total dispersion due to both the prisms will be zero, if

$$D_v - D_r = D'_v - D'_r \quad \text{or} \quad (\mu_v - \mu_r)A = (\mu'_v - \mu'_r)A'. \quad ...(2)$$

Thus by combining two (Fig. 39) prisms of different materials having refracting angles which satisfy the relation (2), the dispersion between the violet and red rays vanishes and they come out of the prism in a direction parallel to one another.

Note. It is to be noted that if the combination be achromatic for two colours it will not, in general, be achromatic for all other colours owing to the irrationality of dispersion in prisms of different materials. By increasing the number of prisms perfect achromatism may be obtained.

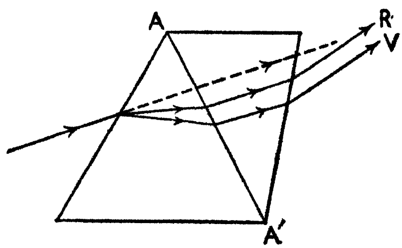


Fig. 39

(B). Again since the refractive indices of the two prisms are not the same and since their angles are also different, the deviation of the mean ray produced by the first prism is not annulled by that produced by the second prism and the resultant deviation is expressed by the relation $D - D' = (\mu - 1)A - (\mu' - 1)A'$.

If the two prisms are made from the same glass, *i.e.*, $\mu'_v = \mu_v$, $\mu'_r = \mu_r$ and $\mu' = \mu$ and if $A = A'$ then $D - D' = O$.

That is, both dispersion and deviation are annulled when the prisms are made of the same material and of the same angle.

63. Dispersion without Deviation : It is also possible to produce dispersion without deviation by combining two prisms of different substances (Fig. 40) having different refracting angles.

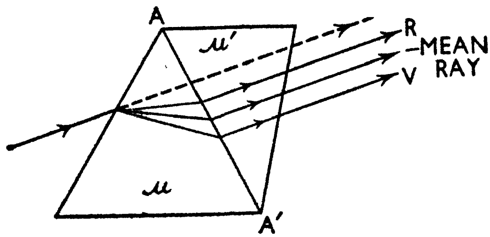


Fig. 40

We have seen in Art. 62, (B) that the resultant deviation of the mean ray through the two prisms having different refractive indices and different angles is expressed as

$$D - D' = (\mu - 1)A - (\mu' - 1)A'.$$

Thus if $\dfrac{A}{A'} = \dfrac{\mu' - 1}{\mu - 1}$, the deviation of the mean ray $D - D' = 0$

and the dispersion between the violet and red rays is equal to

$$(D_v - D_r) - (D'_v - D'_r) = (\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A'$$

$$= \frac{(\mu' - 1)}{(\mu - 1)} . (\mu_v - \mu_r)A' - (\mu'_v - \mu'_r)A'$$

$$= (\mu' - 1)A' \left\{ \frac{\mu_v - \mu_r}{\mu - 1} - \frac{\mu'_v - \mu'_r}{\mu' - 1} \right\} = (\mu' - 1)(\omega - \omega')A'.$$

This principle has been utilised in the Direct Vision Spectroscope for examining the nature of the spectrum of light coming from the source under examination.

64. Irrationality of Dispersion ; Anomalous Dispersion : If the spectra formed by different substances be examined, it will be seen that the dispersion between any two colours will not be, in general similar, the dispersion depending on the nature of the medium. This fact is known as *irrationality of dispersion*. This is found in substances having surface colours. Sometimes the order of the colour is changed. For example, a solution of Fuchsine in alcohol deviates red more than the violet. This phenomenon is known as *anomalous dispersion*.

65. Secondary Spectrum : If a beam of white light be made to pass through a combination of prisms, the emergent beam instead of being colourless shows a residual colour. This residual spectrum is called a *secondary spectrum.*

66. Spectroscope and Spectrometer : The **spectroscope** is an instrument for producing dispersion of rays of light so as to form a spectrum, and for observing the spectrum so formed.

The **spectrometer** which has been already described, is a similar instrument provided with suitable arrangements for measuring the deviations of dispersed rays.

66(a). Direct-vision Spectroscope : The principle of dispersion with no deviation is a great advantage in the construction of a pocket Direct-vision Spectroscope. It consists (Fig. 41) mainly of five prisms, two of flint glass F, and the other three of crown glass C,

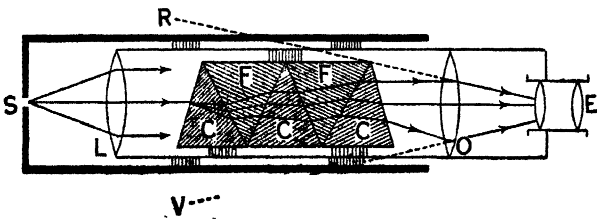


Fig. 41

and they are placed in such a way that the flint glass prisms lie between the crown glass prisms with their edges pointing in a direction opposite to those of the other two prisms.

This combined system is placed in a cylindrical tube fitted with a convex lens L at one of its ends which again slides freely inside another cylindrical tube provided with a narrow adjustable slit S. By adjusting the position of the slit in front of the lens a parallel beam of light is made to fall on the system of prisms and since the dispersive powers of crown and flint glass prisms are different, the deviation of light is got rid of, only a dispersed patch in which all the colours are separated, being obtained.

The deviation of the mean ray, say yellow, through the crown glass prisms C is the same but opposite in direction to that in the flint glass prisms F, but the deviations of the other rays, though opposite in directions are not the same through both the prisms. There is, therefore, some amount of dispersion left which causes the spectrum to lie in approximately the same line as the incident beam for the mean ray remains undeviated after refraction through the prisms.

67. Dispersion in a lens : We know that in a lens the relation between the conjugate distances is given by,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

If f_r, f_v and μ_r, μ_v are respectively the focal lengths and refractive indices for red and violet rays.

$$\frac{1}{f_v}=(\mu_v-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=\frac{\mu_v-1}{\mu-1}\cdot\frac{1}{f} \quad (1)$$

$$\frac{1}{f_r}=(\mu_r-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=\frac{\mu_r-1}{\mu-1}\cdot\frac{1}{f} \quad (2)$$

$$\therefore \frac{f_r}{f_v}=\frac{\mu_v-1}{\mu_r-1} \text{ i.e., } f_r>f_v, \text{ since } \mu_v>\mu_r.$$

Thus the violet rays are focussed nearer to the lens than the red rays.

68. Chromatic Aberration : Objects seen through lenses or images formed by them on a screen appear coloured at their edges,

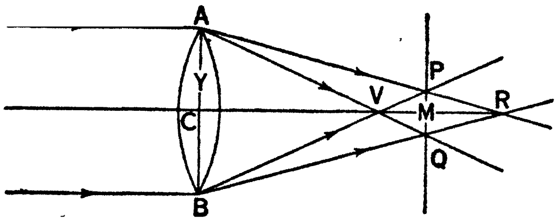


Fig 42

the nature of coloration depending on the position of the screen with respect to the lenses. Like prisms, lenses can, not only refract rays of light, but can decompose them. This is due to the fact that white light contains seven different colours having different refractive indices of which violet is the most refrangible and red the least.

So when a beam of white light is made to fall on a convex lens AB parallel to the principal axis, it is split up and the constituent colours converge to different points on the axis, the violet converging to a point V nearest the lens, and red to a point R furthest away from the lens, while all the other colours such as indigo, blue, green yellow and orange converge to different points between the violet and red foci.

Thus the emergent rays instead of forming a white image at a single point give rise to a series of coloured points on the axis, the

iolet image being nearer to the lens than the red one. So instead f having a common focus for all the colours we get different foci or different colours. This divergence or separation of the foci from he common focus due to which the image of an object formed by single convex lens becomes coloured, is what is known as **romatic aberration.**

If a screen be held at V a circle of light will be seen on the screen with red t the outer edge and violet at its centre. Similarly, when the screen is placed t R a circle with violet at the outer edge and red at the centre will be observed. f the screen is placed at M the circle with PQ as diameter will be the nearest pproach to a point focus where the border of the image shades off from the red violet.

From (1) and (2) we have

$$\frac{1}{f_v}-\frac{1}{f_r}=\frac{\mu_v-\mu_r}{\mu-1}\cdot\frac{1}{f} \quad \text{or } f_r-f_v=\frac{\mu_v-\mu_r}{\mu-1}\cdot f=wf.$$

Thus the chromatic aberration for parallel rays, (f_r-f_v) is equal to the mean focal length f multiplied by the dispersive power ω of the material of the lens.

69. Achromatic combination of lenses: To correct the defect of Chromatic Aberration in a lens, a second lens is used in combination with the first one.

Let f_r and f_v denote the focal lengths of the red and violet rays and μ_r and μ_v the refractive indices for red and violet rays when the first lens is considered.

Then, for red rays, $\frac{1}{f_r}=\left(\mu_r-1\right)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$

$=\frac{\mu_r-1}{\mu-1}\left(\mu-1\right)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=\frac{\mu_r-1}{\mu-1}\cdot\frac{1}{f}$ where f is the focal length corresponding to the mean rays for which the refractive index is μ.

Then, for violet rays, $\frac{1}{f_v}=\frac{\mu_v-1}{\mu-1}\cdot\frac{1}{f}$.

When the second lens is used in combination with the first one, the combined focal length for red rays is given by

$$\frac{1}{F_r}=\frac{\mu_r-1}{\mu-1}\cdot\frac{1}{f}+\frac{\mu'_r-1}{\mu'-1}\cdot\frac{1}{f'}$$

For violet rays, $\frac{1}{F_v}=\frac{\mu_v-1}{\mu-1}\cdot\frac{1}{f}+\frac{\mu'_v-1}{\mu'-1}\cdot\frac{1}{f'}$

where μ, μ', f, f' denote respectively the same quantities as before for the first and the second lens.

Now to correct the lenses for chromatic aberration *i.e.*, to br all the variously coloured rays to the same focus, the combinat must have the same focal length for the rays of all colours.

Thus $F_r = F_v$

$$i.e.\quad \frac{\mu_r-1}{\mu-1}\cdot\frac{1}{f}+\frac{\mu'_r-1}{\mu'-1}\cdot\frac{1}{f'}=\frac{\mu_v-1}{\mu-1}\cdot\frac{1}{f}+\frac{\mu'_v-1}{\mu'-1}\cdot\frac{1}{f'}$$

$$\text{or}\quad \frac{\mu_v-\mu_r}{\mu-1}\cdot\frac{1}{f}+\frac{\mu'_v-\mu'_r}{\mu'-1}\,\frac{1}{f'}=0\quad\text{or}\quad\frac{\omega}{f}+\frac{\omega}{f'}=0$$

where ω and ω' are the dispersive powers of the lenses of refract indices μ and μ' respectively.

Since $\mu_v > \mu_r$ and $\mu'_v > \mu'_r$ while μ and μ' are both greater tl unity, it follows that f an f' will have opposite signs. That is, of the lenses must be **convex** and the other **concave** and the ra of the focal lengths must be equal to the ratio of the dispers powers of the glasses of which the lenses are made.

The faces in contact of the two lenses forming the combinat should have the same radius of curvature and they may be cemen together with Canada balsam to reduce reflection losses.

Errors due to spherical aberration is lessened if the combinat is plano-convex.

For this, a *plano-concave* lens is coupled with the *convex l* to form the combination.

Such a combination is called an **achromatic doublet.**

Note: If the combination be achromatic for two colours it will no general, be achromatic for all other colours due to irrationality of dispersion.

The two ends of the spectrum are superposed but there is a remaining co defect called the "*secondary spectrum*".

By the addition of a third lens its achromatism may be made more pe and the arrangement is called *apochromatic.*

69(a). By Calculus: We know that when two lenses of fo lengths f_1 and f_2 are placed in contact, $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$, where the focal length of the combination.

For achromatism, $F_r = F_v$ or $F_r - F_v = 0$ *i.e.*, $dF=0$

or $d\left(\frac{1}{F}\right)=0$, where F_r and F_v are the equivalent focal leng for red and violet light respectively.

So $d\left(\frac{1}{f_1}\right)+d\left(\frac{1}{f_2}\right)=0.$ But $\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{f_2}\right).$

$$\therefore\ d\left(\frac{1}{f_1}\right)=d\left[(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\right]=d\mu.\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$=\frac{d\mu}{\mu-1}(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=\frac{d\mu}{(\mu-1)f_1}=\frac{\mu_v-\mu_r}{\mu-1}\cdot\frac{1}{f_1}=\frac{\omega_1}{f_1}.$$

Similarly $d\left(\frac{1}{f_2}\right)=\frac{d\mu'}{(\mu'-1)f_2}=\frac{\mu'_v-\mu'_r}{\mu'-1}\cdot\frac{1}{f_2}=\frac{\omega_2}{f_2}$

where ω_1 and ω_2 are dispersive powers of the material of the enses.

Hence for perfect achromatism the condition is, $\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0.$

70. Achromatism of two thin lenses separated by a distance : Let f_1 and f_2 be the focal lengths of the two lenses separated by a distance a, and F the focal of the combination.

$$\text{Then}\quad \frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{a}{f_1f_2}$$

Now, for the condition of achromatism, $d\left(\frac{1}{F}\right)=0$

$$\therefore\quad d\left(\frac{1}{F}\right)=d\left(\frac{1}{f_1}\right)+d\left(\frac{1}{f_2}\right)+d\left(\frac{a}{f_1f_2}\right)=0$$

$$0=d\left(\frac{1}{f_1}\right)+d\left(\frac{1}{f_2}\right)+\frac{a}{f_2}\ d\left(\frac{1}{f_1}\right)+\frac{a}{f_1}\ d\left(\frac{1}{f_2}\right)$$

As in article 69(a) we have, $d\left(\frac{1}{f_1}\right)=\frac{\omega_1}{f_1}$ and $d\left(\frac{1}{f_2}\right)=\frac{\omega_2}{f_2}$

Hence $\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}+\frac{a}{f_2}\cdot\frac{\omega_1}{f_1}+\frac{a}{f_1}\cdot\frac{\omega_2}{f_2}=0$

or $\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}+\frac{a}{f_1f_2}(\omega_1+\omega_2)=0$ ... ...(1)

This is the general condition for achromatic combination.

If the two lenses have the same dispersive power, *i.e.*, if $\omega_1=\omega_2$

The eqn. (1) may be written as $\frac{1}{f_1}+\frac{1}{f_2}+\frac{2a}{f_1f_2}=0$

or $f_1+f_2+2a=0$ or $a=-\frac{1}{2}(f_1+f_2)$. **Thus distance apart is half the sum of the focal lengths.**

QUESTIONS

1. Explain what do you mean by the dispersion of light and .the dispe power of a material. [C. U. 1928, '38, '49, '54,

2. Write a short note on Anomalous Dispersion. [C. U. 1

3. What is chromatic aberration of a lens? [C. U. 1944, '55

What do you mean by the achromatic combination of two lens Establish the condition of achromatism of two lenses in contact.

[C. U. 1938, '40, '42, '44, '46, '49, '52,

4. Distinguish between a spectroscope and a spectrometer.

Explain the construction of a Direct Vision Spectroscope.

[C. U. 1925, '41,

5. Explain the dispersion produced by a single lens and show how the de may be corrected.

Why is such correction unnecessary in the case of a simple convex used as a magnifying glass held close to the eye? [C. U. 19

EXAMPLES

1. Calculate the focal lengths of a convex lens of crown glass of dispe power 0·012 and a concave lens of dispersive power 0·020 so that they form placed in contact, an achromatic converging combination of focal length 30 cn [C. U. 19

$$\text{We know, } \frac{\omega}{\omega_1} = \frac{-f}{f_1}$$

where ω and ω_1 are the dispersive powers and f and f_1 are the focal lengths convex and concave lenses respectively.

Therefore we have $\frac{\cdot 012}{\cdot 020} = \frac{-f}{f_1}$ or $f = -\frac{3}{5} f_1$

But we have for combination of lenses $\frac{1}{F} = \frac{1}{f} + \frac{1}{f_1}$

Using proper signs for numerical values, we have

$$-\frac{1}{30} = -\frac{5}{3f_1} + \frac{1}{f_1} = \left(1 - \frac{5}{3}\right)\frac{1}{f_1} = -\frac{2}{3f_1} \qquad \therefore\ f_1 = 20 \text{ cms.} \qquad \therefore\ f = -12 \text{ cms.}$$

2. An achromatic converging combination of focal length 60 cms. is form with a convex lens of crown glass and a concave lens of flint glass placed contact with each other. Calculate their focal lengths, if the dispersive power crown glass be 0·03 and that of the flint glass 0·05. [C. U. 19

Since the combination is achromatic, we have,

$$\frac{\cdot 03}{f_1} + \frac{\cdot 05}{f_2} = 0,$$ where f_1 and f_2 are the focal lengths of convex and the concave lenses respectively

$$\text{or } \frac{1}{f_1} = -\frac{5}{3f_2} \quad \ldots(1)$$

Again for the combination we have $-\frac{1}{60} = \frac{1}{f_1} + \frac{1}{f_2}$

$\therefore$ From (1) we have $-\frac{1}{60} = -\frac{1}{f_2}\left(\frac{5}{3} - 1\right)$ $\qquad f_2 = 40$ cms. $f_1 = -24$ cms.

3. **The refractive indices of crown glass for blue and red light are 1·523 and 13 respectively and for dense flint glass the corresponding figures are 1·773 1·743. Calculate the dispersive powers of the two materials.**

For the crown glass, the mean $\mu = \frac{1.523+1.513}{2} = 1.518$

flint glass, $\mu' = \frac{1.773+1.743}{2} = 1.758$

Dispersive power of crown glass $= \frac{1.523-1.513}{1.518-1} = .0193$

" " " flint glass $= \frac{1.773-1.743}{1.758-1} = 0.0396$

4. A prism of 5° is made of crown glass. What must be the angle of a t glass prism combined with it to give (a) no dispersion (b) no deviation ?

The condition for no dispresion is

$$(\mu b-\mu r)A=(\mu' b-\mu' r)A', \quad 5(1.523-1.513)=(1.773-1.743).A'$$
$$5\times .01 = .03A' \quad \therefore \quad A'=1.67°.$$

(b) The condition for no deviation is $\frac{A'}{A}=\frac{\mu-1}{\mu'-1}$ $\quad A'=\frac{\mu-1}{\mu'-1}.$

Hence $A'=\frac{(1.518-1)\times 5}{(1.758-1)}=\frac{2.59}{.758}=3.42.$

The mean refractive indices of two specimens of glass are 1·52 and 1·66 spectively ; the difference in the indices for the same two lines of the spectrum e ·013 for the first and ·022 for the second ; find the focal length of a lens of e second glass, which when combined with a convex lens of 50 cm. focal ngth of the first, will make an object glass achromatic for the two lines.
[C. U. 1944] [Ans. $f=66\frac{2}{3}$ cm.]

6. The dispersive powers of crown and flint glass are 0·03 and 0·05 respective- If the difference in the refractive indices of red and blue colours be 0·014 crown glass and 0·023 for flint glass, calculate the angles for a deviation 10°. [C. U. 1956]

From the condition of achromatism, we have, $\frac{A}{A'}=\frac{\mu' b-\mu' r}{\mu b-\mu r}$ ...(1)

Total deviation of the mean ray for the two prisms

$$\delta=(\mu-1)\ A-(\mu'-1)\ A' \quad ...(2)$$

Dispersive power of the crown glass $=\frac{\mu b-\mu r}{\mu-1}= .03$

" " flint glass $=\frac{\mu' b-\mu' r}{\mu'-1}= .05$

$$\mu-1=\frac{\mu b-\mu r}{.03}=\frac{.014}{.03}=\frac{14}{30}; \quad \mu'-1=\frac{\mu' b-\mu' r}{.05}=\frac{.023}{.05}=\frac{23}{50}.$$

From (2) we have, $\delta=(\mu-1)A-(\mu'-1)\ A'=10=\frac{14}{30}\ A-\frac{23}{50}\ A'$ ...(3)

From (1) we have, $\frac{A}{A'}=\frac{\mu' b-\mu' r}{\mu b-\mu r}=\frac{.023}{.014}$ $\quad =\frac{23}{14}.A'$ ... (4)

Substituting the value of A in (3) we have

$$10=\frac{14}{30}\cdot\frac{23}{14}A'-\frac{23}{50}A'=\frac{23}{30}A'-\frac{23}{50}A'$$

$$=23A'\left(\frac{1}{30}-\frac{1}{50}\right)=\frac{23A'\times 20}{1500}=\frac{46A'}{150} \qquad A'=\frac{1500}{46}=32{\cdot}6^{\circ}$$

Substituting the value of A′ in (4) $A=\frac{23\times 32{\cdot}6}{14}=53{\cdot}55$

7. The dispersive powers of crown and flint glass are 0·036 and 0·064 respectively. Find the focal lengths of two thin lenses which will form a convergent achromatic lens of focal length 50 cms. when put in contact with each other. [C. U. 1957]

Since the combination is achromatic, we have the condition

$$\frac{\omega}{\omega_1}=-\frac{f}{f_1}$$

where ω and ω_1 are the dispersive powers of the lenses having focal lengths f and f_1 respectively.

$$\frac{{\cdot}036}{{\cdot}064}=-\frac{f}{f_1} \qquad \text{or} \quad f=-\frac{9}{16}f_1$$

Again for the combination of lenses we have

$$\frac{1}{F}=\frac{1}{f}+\frac{1}{f_1} \qquad \text{or} \quad -\frac{1}{50}=-\frac{16}{9f_1}+\frac{1}{f_1}=-\frac{7}{9f_1}$$

$$\text{or} \quad 9f_1=350 \qquad \text{or} \quad f_1=38{\cdot}9 \text{ cms.}$$

$$f=-\frac{9}{16}\times 38{\cdot}9=-21{\cdot}88 \text{ cms.}$$

CHAPTER VI

SPECTRUM—ITS STUDY

71. Introductory : The spectrum as obtained by allowing a beam of white light to pass through a prism is not pure, *i.e*, its constituent colours are not separated but they overlap one another and the colours again do not occupy equal spaces in the spectrum, violet occupying the greatest and orange the least.

If we carefully study the different parts of the spectrum we will see that **luminous effect** is prominent in the yellow but it diminishes as the red or violet end of the spectrum is reached in turn. Two other effects such as **Heating effect** and **Chemical effect** are observed in different degrees in different parts of the spectrum. By holding a thermopile or a sensitive thermometer in various parts of the spectrum it is found that the heating effect gradually diminishes as it is moved from the red to the violet end of

spectrum but if the thermopile be moved beyond the red part, heating effect is seen to increase for some distance. This sible portion of the spectrum beyond the red is called the ra-**red** spectrum.

'he chemical effect in the spectrum becomes marked when certain salts are mposed by the action of rays of different colours in the spectrum and this t is very feeble at the red but gradually increases to the violet and extends considerable distance beyond it. This invisible portion of the spectrum nd the violet is called the **ultra-violet** spectrum and the rays from this are very active for chemical action and so they are called **actinic-rays.**

72. **The Infra-red and Ultra-violet Spectra** : The portion he spectrum beyond the visible red is remarkable for its *heating* ;t and is called the *Infra-red* spectrum.

'his was discovered by Herschel in 1800 by placing a delicate thermometer a blackened bulb in this region. The rise in temperature indicated the rption of heat.

This region has been thoroughly investigated by Langley by means of a ate instrument known as **Bolometer.**

Since glass is a good absorbant of these rays, prisms of rock-salt or flour-spar generally used for studying this region of the spectrum.

Because of the ease with which Infra-red rays penetrate the atmosphere, they used for photographing of landscapes and objects situated a great distance y.

The portion of the spectrum beyond the visible violet is known the *Ultra-violet* spectrum. The wave-lengths in this region are short to excite the nerves of the eye but are capable of omposing silver salts of photographic emulsions. This was first erved by Scheel.

.... glass absorbs ultra-violet rays, the glass prism and lenses of the imator and telescope of the ordinary spectrometer should be replaced by prism lenses of *quartz* for studying the ultra-violet spectrum.

Ultra-violet waves have a very penetrating influence and are used in medicine. y cause certain substances such as quinine, paraffin oil, etc., to fluoresce and iso gases.

73. **The Complete Spectrum** : The radiation from the sun d other luminous sources is not limited to the region of visible ectrum whose wave-lengths range only about 7.10^{-5} cm. (7000°A) the extreme red to about $3{\cdot}9.10^{-5}$ cm. (3900°A) but the infra-radiation has been detected and studied up to wave-lengths as as 10^{-2} cm., while in other direction radiations beyond the ible violet have been studied up to wave-lengths as short as $^{-6}$ cm.

Wave-lengths are measured in units, known as Angstrom units . U.), one Angstrom unit being equal to 10^{-8} cms.

Beyond ultra-violet radiations, other shorter radiations known X-rays have been found. These X-rays consist of wave-lengths

ranging from 10^{-7} cm. to 10^{-9} cm. Beyond X-rays, radiation known as γ-rays are emitted by radioactive atoms and have been detected and they have wave-lengths as short as 10^{-10}cm.

Beyond infra-red radiations, Electromagnetic Hertzian Waves known as Radio Waves of wave-lengths ranging from 10^{-1} cm. to 10^{7} cm. have been found.

These radio waves are produced by oscillations of electricity in the transmitting aerial.

All these together with visible light are radiations of the same nature—electromagnetic vibrations in ether.

They all travel in empty space with the same velocity and exhibit the characteristics of a transverse wave motion.

They shew the phenomena of polarisation, interference, reflection refraction etc.

These radiations are all alike in nature but they differ only in their wave-lengths.

74. Absorption of Light: When light is allowed to fall on the surface of a transparent substance, part of the light is reflected part is absorbed, and the rest is transmitted unchanged. The amount of light reflected or absorbed depends on the nature of the substance over which it falls. Lamp-black and platinum black do not reflect any light, neither they transmit it, but they absorb waves of all lengths, except the very longest. This kind of absorption is called **general absorption**.

Certain substances instead of absorbing the whole amount of light falling on them, absorb a part and the amount of this absorption is generally different for different wave-lengths. This kind of absorption is called **Selective Absorption**.

To examine the character of absorption a beam of white light is passed through the given substance and the transmitted light is examined by a spectroscope. If the substance exhibits selective absorption, *i.e.*, if it absorbs light of one wave-length more strongly than it does of other wave-lengths, the spectrum will be crossed by a number of dark lines corresponding to the colours which have been absorbed.

A piece of red glass when interposed between the source of light and the slit of the spectroscope only red colour will be visible, the other colours being completely absorbed. A dilute solution of human blood produces well-marked absorption bands in the yellow and green part of the spectrum.

75. Causes of dark lines in the Solar Spectrum: If we carefully study the solar spectrum, we will notice that a large number of dark lines cross the whole length of the spectrum. The existence of these dark lines were first observed by Wollaston in 1802, but Fraunhofer made a systematic study of these lines, mapped them and denoted these marked lines by several letters such as A, a, B, C, D, E, *b*, F, G, H. These lines are known as

Fraunhofer lines. Fraunhofer counted in the spectrum more han 600 dark lines, more or less distinct, distributed irregularly rom the extreme red to the extreme violet colour.

The lines A, B and C are in the red, D in the orange-yellow, E n the green, F in the greenish-blue, G in the indigo and H in he violet part of the spectrum. The positions of these dark lines are fixed and definite.

Most of these lines have been found to correspond to lines in the spectra of lements present on the earth. Consequently we can easily conclude that the ubstances or elements whose bright lines correspond in position to the dark lines n the solar spectrum compose the outer atmosphere of the Sun.

The explanation of the dark lines was first given by Kirchhoff who from several xperiments came to the conclusion that the vapour of an element absorbs those ight waves which it would emit if it were incandescent.

From this theory, the existence of dark lines is easily explained. The sun is issumed to consist of an incandescent solid or liquid nucleus surrounded by a cooler envelope in which practically vapours of all terrestrial elements are present.

The inner nucleus is called the *photosphere* and the surrounding cooler envelope, the *chromosphere.*

According to Kirchhoff's Law, white light emitted by the Sun is robbed in passing the enveloping layer, of those waves which correspond to the waves the element would emit if they were incandescent. In consequence of the absorption of these waves dark lines are observed.

76. Different kinds of Spectra : There are two kinds of spectra—(1) Emission spectra, (2) Absorption spectra.

(A). Emission Spectra : When a body is heated to incandescence, the spectrum produced by it is called an *emission spectrum.*

Emission spectra may be divided into three characteristic classes :—

(1) *Continuous Spectrum* : It is an unbroken band of light in which all the spectral colours are present. Electric light, luminous Bunsen flame and white hot solid body give rise to this kind of spectrum.

(2) *Line Spectrum.* It consists usually of a number of bright lines separated by dark spaces. Such a spectrum is given by vapours or gases of elementary substances in the incandescent state, each giving its characteristic line or lines.

A bit of metallic salt such as NaCl when introduced into a colourless Bunsen flame gives rise to this kind of spectrum. A vacuum tube containing a gas and made luminous by an electric discharge with an induction coil, gives rise to a number of isolated bright lines. The colour and position of these lines will differ with different gases.

(3) ***Fluted or Band Spectrum*** **: It consists of a number of luminous bands sharply defined at one edge and shading off gradually at the other edge.**

It has been found by examination that each fluting consists of a large number of lines at the bright end and more widely spaced at the other.

Chemical compounds such as Cyanogen give rise to fluted spectra while simple substances give line spectra. Antimony Fluoride gives a band spectrum. Gases whose molecules contain more than one atom give a band spectrum

(B). Absorption Spectra : It is not a continuous spectrum but it is a spectrum of all colours crossed by a number of dark bands known as *absorption bands.*

If we examine the light coming out from the sun, the spectrum obtained in this case is a continuous spectrum crossed by a number of *dark lines.*

Again if white light be passed through a piece of red glass and examined, the spectrum of the transmitted light consists of the red portion only, there being *dark bands* in the remaining portion. These are known as *absorption bands.*

76(a). Stellar Spectra : The spectra of some of the stars resemble in appearance the solar spectrum and contains a number of dark lines corresponding to a number of elements present in the earth, while others are closely allied to the fluted spectra of some of the elements.

The spectra of nebulæ consist entirely of bright lines and resemble the spectrum formed by an incandescent gas. Consequently nebulæ are purely masses of incandescent gases.

The star *Orionis* gives a continuous spectrum with narrow hydrogen lines, while the star *Ceti* gives a spectrum of bright lines and flutings.

In the case of light, say of yellow colour, emitted by a source which is stationary, the spectrum will consist of a bright yellow line but if the source be made to move with a high velocity, the spectral line will be displaced either towards the violet or towards the red end of the spectrum depending on the direction of the source either towards or away from the spectrometer.

This displacement in spectral lines in the case of spectra of stars and other celestial bodies is generally noticed and explained by a principle known as **Doppler's Principle.**

To explain the displacement of the lines in the spectra of stars we are to consider the motion of the star along the line of sight. We know that the motion of the star towards the earth along the line of sight will modify the lengths of the waves sent out by the star and that the wave-lengths of these radiations will be smaller than if it were stationary and consequently the bright line in the spectrum will be displaced towards the violet end, its colour at the same time changing. Similarly if the star moves away from the earth, the spectral lines will be displaced towards the red end of the spectrum for the wave-lengths of the radiations increase as the star moves away from the earth.

If we know the amount of shift in the lines we can calculate the velocity as well as the direction of motion of the star. (See Doppler's principle in sound).

Dr. Huggins observed that the F line (of Hydrogen) in the spectrum of **Sirius** is slightly shifted towards the red end of the spectrum proving that the star **Sirius** is moving away from the earth with a velocity of about 29 miles per sec.

77. Study of spectra of incandescent gases : The experiment is performed with a vacuum tube and a spectrometer. The vacuum tube contains a gas under reduced pressure and is provided with two electrodes of short aluminium wires fused to two pieces of platinum wires, the platinum wires being sealed into the tube with a small loop outside.

An electric discharge from an induction coil is passed through the tube and the gas within the tube becomes luminous. Light from the vacuum tube illuminates the slit of an adjusted spectrometer and a spectrum consisting of lines of different colours separated from one another by dark intervals are observed. If the tube contains **Hydrogen**, red, blue, green and violet lines are observed. If the gas be Helium, red, yellow, green, blue and violet lines are observed.

78. Study of Ultra-Violet and Infra-red spectrum : To study the ultra-violet part of the spectrum it is necessary that the glass prism and lenses of the collimator and the telescope of an ordinary spectrometer must be replaced by prism and lenses of quartz since glass is found to absorb ultra-violet rays.

The ultra-violet spectrum can not be seen by the naked eye and so it is photographed on a sensitised plate placed at the focal plane of the telescope, the whole instrument being enclosed in a light-tight box with the exception of the slit. The plate is developed and the lines are observed.

For studying the Infra-red spectrum the experimental arrangement is precisely the same as in the case of ultra-violet spectrum, for infra-red spectrum the glass prism and the lenses of the collimator and the telescope should be made of rock-salt or flour-spar.

79. Flourescence : It is the phenomenon observed in many substances which become luminous when light of certain wave-lengths falls upon them. The name is derived from flourspar, the substance which first exhibited this peculiar emission of light. It is not a case of reflection, for the light emitted is generally of longer wave-length than that which excites it.

This phenomenon is noticed strongly in many of the aniline dyes, such as *eosine, fuchsine* and *flourescene* and also in *sulphate of quinine* and *paraffin oil.*

The flourescence is most brilliant at the surface of incidence of the white light, the brilliancy gradually decreasing with the thickness of the solution through which the light has passed.

The phenomenon of flourescence can be successfully applied to the study of the ultra-violet region of the spectrum.

When a spectrum is projected on flourescent substances, the blue flourescent light is emitted by the parts of the surfaces on which the blue, violet and ultra-violet portions of the solar spectrum fall. Stokes investigated the ultra-violet solar spectrum in this way and mapped the positions of the principal Fraunhofer lines in it.

Sulphate of quinine and flourspar flouresce with a blue light, chlorophyll with a red light and flouroscene with a green light.

Flourescent powders are coming into use with mercury vapour lamps to convert the intense ultra-violet light into visible violet.

80. Phosphorescence : Flourescence ceases instantaneously with the cessation of the exciting light. Many substances continue to emit light when placed in a dark room after being exposed to light of short wave-lengths. This phenomenon is called phosphorescence. Diamond, sulphides of Calcium, Barium and Strontium are prominent amongst the phosphorescent substances. The ultra-violet light is most active in producing phosphorescence.

The term phosphorescence is rather misleading, since the glow emitted by *phosphorous* is due to slow chemical action but the glow of a phosphorescent substance is not due to chemical action but is due to flourescence which persists after the exciting light is removed.

81. Calorescence : The action is converse to that which occurs in the case of flourescence. The absorption by a body of radiatian of one wave-length and its consequent emission of radiation of shorter wave-length is called *calorescence.*

Tyndall showed that if the *infra-red* rays from an electric arc are passed through a solution of iodine in carbon-bisulphide, the luminous rays would be absorbed and the transmitted infra-red rays when focussed on a piece of paper or a cigar will burst into flame.

QUESTIONS

1. Write a short note on spectrum and its teachings, [C. U. 1930, '47, '49, 50]
2. Write an essay on spectroscopy and spectrum analysis. [C. U. 1956]

3. Give a general account of the spectrum. Explain how it enables us to draw conclusions regarding the presence of familiar elements in the sun. Compare the physical properties of the visible and non-visible portions of the spectrum. [C. U. 1951

4. Write a short account of Fraunhofer's lines. [C. U. 1926, '28, '50

5. Write short notes on (a) Flourescence (b) Phosphorescence (c) Caloresence. [C. U. 1958]

6. Describe and explain the difference noticed between the spectrum of sunlight and that of a white-hot solid body. [C. U. 1932, '48]

7. Describe an apparatus you require and the method you would adopt to observe the emissson spectrum of a gas such as hydrogen.

What modification of the apparatus would be necessary if you wished to observe the ultra-violet part of the spectrum ? [C. U. 1948]

CHAPTER VII

THE EYE AND THE CAMERA

82. Eye as an Optical Instrument :

The eye may be considered as a light-tight enclosure with a lens at one end and a screen at the other, the lens producing on the screen real images of external objects. The screen is made up of nerve fibres which are excited when the inverted image of an external object falls on them and the vibrations are conveyed to the brain producing sensation of vision. The eye is a very complex piece of apparatus and the description of its different parts are given below.

Figure 43 which represents a vertical section of the eye, consists of the following parts—

(1) The *sclerotica* S. It is a dense, opaque and horny layer external to the eye ball. It is called the white of the eye and its function is to protect the eye.

(2) The *Cornea* C. In front of the eye the sclerotica merges into a transparent meniscus known as corn ea.

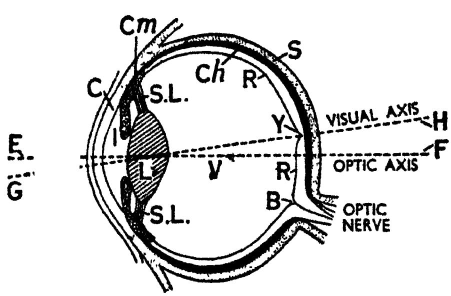


Fig 43

(3) The *Choroid* Ch. It is the black inner coating of the Sclerotica saturated with a black pigment and its function is to absorb the superfluous light which would otherwise produce a general illumination of the interior of the eye.

(4) The *Iris* I. It is a contractile diaphragm with a circular aperture near the centre. The circular aperture is known as the pupil of the eye.

The function of the Iris is to adjust and admit suitable quantities of light into the eye.

(5) The *Retina* R. It is a semi-transparent sensitive membrane of nerve-fibres forming the inner coating of the eyeball. The optic nerves terminate at this membrane containing certain minute structures, called the *rods* and *cones* and carry the sensation of sight to the brain as soon as any image of any external object is formed on it.

There are two distinct spots in the retina, one, the **yellow spot, Y,** the most sensitive part of the retina, and the other, the least sensitive part known as the **blind spot B.**

(6) The *Lens* L. It is a transparent body resembling a double convex lens, being more convex behind than in front. It is suspended behind the Iris by the cilliary ligaments S. L. The function of the lens is to form real and inverted images of external objects on the retina.

The line joining the centre of the cornea and that of the lens is called the **optic axis** of the eye.

The line joining the centre of the lens and the **fovea centralis**, a small depression in the centre of the yellow spot, is called the **visual axis** of the eye.

(7) The *Aqueous Humour and Vitreous Humour.* These are transparent watery liquids of almost the same refractive index and filling up the anterior and posterior chambers respectively which are partitioned by the cystalline lens.

Cornea, aqueous humour and the lens form a single lens combination with air on one side and vitreous humour on the other. The focal length of the combination when the normal eye is looking at a distant object, is 15·5 mm.

83. Accommodation : Least Distance of Distinct Vision :

Accommodation : It is the process by which the eye adapts itself for objects at different distances.

The focussing is done by the eye by altering the curvature of the crystalline lens caused by a change in the tension in the cilliary muscles of the eye lens. When the eye is at rest, distant objects are focussed on the retina. But as the objects move towards the eye, the muscles are put to tension and bring back the image on the retina which otherwise would have been focussed behind the retina. This process by which the eye sees objects at all distances is called **accommodation.**

But there is a limit up to which this accommodation can be brought about and the normal eye exhausts its power when the object is placed at its *near point i.e.*, at a distance of about 25 cms. from the eye. This distance is called the **Least distance of distinct vision.**

The point at the greatest distance beyond which distinct vision is not possible is called the **far point** and the closest distance at which objects can be seen is called the **near point.**

84. Defects of vision : (A) Short-sight or Myopia : It is the defect in the eye which can not see distant objects distinctly. The eye ball is a littte bit elongated or the focal length of the lens is too short so that the images of distant objects are not formed exactly on the retina R (Fig. 44). For a normal eye the ball is nearly spherical and the images of distant objects are formed exactly on the retina and so the images are distinctly seen.

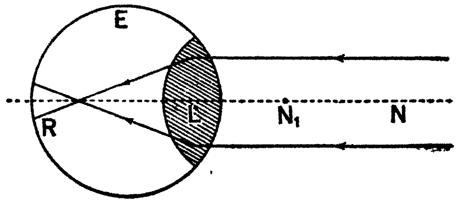


Fig. 44

In the short-sighted eye, the images are formed before the retina, and consequently no vision is produced. So to correct this defect a concave lens L_1 of suitable focal length should be used in front of the eye to get a clear impression of the image.

The focal length of the lens should be equal (Fig. 45) to the **distance** of the **Far point** which is situated at the greatest distance from the short-sighted eye at which distinct vision is possible ; for when

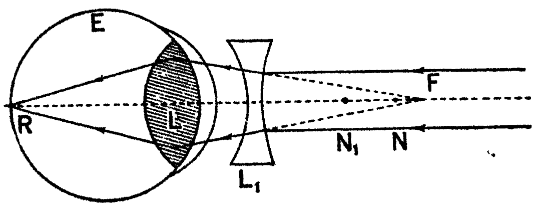
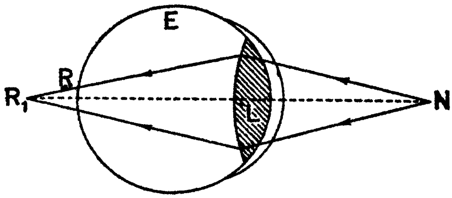


Fig. 45

the concave lens is held close to the defective eye, distant objects will form images either at its focus or at points between the lens and its focus and consequently they will be distinctly seen.

(B). Long-sight or Hypermetropia : It is the defect in the eye which can not see distant objects without accommodation. In

this case, the eye ball is too short or the focal length of the eye lens has been increased. So the images of distinct objects are formed behind the retina and consequently vision is not possible. By the process of accommodation the image is formed on the retina and the limit of near vision is reached sooner than it would be in a normal eye.



Fiq. 46

So to correct this defect a convex lens (Fig. 46) should be use in front of the eye to form the image on the retina to get a clear view of the object.

When the convex lens L_1 (Fig. 47) is held before the long-sighted eye, the image of the object placed inside the distance of distinct vision D will be formed further away than the near point N_1 of the long-sighted eye, but if the image be formed within the near point, nothing will be seen. For vision, the image of the object must be formed at least at a distance equal to the distance of the near point from the eye. Thus if D be the distance of the object placed at the least distance of distinct vision for a normal eye and d, that of the image, *i.e.*, of the near point N_1 from the eye, then we have.

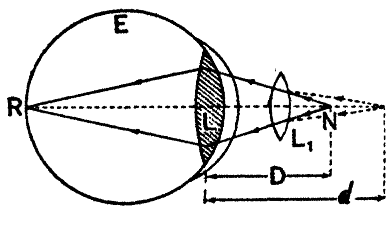


Fig. 47

$$\frac{1}{d}-\frac{1}{D}=-\frac{1}{f}\quad \begin{cases}\text{Since the focal length of the convex}\\ \text{lens is negative.}\end{cases}$$

$$\text{or}\quad \frac{1}{f}=\frac{1}{D}-\frac{1}{d}.$$

Thus the focal length f of the convex lens is determined.

The spectacles with convex lenses are generally worn by old people for the least distance of the distinct vision of their eye is greater than the distance of the distinct vision for a normal eye. So in order to read books held at a distance of 25 cms. from the eye they should wear spectacles with convex lenses of focal lengths determined by the above expression.

(C). Presbyopia : It is sometimes called **far-sight.** A person whose near point is further from the eye than the normal 25 cms., is said to be far-sighted if his far point is normal. This is met with

advancing age. This is a defect of the eye and is due to the radual loss of its power of accommodation and caused by the rogressive hardening of the cortical layers of the cystalline lens hich is unable to swell out and becomes more convex when near bjects are looked at.

It is sometimes mistakenly called long-sightedness.

When the loss of power of accommodation is complete *i.e.*, when there is nly one point of distinct vision, concave lenses should be used for viewing istant objects and for viewing nearer objects weak lenses are to be used.

In order that near objects should be seen, light from them must be rendered allel before reaching the eye. Hence in order that such an eye shall be able read ordinary type placed at a distance of 30 cms. from the position where the ectacles are worn, a convex lens of -30 cms. focal length must be used.

Two lenses, one for viewing near objects and the other for viewing more stant objects are necessary.

These two lenses are mounted in one frame in the form of half lenses. The e looks through the upper part for viewing distant objects and through the wer part for viewing near objects. Such lenses are called **bifocal lenses.**

The gradual loss of accommodating power of the eye-lens explains why young ople suffering from short-sightedness have their sight improved as they grow older.

(D) **Astigmatism** : It is a common defect of the eye. In stigmatism the surfaces of the cornea and the lens are not the urfaces of revolution about the axis of the eye and the vertical ection of the cornea and the lens is more curved than the orizontal section, so the image of a horizontal line is formed earer the lens than the image of the vertical line.

If the curvatures in the horizontal and vertical sections of the ornea are widely different there will be no sharp image, each point iving two *focal lines*. The eyes try to focus both at the same ime with the result that muscular strain is caused. This defect is remedied by using lenses with cylindrical surfaces in order to vary the focal length in one plane.

The lenses are known as *cylindrical* or *toric* lenses. A toric lens is one whose surfaces are parts not of a sphere but of a *tore* or *anchor-ring*.

85. Binocular Vision : When we look at a small object by neans of both eyes, an image of the object is formed on corresponding parts of either retina and its two identical impressions of the ages combine in the brain and enable us to see the object with great lustre and obtain a wonderful appearance of relief in the object.

The advantage of having two eyes is to get an idea about the distance of near objects by the muscular effort required to turn the eye ball so as to converge the axes of both eyes towards the object. The two eyes also help us to obtain an appearance of depth or solidity in the object. This effect is observed in an instrument known as **Stereoscope.**

86. Stereoscope : The instrument consists of a box divided into two compartments each fitted with a half convex lens with its anglé turned inward.

Two photographs of the same object which are not exactly similar are taken by two cameras taking the place of two eyes at their actual distance.

These pictures are mounted on a card and introduced into the stereoscope in such a way as to present them simultaneously before the two lenses through which observation is made.

The virtual images of the two slightly different objects formed by the two prismatic lenses combine into a single image possessing the characteristics of an object as seen by naked eyes.

If the two pictures are exactly similar, the result of the combination of the images would be to present a flat appearance.

87. Camera. Its defects : The simplest application of lenses for optical purposes is found in the Photographic Camera. It is used for producing a real and permanent image of an external object.

It consists of the following parts :

(1) *The camera lens* L : It is an achromatic combination of lenses for producing a real image. It is provided with a *shutter* and a *stop or diaphragm.*

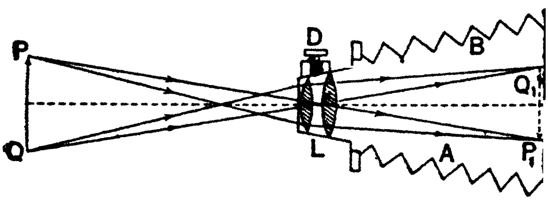


Fig. 48

(2) *The light tight chamber* AB with extensive sides in the form of bellows.

At one end of it the lens system L is fixed and at the other end a translucent screen E is fitted, on which image is first formed.

The bellows serve to exclude all light that does not contribute to the formation of the image.

(3) A *rack and pinion arrangement* at the base of the box for securing the image on the ground-glass plate by moving the lenses or the plate backwards and forwards.

The object PQ whose photograph is to be taken is placed in front of the lens of the camera at a certain distance away and a distinct, real and inverted image Q_1P_1 is formed on the plate by moving the lens or the plate backwards and forwards by the rack and pinion arrangement.

The diaphragm is adjusted to admit sufficient light for necessary illumination of the image and it helps to cut off any light that

es not pass through the central portion of the lens. This is to oid *spherical aberration.*

When the image is sharply focussed on the plate the ground-ıss plate is then replaced by a sensitive photographic plate which then exposed.

The plate is then treated with the fixing solution (hypo) for ing the image on the plate. The plate is then washed with ıter for removing traces of the hypo solution.

Thus the negative is prepared in which white portions of the ject appear black and *vice versa.*

From the 'negative', copies of photograph are prepared on a nstised paper by placing the paper in contact with the negative a frame and allowing light to act on the paper through the gative for a certain time.

The image formed on the paper is fixed on it by treating it th hypo solution and thus photos are prepared.

Defects :

(1) *Spherical aberration* :

This is avoided by using a diaphragm with a central aperture ıced in front of the lens system.

To reduce aberration to a minimum the refraction must be ided as evenly as possible (equally) between the refracting rfaces of the lens.

Hence the lens should be plano-convex and the convex side ould be presented to the source.

(2) *Chromatic aberration* :

The lens system should be corrected for chromatic aberration. iis is effected by using for each component of the lens system a nvergent crown glass lens combined with a divergent flint glass

ıs satisfying the condition $\frac{\omega_1}{f_1}+\frac{\omega_2}{f_2}=0$

(3) *Astigmatism* :

A narrow pencil of rays striking the lens obliquely is not ought to a point but to *two focal* lines. This effect is called *tigmatism.*

This defect is corrected by the use of cylindrical, sphero-lindrical and toric lenses.

(4) *Curvature of the field* :

It is not possible to obtain the flat image of a flat object by ing a curved optical surface.

To have a sharp image on a screen the simplest plan is make the screen curved as is done in camera obscura.

This defect is reduced to a minimum by using a compl lens system.

(5) *Distortion* :

It is possible to have the image sharp and flat but r geometrically similar to the object.

The image of an object is said to be distorted when differe portions of the image are differently magnified.

This defect is reduced to a minimum by using a stop (apertu between the two systems of the combination.

(6). *Comma* :

The image of a non-axial point source produced by a lens often egg-shaped. This image may be regarded as arising fro the overlapping of a series of images formed by the central zo and a series of concentric zones of the lens.

For no comma, $\frac{\sin\theta}{\sin\theta'}=\text{const.}$, where θ and θ' are the angl made by the conjugate rays with the common axis of the whc system.

QUESTIONS

1. Write a short account of 'human eye.' [C. U. 1929, '30, '50, '

2. What are the functions of the eye? Show how the principal defe arise and show how they can be compensated for, by suitable spectacles. [C. U. 1929, '

What are the advantages, if any, of having two eyes? [C. U. 193

3. Describe the principle of working of a camera.

What are the defects in the image formed by a camera and explain how th are minimised. [C. U. 19

CHAPTER VIII

OPTICAL INSTRUMENTS

88. Apparent size of an object ; Visual angle : The size of the image on the retina determines the apparent size of the object. It depends on the angle subtended at the eye by the object, known as the *visual angle.*

As the object approaches the eye, the visual angle increases and with it the apparent size of the object. When the object is placed at the near point, the apparent size becomes as great as possible and the details of the object stand out clearly. If the object be nearer to the eye than its near point, the impression on the retina is large but indistinct.

89. Magnification and Magnifying Power : *Magnification* has been defined before as the ratio of the size of the image to that of the object. It is expressed as $m=\frac{I}{O}$, where m is the magnification and I and O are respectively the sizes of the image and the object.

The term magnifying power is usually used in the case of optical instruments which act as an aid to vision.

The optical instruments are designed to increase the apparent size of objects—*i.e.*, the angle they subtend at the eye. Since the apparent size of an object depends on the visual angle at the eye, we are solely concerned with the angular magnification. So the *magnifying* power of the optical instruments is given by

$$\text{Magnifying Power}=\frac{\text{angle subtended at eye by image}}{\text{angle subtended at eye by object}}$$

For any particular instrument in *normal adjustment,* the object and the image are usually considered to be at the same distance from the eye.

90. Microscope : Simple Microscope : A convex lens used to get a magnified image of an object when placed inside its focal length is called a Simple Microscope.

The magnifying power of a lens or a simple microscope is defined as the ratio of the angle which the image subtends at the eye when formed at the least distance of distinct vision (near point)

to the angle which the object would subtend at the eye, if it were placed in the position of the image and viewed directly. In defining the magnifying power, the eye is supposed to be placed close to the lens.

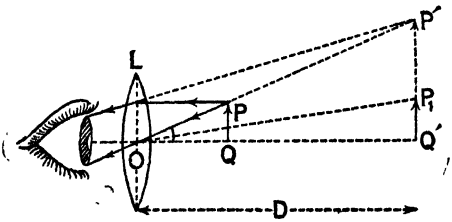


Fig. 49

Let u be the distance of an object PQ placed within the focal length f of a convex lens L, and let the image P′Q′ be at the distance of distinct vision D.

Then, according to the definition the magnifying power,

$$m=\frac{\angle P'OQ'}{\angle P_1OQ'}=\frac{\tan P'OQ'}{\tan P_1OQ'}, \text{ since angles are small}$$

or $$m=\frac{P'Q'/D}{P_1Q'/D}=\frac{P'Q'}{P_1Q'}=\frac{P'Q'}{PQ}=\frac{D}{u},$$ where P_1Q' is the size of the object at the least distance of distinct vision.

Then, according of the formula of the lens, $\frac{1}{D}-\frac{1}{u}=\frac{1}{f}$

or $1-\frac{D}{u}=\frac{D}{f}$ or $\frac{D}{u}=1-\frac{D}{f}$

Hence $m=1-\frac{D}{f}$ $\quad\quad \frac{D}{u}=m$, and the lens being convex, f is negative. Thus we have $m=1+\frac{D}{f}$.

The image may be formed anywhere between *near point* and *infinity*.

With the image formed at the *near point* the eye must exert its maximum accommodation. The eye-strain can be avoided by forming the image at the *far point* of the eye.

When the object is at the focal length of the lens, the eye is receiving parallel rays and the angular magnification is given by

$$m=\frac{\text{Angle subtended at eye by image at infinity}}{\text{Angle subtended at eye by object at near point}},$$ which can be seen to be $\frac{D}{f}$ numerically.

So the magnification m increases if f diminishes and consequently to obtain a high magnification a convex lens of a very short focal length is to be used.

If the eye be placed at a distance a from the lens and if the image is formed at the least distance of distinct vision,

The magnification $m = \dfrac{D - a + f}{f} = 1 + \dfrac{D - a}{f}$.

Since in this case the distance of the image from the lens is not equal to D but equal to $D - a$ and f is negative.

Thus if the eye is not close to the lens the near point magnification is reduced but the infinity magnification is unchanged.

91. Achromatism of Magnifying Glass: A magnifying glass is free from chromatic errors if the eye is placed very close to the lens and the object viewed be a small white object.

Let an object of linear dimension O be situated at a distance u from the lens and let v_r and v_b be respectively the distances of the red and blue images from the lens.

Then $\dfrac{I_r}{O} = \dfrac{v_r}{u}$, where I_r is the size of the red image.

Again the angle α subtended by the red image is

$$\alpha = \frac{I_r}{v_r} = \frac{v_r}{u} \cdot O \div v_r = \frac{O}{u}$$

Similarly the blue image also subtends an angle equal to $\dfrac{O}{u}$ so that the retinal images are equal in size.

Thus the images of all colours are of the same size and they subtend the same visual angle at the eye. Hence the image seen by the eye is perfectly achromatic.

If the object is large, the marginal parts of the image show traces of colour due to the chromatic effects of spherical aberration.

92. Magnifying power of a telescope or of a microscope: It is defined as the ratio of the angle which the image seen through the instrument subtends at the eye to the angle which the object subtends at the eye when viewed directly.

93. Compound Microscope: In order to obtain a much greater magnification than can be obtained with a single convex lens, two convex lenses are used in combination in a tube so that they have a common axis and are placed at a certain distance from one another.

The one which is placed near the object is called the **objective** and has a short focal length and the other near the eye is called the **eye-piece** and has a comparatively large focal length.

An object is placed just beyond the focus of the objective so that a real, inverted and magnified image is formed on the opposite side and inside the focal length of the eye-piece. When the eye is placed close to the eye-piece, a virtual, inverted and much more magnified image is observed.

In Figure 50 PQ is the small object placed in front of the object glass O and P′Q′ is the real image formed by it within the focal length of the eye-piece O′ and P″Q″ is the magnified virtual image formed by the eye-piece.

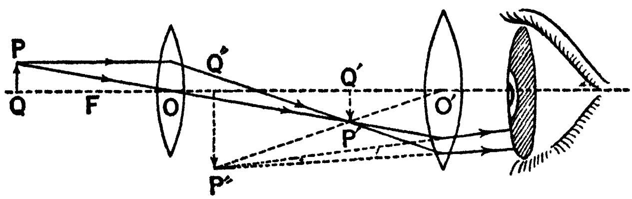


Fig. 50

For **normal vision**, the eye-piece is so adjusted that the image P′Q′ is at the focal plane of the eye-piece so that rays from P′ of the image P′Q′ will emerge as a parallel beam after refraction through the eye-piece. In this case a highly magnified image is seen without any strain in the eye.

Magnification. Let u be the distance of the object of length O situated in front of the objective and let the image formed by it be at a distance v on the other side. Then the length of the image is equal to $\frac{v}{u}$. O.

That is, the objective magnification $m_0 = \frac{v}{u}$

If this image be within the focal length of the eye-lens so that the final image is formed at the least distance of distinct vision, the magnification m_e produced by the eye-lens is equal to $1+\frac{D}{f_e}$, where D is the distance of distinct vision and f_e, the focal length of the eye-lens.

The angular magnification of the eye-piece for near point vision when the image is formed at the near point of the eye is given by

$$m_e = \left(1+\frac{D}{f_e}\right)$$

Therefore the final magnification for near point vision is

$$m_n = m_o \times m_e = \frac{v}{u}\left(1+\frac{D}{f_e}\right) = \frac{v}{f_o}\left(1+\frac{D}{f_e}\right)$$

Here u is approximately equal to f_o, the focal length of the objective.

If again the image is formed at infinity the angular magnification for infinity vision for normal eye is $m_e = \frac{D}{f_e}$

Therefore the final magification for infinity vision is

$m_\infty = m_o \times m_e = \frac{v}{f_o} \cdot \frac{D}{f_e} = \frac{LD}{f_o f_e}$, where v is approximately equal to the tube length L.

There are therefore three ways of increasing the magnification m: (1) by reducing f_o (2) by reducing f_e and (3) by increasing v *i.e.*, by increasing the tube length.

A compound microscope must therefore comprise a short-focus objective and short-focus eye-piece.

94. Modern Microscope: It is an improvement on the ordinary compound microscope in securing (1) the greater brightness of the image. (2) the greater resolving power, (3) elimination of spherical and chromatic aberrations.

It consists of the following parts :—

(*a*) A special type of *objective* known as **oil immersion objective** or Abbe's apochromat.

(*b*). The **sub-stage condenser** placed below the stage of the microscope and directly opposite to the objective.

(*c*). A **compound eye-piece,**

(*a*). **Microscope Objective**: It consists of ten small lenses with a combined focal length not exceeding 2*mm*., its numerical aperture being equal to 1·4. The purpose of this objective is (1) to ensure that a good deal of light shall enter the microscope, (2) to eliminate spherical and chromatic aberrations, (3) to increase its resolving power.

In this objective the first lens is a plano-spherical convex lens with the plane face facing the object placed below the cover-slip on the bed of the microscope. The space between the hemispherical lens and the coverslip is filled with a liquid (ceder-wood oil) having the same refractive index as the glass of the first lens in such a way that the plane face of the lens is kept immersed in the liquid.

(*b*) **Sub-stage condenser**: It consists of a number of lenses as in the objective but placed in the reverse order. To get a bright aerial image, the condenser is focussed on the objects which are

usually transparent sections mounted in Canada balsam on a glass slide, under a thin cover-slip. The light is first reflected from an adjustible plane mirror to the condenser which then focusses it on object at a large converging angle. This type of illumination is called bright ground illumination.

In this arrangement the brightness of the object as well as the resolving power is increased and at the same time aberrations are reduced.

(*c*) **Compound eye-piece** : It consists of two plano-convex lenses, a field lens and an eye-lens, the former being placed nearer the objective. The combination is designed to be as free as possible from spherical and chromatic aberrations. Thse two lenses are placed at a distance apart inside a draw tube which slides in a wider tube fitted at the other end with the objective.

Two types of compound eye-pieces are usually used. When no measurement is required, Huyghens' eye-piece is used with advantage but when measurement is necessary, Ramsden's eye-piece is used.

For *a detailed description of Huyghen's and Ramsden's eye-pieces consult Articles on Compound Eye-pieces.*

95. Telescopes. Refracting Telescopes :

(A) Astronomical Telescope : It consists of two convex lenses mounted in a tube so as to have a common axis. The object glass O has a large focal length and the eye-piece E which has a short focal length is placed at a distance equal to its own focal length from the focus of the objective so that the distance between the lenses is equal to the sum of the focal lengths of the lenses.

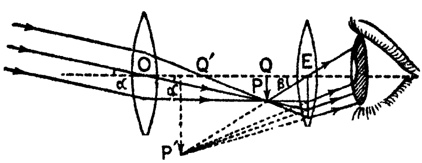


Fig. 51

Since the instrument is used to view a distant object the rays falling on the objective from the object may be considered parallel and so the image PQ is formed at the focus of the objective *i.e.*, at the focus of the eye-piece.

The rays from the image diverge and fall on the eye-piece and emerge as a parallel pencil. If an eye be placed close to the eye piece to receive the parallel rays a virtual magnified image will be seen at infinity. Since the first image is inverted, the virtual image is also inverted with respect to the object. This telescope is used to view astronomical bodies such as sun, moon, stars and planets etc.

Magnification : In observing distant objects of images, it is their apparent size only that concerns us and this is determined by the angle the image subtends at the eye.

Magnification is therefore determined by the ratio of the angle subtended by the image at the eye to the angle subtended by the object at the eye when viewed directly.

Let β be the angle subtended by the image at the eye and α, the angle subtended by the object at the objective *i.e.*, at the eye-piece since the object is at a great distance from the objective.

$$\text{Magnification } m=\frac{\beta}{\alpha}=\frac{\tan\beta}{\tan\alpha}=\frac{PQ/EQ}{PQ/OQ}=\frac{OQ}{EQ}=\frac{F}{f}$$

(for **normal vision**)

where F and f are respectively the focal lengths of the objective and the eye-piece.

For *near point magnification*, the final image is formed at the near point of the eye *i.e.*, at the least distance of distinct vision D from the eye-piece E. So the magnification $m_n=\frac{F}{EQ}$

In the formula for the eye-piece lens, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

Put $v=+D$ and $u=EQ$, and f is $-ve$.

$$\text{Therefore}\quad \frac{1}{D}-\frac{1}{EQ}=-\frac{1}{f}\qquad\text{or}\qquad \frac{1}{EQ}=\frac{1}{f}+\frac{1}{D}$$

$$\therefore\quad m_n=\frac{F}{EQ}=F.\left(\frac{1}{f}+\frac{1}{D}\right)=\frac{F}{f}\left(1+\frac{f}{D}\right)$$

Thus when the eye-piece is adjusted to see the image at the least distance of distict vision D, the magnifying power is slightly increased.

95 (B). Terrestrial Telescope (Galileo's):

Since the image of the object is inverted it is very dis-advantageous to see terrestrial objects with the holp of the astro-nomical telescope. So a concave lens is used in the place of the convex lens as eye-piece and the distance between the lenses—the objective and the eye-piece, is equal to the difference between their focal lengths. The image of the distant object would have been formed at the focus of the objective but before reaching the focus the rays are intercepted by the concave lens and since the focus of the eye-piece coincides with the focus of the objective, the

emergent rays will form a parallel pencil capable of producing normal vision in the eye placed to receive them. The image is erect and magnified. The magnification formula is deduced in the same way as done in the case of an astronomical telescope. In Fig 52 O is the convex-lens, O′, the concave lens as eye-piece, P′Q′, the image due to the convex lens and P″Q″ the final image which is seen erect.

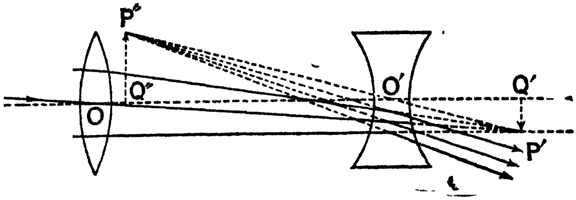


Fig. 52

Magnification (M) :

$$M = \frac{\angle P''O'Q''}{\angle P'OQ'} = \frac{\angle P'O'Q'}{\angle P'OQ'} = \frac{\tan P'O'Q'}{\tan P'OQ'} = \frac{P'Q'/O'Q'}{P'Q'/OQ'} = \frac{OQ'}{O'Q'} = \frac{F}{f}$$

where $OQ' = F$, the focal length of the objective
$O'Q' = f$, the focal length of the eye-piece.

In Figure 52 the image of the object is not viewed at infinit but at some distance in front of the eye and so the rays emergin out of the eye-lens are not parallel.

For *near point* magnification, the final image is formed at tl near point of the eye, i.e. at the least distance of distinct visio D from the eye-piece O′.

So the magnification $m_n = \dfrac{F}{O'Q'}$

In the formula for the eye-piece lens, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

Put $v = +D$ and $u = -O'Q'$ $\therefore$ $O'Q' > f$

$$\therefore \quad \frac{1}{D} + \frac{1}{O'Q'} = \frac{1}{f} \text{ or } \frac{1}{O'Q'} = \frac{1}{f} - \frac{1}{D}$$

$$\therefore \quad m_n = \frac{F}{O'Q'} = F\left(\frac{1}{f} - \frac{1}{D}\right) = \frac{F}{f}\left(1 - \frac{f}{D}\right)$$

Thus, when the eye-piece is adjusted to see the image at th least distance of distinct vision D, the magnifying power is slightl diminished.

96. Object Glass : Since the rays incident on the object glass of a telescope are centrical, the errors due to chromatic aberration

are more important than those due to spherical aberration. For removing the defect of chromatic aberration the object-glass of the telescope generally, consists of a combination of two lenses, one concave and the other convex. The concave lens is made of flint glass and placed behind the convex lens of crown glass, the two being cemented together by canada balsam. This combination is used for minimising the chromatic aberration, and the focal lengths of the lenses should be so chosen that they satisfy the relation $\omega_1/f_1 + \omega_2/f_2 = 0$, where ω_1, ω_2 and f_1. f_2 are respectively the dispersive powers and the focal lengths of two lenses.

Again for removing the defect of spherical aberration a plano-convex lens is used in the telescope objective with convex surface facing the incident parallel rays.

The best form for a converging lens for a telescope objective is a bi-convex lens of crown glass ($\mu = 1.5$) having radii of curvature of its surfaces in the ratio 1 : 6. Such a lens is termed a *crossed lens*. To produce the smallest amount of aberration the more strongly curved surface of the lens should face the incident parallel rays.

The aperture or rather the diameter of the object glass should be large so as to allow more light to pass through the instrument and increase the brightness of the image.

For the objective of a large diameter the resolving power will be great and consequently double stars which are not separately visible by a telescope of a small objective are resolved by a telescope having a large objective, and viewed as separate stars due to the reduction in size of tha images.

The focal length of the objective is made long and so the magnification which depends on $\frac{F}{f}$ (F = the focal length of the objective and f, that of the eye-piece) becomes large.

97. Field of view: If we look into the eye-piece of a telescope or a microscope or any other special instrument which is used as an aid to vision, we find a circular patch of light uniformly illuminated all throughout. This uniformly illuminated area is called the *field of view* of the instrument.

In Figs. 51 and 52 we have noticed that the rays from the extremities of the image are refracted through the peripheral portions of the eye-lens in such a way that they could not enter such a small aperture as that of the eye placed behind the eye-lens.

Only those parts of the mage near the axis will only be seen and the field of view is limited

This defect is rectified by using a field-lens.

Besides the defects of spherical and chromatic aberrations in a single lens when used as an eye-piece for optical instruments, the main defect of such a lens is its limited field of view.

So to enlarge the Field of view another lens known as the **field lens** (Fig. 53) is placed in front of the eye-lens. The section of the field lens is described below.

The real image due to the objective AO is formed in the principal plane of the field lens F which simply deflects the rays so that the image is practically formed at the second principal focus of F. If the eye be placed near this focus, rays from all points of the image I will enter the pupil of the eye.

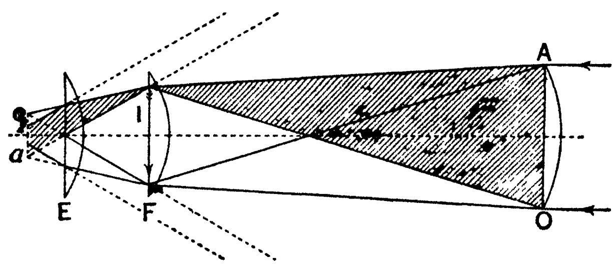


Fig. 53

If the eye-lens E has a focal length equal to that of F and so placed that its principal plane passing through the second principal focus of F, the entire portion of the image will be seen at infinity.

In Fig. 53 *oa* is the image of the objective formed by the field-lens.

The field lens F and the eye-lens **E** from a **Compound eye-piece.**

The compound eye-piece described here is **Kellner's eye-piece.** Although this eye-piece enlarges the field of view it does not eliminate completely the defect of spherical and chromatic aberrations.

98. Compound Eye-pieces. The rays falling on the eye-piece are excentrical. So with excentric pencils the errors due to spherical aberration are more important than those arising from chromatic aberration.

If in telescopes or microscopes the eye-piece consists of a single convex lens, due to spherical aberration the images formed by an eye-piece of short focus would be blurred and much distorted. To reduce this defect and to observe the coincidence of a point of the image with the intersection of the cross wires, compound eye-

pieces have been designed of which two types need a special consideration.

Advantages : (1) To increase the field of view. (2) To avoid spherical aberration, (3) To avoid chromatic aberration.

99. Effects of Spherical Aberration : The image of a linear object formed by oblique centric pencils in a lens is generally *curved* and by excentric pencils it is *curved* and at the same time *distorted.*

The distortion due to spherical aberration is diminished by decreasing the effective aperture of the lens by using a diaphragm with a central aperture. Such a diaphragm is called a *stop.*

According to Raleigh, it is preferable to stop out the central portion of the lens to get a sharply defined real image.

Lenses may also be designed to minimise the spherical aberration.

We know that the greater the deviation of the marginal ray, the greater is the aberration. So the error due to aberration can be reduced by diminishing the deviation of the extreme pencil.

In a single lens this is done by making the angles of incidence and emergence equal. It has been found that a *plano-convex lens* with convex surface facing the incident rays produce nearly equal deviation with both the surfaces.

This is the reason why a plano-convex lens is frequently used in optical instruments.

Two converging lenses can be combined to reduce spherical aberration. The deviation is distributed over the four surfaces instead of two and the lenses share equally in this.

The **condition of equal deviation** in case of refraction through lenses of focal lengths f_1 and f_2 and separated by a distance d when incident rays are parallel to the axis is given by the expression $d = f_1 - f_2$.

Proof : Let L_1 and L_2 be two lenses of focal lengths f_1 and f_2 and separated by a distance d.

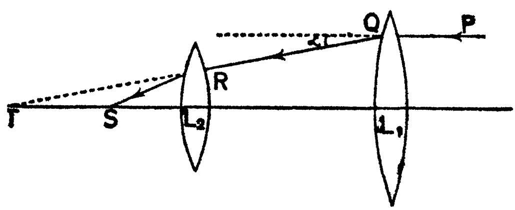


Fig. 54

Let PQ be a ray incident on the lens L_1 in a direction parallel to the axis and let the refracted ray QR be directed towards T, the second principal focus of the lens L_1.

The deviation produced by the lens L_1 is equal to α and is equal to the angle $\angle RTS$.

After refrction through lens L_2 the ray QR undergoes a deviation and cuts the axis at S.

If the lenses produce equal deviations, the $\angle SRT = \angle RTS$ and therefore $ST = RS$.

When the total deviation is small *i.e.*, in the limit when PQ is very close to the axis, $RS = SL_2$ so that $SL_2 = \frac{1}{2} TL_2$

Considering the socond lens L_2, T is the virtual object and S its image.

Let $TL_2 = u$, then $SL_2 = v = \frac{u}{2}$. Therefore, $-\frac{1}{\frac{u}{2}} + \frac{1}{u} = -\frac{1}{f_2}$

$$\text{or} \quad \frac{1}{u} = \frac{1}{f_2} \quad \text{or} \quad u = f_2$$

But $TL_2 = TL_1 - L_2L_1 = f_1 - d$. That is $f_2 = f_1 - d$
or $d = f_1 - f_2$.

100. Achromatism in lenses : As has already been pointed out that if the combination be achromatic for two colours it will not in general be achromatic for all the colours owing to irrationality of dispersion in lenses of different materials.

Perfect achromatism for any two colours may be obtained if the images of the colours produced by the combination are of the same size and coincident in position for all positions of the object.

A combination of three lenses of different glasses can be made achromatic for three colours on the same principle as the doublet.

Thus increasing the number of lenses perfect achromatism may be obtained.

When two lenses are separated by a distance it is impossible to have perfect achromatism unless they are *achromatic* themselves.

But partial achromatism may be obtained by making the sizes of the coloured images the same. This is done by making the emergent rays of the colours for which the system is achromatic, parallel to each other.

It can be observed that two lenses of focal lengths f_1 and f_2 separated by a distance d may be partially achromatic if the distance d between them be equal to the mean of their focal lengths. That is, $d = \frac{f_1 + f_2}{2}$.

Note: The result holds for all colours. This does not mean that the principal foci for all colours coincide as the system of lenses is a combination and not a thin lens.

It means that rays of all colours initially parallel to the axis are deviated through the same angle so that each coloured image subtends the same angle at the eye and an eye looking at a virtual image through the combination will see all the coloured images as if superposed.

In this case the eye-piece is achromatic in the sense in which ordinary magnifying glass is achromatic.

101. Ramsden's (Positive) Eye-piece: It consists of two plano-convex lenses E and F of (Fig. 55) equal focal lengths placed with their curved surfaces turned towards each other and separated by a distance equal to two-thirds of the numerical value of the focal length of either.

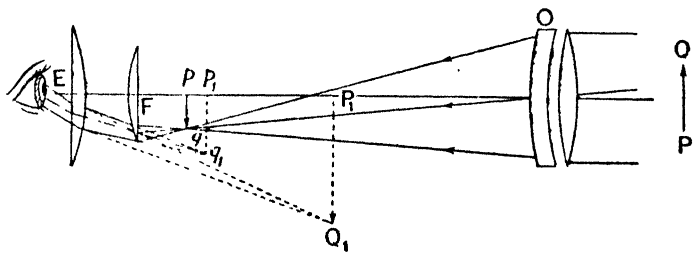


Fig. 55

Rays coming from a distant object PQ and refracted by the object glass O are then brought to a focus at q in front of the field-lens F. After refraction through the field-lens F, these rays appear to diverge from q_1 and finally after refraction through the eye-lens E, they appear to the observer to come from Q_1.

The object glass of the telescope or microscope forms a real image pq in front of the eye-piece lenses and so a cross-wire can be placed in the plane of the image to make micrometric measurements or to observe coincidence. Since the image and the cross-wire are observed through both the lenses of the eye-piece any distortion will affect the image and cross-wire equally.

It is called a positive eye-piece, since the image due to the objective is formed on the positive side of the field-lens (the lens facing the objective) of the eye-piece.

The **position of the field-lens** F from the focal plane of the objective is calculated in the way described below.

The condition for rays emerging out parallel from the eye-lens is that the image p_1q_1, must lie in the focal plane of the eye-lens E.

Hence $Ep_1 = f$. But $EF = \frac{2}{3}f$ $\therefore$ $Fp_1 = \frac{1}{3}f$.

Now considering the field-lens, pq is the real image and p_1q_1 the virtual image.

We have $\frac{1}{Fp_1} - \frac{1}{Fp} = -\frac{1}{f}$; $\frac{3}{f} - \frac{1}{Fp} = -\frac{1}{f}$ $\therefore$ $Fp = \frac{1}{4}f$

Here, Fp is the distance of the field-lens from the focal plane of the objective on which the image pq is formed.

In *Ramsden's eye-piece*, spherical aberration is minimised by using plano-convex lenses in the compound eye-piece and allowing the deviation to spread over four surfaces.

Chromatic aberration is minimised by using a combination of crown and flint glass lenses for the eye-lens.

102. Huyghens' (Negative) Eye-piece : It consists of two plano-convex lenses E and F with (Fig. 56) their curved faces turned towards the objective. The focal lengths of the field and the eye-lens are in the ratio 3 : 1 and the distance between them is numerically equal to twice the focal length of the eye-lens (the lens placed in front of the eye).

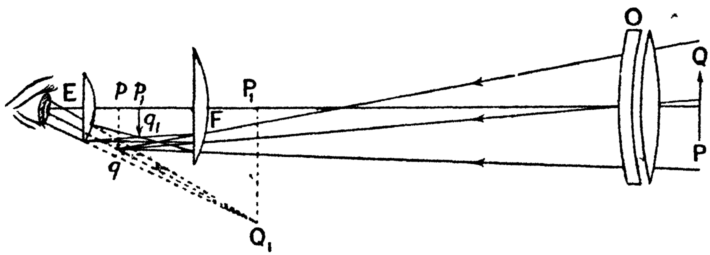


Fig. 56

In Fig. 56 PQ is the object, p_1q_1 is the image after refraction through the field-lens F. P_1Q_1 is the image of p_1q_1 after refraction by the eye-lens E.

It is to be noted here that pq is the image of the object PQ when uninterrupted by the field-lens F.

The position of the field-lens F from the focal plane of the objective is calculated as below.

If the rays are emerging out parallel from the eye-lens the image p_1q_1 must lie on the focal plane of the eye-lens E.

Hence $Ep_1 = f_2$; $Fp_1 = f_2$, where f_2 is the focal length of the eye-lens.

Considering the field-lens F, p_1q_1 is the real image of the virtual object pq.

$$-\frac{1}{f_2}+\frac{1}{Fp}=-\frac{1}{3f_2} \text{ or } \frac{1}{Fp}=\frac{1}{f_2}-\frac{1}{3f_2}=\frac{2}{3f_2} \text{ or } Fp=\tfrac{3}{2}f_2$$

That is, the field-lens is placed in front of the focal plane of the objective at a distance from the latter equal to $\frac{1}{2}.\ 3f_2$ or half the focal length of the field-lens which is equal is $3f_2$.

Since, in this eye-piece the real image is formed between the field and eye-lens, it is not advantageous to place a cross-wire in the plane of the image, for the image may be distorted due to the field-lens, but the cross-wire being on the eye-side of this would not suffer such distortion.

It is called a negative eye-piece since the image due to the objective is formed on the negative side of the field-lens.

This eye-piece has been so designed as to minimise the *effects of spherical aberration.* The condition for attaining this is that the rays initially parallel to the axis should suffer equal deviation at the two lenses and the expression satisfying this is given by

$$d=f_1-f_2 \qquad \ldots \qquad (1)$$

where f_1 is the focal length of the field-lens and f_2 that of the eye-lens.

The condition for minimum chromatic aberration for two colours forming images subtending equal angles at the eye is

$$d=\tfrac{1}{2}(f_1+f_2) \qquad \ldots \qquad (2)$$

Combining (1) and (2) we have $f_1=3f_2$.

Thus the condition for minimum spherical and chromatic aberration is that both the lenses are plano-convex with the convex side facing the incident light and that the focal length of the field lens is three times the focal length of the eye-lens.

103. Achromatism of Huyghen's Eye-piece : Huyghen's eye-piece consists of two plano-convex lenses of focal lengths $f_1=3f_2$ and f_2 separated by a distance d which is equal to twice the focal length of the eye-lens. That is, $d=2f_2$.

But the condition of achromatism of two lenses of focal lengths f_1 and f_2 separated by a distance d is $d=\frac{f_1+f_2}{2}$.

Now for Huyghen's eye-piece $d=\frac{3f_2+f_2}{2}=2f_2$

Thus Huyghen's eye-piece satisfies the condition of achromatism with respect to focal length. The focal length f of the combination is the same for all colours.

Again the deviation of a ray parallel to the axis situated at a distance h from it is $\frac{h}{f}$, which is consequently the same for all colours.

The emergent rays of different colours are thus parallel to each other and consequently the various coloured images subtend the same angle at the eye and no colour effect is detected.

Note : Certain eye-pieces are fitted with frames carrying cross-wires which can be moved by means of a micrometer screw with a graduated head. The angular distance between two points, say two stars which are not wide apart is measured by dividing the distance (in terms of micrometer graduation) through which the micrometer screw is moved to bring the cross-wire into coincidence with the two images one after another, by the focal length of the objective.

104. Reflecting Telescopes : In all reflecting telescopes the image of a distant object is formed by a concave mirror of large radius of curvature and made of *speculum* (alloy of copper and tin) at the principal focus of the mirror and the image thus formed is viewed in the magnified form by an eye-piece directly or after one reflection at a plane or a spherical mirror.

There are three different kinds of reflecting telescopes: (1) Herschel's telescope ; (2) Newtonian telescope and (3) Gregory's telescope.

In Herschel's telescope, the axis of the mirror is inclined to the direction of the incident rays and the image which is thrown on one side of the axis is viewed by an eye-piece.

104(a). Newtonian Telescope : In Newton's telescope the image is thrown on one side of the axis of the mirror by means of a plane mirror inclined to the axis at 45° and viewed by an eye-piece. The plane mirror is sometimes replaced by a totally reflecting prism.

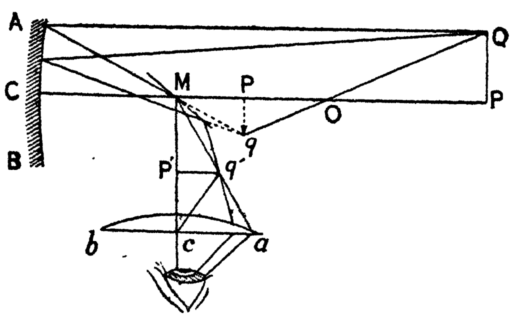


Fig. 57

The adjoining Figure 57 represents the arrangement of the concave mirror, plane mirror and the eye-piece. Rays coming from a distant point are reflected by the concave mirror ACB made of speculum metal, an alloy of copper and tin and then brought to a focus at q' after second reflection from the plane mirror at M inclined at 45° to the axis of the concave mirror

From q' the rays proceed and fall on the eye-piece *bca* and the image $p'q'$ is observed.

The image is magnified and the magnifying power of the telescope is found out to be given by $m=\frac{F}{f}$,

where F is the focal length of the concave mirror and f, the focal length of the eye-piece.

In Gregory's telescope, the concave mirror is pierced with a small central hole to which a tube containing the eye-piece is fitted. A second concave mirror is placed on the axis of the pierced mirror with their concavities turned towards each other and at a distance greater than the sum of the focal lengths of the two mirrors.

Rays from a distant object are reflected first from the pierced mirror and then from the second concave mirror.

The image formed is erect and viewed by the eye-piece.

105. Telescopes—Their advantages and disadvantages :

(1) The magnifying powers of the two telescopes, astronomical and Galilean are the same but Galilean telescope of the same power is shorter by twice the focal length of the eye-lens, since the distance between the two lenses is the difference, instead of the sum of their focal lengths as in the case of an astronomical telescope.

(2) In the astronomical telescope, special advantage can be secured by placing cross-wires in the position of the real image formed by the objective for accurate observation of stars, planets etc., but no such advantage can be obtained in the Galilean telescope in which the image is virtual.

(3) While viewing objects through a telescope or a microscope, we generally meet with two kinds of defects in the images. The images are coloured and distorted. These defects arise from two causes (*a*) Chromatic aberration (*b*) Spherical aberration, and are generally observed in the telescopes or microscopes in which each of objective and eye-piece, consists of a single lens only.

(*a*) To do away with the defects of chromatic aberration in the objective or eye-piece, each consisting of a single convex lens only, it is advisable to use a concave lens in combination with the convex one so that the ratio of their focal lengths is equal to the ratio of the dispersive powers of the glasses of which the lenses are composed.

(*b*) The defect of spherical aberration in the objective and the eye-piece may be avoided by using a plano-convex lens instead of a double-convex lens for the objective, and a compound eye-piece,

either Ramden's or Huyghen's for the eye-lens. Due to spherical aberration, the images formed especially with large magnifying powers, *i.e.*, with eye-piece of short focus would in the case of a single lens be much distorted and it would be impossible to get the central and the peripheral parts of the image in focus at the same point.

(4) In **reflecting telescopes**, the defect of chromatic aberration is entirely avoided for the rays do not undergo refraction and therefore no dispersion is produced.

The defect of spherical aberration is also avoided by making the concave reflector in the form a paraboloid of revolution.

The aperture of the reflecting telescope can be made much wider than that of the refracting telescope.

Inspite of all these advantages, the image as formed by the reflecting telescope is not so bright as that obtained by the refracting telescope.

106. Opera Glass : The opera-glass consists of two Galilean telescopes mounted together (Fig. 58) in frames with parallel axes and is termed **Binoculars** as they are used in pairs, one for each eye of the observer.

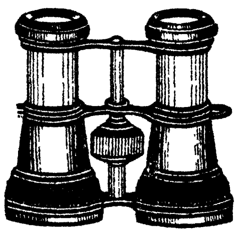


Fig. 58

The focussing is done by turning a screw attached to two uprights carrying the eye-pieces which slide in or out of the tube fitted with the object glasses of the telescope.

Since the tube lengths of the telescopes are short, the magnification depending upon it cannot be high, but the use of telescopes for the two eyes gives a good stereoscopic effect in the image.

The best form of **binoculars** is the **prism-binoculars** as shewn in Figure 59. In it, two total reflecting prisms are used to increase the image size to three times the object size and thereby obtain a much higher magnification than in ordinary binoculars.

In Figure 59 L is the object glass, P_1 and P_2, the total reflecting prisms and L_1 and L_2, eye-piece lenses for viewing the image in the erect position.

Rays from the object pass through the object glass, are reflected internally from the two prisms and then after traversing the length of the instrument pass through the eye-piece lenses.

The image formed by the first prism P_1 whose refracting edge is vertical is inverted **laterally** but not **vertically** and the image by the second prism P_2 whose refracting edge is horizontal *i.e.*, at right angles to that of P_1 is inverted **vertically** but not **late-**

rally. The result is that the rays emerge parallel to the original direction and the final image is seen erect through the eye-piece L_1 L_2.

This is essentially an astronomical telescope put in a compact form. In prism-binoculars two such telescopes are used as in opera-glass, one for each eye to get a good streoscopic effect.

107. Periscope : It is an instrument by means of which ships or other objects on the surface of the sea are viewed with advantage by the crew of a submarine under water.

To understand the principle of the instrument, Figure 60 is to be referred to.

The instrument consists of two telescopes in a water-tight brass tube with two 45° prisms, one at A and the other at B for reflecting the rays of light to a desired direction. Of the two telescopes the upper one is inverted with the short focus lens towards the object.

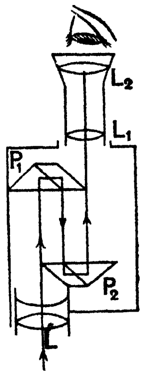


Fig. 59

Rays from the object pass through a thick plate glass window fitted to the tube at the top, are reflected downwards by the prism A and form the diminished image of the object at p due to the eye-piece.

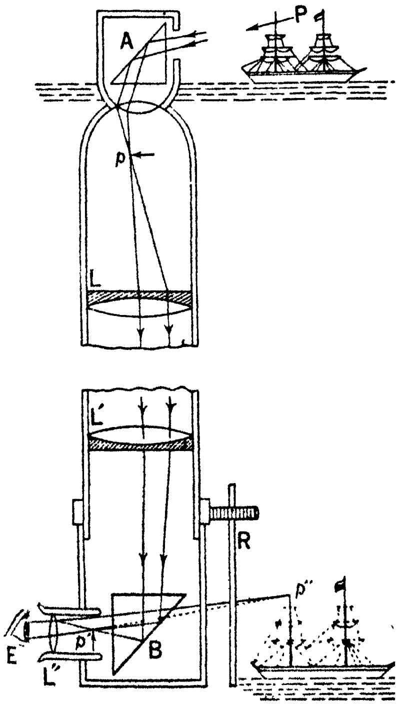


Fig. 60

The image is formed at the focus of the object-glass L and the rays emerging from the latter are rendered nearly parallel to the axis of the tube. These parallel rays are refracted through the object glass L′ of the lower telescope and then reflected by the prism B before they came to a focus and the real image at p' which is then viewed in the magnified form at p'' by the eye-piece lens L″.

By the side of the tube there is a gearing arrangement by which the upper part of the tube containing the prism can be rotated to enable the observer to look out in any direction.

The magnification produced in the image is about $1\frac{1}{4}$.

108. Sextant : The instrument is employed for measuring the angle between any two distant objects from the position of the observer. It is used in surveying and by mariners at sea for measuring the altitudes of heavenly bodies such as the sun, stars and planets.

It consists of a graduated circular arc AB (Fig. 61) with the centre at Q and fitted to the radial arms QA and QB. There is a movable arm QV known as the index bar which rotates about Q and carries a vernier at V for taking reading of the position of the plane mirror M_1 fitted perpendicularly to the frame on the movable arm at the centre Q.

The mirror M_2 is fixed perpendicularly to the arm QA and silvered on the half of its surface. The planes of the two mirrors are parallel when the vernier reads zero.

A small telescope T is fitted to the arm QB in such a way that the axis passes through the middle point of the mirror M_2. So when an observer looks through it, he will see any object, say S directly through the unsilvered portion of the mirror M_2, along SM_2T. When the two mirrors M_1 and M_2 are parallel two images of the object S will be seen side by side one directly through M_2 and the other by reflection first along QM_2 and then along M_2T.

Now to determine the angle between the objects S and P the arm carrying the mirror M_1 is rotated so that the rays PQ from the object pass into the telescope T after reflection at M_1 and M_2 along the path QM_2T and the observer sees the image of the object along-side the image of S as seen directly. In this case it is the direction of the reflected ray that is fixed while the direction of the incident ray is changed.

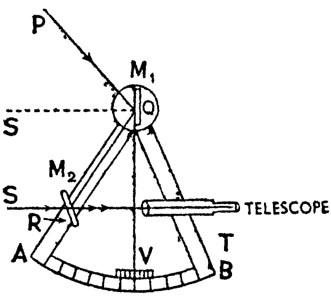


Fig. 61

So if the angle through which the mirror M_1 is turned is α as given by the vernier reading, the angle between the objects S and P which is equal to SQP is equal to 2α

To facilitate reading of the angle, each degree of the scale is marked as a whole degree. So the reading gives directly the value of 2α.

QUESTIONS

1. Describe the different parts of a modern microscope On what factors does its magnifying power depend ? [C. U. 1948, '5

2. Define the magnifying power of a microscope. [C. U. 192

Deduce the expressions for the magnifying power of (*a*) Simple microscope (*b*) Compound microscope.

3. Describe the construction of the refracting astronomical telescope and show by a good diagram the trace of an oblique pencil of parallel rays through the instrument. [C. U. 1930, '31, '35, '3

4. Define the magnifying power of a telescope.

Prove that for infinity vision, the magnifying power of an astronomical telescope is equal to the ratio of the focal length of the object glass to the focal length of the eye-piece. [C. U. 1926, '31]

How is the magnifying power for infinity practically determined? [C. U. 1935]

5. Why is the objective of an astronomical telescope large in diameter and of long focal length? Why does the objective consist usually of a combination of lenses? [C. U. 1930]

How can you use it to measure angular distances between stars? [C. U. 1931]

6. Describe the Galilean telescope and draw a diagram to show the image of a distant object indicating clearly the path of the light. [C. U. 1921]

7. A Galileo's telescope is adjusted for normal vision; calculate its magnifying power. What change in the adjustment is necessary for a short-sighted person and what will be its magnifying power when the distance of distinct vision is D. [C. U. 1943]

8. What is a compound eye-piece?

What are the advantages of using a compound eye-piece? [C. U. 1956, '58]

Describe and point out the respective merits of Ramsden's and Huyghen's eye-pieces and state their uses in optical instruments [C. U. 1945, '53, '56, '58]

Trace a pencil of rays from a distant object through such a telescope fitted with eye-piece and adjusted for normal vision. [C. U. 1945, '56]

9. Explain the achromatism attained in a Huyghen's Ramsden's ocular with lenses made from the same kind of glass. [C. U. 1942]

10. Write short notes on (a) Prism binocular.

(b) Periscope and its use. [C. U. 1947, '58]

11. Describe the construction and use of a Sextant. [C. U. 1941]

For what purpose you use a Sextant?

Explain how you would use it to measure the angle between the sun and the horizon. [C. U. 1948]

12. State the optical arrangements in a reflecting telescope and explain in what respects this is superior to a refracting telescope. [C. U. 1932]

EXAMPLE

1. The focal lengths of objective and eye-piece of a compound microscope are 1 cm. and 5 cm. respectively. An object is placed 11 mm. from the objective and the final image is 25 cm. from the eye. Find the magnification and separation of lenses?

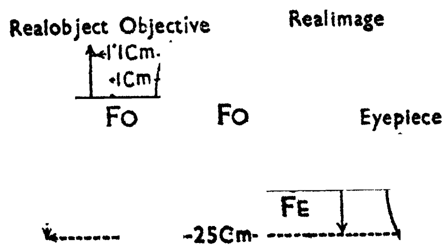


Fig. 62

For the objective, using for real image, the corrected relation is

$$-1/v-1/u=-1/f$$

or $1/v+1/u=1/f$, $1/v+1/1{\cdot}1=1$.
where $u=1{\cdot}1$ cm. $f=1$ cm. whence $v=-11$cm.

$\therefore\ m_o=-v/u=-11/1{\cdot}1=-10.$

For the eye-piece the final image is at 25 cm. in front of the lens, so using $1/v-1/u=-1/f$ or $-1/v+1/u=1/f$ we have $-1/25+1/u=1/5$, whence $u=\frac{25}{6}=4\frac{1}{6}cm.$

Angular magnification of the eye-piece $m_e=v/u=25\div4\frac{1}{6}=6.$

$\therefore$ Total magnification $=m_o\times m_e=-10\times6=-60$

Distance of separation of the lenses $=11+4\frac{1}{6}=15\frac{1}{6}cm.$

CHAPTER IX

VELOCITY OF LIGHT

109. Velocity of Light :

(a) Romer's Method (Astronomical Method) : Romer, a Danish astronomer, while determining the velocity of light by his observations of the eclipses of one of the satellites of Jupiter, found that the period or the actual interval between two successive eclipses, as observed from the earth is not constant but varies according to the relative positions of the Earth and Jupiter in their respective orbits.

Romer accounted for this discrepancy by supposing that light took an appreciable time to travel from the Jupiter to the Earth and on this supposition he succeeded in determining the velocity of light.

The adjoining Figure 63 will explain this.

Let S be the Sun and let E_1, E_2, J_1, and J_2 be the positions of the Earth and the Jupiter in their respective orbits at different instants of time.

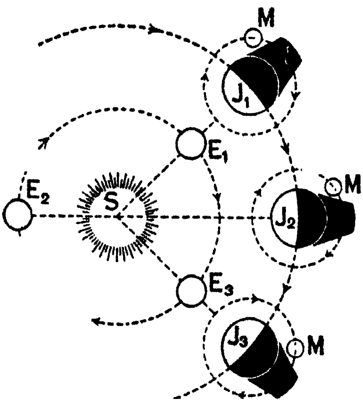


Fig. 63

Suppose the first eclipse of the satellite M takes place when the Earth and Jupiter are in conjunction *i.e.*, when they are respectively at E_1 and J_1, being on the same side of the sun. Since light takes time to travel through a certain distance, the first eclipse will be observed on the earth $\frac{E_1J_1}{V}$ secs. later than its actual time of occurrence where E_1J_1 is the distance between the Jupiter and the Earth and V, the velocity of light.

At each successive eclipse the Earth will move further and further away from the Jupiter due to their different rates of revolution in their orbits and consequently the distance between them will increase untill it becomes maximum when the Earth and Jupiter will be in opposition *i.e.*, at E_2 and J_2, being on opposite of the sun.

Let *n* eclipses occur during the interval *i.e.*, between that which occurs when the Earth and the Jupiter are at conjunction *i.e.*, at

E_1, J_1 and that which occurs when they are in opposition *i.e.*, at E_2, J_2. Then the actual interval between the first and the last of these eclipses is nt where t is the actual period of the eclipse.

Then the observed interval $T_1 = nt + \dfrac{E_2J_2 - E_1J_1}{V} = nt + \dfrac{d}{V}$ (1)

where $E_2J_2 - E_1J_1 = d$, the diameter of the earth's orbit.

Again when the Earth and Jupiter come to the position of conjunction, the observed interval T_2 is less than the true interval by $\dfrac{d}{V}$, since the distance between the Earth and the Jupiter is less than that when they were in opposition. Thus, we have $T_2 = nt -$ (2)

Therefore from (1) and (2) we have $V = \dfrac{2d}{T_1 - T_2}$

Romer measured T_1 and T_2 and found that $T_1 - T_2 = 33{\cdot}3$ minutes and taking d equal to 191×10^6 miles from astronomical data, the velocity of light was calculated as 190000 miles per sec. Latest experiments showed that velocity of light in vacuum is 186000 miles per second.

(b) Fizeau's Method (Terrestrial Method) : The method adopted by Fizeau in determining the velocity of light depends on the eclipsing of a source of light by the teeth of a rapidly roating wheel.

The principle of the experiment performed by Fizeau can be understood by the following Figure (Fig. 64). A bright source of light placed at S sends out rays which after traversing the lens L fall on a glass plate P placed at an angle at 45° and then after reflection converge to a point, say F which is the focus of the lens L_1. The rays of light emerge from the lens L_1 in a parallel direction and after traversing a distance of about 4 miles fall on the lens L_2 and are brought to a focus on the surface of a spherical mirror M having its centre of curvature in a position coincident with centre of the Lens L_2. The rays are therefore reflected back from the mirror along their own paths, form a parallel beam and after traversing the lens L_1 form an image of the source at F, which is viewed by the eye-piece T.

A toothed wheel W which can be rapidly rotated round an axle is so arranged that its teeth pass one after another through the point F. If the wheel rotates slowly the light passing through F will be alternately stopped by a tooth and allowed to pass out

between two consecutive teeth. Thus the image as seen through the eye-piece will alternately appear and disappear. If the speed of the wheel is incroassed, then due to persistence of vision a permanent image will be seen.

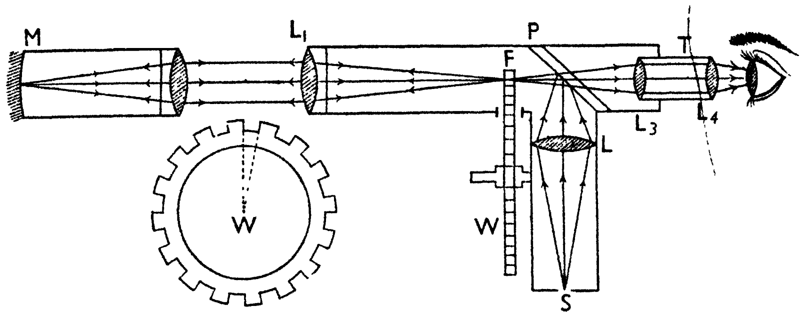


Fig. 64

Now if the speed of the wheel be so adjusted that the light which has escaped between two consecutive teeth and after reflection at the mirror M, returned to F is stopped by the tooth which has in the mean time moved through a distance equal to half that between two teeth, the image in this case will not be visible. If the wheel makes n revolutions per sec. and if m be the number of teeth in the wheel, the time required for the wheel to turn so that a tooth may exactly occupy a position previously occupied by a space is easily determined. Since in the time thus determined the light has travelled from F to M and back again to F, the velocity is obtained by dividing the distance traversed, by the time taken to do that.

Let T be the time taken by light to travel from F to M and back again to F and let MF be equal to D, then the velocity of light V is given by $V=\frac{2D}{T}$.

If n be the number of revolutions of the wheel per second, then its anguler velocity *i.e.*, the angle turned through per second is $2\pi n$.

Therefore the time taken by the wheel to rotate through θ is equal to $\frac{\theta}{2\pi n}$ and this must be equal to T.

Thus the first eclipse will occur when $T=\frac{2D}{V}=\frac{\theta}{2\pi n}$

When the speed is so adjusted that the next space comes at F in time T the bright image will be seen and for this the wheel rotates through an angle 2θ. When again, the speed is increased and adjusted such that the second eclipse or darkness occurs in time T, then for the second eclipse, $\frac{2D}{V}=\frac{3\theta}{2\pi n_3}$

In this case the wheel rotates through 3θ and makes n_3 revolutions per sec.

Similarly for the pth eclipse, $\frac{2D}{V}=\frac{(2p-1)\theta}{2\pi n_{2p-1}}$

$$\therefore \quad V=\frac{4\pi n_{2p-1}D}{(2p-1)\theta}$$

If m be the number of teeth on the wheel and if the angular width of a space and a tooth be equal, then $2m\theta=2\pi$

i.e., $\theta=\frac{\pi}{m}$ $\quad\therefore\quad V=\frac{4mn_{2p-1}D}{2p-1}$, whence V can be found out.

Velocity determined $=3{\cdot}13\times10^5$ kilometres per sec.

Discussions.

(1) Since the distance between the lenses L_1 and L_2 is very large, say 4 miles, it is not possible to perform the experiment in the laboratory. A large open air space is necessary.

(2) The intensity of light reaching the eye is diminished due to absorption of light in the medium and also due to reflection at glass plate.

(3) There is a general illumination of the field due to light reflected from the intercepting teeth of the rotating wheel.

(4) The speed of rotation of the wheel at which complete extinction of light takes place cannot be determined with sufficient accuracy.

The error (3) is avoided by bevelling the teeth and by throughly blackening the inner wall and the teeth of the wheel so that the light is reflected towards the side of the telescope and absorbed by the wall.

The error (4) is avoided by making arrangement for automatically recording the rotations of the wheel so that its speed at any instant is determined with accuracy.

Fizeau's wheel had 720 teeth and 720 spaces and the distance between the wheel W and the reflector M was about 8633 metres.

The first eclipse occurred when the wheel rotated $12{\cdot}6$ times per second.

The time taken to travel 2×8633 metres was $\frac{1}{12{\cdot}6\times2\times720}$ secs.

Therefore the velocity of light in air was $2 \times 8633 \times 12{\cdot}6 \times 2 \times 720$ metres per sec. of $3{\cdot}13 \times 10^{10}$ cm/sec.

(c) Foucault's Method : By this method the velocity of light can be determined **inside a laboratory.** Light from an illuminated

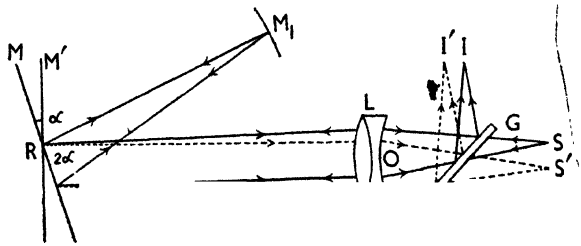


Fig. 65

slit S falls on a (Fig. 65) transparent glass plate and after passing through it falls on an achromatic lens L and being reflected from the plane mirror converges to a point M_1 on the concave mirror whose centre of curvature lies on the axis of R.

The plane mirror R can be rotated round an axis perpendicular to the plane of the paper.

When the mirror R is stationary, the light reflected normally at the concave mirror M_1 will retrace its path and after passing through the lens form an image coincident with S. The returning light will appear to diverge from a virtual image *s* (not shewn in the figure) behind the plane mirror R and form an image at I after reflection from the glass plate. If during the time taken by the light to travel from R to M_1 and back again the mirror has moved through a certain angle α, the reflected light makes an angle 2α with its original path, appears to proceed from the virtual image s' behind the mirror (not shown in the Figure) and forms the real image at S′, or at I′ where dotted rays meet after reflection from the plate glass.

The velocity of light V is then given by $\frac{2RM}{T}$, where T is the time taken by the light to travel from R to M_1 and back again *i.e.* through $2RM_1$.

During the time T, the mirror R has turned through an angle α and if we know the angular velocity of the rotating mirror R, the time taken by the mirror to turn through α is obtained and is equal to T.

If the mirror makes n revolutions per second the angular velocity $=2\pi n$ radians per second.

$\therefore$ the time to turn through $\alpha = \dfrac{\alpha}{2\pi n} = T$

To determine the angle α the images s and s' (not shown in the Figure) may be considered as the virtual images of the point M_1 for the two positions of the mirror and are situated on the circumference of a circle having its centre on the axis of rotation of the mirror and passing through M_1. The arc ss' subtends angle 2α at the centre of the circle. Therefore $ss' = 2\alpha \times D$ where

$$RM_1 = D \text{ or } \alpha = \frac{ss'}{2D} \qquad (1)$$

Now, with regard to the lens, instead of considering M_1 as the object for the two positions of the mirror, let us consider s and s' as the object and S and S$'$ as their corresponding images.

Then $$\frac{ss'}{SS'} = \frac{D+a}{b}$$

where a is the distance between the lens and the mirror and b, the distance between the lens and the line joining the images S,S.

Therefore $$ss' = \frac{(D+a)}{b}x$$

where x is distance between S and S$'$

Therefore from (1) $\alpha = \dfrac{(D+a)x}{2Db}$.

Therefore, $$T = \frac{\alpha}{2\pi n} = \frac{(D+a)x}{2Db \times 2\pi n}$$

Hence, the velocity of light $V = \dfrac{2D}{T} = \dfrac{8\pi D^2 bn}{(D+a)x}$

Velocity determined $= 298000$ kilometres per sec.

Note : In Foucault's experiment, the radius of curvature of concave mirror M. was 20 metres and the shift x was small—about ·7 mm.

Foucault's method can be used to compare the velocities of light in different media by introducing the substances between the fixed concave mirror and the revolving mirror R.

(d) Michelson's Method : Michelson modified Foucault's method by placing the lens between the fixed concave mirror and

the revolving plane mirror and thereby obtained a greater displacement of the image by increasing the distance between the two mirrors without diminishing the brightness of the image.

The experimental arrangement of the method is shown in Figure 66.

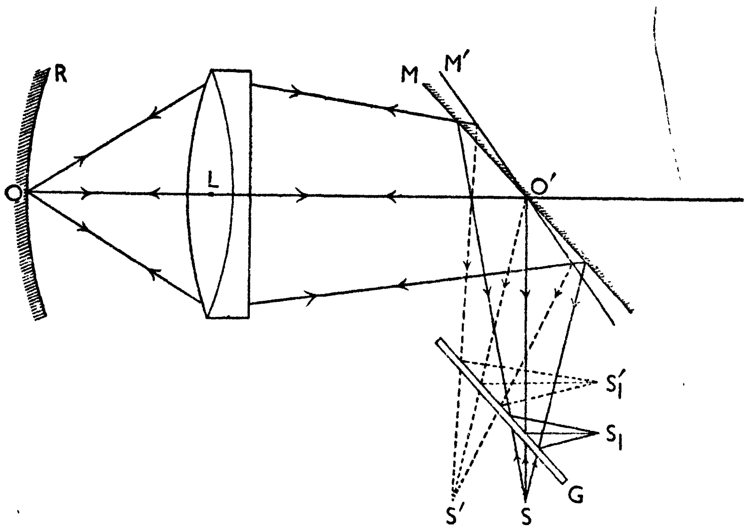


Fig. 66

Light from an illuminated slit S passes through a glass plate G falls on the mirror M, capable of rotation about a vertical axis, and then reflected from M passes through an achromatic lens L and gets reflected from a concave mirror OR whose centre of curvature is coincident with the centre of the lens.

The incident ray is, therefore, reflected normally at the mirror OR, retraces its path and after passing through the lens and after reflection from M and G forms an image at S_1.

If the mirror M be rotated at a high speed, the image will be displaced to S'_1. For, if during the time taken by the light to travel from O′ to O and back again the mirror has moved through a certain angle α, the reflected light makes an angle 2α with its original path, appears to proceed from S′ and forms the image at S'_1.

Let $SO' = C$, $O'L = d$ and $OL = D$.

Let V be the velocity of light and T the time taken by the light to travel from M to P and back.

We have, therefore $T = \dfrac{2(D+d)}{V}$ ... (1)

During the time T the mirror M has turned through an angle α and therefore $\alpha = \omega T = 2\pi n T$ where ω is the angular velocity and n, the number of revolutions of the mirror M per second.

Since the reflected ray turns through double the angle α through which the mirror is turned in time T

we have $SS' = S_1S_1' = 2\alpha C = 4\pi n$ T.C.

If the displacement $SS' = S_1S_1' = x$

we have $x = 4\pi n TC$ or $T = \dfrac{x}{4\pi n C}$

Substituting the value of T in (1) we have $V = \dfrac{8\pi n C(D+d)}{x}$

Note: Due to the difficulty of constructing a concave mirror of wide aperture, the concave mirror was replaced by a plane mirror causing a loss of definition of the image.

In Michelson's experiment, $D = 1865$ ft., $d = 135$ ft., $x = 11{\cdot}2$ cms. and $n = 258$ revolutions per second.

The value of V, the velocity of light found out by this method is 3×10^{10} cms. per second.

The present value is $2{\cdot}99735 \times 10^{10}$ cms. per second.

(e) **Michelson's latest method**: In this method Michelson modified Newcomb's method by using octagonal rotating mirrors instead of four-mirrors of Newcomb's apparatus.

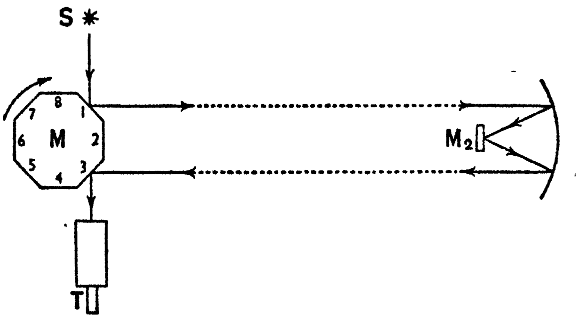


Fig. 67

Light from a source S falls at an angle of 45° on one of the faces of the octagonal mirror M mounted on the shaft of a variable speed motor. When the mirror M is stationary light is reflected

from the face 1 of the mirror M to a distant mirror M_2 after being further reflected from the concave mirror. The light reflected from M_2 falls on the face 3 of the mirror M at angle of 45° and is reflected into the telescope T.

As the octagonal mirror is rotated, at first a flickering light is seen. If the speed of the rotating mirror is adjusted so that the image occupies the same position as that occupied by the image when the mirror was stationary. This happens when the time taken by light to move from face 1 of the mirror M to the distant mirror M_2 and then back to the face 3 of the mirror M is the same as the time, taken by the face 1 to occupy the position of the face 3, *i.e.*, the time taken by the mirror M to rotate through 45°.

If D be the distance between the octagonal mirror M and the distant reflector M_2, and if n be the number of revolutions of the mirror M per second, the velocity of light is given by

$$\frac{2D}{V}=\frac{\theta}{\omega}=\frac{45^\circ}{2\pi n}=\frac{\pi}{4\times 2\pi n}=\frac{1}{8n} \quad \text{Thus } V=16\,Dn$$

In the experiment $D=70{\cdot}85$ km., $n=528$ rev./sec.

Velocity of light $V=299{,}796$ km./sec. in vacuo.

110. Comparison of velocities in different media: In Foucault's experiment (Fig. 65) a second mirror M_1' of the same radius of curvature as M_1 is placed on the opposite sides of the axis of the lens L with their axes on the centre of the revolving mirror M.

A tube containing the substance, say water, is placed between the mirrors M_1' and M. Two images are seen and the image formed by light which has traversed water is more displaced shewing that light travels more slowly in water than in air.

QUESTIONS

1. Describe carefully Fizeau's method for finding the velocity of light, giving a neat diagram. What are its disadvantages, and how have some of these been subsequently minimised. [C. U. 1928, '36, '38, '42, '47, '51]

What value did Fizeau obtain and what is the present accepted value in C. G. S. units. [C. U. 1936]

2. Describe an apparatus which can be fitted up in a laboratory for determining the velocity of light. Mention some of its defects and state how they were subsequently minimised. [C. U. 1953, '55, '58]

Explain how the measurements of velocity of light in water shows that the emission theory of light can not be accepted as true. [C. U. 1952, '53]

EXAMPLES

1. In Fizeau's method of determining the speed of propagation of light by means of a toothed wheel, the wheel has 120 teeth and an equal number of spaces of equal width. If the distance of the mirror be 12 km., at what speed (in revolution per minute) of the wheel the first eclipse occurs? [C. U. 1947]

We know that $V=\dfrac{4mn_{2p-1}.D}{2_p-1}$

Here $m=150$, $p=1$, $D=12\times1000\times100$ cm., $V=3\times0^{10}$ cm./sec.

$\therefore$ speed $n=\dfrac{3\times10^{10}}{4\times150\times12\times10^{5}}=2500$ per min.

CHAPTER X

RAINBOW

111. Introduction : Rainbows are generally seen when sun shines on falling rain-drops or on the spray from a cascade or a fountain, the back of the observer being turned towards the sun. They consist of seven brilliant arcs shewing successively all the colours of the spectrum.

Rainbows are produced by refraction and dispersion of sun's rays when the latter fall on water drops suspended in air. However, only those rays which suffer minimum deviation after one or more internal reflections contribute to form rainbows.

Sometimes a single bow is seen but there are usually two and in favourable circumstances several bows may be seen.

Of all the bows the brightest one is known as the **Primary bow.** A large and a fainter bow external to the primary bow is known as the **Secondary bow.**

Other fainter bows known as **Supernumerary bows** are sometimes seen within the primary bow.

A brief description of the formation of **Primary** and **Secondary** bows is given below.

In Figure 68 O_1, O_2, O_3 and O_4 are the positions of the rain-drops arranged along a vertical line and so situated that sun's rays falling on the drops suffer refraction and internal reflection (not total reflection) and emerge out in a direction of ***minimum deviation*** and form rainbows.

In the case of a *primary bow*, besides two refractions there is one internal reflection in the drops as at O_1 and O_2, and the emergent violet and red rays O_1V and O_2R undergoing minimum deviation, make respectively angles of 40° and 42° at the observer's eye at E.

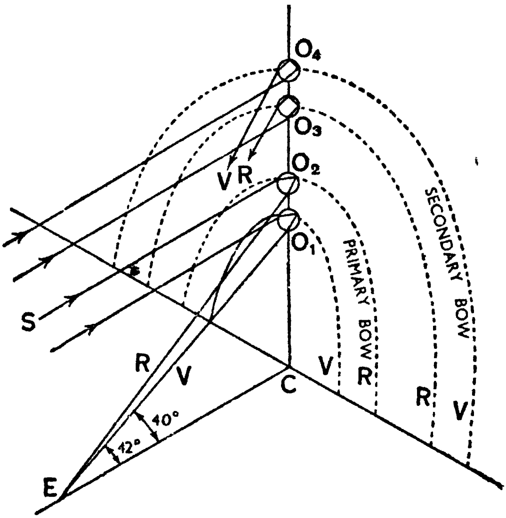


Fig. 68

Besides O_1 and O_2 there are other drops similarly situated on the circular arcs which are obtained by describing cones with E as apex and EC as axis and semi-vertical angles equal to 40° and 42° for violet and red rays respectively. Thus the primary bow consists of a coloured arc with red on the outside and violet on the inside, the other colours occupying intermediate positions.

Similarly in case of a *secondary bow*, sun's rays falling on the drops, as at O_3 and O_4, suffer two refractions and two internal reflections and the minimum deviated emergent red and violet rays make respectively angles of 51° and 54° at the observer's eye at E.

In this case also a coloured arc is observed with violet on the outside and red on the inside.

The secondary bow is much less intense than the primary bow.

112. Deduction of Deviation The formation of the rainbow is easily explained by considering the decomposition of white light of the sun when it passes into the rain or water drops in air.

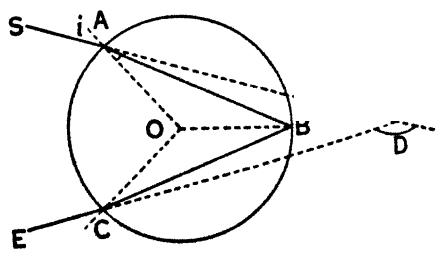


Fig. 69

Solar rays get refracted while passing into the drop, suffer successive internal reflections (not total reflection) and emerge out after a second refraction in directions depending upon the angle of incidence. The emergent rays which undergo

minimum deviation are effective in producing the phenomenon of rainbow.

Consider a ray SA incident on a drop of water at A and undergoing refraction along AB (Fig. 69). If it suffers only one internal reflection at B and emerges along CE, the resultant deviation D, which is equal to the sum of the deviations at A, B and C is given by

$$D=(i-r)+180-2r+(i-r)=180+2i-4r$$

Note: The deviation is maximum and equal to 180° when the rays strike the drop normally. In other positions the deviation will vary between 180° and a minimum value of about 138° for one internal reflection.

For two refractions and two internal reflections the resultant deviation is also given by

$$D=2(i-r)+2(180-2r)=360+2i-6r.$$

If we plot curves showing deviations corresponding to various angles of incidence in the two above cases, *i.e.*, when there is one internal reflection and when there are two internal reflections, the minimum deviation when there is one internal reflection is equal to about 138°, and its value when there are two internal reflections is equal to about 232°.

Since in the first case no ray in deviated by less than (180° − 42°) or 138° it follows that the rays emerging from the spherical drops of water are all contained in the right circular cone half the vertical angle of which is 42° and they have come from drops which lie on the cone whose semi-vertical angle is 42°.

Similarly it can be shown that the rays which have suffered two internal reflections have come from drops which lie on a cone whose semi-vertical angle is equal to about

$$180° - (360° - 232°) \text{ or } 52°$$

The primary bow is formed by rays undergoing one internal reflection and two refractions, while the secondary bow is formed by rays undergoing two internal reflections and two refractions.

The general expression for the resultant deviation for two refractions and n internal reflections is given by

$$D=2(i-r)+n(180-2r)$$

112 (a). Calculation of the Minimum Deviation for n Internal Reflections : Except when the deviation is 180°, we have

For n internal reflections, $D=2(i-r)+n(\pi-2r)$;

$$\frac{dD}{di}=2-2\frac{dr}{di}-2n\frac{dr}{di}, \text{ For D to be minimum, } \frac{dD}{di}=0.$$

$$\therefore \quad 0=2-2(n+1)\frac{dr}{di} \text{ or } \frac{dr}{di}=\frac{1}{n+1}$$

Now $\sin i = \mu \sin r$, $\cos i = \mu \cos r . \dfrac{dr}{di} = \mu \cos r . \dfrac{1}{n+1}$

$$\therefore \quad (n+1)^2 \cos^2 i = \mu^2 \cos^2 r$$
$$(n+1)^2(1-\sin^2 i) = \mu^2(1-\sin^2 r) = \mu^2 - \sin^2 i$$
$$(n+1)^2 - (n+1)^2 \sin^2 i = \mu^2 - \sin^2 i$$
$$(n+1)^2 - \mu^2 = \sin^2 i\{(n+1)^2 - 1\}$$
$$\sin^2 i = \frac{(n+1)^2 - \mu^2}{n(n+2)}$$

For one internal reflection, *i.e.* when $n = 1$, $\sin^2 i = \dfrac{4-\mu^2}{3}$

For two reflections, *i.e.* $n = 2$, $\sin^2 i = \dfrac{9-\mu^2}{8}$

For red and violet light the values of μ are 1·33 and 1·34 approximately. This gives

$i = 59°·6$ and $D = 137°$ for red (approx.)
$i = 58°·8$ and $D = 139°$ for violet (approx.)

(A) For *two internal reflections*, the value of i when the deviation is minimum is given by $\sin i = \sqrt{\dfrac{9-\mu^2}{8}}$ or $\cos i = \sqrt{\dfrac{\mu^2-1}{8}}$

(B) For *n internal reflections* the minimum deviation occurs when $\sin i = \sqrt{\dfrac{(n+1)^2-\mu^2}{n(n+2)}}$ or $\cos i = \sqrt{\dfrac{\mu^2-1}{n^2+2n}}$

113. Primary Bow : Let the circle MM_1R represent a section of a drop of a water (Fig. 70) into which solar rays penetrate.

Let $S_1\ M_1$ be a particular ray of a pencil of parallel rays which fall on the drop.

The ray $S_1\ M_1$ is refracted along M_1R_1 and after reflection at R_1 travels along R_1N_1 and emerge after refraction along N_1P_1.
In Figure 70 the paths of a number of incident rays after refraction and one internal reflection have been shown.

If we study the directions of the rays incident near M_2, it would be seen that they are less deviated by two refractions and one reflection than the rays incident at any other point.

It is also seen from Figure 70 that the rays incident near M_2 form a parallel pencil on emergence while in the case of rays incident at any other point form a divergent pencil on emergence.

So if we look at the rain drop from a distance the rays which form a parallel beam N_2P_2 after emergence will impress the retina more but the rays emerging out as a divergent pencil such as N_3P_3 will produce very little impression on the retina. This is because the loss of intensity of illumination with distance in the case of parallel beam is less than in the case of a divergent beam.

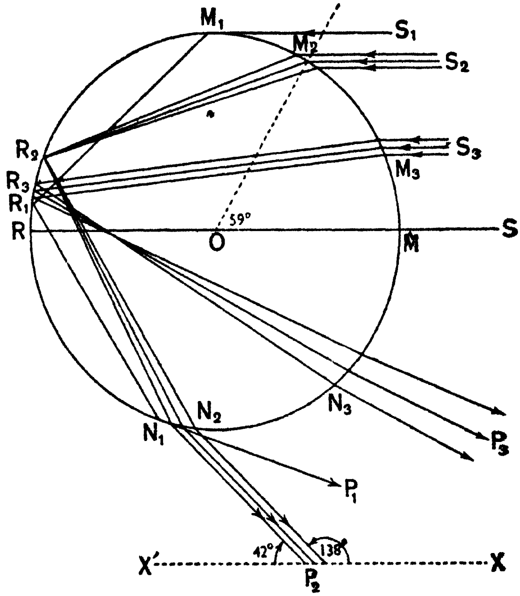


Fig. 70

Let XX′ be drawn parallel to the incident beam so that the angle N_2P_2X' is 42° and N_2P_2X is 138°.

Now if with our eyes at E (Fig. 71) as apex and the direction of sun's ray SES′ as axis we describe a cone of which the angle between the generating line and the axis is 40°, all the rain drops which are on the circum-

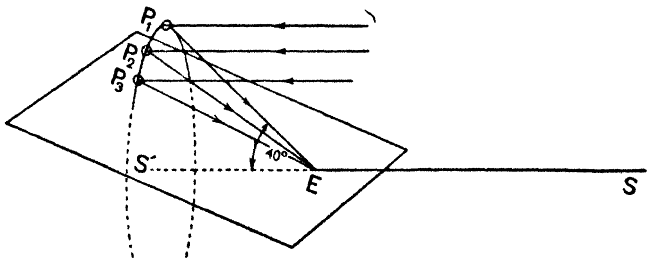


Fig. 71

ference of the base of the cone will be so situated that the pencil of parallel rays which has undergone minimum deviation can

enter the eye and so the drops will be visible as bright points.

Since sun light consists of seven different colours the rays which emerge out from the drop will consist of different sets of rays having different minimum deviations. For red rays the angle between the incident light and the pencil of red rays which emerge parallel and undergo minimum deviation is 42° and that between the incident rays and the emergent violet rays is about 40° and the angles for all the other intermediate colours will be between those for the red and the violet (Fig. 72).

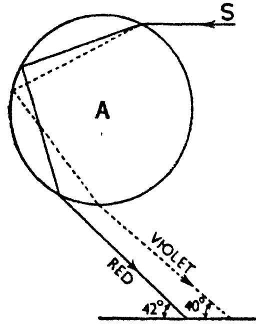


Fig. 72

So if we look at the rain-drops on which solar rays are falling and construct a series of curves having the semi-vertical angle of 42° for the outer red curve and 40° for the inner violet curve, the result is a series of circular arcs shewing all the spectral colours arranged in order of descending wave-lengths from red to violet and constitute a *rainbow*. (Fig. 68)

114. Secondary Bow : To explain the formation of a **Secondary Bow**, we have to take into account two refractions two internal reflections and also the minimum deviation of all the different sets of parallel rays of different colours.

In this case the angles of minimum deviation for the emergent red and violet rays are 231° and 234° respectively, and therefore the angles between the incident light and the least deviated red and violet rays are 51° and 54° respectively (Fig. 73).

So if we look at the drops of rain in which two internal reflection take place we will get a series of circular arcs showing all the spectral colours and in which violet appears on the outside and red inside.

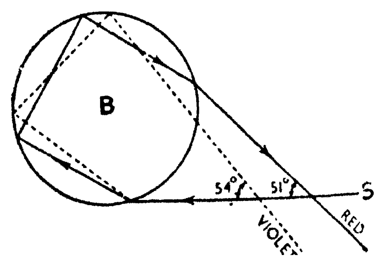


Fig. 73

For *three or four internal reflections*, the bow will be between the sun and the observer.

The light after three or four reflections will be very much weakened and the sun, except under favourable circumstances will by contrast make them unobservable.

For *five internal reflections* the bow will be seen again opposite to the sun. But it is seldom visible on the account of loss of light due to so many reflections.

In the morning, the rainbow will be observed in the western sky and in the afternoon in the eastern sky.

The *supernumerary bows* as found in the Primary bows are due to phase difference in the beams on emergence from the rain drops and a series of positions of maximum and minimum brightness is obtained.

Note: If we advance towards the bow, a new bow will be seen satisfying the condition of the previous bow and there will be no change in the semi-vertical angle of the cone of light reaching the observer.

If a man ascends vertically from the ground the rainbow will gradually appear circular in shape.

QUESTIONS

1. Explain the formation of a primary rainbow. What is the semi-vertical angle of the cone of light reaching the eye of the observer, and in what order are the colours arranged?

Will there be any change in the angle if the observer advances towards the rainbow by some appreciable distance. [C. U. 1937, '43, '45, '46, '49, '52]

2. In what part of the sky is a rainbow seen in the early morning and how would its appearance change to a man ascending vertically from the ground in a balloon? [C. U. 1949]

EXAMPLES

1. A ray of light is incident on the surface of a transparent sphere of refractive index 1·32. After refraction, it is internally reflected (not total) and then refracted out of the sphere. Find the deviation suffered by the ray and the angle of incidence to the nearest degree so that the deviation may be a minimum. [C. U. 1949]

For one internal reflection $n=1$; $\sin^2 i=\dfrac{4-\mu^2}{3}=\dfrac{4-(1\cdot32)^2}{3}=\cdot7525$

or $\sin i=\cdot8674$ i.e., $i=60^\circ$ (approx.)

$\therefore\ D=2(i-r)+(180-2r)=2(60-r)+(180-2r)$

Since $\dfrac{\sin i}{\sin r}=1\cdot32$, $\sin r=\dfrac{\sin i}{1\cdot32}=\dfrac{\cdot8674}{1\cdot32}=\cdot6561$

$r=41^\circ$ from log table $\qquad D=136^\circ$

2. A ray of light is incident at an angle of 60° on a sphere of material whose index of refraction is $\sqrt{3}$ and emerges after one reflection. Show that its direction on emergence is parallel to the original direction.

(The problem is solved if the total deviation is proved to be equal 180).

CHAPTER XI

PHOTOMETRY

115. Introductory : The subject of photometry deals with the intensity of light emitted by a source and the intensity of illumination on a given area.

The following terms, *viz luminous flux*, *illumination*, *luminous intensity* and *brightness* are the four most important photometric quantities.

To understand all these terms it is better to have some idea about solid angles.

116. Solid Angles : If from numerous points on the boundry of an area, say S′ (Refer Fig. 77) straight lines are drawn to a point say O, these lines generate a cone with the vertex at O and the angle at O subtended by area S′ is the *solid angle* at O.

If with O as centre a sphere is described with radius $OA = r$, cutting the cone in the area S the solid ang'e ω at O is given by $\omega^2 r = S$.

If the radius of the sphere be unity, *i.e.* $r = 1$ cm., the *area* cut off on the sphere of unit radius by the cone is a measure of the *solid angle*. Refer Figure 77.

Unit solid angle is the angle subtended at the centre of a sphere of radius r by a portion of surface of area r^2.

If S represents the whole surface area of the sphere of radius r the total solid angle subtended at the centre is

$$w = \frac{S}{r^2} = \frac{4\pi r^2}{r^2} = 4\pi.$$

117. Photometric quantities :

(a) Luminous Flux : It is the quantity of light energy which flows through a given surface in unit time.

(b) Illumination or the Intensity of Illumination : The luminous flux falling on unit area of a given surface is called its illumination.

The intensity of illumination at a point is measured by the amount of light falling perpendicularly on a unit area placed at the point, in one second.

If Q be the amount of light falling perpendicularly on an area A at a point P, the intensity of illumination I at P is given by

$$I = \frac{Q}{A}.$$

It can be proved that the intensity of illumination I at a point ıries inversely as the square of the distance d of the point from ıe source. That is, $I \propto \frac{1}{d^2}$.

This law is known as *Inverse Square Law.*

117(a). Units of Photometry : The unit of luminous flux is ımen. It is the flux emitted per unit solid angle by a uniform)int source of one candle power.

The unit of illumination is *one lumen* per unit area.

One lumen per sq. cm. is **Phot.** One lumen per sq. m. is called **lux. 1 Phot. = 10000 lux or one metre-candle.**

One lumen per sq. ft. is called a **foot-candle.**

These units, the metre-candle and the foot-candle are defined ; the intensity of illumination produced by a standard candle at distance of one metre (in the case of *metre-candle*) or one foot n the case of *foot-candle*) from a screen.

118. Inverse Square Law : Consider a source of light S mitting light in all directions at the ate of Q per sec.

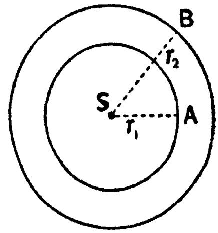


Fig. 74

Imagine a spherical surface A (Fig. 74) ırawn with S as centre and with adius equal to r_1.

Since the spherical surface A is ıniformly illuminated, the intensity of llumination at a point on A is

$$I_1 = \frac{Q}{4\pi r_1^2} \qquad ...(1)$$

If another sphere B with radius r_2 be drawn instead of A round S as centre, the intensity of illumination at a point on B is

$$= \frac{Q}{4\pi r_2^2} \qquad (2).$$

Since the same amount of light Q falls on B

$$\text{dividing (1) by (2), } \frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}, \qquad (3)$$

This expression proves that the intensity of illumination at a point due to a source varies inversely as the square of the distance of the point from the source.

119. Luminous Intensity or Illuminating Power : The luminous flux per unit solid angle is called the **luminous intensity** of the source.

The Luminous intensity is also called the illuminating power of the source and may be defined as the intensity of illumination at unit distance from the source.

The unit of power is a lumen and it is usually given in *lumen per watt*, or *candle power per watt.*

If in the expression (1) $r_1 = 1$, the intensity of illumination I_1 at unit distance from the source is called its *Illuminating Power* P of the source.

If in expression (3) $r_1 = 1$, $r_2 = r$, $I_2 = I$, and $I_1 = P$ we have

$$\frac{P}{I} = \frac{r^2}{1^2}. \quad \therefore \quad I = \frac{P}{r^2}.$$

120. Standards of Illuminating Power : The British standard of illuminating power is called the candle-power.

(A) Candle Power : It is defined as the illuminating power of a sperm candle weighing one-sixth of a pound having a diameter of $\frac{7}{8}$ inch. and burning at the rate of 120 grains per hour.

(B) Standard Candle : It is a candle made of spermaceti wax weighing one-sixth of a pound and burning at the rate of 120 grains of wax per hour.

This standard is useful for rough purposes, but not for scientific purposes.

The most trustworthy standard is the Vernon **Harcourt Pentane Lamp** in which air is drawn over liquid pentane, a very highly inflammable hydrocarbon distilled from petroleum. The mixture of air and pentane vapour passes to a burner of steatite pierced with 32 holes through an Indian rubber tubing and is ignited.

When burnt under standard conditions of temperature and pressure, the illuminating power of this lamp is fairly constant and is equal to that of ten standard candles.

The most convenient standard for use in the laboratory is the **electric incandescent lamp**, which gives a constant candle power when run at a definite voltage.

(C) International Candle : It is an incadescent electric lamp when run at a definite voltage. It is one-tenth of the intensity due to the flame of a Vernon Harcourt Pentane Lamp burning under definite condition of humidity and pressure, in a direction at right angles to the flame.

121. Photometers : These are devices for measurement of illuminating power of a source of light. The principle utilised in these instruments is to adjust the distances of the two sources whose powers are to be compared from the screen until they both produce the same intensity of illumination at the screen.

If P_1 and P_2 are the powers of the two sources and d_1 and d_2, their respective distances from the screen when they produce the same illumination,

$$\text{we have } \frac{P_1}{d_1^2}=\frac{P_2}{d_2^2} \text{ or } \frac{P_1}{P_2}=\frac{d_1^2}{d_2^2}$$

121(a). Bunsen's Grease-Spot Photometer : It consists of a screen of white unglazed paper with an oil or grease-spot in the middle and held between two sources of light L_1 and L_2 whose illuminating powers are to be compared. The heights and the positions of the sources and the unglazed paper are so adjusted that on looking from either side the grease-spot cannot be readily distinguished from the rest of the paper.

In this position the intensities of illumination at a point, both of the glazed and the unglazed surfaces are equal.

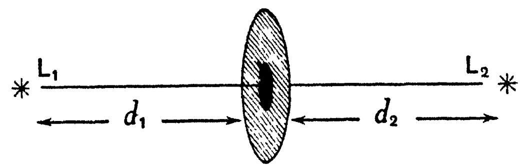


Fig. 75

Let L_1 and L_2 be the two sources whose powers are to be compared and let their respective distances from the screen be d_1 and d_2 (Fig. 75).

Let q_1 and q_2 be the quantities of light falling on unit area of the screen from the sources L_1 and L_2 respectively and let the fractions a and b of unit quantity of incident light represent th light transmitted by the unit areas of the glazed and unglazed parts of the screen, respectively.

Then $(1-a)$ and $(1-b)$ represent the fractions of light reflected from the unit areas of the glazed and unglazed portions.

Thus the quantity of light reaching the eye placed on the same side as the source L_2 from unit area of the greased portion is

$$q_1a+q_2(1-a)$$

Similarly the quantity of light reaching the eye from unit area of the ungreased portion is $q_1b+q_2(1-b)$.

Then, when the two portions appear equally bright we have

$$q_1a+q_2(1-a)=q_1b+q_2(1-b)$$
$$q_1(a-b)=q_2(a-b) \quad \therefore \quad q_1=q_2.$$

Hence the intensities due to the two sources at the screen are equal, and we have $\quad d_1$

Here P_1 and P_2 are the powers of the sources L_1 and L_2.

Note: In an improved form of the grease-spot photometer two plan mirrors are atranged inclined to each other on the same stand carrying th screen in such a way that both sides of the grease-spot are viewed at the sam time and very nearly under the same conditions.

121(b). Lummer-Brodhun Photometer: This photometer consists of a system of total reflection prisms E, F, C, D (Fig. 76) with a slab of magnesium carbonate placed in the position AB. The whole system is enclosed in a metal box fitted with a telescope T in order to receive light diffused from the left and right sides of the slab. The directions of the rays of light from the two sides after their passage through the system of prisms are shown in the figure.

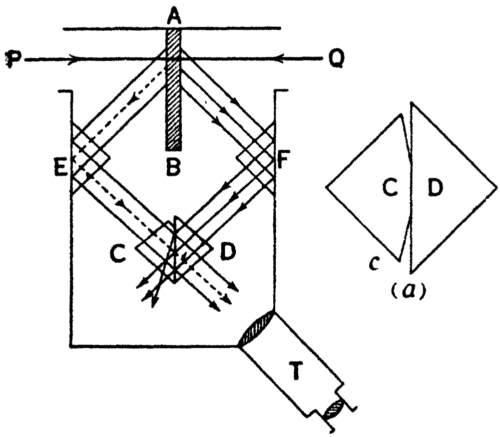


Fig. 76

The two prisms C and D [Fig. 76(*a*)] are placed with their hypotenuse faces in contact. The hypotenuse face of the prism C is slightly ground away, except for a circular portion in the centre.

Two sources whose powers are to be compared are on the opposite sides of the slab AB at P and Q.

Of the rays which reach the prism C from the left-hand surface of the slab, a certain portion will pass through the common parts of the prisms C and D while the rest will be totally reflected from the ground-away parts of C.

Again, of the light which comes from the right-hand surface of the slab, a certain portion passes straight through the prisms C and D and the rest totally reflected as before.

When these rays are received by the telescope, the field of view will be divided into two parts, the central part being illuminated by light from the left-hand surface of the slab, while the surrounding portion by light from the right-hand part of the slab.

It is to be noted that both portions of light have traversed equal thickness of glass so that absorption within the glass affects them equally.

The positions of the two sources are so adjusted that the central and the surrounding portions of the field of view of the telescope appear equally bright.

In this case, the intensity at the slab due to the two sources are the same and hence we have $\frac{P_1}{P_2}=\frac{d_1^2}{d_2^2}$

Photometeric measurements depending on the judgment of equal brightness or equal contrast are unreliable when the sources differ in colour.

To compare the powers of sources of widly different colours a photometer, known as **Flicker Photometer** is used.

122. Brightness or Intrinsic Luminosity: The luminous ntensity of unit area of the surface whether self-luminous or artificially illuminated, is called its brightness in that direction.

If P be the luminous intensity of a surface of area s, then the brightness or the intrinsic luminosity i is expressed as $i = \frac{P}{s}$.

123. Apparent Brightness: The illumination of the image formed on the retina determines the **apparent brightness** of the object.

Let s be the area of the object of brightness i in lumens per cm^2 situated at a distance R from the eye. Let the area of the pupil be e and that of the image be A.

Then the intensity of the object (*i.e.*, the source) is $i.s$ lumens.

The solid angle subtended by the pupil at each point on the object is $\frac{e}{R^2}$. The flux q through the pupil is $q = \frac{ise}{R^2}$

The flux on the area A of the ocular image $= \frac{ise}{R^2} \div A = \frac{ise}{AR^2}$.

But we have $\frac{A}{s} = \frac{r^2}{R^2}$, where r is the distance of the ocular image from the pupil. Therefore the apparent brightness B_a is given by

$$B_a = \frac{q}{A} = \frac{esi}{R^2} \cdot \frac{R^2}{sr^2} = \frac{ei}{r^2} \frac{lumens}{cm^2} \qquad \text{since } A = \frac{sr^2}{R^2}$$

Hence the apparent brightness of a surface is independent of its size and of its distance and consequently of its inclination to the line of sight.

In other words an object appears equally bright at all distances from the eye.

The dimness of distant objects when seen through the atmosphere is due to the partial opacity of the latter.

If the source is a mere point such **as a distant star** the apparent brightness depends simply on the light collected by the eye, and the ocular image is also a point. In this case the value of q *i.e.* the quantity of light falling on the pupil is diminished with the increased distance and the area of the ocular image is also diminished. But since the diminution of q is not compensated by the diminution of A, the point source will appear fainter as it recedes from the eye.

Note: Illumination of the surface is determined by the light received. Brightness, however, is concerned with the light emitted by the surface.

A piece of black velvet lying on fresh snow may well have the same *illumination* as the snow but its *brightness* will be widely different.

124. Glare : It is defined as dazzling or annoying brigtness of a powerful source of light as an electric lamp or the head light of motor cars.

The glare of a lamp is diminished by covering the lamp by a globe of glass or porcelain.

125. Oblique Illumination : Let O be a point source of light, while S is a small area of a spherical surface with radius $OS = r$. Let S' be also a small element of area inclined to S at angle θ, both being sections of the cone *oab*,

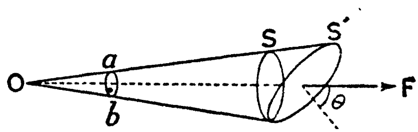


Fig. 77

Let I' and I be the intensities over the areas S' and S respectively and let Q be the quantity of light emitted per second within the cone *oab*.

Then, we have $I' = \frac{Q}{S'}$ and $I = \frac{Q}{S}$

Again since $S' \cos\theta = S \quad \therefore \quad S' = \frac{S}{\cos\theta}$

$$\text{So } I' = \frac{Q}{S}\cos\theta = I\cos\theta$$

If P be the illuminating power of the source at O,

$$\text{then } I = \frac{P}{r^2}$$

$$I' = \frac{P}{r^2}\cos\theta$$

This is known as **Lambert's Cosine Law.**

Thus the intensity of illumination varies directly as the cosine of the angle of incidence and inversely as the square of the distance of the illuminated object from the source.

126. Fraction of light transmitted by a transparent plate : Let two sources of light L_1 and L_2 produce equal brightness on the two sides of a screen S placed at distances d_1 and d_2 respectively from the sources L_1 and L_2.

In this case, $\frac{P_1}{d_1^2} = \frac{P_2}{d_2^2}$, where P_1 and P_2 are the powers of the sources L_1 and L_2.

Let a transparent glass plate be placed between L_1 and S. So
L_1 has to be moved nearer to a distance d_1' to restore equality
illumination. The effective power of L_1 has been reduced from
P_1 to P_1'.

Therefore, in this case $\frac{P_1'}{d_1'^2} = \frac{P_2}{d_2^2}$

Thus $\frac{P_1}{d_1^2} = \frac{P_1'}{d_1'^2}$ Or $\frac{P_1'}{P_1} :: \frac{d_1'^2}{d_1^2}$, Or $P_1' = \frac{d_1'^2}{d_1^2} . P_1$

Thus the plate transmits $\frac{d_1'}{d_1^2}$ th part of the luminous flux it
receives.

The fraction $\left(1 - \frac{d_1'^2}{d_1^2}\right)$ is cut off being caused by absorption,
reflection and scattering at the surface of the plate.

QUESTIONS

1. Define the following photometric quantities :—
   Luminous flux, illumination, luminous intensity and brightness.
2. What do you understand by the terms : lumen, lux, phot, and metre-candle.
3. State and verify Inverse Square Law of illumination.
4. Describe the Lummer-Brodhun photometer and explain how the illuminating powers of two sources of light are compared with its help.
5. What is intrinsic luminosity ? Shew how an object appears equally bright at all distances from the eye.

CHAPTER XII

PHYSICAL OPTICS

127. Introductory : Light is classified into two branches namely (1) Geometrical Optics, and (2) Physical Optics. The first already studied in details, refers to geometrical properties of rays light. The latter refers to wave theory of light.

127(a). Theories of Light : There are a number of theories light which in the order of their discovery are as follows :—
(1) Corpuscular theory (2) Wave theory (3) Electromagnetic theory and (4) Quantum or Photon theory.

128. The Corpuscular Theory : To explain the propagation nature of light Sir Issac Newton developed a theory, known the corpuscular theory which assumes that light consists of a

swarm of material particles or luminous corpuscles of extrem minuteness. They are projected from a luminous body witl enormous speed in all directions and in straight lines and cau the sensation of vision when they impinge on the retina. Thes particles move in straight lines as long as they travel throug interstellar space but on approaching to within a small distanc of a material medium, the path of the corpuscles is modified, an the nature of the modification varies according as the corpuscle in a condition favourable to reflection or refraction.

128(a). Rectilinear propagation of light: It is easily seer that the rectilinear propagation of light is directly explained on the corpuscular theory, since it is assumed according to this theory that the light particles, while travelling in a uniform medium are not subjected to any force and hence in accordance with Newton's first law of motion, they travel with uniform constant velocity in a straight line.

128(b). Reflection of light: In the case of **reflection** a corpuscle on approaching a reflecting medium is supposed to experience a force of *repulsion* acting normal to the surface so long as it remains within a very small distance from the latter. If the velocity of the light corpuscle be resolved into components parallel and perpendicular to the surface, the former component is unaltered by the repulsive force, but the latter graduall diminishes and is ther reversed to its former value In Figure 78 the corpuscle travelling along PE begins to be repelled when it enters the zone of repulsive force between AB and A_1B_1. The component gradually be comes less and the path curves round until i becomes parallel to surface. The repulsion then gives the corpuscle an upward velocity which goes on increasing until FD = EC and the parallel components EC and FH always remaining equal. The corpuscle finally takes th direction FQ which must be inclined to normal FN_2 at the sam angle as the path PE to normal EN_1. Hence $\angle i = \angle r$.

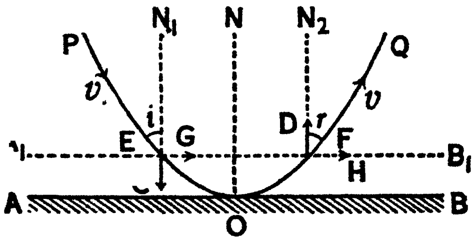


Fig. 78

128(c). Refraction of light: In the case of **refraction**, th corpuscle experiences forces of attraction towards the more refrac ting medium AB and due to which the vertical component of th velocity is increased as the corpuscle passes through a thin laye bounded by two planes A_1B_1 and A_2B_2 (Fig. 79) on both sides (

ıe surface, while the horizontal component QE or RF of the elocity, parallel to the surface, remains unaffected. After passing ırough the layer the corpuscle ravels in a straight line.

If i and r are the angles hich the initial and final aths of the corpuscle in the vo media make with the normal the surface of separation nd if v and v' are respectively he velocities in the rarer and he denser medium, then since he horizontal velocity remains ınaffected, we have QE=RF. But QE$=v \sin i$ and RF$=v'$ in r.

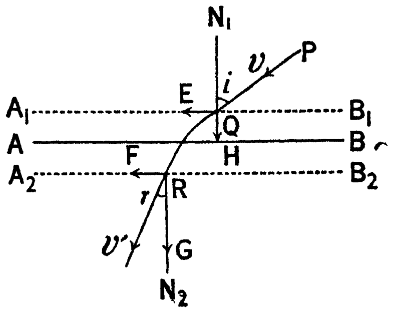


Fig. 79

$$v \sin i = v' \sin r \quad \text{or} \quad \frac{\sin i}{\sin r} = \frac{v'}{v} = \text{a constant.}$$

Thus the sine of the angle of incidence bears a constant ratio o the sine of the angle of refraction.

But since i is greater than r, v' is greater than v.

That is, the velocity of light in the denser medium is greater than that in the rarer medium—a fact in direct opposition to Foucault's experimental result.

Thus the corpuscular theory becomes untenable.

Note: Since reflection involves a repulsive force on the corpuscle and refraction an attraction on the corpuscle, it is obviously not possible to explain in any simple way the experimental fact that a beam of light incident on a transparent medium is partially reflected and partially refracted or transmitted.

129. Wave Theory or Undulatory Theory: Although the **corpuscular theory** explains the rectilinear propagation of light, formation of shadows, reflection and many other optical phenomena but it breaks down as it begins to explain the phenomena of refraction, double refraction, interference, polarisation and diffraction.

So to explain all these optical phenomena, the **Wave Theory** was propounded. This theory supposes that a luminous body by its rapid vibratory movement sets up disturbances or waves in a mysterious, all pervading medium, known as *ether*, which travel with the velocity of light in all directions in the form of transverse waves and fall upon the eyes.

Huyghens first stated the **wave theory** in a definite form and he satisfactorily explained by it the phenomena of reflection, refraction and double refraction but could not account for the rectilinear propagation of light and the theory of shadows which were so

easily explained by the corpuscular theory. Thus the wave theor fell into disrepute and remained lifeless for about a century.

The theory was then revived by **Dr. Young** by his discover of the principle of interference which remained inexplicable by th corpuscular theory.

Although Dr. Young brought the wave theory into prominence, neith Huyghens nor he could account for the *polarisation of light* as they erroneousl conceived the wave disturbance in *ether* to be of the longitudinal type.

It was not until **Fresnel** discovered the transverse character of the wa that the objections against the wave theory were removed and not only tl polarisation and diffraction of light but many other optical phenomena we satisfactorily explained.

Besides explaining the phenomena of interference, diffraction, polarisatic and double refraction on the wave theory, it gives a satisfactory explanation (the colour of the spectrum and explains Lippmann's colour photography on tl principle of the formation of stationary waves.

Huyghen's wave theory however had many drawbacks an difficulties. It assumed the existence of a hypothetical elastic flui known as ether in which particles of matter are embeded lik bodies in a jelly. For the propagation of transverse disturbanc in ether, it must have high rigidity and very low density and b incompressible.

What such a medium is no body knows. The result o **Michelson—Morley's** experiment with regard to the relative motio of ether and matter completely upset the idea of the existence o ether. Somebody even doubted the existence of ether.

130. Electromagnetic Theory: In 1893 **Clerk Maxwel** introduced a new form of wave theory known as the **Electro magnetic Theory of Light**.

According to this theory, all sorts of radiation are electrc magnetic and the radiation consists of an electric force and of magnetic force set up in the surrounding medium at right angle to the direction of the propagation.

According to this theory, light is an electromagnetic phenomeno and consists of waves having the same velocity as that of light.

131. Quantum or Photon theory of Light: The electro magnetic theory of light satisfactorily explained the propagation o light, interference, diffraction, polarisation and many other optica phenomena but it failed to explain the process by which light i emitted and absorbed by material bodies.

In 1900 **Planck** developed a theory known as the *quantum theory* in whicl he assumed that whenever an atom emitted energy in the form of radiation i did so in definite amounts of energy or packets at a time. Each of thes minimum amount of energy is called a *quantum or photon*.

In 1905 **Einstein** extended the conception of Planck's quantum theory and ıccording to him the energy of radiation emitted from any source does not pread out in waves over ever-widening surfaces but is shot out like bullets.

In the classical wave theory the emission or absorption of light was upposed to be continuous but in the quantum theory the emission or absorption ıf light takes place in definite packets at a time.

Thus a complete dualism now exists between Wave theory and Quantum heory and as yet the two conflicting views have not been replaced by a single ınified theory.

132. Light waves differ from sound waves in the following respects: (1) Light waves are formed in the medium of ether, whereas sound waves are formed in a material medium. (2) Light waves are transverse while sound waves in air are longitudinal. (3) Light waves are very short (of the order of 5890×10^{-8}cm) while sound waves are long (of the order of a few feet). (4) Light waves affect the sense of vision whereas sound waves affect our sense of hearing.

133. Wave Theory : For the propagation of waves or disturbances caused by the *to and fro* motion of particles we must require a medium and the medium must be continuous and possess a definite density and elasticity. For the transmission of light waves, a hypothetical medium known as *Ether* has been supposed to exist and pervade the atmosphere and every body in the universe.

A source of light such as the sun, the stars, a gas-flame contains innumerable material particles which vibrate to and fro, and produce waves in the medium of ether and if the properties of ether are similar in all directions the ethereal waves will spread out from the luminous body as centre in the form of spherical waves which will travel out with increasing radius in all directions with the same velocity. **The wave-front at any instant is the locus of all points in the medium** over which the waves pass and at which the particles are in the same phase of vibration. The wave-fronts become different at different instants of time.

So the disturbance created at the source is propagated onwards as the disturbance on any element causes a subsequent disturbance of the other elements. Thus we may regard the disturbance of the elements of one wave-front as the centres of displacement in the succeeding wave-fronts.

133(a). Huyghens's Principle : According to Huyghens each point or a particle in a wave-front becomes an independent source of disturbance which originates a secondary wave or wavelet. These wavelets will touch a spherical surface having its centre at the source and this spherical surface will be the new wave-front in which the particles are also in the same phase of vibration.

In Figure 80, spherical waves spread out in the medium of ether from the luminous point L as centre and let PQ represent a portion of a particular wave-front. Each particle in the wave-front PQ such as *a*, *b*, *c*, *d* etc. acts as a new centre of disturbance and generates secondary wavelets. Let all these wavelets be represented at any instant by the circular arcs in the figure. All these touch P′Q′, a portion of a big sphere with L as centre. P′Q′ is thus the new wave-front. In this way the disturbance created at the source is propagated onwards.

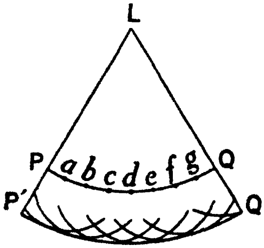


Fig. 80

Now if the distance of any wave-front from the source be very great a portion of this wave-front will be considered plane. Thus waves coming from the sun or stars and reaching the surface of the earth are plane waves. This process for the transmission of waves in any medium is based upon a theory known as the *Wave Theory of Light.*

134. Rectilinear Propagation of Light on the Wave Theory : Let us consider a plane wave-front ABCD perpendicular to OP where OP is the direction of propagation (Fig. 81) and P, the point at which we are to calculate the effect of the waves. Now all the particles of ether in the wave-front will act as centres of disturbance and produce waves which will pass through the given point P at a given instant and the resultant disturbance at P will be due to the combined effects of all these waves reaching P. The disturbances or the waves from all the particles in the wave-front travel with the same velocity but reach P in different phases since some of them are originated from particles which are more distant from O than others.

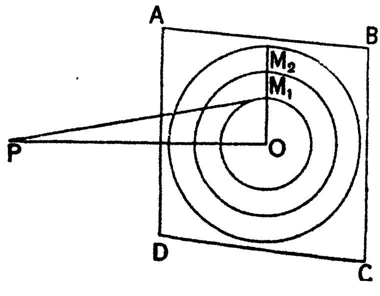


Fig. 81

From P draw PO perpendicular to the wave-front and let PO be equal to *d*. Then with P as centre and radii equal to $d+\frac{\lambda}{2}, d+\frac{2\lambda}{2}, d+\frac{3\lambda}{2}$ etc., draw spheres cutting the plane ABCD in circles M_1, M_2, etc. Join PM_1, PM_2, etc. Then since $PM_1 - PO = \frac{\lambda}{}$

(where λ is a wave-length for the transmitted waves), the waves sent out from all the particles on the circle M_1 and also from O will reach P in opposite phases and therefore they will interfere (see Interference). Again if we consider the waves coming out from all the particles included in the circle M_1 we will see that all these waves will interfere with the waves coming out from the particles in the space between the circles M_1 and M_2. These spaces are called **half-period zones.**

Now the resultant effect of all the waves sent out from all the particles in any zone to P will depend on (1) the area of the zone (2) the inclination to the wave-front or the zone of the line joining P to the zone (3) the distance of the zone from P.

Let us calculate the areas of the successive zones.

The area of the circle $M_1 = \pi OM_1{}^2$

But $OM_1{}^2 = PM_1{}^2 - OP^2 = \left(d + \frac{\lambda}{2}\right)^2 - d^2 = d\lambda$

$\therefore$ The area of the circle $M_1 = \pi OM_1{}^2 = \pi d\lambda$

The annular space between M_1 and M_2 *i.e.*, the second half period zone has area equal to the difference between the areas of the circles M_2 and M_1.

The area of the circle $M_2 = \pi OM_2{}^2$

But $OM^2{}_2 = PM_2{}^2 - OP^2 = (d+\lambda)^2 - d^2 = 2d\lambda.$

$\therefore$ The area of the second half-period zone

$$= 2\pi d\lambda - \pi d\lambda = \pi d\lambda.$$

Thus we see that the areas of all the half-period zones are equal to one another.

Thus as we proceed from O, the inclinations and the distances of two consecutive zones of equal area from P change very rapidly and as we move further away from O, the change becomes very small. So for consecutive zones of equal area situated near O the effects at P will be considerable but for other consecutive zones situated at a greater distance away from O the effects will be practically inappreciable.

Thus if a small obstacle be placed round O to cover up the first few half-period zones no effect will be produced at P and so the effect of a plane wave is confined to the first few half-period zones only in front of the point at which the action is to be considered. *This proves that light rays propagate in straight lines.* The propagation of light can however be proved to be **approximately** rectilinear since the wave-length of light is extremely short and it bends round the corner of an obstacle or in other words undergoes diffraction.

134(a). Resultant Effect at P :

Let d_1, d_2, d_3, d_4 etc. be the numerical values of the displacements at P due to wavelets from the 1st, 2nd, 3rd, 4th etc. half-period zones or elements.

Due to the inclination of the wave normal to the line joining P to the element in question, the amplitude, rather the displacement at P by any zone will gradually decrease as its distance from P increases.

Thus d_1, d_2, d_3, d_4, etc. will be in descending order of mangnitude.

Again considering the zones separately the phase of the resultant of the wavelets from the first zone differs by π from that of the resultant of the wavelets from the 2nd zone and so the displacement d_2 is taken to be opposite in sign to the displacement d_1. Similarly it can be shown that d_3, d_5, d_7, etc. are opposite in sign to d_4, d_6, d_8 etc.

Thus indicating the phase difference between the displacements at P due to odd and even elements by prefixing a minus sign to d_2, d_4, d_6 etc., the resultant displacement at P is written as

$$D = d_1 - d_2 + d_3 - d_4 + d_5 - d_6 + \dots\dots\dots$$

$$\frac{d_1}{2} + \left\{\frac{d_1 + d_3}{2} - d_2\right\} + \left\{\frac{d_3 + d_5}{2} - d_4\right\} + .$$

$$= \frac{d_1}{2}. \quad \text{Since } \frac{d_1 + d_3}{2} = d \quad \text{and} \quad \frac{d_3 + d_5}{} = d_4 \text{ approx.}$$

Thus the resultant effect at P due to the entire wave-front ABCD is equal to half the effect of the first half-period-zone.

135. Law of Reflection on the Wave Theory : Let a plane wave-front AB perpendicular to the plane of the paper be incident at the time zero, on the reflecting surface AA$'$ also perpendicular to the plane of the paper.

As the wave-front strikes in succession different points on the surface AA$'$, those points will act as centres of disturbances and will generate secondary wavelets of different radii. A will then act as a centre of disturbance and a spherical wavelet will proceed outwards round A as centre with the velocity of the waves. When the point B of the wave-front reaches A$'$ the wavelet generated at A will in the mean time diverge into a spherical wave of radius AB$'$ = AD and the wavelet from P will also reach a distance PM$'$ = PN, where D and N are the points at which the waves from A and P would have reached had there

been no reflecting surface. It is to be noted that the straight lines AD, MN and BA$'$ have been drawn perpendicular to AB.

Through A$'$ draw a plane perpendicular to the plane of the paper and touching the reflected wave diverging from A at the point B$'$. Since the velocity of the incident wave is equal to the velocity of the reflected wave, the radius of the wave from A *i.e.*, AB$'$ at the instant the light from B reaches A$'$ is equal to A$'$B and is also equal to AD. Hence the triangles AA$'$B$'$ and AA$'$B or AA$'$D are equal in all respects and so the angles AA$'$B$'$ and AA$'$D are equal to one another.

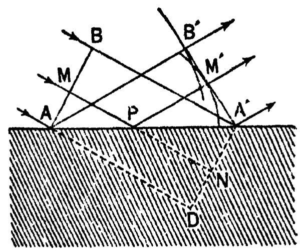


Fig. 82

Again if from P a perpendicular PM$'$ be drawn on A$'$B$'$ we have PM$'$=PN from the equality of the two right-angled triangles PA$'$M$'$ and PA$'$N and therefore the reflected wave diverging from P will touch the tangent plane A$'$B$'$ at M$'$. Similarly it can be shown that the tangent plane A$'$B$'$ will touch all the wavelets diverging from different points on AA$'$ when B reaches A$'$ and is called the reflected wave-front at the instant.

Since the triangles AA$'$B and A$'$AB$'$ are equal in all respects, the angle A$'$AB=the angle AA$'$B$'$. But the angle A$'$AB is equal to the angle between the normal to the wave-front or the incident ray and the normal to the surface AA$'$ and therefore it is the angle of incidence. Similarly AA$'$B$'$ is the angle of reflection but the angles AA$'$B and A$'$AB have been shown to be equal to one another and therefore the law stating the equality of the angles of incidence and reflection is verified.

Again the normal to the wave-front AB *i.e.*, the incident ray such as BA$'$ or MP, the corresponding reflected ray such as AB$'$ or PM$'$ being normal to the reflected wave-front, and the normal to the surface, all lie in the same plane. Thus the laws of reflection are completely verified.

136. Laws of Refraction on the Wave Theory : Let the plane wave-front AB strike the surface of separation AA$'$ of the two media (Fig. 83) at A, both the wave-front and the surface of separation being perpendicular to the plane of the paper. Since the density of the two media is not the same, the velocity of light in these two media will be different. Let V and V$'$ be respectively the velocity in the upper (rare) and the lower (denser) medium and *t* be the time taken by the point B of the wave-front to

reach the surface at the point A′. Then $BA' = Vt = AD$, since A′D would have been the position of the wave-front AB at the end of the time t, had it not been obstructed by the second medium.

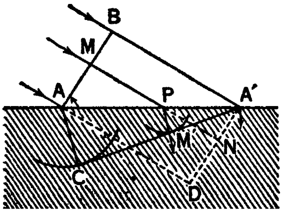


Fig. 83

As soon as the wave-front strikes the surface at A, the point A becomes a centre of disturbance and a spherical wave is generated round it and at the time t it diverges to a wave of radius $AC = V't$. Similarly at the end of time t, any point P on the surface of separation will be the centre of a refracted wave of radius PM′ such that $PN : PM' = V : V'$, where N is the point at which the wave from the point P would have reached had it not been obstructed by the surface.

Then we have $\dfrac{AC}{AD} = \dfrac{V't}{Vt} = \dfrac{PM'}{PN}$ or $\dfrac{AC}{PM'} = \dfrac{AD}{PN}$

But from similar triangles AA′D and PA′N we have

$\dfrac{AD}{PN} = \dfrac{AA'}{PA'}$ Therefore $\dfrac{AC}{PM'} = \dfrac{AD}{PN} = \dfrac{AA'}{PA'}$

Hence a plane A′C drawn* perpendicular to the plane of the paper through A′ and touching the wave diverging from A will also touch the refracted waves diverging from P or any other point on the surface. Thus the plane A′M′C is the position of the refracted wave-front at the end of the time t.

Since the triangles A′AB and AA′D are equal in all respects the angle A′AB = the angle AA′D and each of them is the angle of incidence. Similarly the angle AA′C is the angle of refraction. Let i and r denote respectively the angles of incidence and refraction. Then

$$\frac{\sin i}{\sin r} = \frac{\sin AA'D}{\sin AA'C} = \frac{\dfrac{AD}{AA'}}{\dfrac{AC}{AA'}} = \frac{AD}{AC} \cdot \frac{Vt}{V't} = \frac{V}{V'} = \mu \text{ (Constant).}$$

Thus the ratio $\dfrac{\sin i}{\sin r}$ is a constant quantity called refractive index. Again from the figure the incident and refracted rays and the normal to the surface of separation lie in the same plane.

136(a). Physical significance of the refractive index : The refractive index of one medium with respect to another medium

expresses the ratio of the corresponding velocities of light in the two media.

If the two media be represented by 'a' and 'b', the refractive index $_a\mu_b$ expresses the refractive index of the medium 'b' with respect to the medium 'a' and is equal to the ratio of the velocity of light V_a in the medium 'a' to the velocity V_b in medium 'b'. That is $_a\mu_b = \frac{V_a}{V_b}$.

137. Total Reflection by Wave Theory : Let XY be the surface of separation between the two media (Fig. 84) of refractive indices μ_1 and μ_2, and let AB be the trace of a plane wave-front moving with a velocity v_1 in the first medium and let it be refracted in the second medium with a velocity v_2.

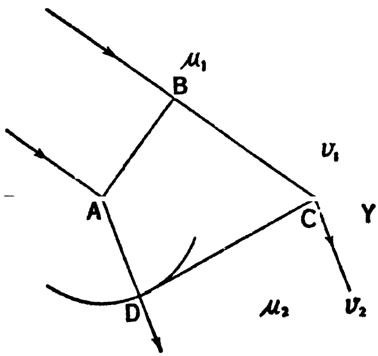


Fig. 84

If $v_1 > v_2$, the plane refracted wave-front CD can be drawn touching the secondary wavelet at D originated from A.

But if $v_1 < v_2$, *i.e.* when the wave-front AB strikes a less dense medium the radius AD of the refracted wave may be either less, equal to or greater than AC depending upon the angle of incidence. If AD be greater than AC no tangent plane can be drawn from C to the semicircle and consequently there will be no refracted wave and the light will be totally reflected.

The limiting case in which a refracted wave is formed occurs when AC = AD. In this case $\sin BAC = \frac{BC}{AC} = \frac{v_1 t}{v_2 t} = \frac{v_1}{v_2} = {_1\mu_2}$

Here the angle BAC is the critical angle.

138. Refraction through a lens : Let the spherical wave-front RML generated from the luminous point P situated on the axis, be incident on the lens at M (Fig. 85). Since the medium of the lens is optically denser than that surrounding it, the central part of the incident wave-front RML is retarded with respect to its peripheral region and the wave-front SNT on emergence becomes curved in the opposite direction with centre at Q which is the image of P.

In this case it is assumed that the surfaces of the lens are spherical and that the lens is thicker at the centre than at the edges and its aperture is small.

Since the difference of phase between two corresponding points in the two wave-fronts RML and SNT is the same for all pairs of points, the time taken by the light to travel along the path RAS (in air) is equal to the time taken by the light to travel along the path MDN (in the medium of the lens).

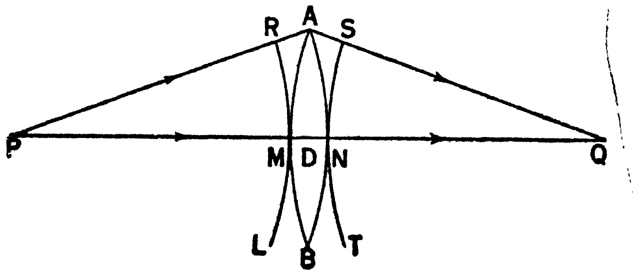


Fig. 85

We have therefore, $t = \frac{RA+AS}{v_0} = \frac{MN}{v}$

where t is the time and v_0 and v are respectively the velocities in the surrounding medium air and in the medium of the lens.

$$\therefore \quad RAS = \frac{v_0}{v}.MN = \mu MN$$

or $PR+RAS+SQ=PM+\mu MN+NQ$ since $PR=PM$ and $SQ=QN$

or $PA+AQ=PM+NQ+\mu MN=PQ-MN-\mu MN=PQ+(\mu-1)MN$

For the relation between conjugate distances consult Article 45 (Light).

QUESTIONS

1. Give a brief account of the evidence in favour of the Wave Theory of Light. [C. U. 1928].

2. Clearly explain Huyghen's principle and deduce the law of refraction from it. What is meant by the refractive index and how is it related to the velocities of light in different media? [C. U. 1935, '57]

What is the significance of refractive index according to wave theory? [C. U. 1957]

3. Explain how the phenomena of reflection and refraction of light are accounted for on the wave theory and point out the physical significance of the refractive index. [C. U. 1925, '21, '47, '55]

4. Explain the phenomenon of total reflection by Wave Theory.

5. Deduce the relation between the conjugate distances in a lens by wave theory.

6. Explain rectilinear propagation of light on the basis of wave theory of light. [C. U. 1955]

CHAPTER XIII

INTERFERENCE

139. Interference: When two systems of waves from two ources of light traverse the same part of a medium, the actual isturbance at any point of the medium is the resultant of the omponent disturbances acting independently of the other. If the aves reach the point in the same phase *i.e.*, the vibrations of he particles constituting the two systems of waves are in the ame direction the effects will be added and the resultant amplitude will be the sum of the amplitudes of the component vibrations t any instant. But if the waves are in the opposite phase *i.e.*, he vibrations are in the opposite directions, the effects will be iminished and the amplitude of the resultant vibration will be the ifference between the amplitudes of the component vibrations. The principle is the principle of **interference.** In a particular case when the amplitudes of the component vibrations are equal in magnitude but opposite in directions *i.e.*, phases are opposite, the amplitude of the resultant disturbance will be zero and the waves will destroy each other leaving the medium at rest. This is called **destructive interference.**

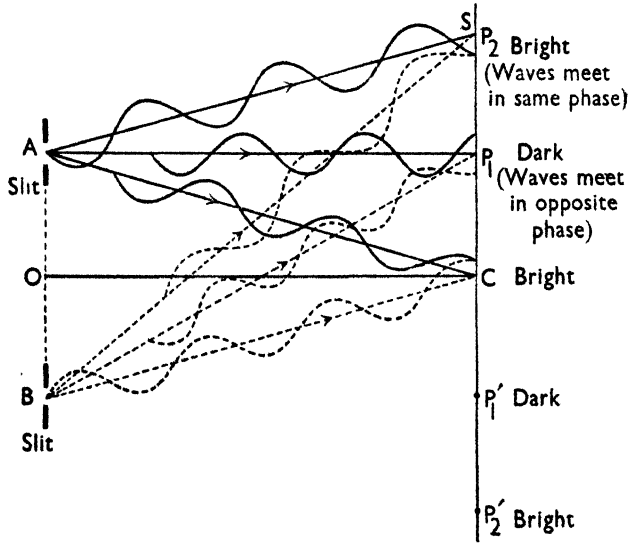


Fig. 86

We know that two sound waves may reinforce each other and produce loudness or destroy one another and produce silence

and also we have just seen that two systems of light waves unde certain conditions will reinforce each other and produce brightnes and under different conditions destroy one another and produc darkness.

The two trains of waves must be **coherent** *i.e.*, they must hav come originally from two sources which are similar in all respects.

140. Mathematical expression for Progressive Waves : W know that the simplest type of transverse wave motion is that i which the particles of the medium execute a simple harmoni motion at right angles to the direction of the propagation c the wave.

The equation for the wave travelling in the x-direction while the mediu is oscillating in the y-direction is given by

$$y=a \sin 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)=a \sin \frac{2\pi}{\lambda}\left(Vt-x\right)$$

where a is the amplitude, T the period and λ the wave-length of the motion.

Here y is the displacement of a particle at a distance x from the source (disturbance at instant t and V the velocity of the wave.

The term $\frac{2\pi}{\lambda}\left(Vt-x\right)$ is called the phase of the motion.

The difference of phase between any two points of a medium traversed b waves at distance x_1 and x_2 from the origin at the instant t is

$$\delta=\frac{2\pi}{\lambda}\left(Vt-x_1\right)-\frac{2\pi}{\lambda}\left(Vt-x_2\right)=\frac{2\pi}{\lambda}\left(x_2-x_1\right)$$

Thus *Phase difference* $=\frac{2\pi}{\lambda}\times$ *Path difference.*

141. Mathematical expression for interference : If tw waves of equal *wave-length* and amplitude, differing in phase ar superposed, the displacements of the individual waves at th instant t are given by

$$y_1=a \sin \frac{2\pi}{\lambda}\left(Vt-x_1\right),\ y_2=a \sin\frac{2\pi}{\lambda}\left(Vt-x_2\right)$$

Let these waves reach a point simultaneously with a difference of phas equal to $\frac{2\pi}{\lambda}e$ where e, the path of difference is equal to (x_2-x_1).

The resultant displacement at the instant t is given by

$$y=y_1+y_2=a \sin \frac{2\pi}{\lambda}\left(Vt-x_1\right)+a \sin \frac{2\pi}{\lambda}\left(Vt-x_2\right)$$

$$=2a \cos \frac{\pi}{\lambda}(x_2-x_1) \sin \frac{2\pi}{\lambda}\left(\frac{2Vt-(x_1+x_2)}{2}\right)$$

$$=2a \cos \frac{\pi}{\lambda} e \sin \frac{2\pi}{\lambda}\left[Vt-\frac{(x_1+x_2)}{2}\right]$$

Thus the amplitude of the resultant wave is given by $2a \cos \frac{\pi}{\lambda} e$. The amplitude will vary with e between O (when $e = \frac{\lambda}{2}, 3\frac{\lambda}{2} \ldots (2n+1)\frac{\lambda}{2}$ and $2a$ (when $e = o, \lambda, 2\lambda \ldots\ldots n\lambda$).

The **intensity** which is proportional to the **square of the amplitude**, will vary between 0 and $(2a)^2$ or $4a^2$ or 4 times the intensity due to a single wave.

142. Young's Experiment : Monochromatic light from an illuminated slit falls through two narrow parallel slits at a certain distance apart, on to a screen placed at some distance away. The diverging beams from the two parallel slits overlap and produce bright and dark bands on the screen on either side of the central bright band.

Let us consider two similar monochromatic waves of equal wave-lengths and amplitude emitted by two neighbouring sources of light. The effect of these waves at any point will be determined by the phases of the waves reaching the point under consideration. If they reach the point in the same phase, crest or trough of one falling with crest or trough of the other *i.e.*, if the paths of the waves from the two sources differ by any number of complete wave-lengths on reaching the point, the point will appear very bright, but if they meet in opposite phase, crest of one falling with trough of the other, so that the paths differ by one half wave or any odd number of half wave-length, the point will be dark. (Fig. 86).

To explain this, let A and B be two narrow parallel slits at a distance d apart (Fig. 87) from which light waves *i.e.*, of the same wave-length and amplitude radiate and let us suppose that they start from A and B in the same phase. If a screen S be placed at a distance D away in front of the sources its illumination will not be uniform but a number of alternately bright and dark bands will be seen if the light is monochromatic.

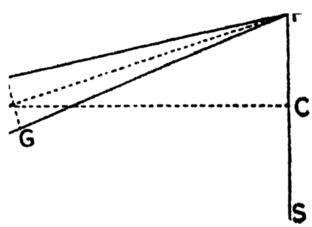


Fig. 87

Join A and B and let O be its middle point. Through O draw OC perpendicular to AB and the screen S. Since AB and BC are equal the light waves starting from A and B in the same phase will reach the point C also in the same phase and reinforce one another. Hence the point C will be very bright.

Let us now determine the nature of illumination at any poin say P situated at a distance CP from C.

Join PA, PB and PO. With P as centre and PA as radiu describe the circular arc AG. If AB is very small in compariso with PA, AG will be sensibly straight and perpendicular to both P and PO. Then BP − AP = the **distance or path retardation** (the wave from B with respect to that from A. We have therefo

for the point P to be **bright** ...$BG = 2n\frac{\lambda}{2}$

... ... **dark** ...$BG = (2n+1)\frac{\lambda}{2}$

Thus due to the interference of light originating from the tw neighbouring sources of light we have alternate bright and dark band on either side of C. These are termed **Interference Bands** or **fringe** and are equidistant from each other. It is never possible to produc interference phenomenon by two separate lamps or sources for neithe the waves sent out by one are necessarily similar to those sen out by the other nor have they any permanent phase relation. I practice we do not use two different sources of light for th production of bright and dark spaces but we generally use tw images of a single source of light as the originators of waves. I the phenomenon of interference, light (considered as energy) i never destroyed but its distribution is altered, the illumination bein compensated at one place at the expense of the other.

143. Calculation of the distance of any bright band from the central band : The two triangles BAG and POC, are similar since AG and AB are respectively perpendiculars to PO and OC and the angle BAG is equal to the angle POC.

$$\frac{PC}{OC} = \frac{BG}{AB} = \frac{BG}{AG} \qquad BG = AG.\frac{PC}{OC} = d\frac{x_n}{D}$$

where $AG = AB = d$ (approx.), $PC = x_n$ and $OC = D$

Hence $x_n = \frac{D}{d}.BG$

For the nth bright band $BG = 2n\frac{\lambda}{2}$ $\therefore \frac{D}{d}.2n\frac{\lambda}{2} = \frac{D}{d}n\lambda$

or $\lambda = \frac{x_n}{nD}$. (1)

This relation gives a method of determining the wave-length of a mono-chromatic light.

The distance between the centres of the nth and $(n+1)$th bright band *i.e.*, the width of a bright band is given by

$$\alpha = x_{n+1} - x_n = \frac{D}{d}\Big((n+1)\lambda - n\lambda\Big) = \frac{D}{d}\lambda \quad \text{or} \quad \lambda = \frac{d}{D}.\alpha \quad \dots \quad (2)$$

Since the expression for the width of a band is independent of the order of the band, then, with a mono-chromatic light the bands are **equi-spaced.** The relation (2) is also used to find λ.

144. Determination of λ, the wave-length by a Bi-prism : The bi-prism CED is an obtuse-angled prism and is made up of two acute-angled prisms placed base to base and hence the name bi-prism. (Fig. 88)

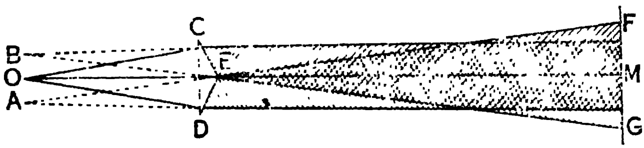


Fig. 88

A narrow illuminated slit at O is placed in the same height as the prism so that the length of the slit is parallel to the edge of the prism.

After refraction through the upper and lower halves of the prisms light will appear to come from the virtual images B and A and will produce interference.

The interference fringes are very narrow and parallel to the edge of the prism. They are focussed in the focal plane of the eye-piece placed at M and seen in a magnified form.

The cross-wire of the eye-piece is made to coincide with the central bright band and then moved by a micrometer screw to some 6th bright band and its position is read off.

In the formula, as deduced in Article 143 $x_n = \frac{D}{d} n\lambda$, where

x_n = distance of the nth band from the central band.

Suppose in actual experiment $n=6$, then x_6 is obtained from micrometer readings, and D is measured from the slit to the focal plane of the eye-piece by the scale of the optical bench on which the slit, the bi-prism and the eye-piece are fitted in uprights.

To measure d *i.e.*, the distance between the two virtual images, a convex lens L is placed between the eye-piece and the bi-prism and the distance between the slit and the eye-piece is made greater then 4 times the focal length of the lens such that for two positions of the lens, two sets of images will be observed by the eye-piece kept at the same place.

Now the distances d_1 and d_2 between the images of A and B for the two positions of the lens are measured with the micrometer screw of the eye-piece.

Then the actual distance d between A and B is given by the expression $d = \sqrt{d_1 d_2}$

Then from the expression $x = \frac{D}{d} n\lambda$ or, $\lambda = \frac{d\,x}{D.n}$

the wave-length λ of monochromatic light is calculated.

Several uprights carrying the slit, bi-prism, the eye-piece lens etc., are fitted on the optical bench so that they can slide along its strong metal bed provided with a scale. The scale serves to determine the positions of the uprights.

In the actual measurement of the wave-length the following adjustments are necessary.

(1) The slit, the bi-prism and the eye-piece should be adjusted to the same height. (2) The line joining the slit to the refracting edge of the bi-prism should be parallel to the scale along the bed of the optical bench. (3) The slit should be narrow and parallel to the edge of the prism. (4) The plane face of the bi-prism should be at right angles to the length of the bed. (5) The cross-wire of the eye-piece should be focussed and placed in the vertical position. (6) Correction for index error should be made in measurement of D, the distance between the slit and the eye-piece.

144(a). White light fringes : The distance x_n of the nth bright band from the central one is (Article 143) given by $x_n = \frac{D}{d} n\lambda$.

For central band $n = 0$, so that x_n is zero for all wave-lengths. For another bright band of definite order number, x_n is greater for light of longer wave-length and less for light of shorter wave-length. Since λ_r for red light is greater than λ_v for violet light, all other bright bands other than the central one will be coloured having violet in the innermost position and red in the outermost position.

145. Colours of Thin films : We have noticed that when ordinary white light falls on a transparent surface such as a soap

bubble or a film of oil spread on the surface of water, brilliant colours are generally observed. The colours vary with the thickness of the film and vanish altogether when the thickness exceeds a certain limit. These colours are produced by the interference of light waves from the upper and the lower surfaces of the film.

146. Retardation: Let the ray AB (Fig. 89) incident on a parallel-sided film of refractive index μ and thickness t be split up into a reflected ray BC and a refracted ray BD. The refracted ray is partly reflected at D along DE to suffer refraction at E and emerge in part from the film along EF.

Since the bounding surfaces of the plate of film are parallel, the ray BC and EF will be parallel and are relatively retarded and they will reinforce or weaken each other according as they are in the same or opposite phases.

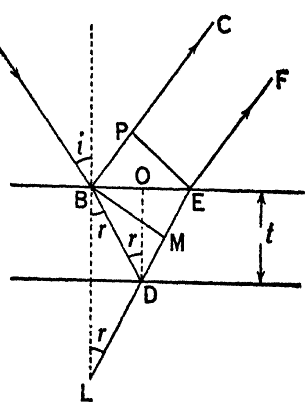


Fig. 89

Calculation of Retardation:—Produce ED to L making DL = DB and join LB.

Then the line LB will be perpendicular to the parallel faces of the film and if the thickness of the film be t then $LB = 2t$.

Draw EP and BM perpendiculars to BC and ED respectively. Then BM and EP are the successive positions of the wave-front **reflected** from the front and the rear surface.

Then the **Path Retardation** δ between the two rays BC and EF is therefore given by,

$$\delta = \mu(BD + DM + ME) - BP = \mu(LM + ME) - BP \qquad \because DB = DL$$
$$= \mu LM \quad (\because \mu.ME = BP) = \mu LB \cos r = \mu\, 2t \cos r \qquad \text{Since } \angle BLM = r$$

Here μ is the index of refraction, and r, the angle of refraction.

Since the reflections at B and D occur under opposite conditions a further difference of path equal to $\frac{\lambda}{2}$ is introduced between the interfering rays, the whole path retardation is $2\mu t \cos r \pm \frac{\lambda}{2}$ (1) when λ is the wave-length in air.

Then the condition for the rays BC and EF to **interfere** is

$2\,\mu t \cos r \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$ **(for dark bands)** or $2\mu t \cos r = n\lambda$ (2)

Similarly, the condition for the rays BC and EF to **reinforce** each other is $2\mu t \cos r \pm \frac{\lambda}{2} = n\lambda$ or $2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$ **(for bright bands)** ...(3), where n is any positive integer including *zero*.

For transmitted light the conditions for brightness and darkness will be opposite to the conditions in the case of reflected light since there is no sudden change of phase if the film is optically denser than the media above and below it.

Thus for bright bands $2\mu t \cos r = n\lambda$

For dark bands $2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$

If $\mu = 1$ and the film be viewed normally *i.e.*, $r = 0$, the conditions

For Reflected Light.

For brightness $2t = (2n \pm 1)\frac{\lambda}{2}$. For darkness $2t = n\lambda$

For Transmitted Light.

For brightness $2t = n\lambda$. For darkness $2t = (2n \pm 1)\frac{\lambda}{2}$

General Discussion : From equation (1) we see that the interference effect *i.e.*, the fluctuation in intensity in the reflected light is due to either of the *two variables* r and t.

If both of them remain *constant i.e.*, if the beam incident on the film is *parallel* and the film is thin and of *uniform thickness*, no fringes will be observed and the film will appear either wholly bright or dark. But with a parallel beam of white light the film will appear coloured.

If one of the variables, say *t is constant and r variable* as in the case of a diverging or a converging beam of monochromatic light incident on a plane-parallel film, fringes will be observed due to varying path difference in different parts of the field. These fringes are known as **curves of equal inclination.**

If again r is constant and t is variable, fringes, known as **curves of equal thickness** will be observed with a parallel beam of monochromatic light. With white light bright bands due to all colours will be observed and the film will show variety of colours.

This is due to the fact that for a particular direction of reflected light, the condition for bright bands for all colours will not be satisfied at the same time due to the difference in their wave-lengths. If this condition is satisfied for red light, it will not be satisfied for other colours. So the film will appear red. For a slightly different direction this condition may be satisfied for violet and not for red or other colours and the film will appear violet. Consequently with white light the film will show a variety of colours.

147. Newton's Rings : Newton studied the colours of thin films in a simple manner and his method is given below.

Theory : A plano-convex lens of known large radius is placed on a piece of glass plate AB so that the convex side touches the plane surface. The thickness of the film of air between the lens and the plate is not constant but increases gradually from the central point E at which the lens touches the plate. Near the point of

contact the thickness of the air film will be very small in comparison with the wave-length of light and so at the point of contact, the central ring is dark when viewed by reflected light. This happens as neutralisation is effected by the path difference $\frac{\lambda}{2}$ introduced by reflection under opposite conditions. Since the lens is a portion of a sphere, concentric circles may be drawn, each of which is the locus of a point whose distance from the plate glass is constant.

In Figure 90, E is the point of contact and R, the radius of curvature of the convex surface of the lens.

When viewed normally by reflected light, the points G and H equidistant from E will lie on a bright or dark circle according as twice the distance GA or HB is equal to an odd or even number of half wave-length of the incident light.

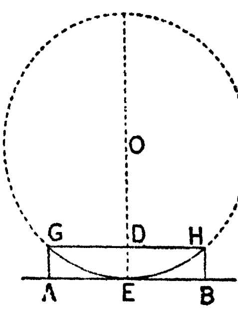


Fig. 90

Let EO be the radius of the circle of which the curved section of the lens is a part. In Figure 90 let $GA = DE = HB = t$ and let the diameter of the ring observed be equal to d.

Then $(2R - t)t = \left(\frac{d}{2}\right)$ or $t = \frac{d^2}{8R}$

For G and H to be situated on a **bright ring**

$2t \;\; \frac{d^2}{4R} = (2n+1)\frac{\lambda}{2}$. For a **dark ring** $2t = \frac{d^2}{4R} = n\lambda$

where n has the values 0, 1, 2, 3 etc. for the 1st, 2nd 3rd, 4th, etc. rings respectively.

Let d_n be the diameter of the nth bright ring and d_{n+m} is the diameter of the $(n+m)$th bright ring.

Therefore $\frac{d_n^2}{4R} = (2n+1)\frac{\lambda}{2}$; $\frac{d^2_{n+m}}{4R} = (2n+2m+1)\frac{\lambda}{2}$

or $\frac{d^2_{n+m} - d_n^2}{4R} = 2m.\frac{\lambda}{2} = m\lambda$ $\qquad \lambda = \frac{d^2_{n+m} - d_n^2}{4Rm}$

Similarly when the radii of the nth and $(n+m)$th **dark** ring are measured

$$\lambda = \frac{d^2_{n+m} - d_n^2}{4Rm} = \frac{r^2_{n+m} - r_n^2}{Rm} \qquad \ldots \qquad (1)$$

where r_n and r_{n+m} are the radii of the nth and $(n+m)$th dark rings.

The reason for measuring the diameters of two rings rather than their radii is because the central spot is rarely well-defined.

Experiment : The lens L is placed on the glass plate P and the combination is placed inside a suitable wooden-box with dark sides (Fig. 91).

A glass plate G is placed above the combination at 45° to the vertical.

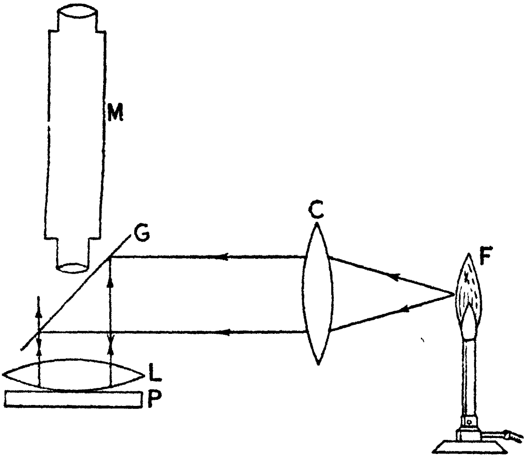


Fig. 91

A parallel beam of yellow light from a flame F is allowed to fall on the plate G by the condensing lens C and is thrown downwards on to the air film. The rings are viewed by the travelling microscope M placed above the plate G and the diameters are measured.

It is to be noted that an extended source of light is necessary for the perception of the general colour of the film.

Note: With mono-chromatic light the central black spot is surrounded by concentric circles separated by dark intervals. But when white light is used the rings will be brightly coloured. The colours of film when viewed by reflected and transmitted light will be complimentary.

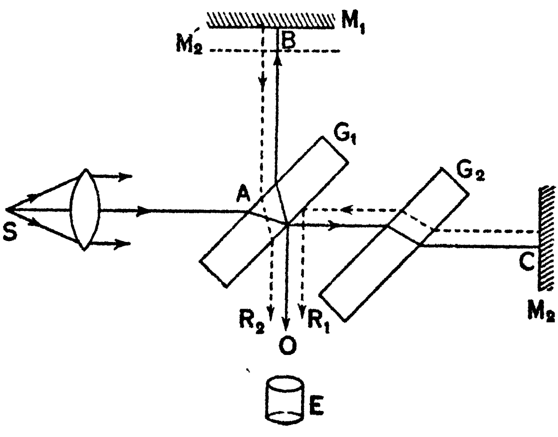


Fig. 92

148. Michelson's Interferometer—Its uses: The schematic arrangement of the interferometer is shown in Figure 92.

Two highly polished mirrors M_1 and M_2 silvered on their front surfaces are placed perpendicular to each other. The mirror M_2 is fixed and the mirror M_1 can be moved backwards along the normal to its surface by a fine micrometer screw.

A parallel-side plate of glass G_1 lightly silvered on its back is placed at an angle of 45° to the light coming out from a lens L at

whose focus an extended source of light S, such as a bright sodium flame is placed.

Part of the incident light passes straight through the plate G_1 and through another plate G_2 of equal thickness and is reflected back from the plane mirror M_2 and on reaching the plate G_1 is partly reflected along R_1. Part of the incident light instead of passing through G_1 is reflected at its back surface and travels to the mirror M_1 meeting the same normally and is reflected back and partly transmitted through G_1 along the direction R_2.

Thus the incident ray has been split up into rays which having traversed different path are finally brought back to traverse the same path.

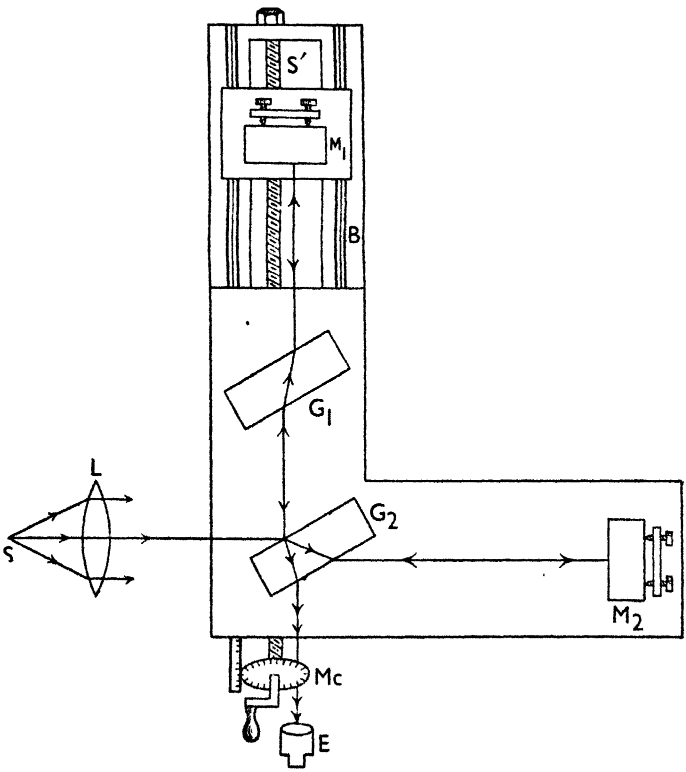


Fig. 93

In Figure 92 the reflected rays from M_2 and M_1 are shown slightly displaced to avoid confusion. In reality the rays R_1 and R_2 coincide and pass on to the eye-piece E along AO.

The object of using another glass plate G_2 of equal thickness as that of G_1 and placed parallel to it is that the two rays traverse exactly the same thickness of glass, otherwise the light reflected from the mirror M_2 would only traverse the glass plate once while the light reflected from the mirror M_1 would do so three times.

If there be any path difference between the two beams, which after traversing the two paths pass into the eye-piece, the two beams will be in a condition to interfere.

But if $AB = AC$, *i.e.* if the path difference is zero, the beams will not interfere. Again if AB is not equal to AC, the mirror M_1 and M_2 (image of M_2 in G_1 and parallel to M_1) enclose an air-film of thickness $(AB - AC)$, a system of circular fringes is observed in the eye-piece E.

If the mirrors M_1 and M_2 are not at right angles such that M_1 and M_2 intersect each other, fringes will be straight and parallel to the line of intersection. Figure 93 shows the sectional sketch of a compact interferometer of Michelson type with all essential details.

148(a). Uses of the instrument :

(1) Measurement of the wave-length of monochromatic light.
(2) Standardisation of metre in terms of wave-length.
(3) Measurement of thickness of a very thin film.
(4) Testing the smoothness of a glass surface.

149. Determination of wave-length of a monochromatic light : To determine the wave-length of a mono-chromatic light, the interferometer is adjusted so that AB and AC are *nearly* equal and that the mirror M_1 and M_2 are perpendicular to each other. Circular fringes are then observed in the eye-piece with a monochromatic light and the cross-wires are adjusted on a particular fringe.

The mirror M_1 is then moved through a distance δ parallel to itself and the number of fringes n that move across the field of view is counted. Then $\delta = n\frac{\lambda}{2}$ ... (1)

since the movement of the mirror through a distance $\frac{\lambda}{2}$ will cause the displacement of each fringe into the position previously occupied by the adjacent fringe.

Thus from the relation (1) the wave-length λ is calculated.

150. Wave-length of light as a standard of length : The standard metre which was based originally as a standard of length

is defined as the distance between the two marks in a metal bar made of an alloy of platinum and iridium kept at Paris.

The arbitrary standard is liable to changes beyond our control. Maxwell and Benoit suggested that the wave-length of a homogeneous monochromatic radiation is an invariable and easily reproducible length and ideally suited for the standardisation of a metre.

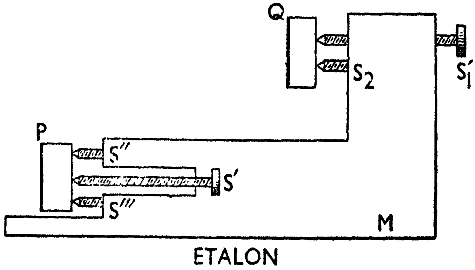


Fig. 94

Michelson modified his interferometer to a slight extent and determined how many wave-lengths corresponding to the red line of cadmium are contained in a standard metre.

For this purpose Michelson constructed a number (10) of substandards called *etalons* beginning from a decimetre and ending in a length of ·05mm, each successive substandard being half the length of the previous one. They consist of (Fig. 94) two accurately parallel plane glass mirrors P and Q silvered on the front and held in a brass holder with spring and screw arrangements to make the mirrors exactly parallel.

The following operations are to be made for the purpose.

(1) Determination of the number of light waves in the shortest etalon.

(2) Comparison of intermediate standards.

(3) Comparison of the longest (10 dm.) etalon with the standard metre.

In Michelson's modified interferometer there are two mirrors, one movable and the other fixed.

The actual procedure in the experiment is to mount on the interferometer the shortest etalon and to find accurately the number of waves between the planes of the two mirrors P and Q. This etalon was then compared with the next one of higher length mounting it by the side of the first etalon. The second one was then compared with the third etalon. The third was then compared with the fourth one until the number of wave-lengths in the largest, *i.e.*, 10 cms. etalon were determined. The 10 cms. etalon was finally compared with standard metre and the number of wave-

lengths of a definite cadmium line covered by one metre was accurately found out.

Michelson found that the standard metre contains 1553163°5 for red cadmium light at 15° C and 760 *mm*, pressure. The error is found to be within 1 in 10^6.

Note : The utility of using a number of etalons is due to the fact that a very large number of fringes would have to be counted if the movable mirror of the Interferometer was moved through a metre. This is an impossible task.

So to fecilitate counting of fringes a series of substandards called *etalons* are constructed.

QUESTIONS

1. What do you understand by the interference of light? Explain the phenomena by illustration. [C. U. 1944, '45, '52, '53]

2. Describe briefly the phenomena of interference in relation to the law of conservation of energy. [C. U. 1952]

3. What do you understand by interference and how are they formed? Describe and explain any arrangement for producing interference fringes. [C. U. 1942, '59]

4. Explain the method of finding the wave-length of mono-chromatic light by a bi-prism giving the adjustments in details. [C. U. 1939, '42, '49, '52, '57, '59]

5. Describe an account of the interference phenomenon which can be seen when a convex lens is pressed against a glass plate. Explain with a diagram how you would exhibit the phenomenon to the best advantages. [C. U. 1946]

6. Explain the colours of thin films.
   Under what conditions are interference effects observed in light? [C. U. 1951]

7. Why is the principal focus of a convex lens placed in a pencil of parallel rays, a point of maximum brightness? [C. U. 1944]

8. Give the theory of Newton's rings. [C. U. 1944]

9. Write a short note on Interferometer and its uses. [C. U. 1948]

10. Write a short note on the wave-length of light as a standard of length. [C. U. 1954, '59]

EXAMPLES

1. A bi-prism is placed at a distance of 5 cm. in front of narrow slit illuminated by a light of wave-length $5{\cdot}89\times10^{-5}$cm. The distance between the virtual images formed by the bi-prism is ·05 cm. Find the width of the fringes on a screen placed 75 cm. in front of the bi-prism.

We have $\lambda=\frac{d}{D}.x\ldots(1)$ where λ = wave-length of light

d = distance between virtual images ; D = distance between slit and screen, and x = distance between consecutive bright bands = width of a fringe.

Here $\lambda = 5{\cdot}89 \times 10^{-5}$ cm.; $d = {\cdot}05$ cm.; $D = 5 + 75 = 80$ cm.

Therefore (1) $x = \frac{D\lambda}{d} = \frac{80 \times 5{\cdot}89 \times 10^{-5}}{{\cdot}05} = 16 \times 589 \times 10^{-5}$ cm.

$= 9424 \times 10^{-5} = {\cdot}094$ cm. approximately.

2. In a Newton's ring experiment with violet light, following data are obtained :—

Radius of curvature of the lower surface = 10 metres
Radius of dth dark fringe = 4 mm.
Radius of $(n+5)$th dark fringe = 6mm.

Assuming the refractive index of air to be unity, find out the wave-length of the light used. (C. U. 1946)

From Art 147, $\lambda = \frac{r^2{}_{n+m} - r_n{}^2}{Rm} = \frac{{\cdot}36 - {\cdot}16}{10 \times 100 \times 5} = \frac{{\cdot}20}{5 \times 10^3} = 4 \times 10^{-5}\text{cm} = 4000\mathring{A}U.$

3. The diameter of the third black ring is 1 cm. When sodium light ($\lambda = 5{\cdot}86 \times 10^{-5}$ cm.) passes normally through the air film. Find the radius of the glass lens. [D. U. 1943]

Note: Angstorm unit (A. U.) $= 10^{-8}$cm., and the micron $(\mu) = 10^{-4}$ cm. are the convenient units of wave-length.

CHAPTER XIV

DIFFRACTION

151. Diffraction: By the geometrical theory of optics we have proved that light rays travel in straight lines and consequently when these rays are allowed to pass by the edge of an obstacle a distinct shadow will be formed and its boundary line will be marked by a straight line drawn from the source to the screen and grazing the edge of the obstacle.

But a careful observation shews that the shadow is not distinct and perfectly dark but the light, gradually fades away continuously and rapidly and at a short distance below the line drawn through the source and the edge of the obstacle complete darkness sets in.

To explain this phenomenon we are to consider the fact that according to the wave theory of light the propagation of light is approximately rectilinear and consequently light waves of extremely small wave-lengths exhibit a bending, though very slight, in passing by the edge of an obstacle.

This bending of light waves round a corner or the edge of an obstacle causing a rapid diminution in the intensity of light within the geometrical shadow is known as **Diffraction.**

Again if we carefully examine the boundary of the shadow we will notice that instead of uniform illumination above the boundary line of the shadow as is suggested by the Geometrical Theory of Optics we will have a series of maxima and minima giving rise to a number of brilliant fringes parallel to the edge of the obstacle. This phenomenon is observed when light passes through a narrow aperture. The bright bands, unlike interference fringes, occur at unequal intervals, the first band being the widest while the rest decrease in regular succession and ultimately merge into uniform illumination at a short distance from the boundary of the shadow.

These rhythmic variations in intensity and the encroachment of light on the geometrical shadow of an opaque obstacle constitute the phenomenon of **Diffraction** of light.

151(a). Explanation of the phenomenon of diffraction at a straight edge : To explain the above phenomenon let us determine the illumination at any point on the screen both inside and outside the geometrical shadow when light diverging from a luminous point passes by the edge of an obstacle.

Let O (Fig. 95) be a luminous point emitting spherical waves, AB an opaque obstacle perpendicular to the plane of the paper and RS, the screen also perpendicular to the plane of the paper.

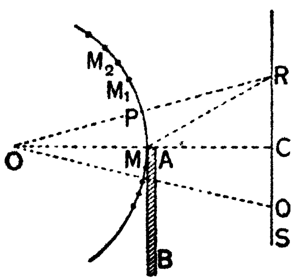


Fig. 95

According to the geometrical theory of optics, the shadow is not distinctly marked by the line OAC, but the light gradually fades away below OAC and passes through many alternate successions of brightness and darkness, forming fringes above OAC.

The intensity of illumination at the point R on the screen is due to a few half-period zones or elements on each side of P in the spherical wave M_2A.

If the point R be far removed from M, none of the effective half-period elements of the wave round P will be intercepted by the obstacle and consequently the illumination at R is not affected in any way. But if R be situated so near to M that the arc PA contains only a portion of the effective half-period elements of the lower half of the wave, the illumination at R may be considered as consisting of two portions, one due to the entire half wave-front above OR and the other due to a portion PA containing a few half-period elements.

If PA contains an even number of half-period elements they will mutually interfere in pair and should have little effect at R. But if PA contains an odd number, the effect would be much greater at R.

Thus the illumination at R is maximum or minimum according as the arc PA contains an odd number or an even number of half-period elements.

That is, for R to be bright, $AR - PR = (2n+1)\frac{\lambda}{2}$

$$\text{dark,} \quad AR - PR = 2n\frac{\lambda}{2}$$

Thus we see that instead of uniform illumination above the line OAC, light passes through many alternate successions of brightness and darkness forming fringes. Due to the unequal widths of the half-period elements, the diffraction bands are of unequal width, the width diminishing from the first band near C.

Now to consider the illumination at any point, say Q (just above S) within the geometrical shadow, let us divide the wave-surface into half-period elements with the centre of the zone at a point determined by the point of intersection of the wave-surface with the straight line joining O to the point Q below C. Now we observe that the lower half of the entire wave-surface and some of the most powerful half-period elements of the upper half are intercepted by the obstacle and that the part of the wave which propagates light to Q is only a fraction of the wave.

So the resultant effect at Q is confined to a few half-period elements near the edge A of the obstacle.

Consequently as Q goes down further into the shadow, the first element is intercepted and then the second, the third, the fourth etc., and thus due to mutual interference of elements in pair the illumination falls off continuously but rapidly within the geometrical shadow.

We know that the effect of any wave-front at any point in front of it is confined to a first few half-period elements around the pole of the point.

The total effect at the point C has been found to be equal to $\frac{d_1}{2}$, where d_1 is the displacement at C due to the wavelets from the first element.

Thus we conclude that as we proceed along CR, a number o bright bands separated by comparatively dark intervals are observe and after a time the illumination becomes uniform.

Owing to the unequal widths of the half-period elements, th bright bands on the screen occur at unequal intervals, the firs band being the widest and the rest decrease in regular succession.

If, instead of monochromatic light, white light is used, brighte bands will be coloured, blue at their inner edges, and red at the outer.

152. Diffraction by a narrow obstacle : In this case *diffraction fringes* are observed *outside* the geometrical shadow o the obstacle on each side.

Inside the shadow, *fringes* are also observed *caused by th interference* of the light waves bending into the shadow round th sides of the obstacle.

153. Distinction between Interference and Diffractio Phenomena :

(1) The phenomenon of interference is due to the interferin action of two distinct sets of waves coming from two coheren sources, whereas the phenomenon of diffraction is due to the mutua interference between the elements of a single wave-front.

(2) In diffraction, the spacing between bands decreases witl higher order bands but in interference it is in general, though no always, constant.

(3). The interference fringes may or may not be of the sam width but diffraction fringes are not of the same width.

154. There two main classes of diffraction phenomena (a) **Fresnel Class** and (b) **Fraunhofer Class.**

(*a*) **Fresnel Class of Diffraction :** In this class the sourc of light and the screen are at finite distances from the obstacle. When the source of light is at a finite distance, the wave-fronts are spherical.

In this case a very narrow slit is illuminated with a strong source of monochromatic light, the obstacle being placed in front of it and the diffraction pattern observed by the micrometer eye-piece.

The slit may be linear or circular according to the nature o the obstacle.

Under these circumstances we will be studying Fresnel class of Diffraction.

(*b*) **Fraunhofer Class of Diffraction :** In this class, th source of light and the screen are removed to infinity. If this i done, the problem of determining the distribution of light an shadow becomes much simpler.

To effect this and to study the Fraunhofer class of diffraction the narrow slit is placed at the focal plane of a convex lens so that plane waves of light can be made to fall on the obstacle. The light diffracted by the obstacle can then be focussed by means of another convex lens and then viewed by an eye-piece.

In this case the phase of vibration is the same at every point of the aperture which is not the case when the source of light is at a finite distance, the wave-front then being cylindrical.

154(a). Fresnel's Pattern :

Diffraction by a Narrow Rectangular Aperture : (1) If the slit is very narrow, there will be a broad flat maximum at the centre, the illumination remaining considerable far out into the geometrical shadow on each side. (2) If the slit be of intermediate width, there will be fringes both within and outside the geometrical shadow. (3) If the slit is wide there will be only minute variations of intensity in the middle part of the pattern.

154(b). Fraunhofer's Diffraction Pattern :

(1) Narrow Rectangular Slit : In this case, diffraction pattern consists of a central bright band which is very intense, bordered by alternate dark and bright bands, the intensity of the latter decreasing very rapidly.

(2) Two Narrow Parallel Slits : In this case, the same diffraction pattern as for a single slit is produced ; but superimposed on it is a system of *interference fringes* due to the superposition of wavelets from corresponding parts of the two slits. The separation of these fringes is determined by the distance apart of the slits.

155. Diffraction through a circular aperture : When a circular aperture is placed in front of a point source of light such as an illuminated pin-hole in a sheet of lead, the diffracted pattern on the screen at any position on the axial line consists of a central spot, dark or bright, surrounded by a number of rings. The explanation is given below. (Fig. 96)

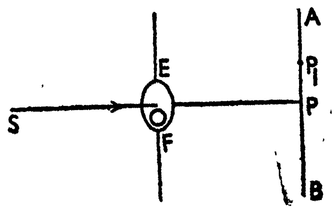


Fig. 96

The illumination at any point, say P on the screen on the axial line is obtained by dividing the wave-front into half-period elements as it diverges through the aperture.

If the aperture exposes an *even* number of half-period elements, the illumination at the point P is *minimum* but if it exposes an odd number of half-period elements, the illumination becomes *maximum*.

Since the area of a half-period element varies as the distance of the point on the axial line from the centre of the aperture the number of half-period elements exposed will be greater as the point approaches the aperture. Consequently, as the point P approaches the aperture along its axis, the intensity of illumination at it will pass through a succession of *maxima* or *minima* according as the aperture exposes *odd* or *even* number of half-period elements.

Thus the diffracted pattern on the screen at any position on the axial line will consist of a central spot, bright or dark, surrounded by a number of rings.

156. Diffraction by an opaque circular disc : When an opaque disc (Fig. 97) is placed in the path of the waves originating from a source the centre of the shadow will be as bright as if the obstacle were absent.

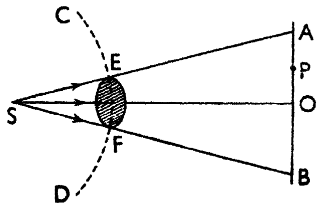


Fig. 97

This is explained by the Theory of Fresnel's half-period zones.

Let us consider a point on the axis of the disc. The resultant effect at the point will be due to the first exposed half-period zone at the edge of the disc and equal to half the amplitude of this zone, the first few half-period zones being covered by the obstacle.

When the point is far away from the disc the effect will be the same as half the amplitude of the first half-period zone which is covered by the disc.

But when the point approaches the disc, a larger number of half-period zones, are covered by the disc since the area of a half-period zone varies directly as the distance of the point from the centre of the disc and the resultant illumination at the point will gradually fall off.

If the point is not on the axis of the disc the diffracted waves meet at the point with a phase difference and due to interferance, circular fringes round a white centre will appear.

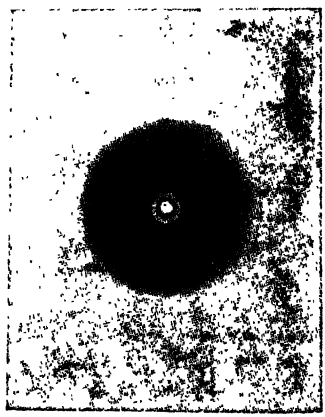


Fig. 98

Besides these, circular exterior fringes will appear near the border of the shadow.

157. Plane Diffraction Grating : It consists of number of parallel equidistant lines (Fig. 99) traced or ruled on an optically plane and parallel glass plate. The number of such ruled lines varies from 4000 to 10000 per inch. Each ruled line acts as an opaque obstacle while the transparent clear space between two consecutive ruled lines acts as a slit. The sum of the width of a clear space and that of a ruled line is called **grating element** or **grating constant**. Any two points in the consecutive clear spaces whose distance apart is equal to the grating constant are said to be corresponding points. If this grating with ruled lines and transparent spaces between them be placed between the source of light and the eye, a bright direct image is observed and on both sides of it several spectral images will be visible which will increase, in breadth and diminish in brilliancy as they recede from the central image.

These spectral images are viewd with advantage if the rays coming out of the grating be allowed to fall on the object-glass of a telescope or on a convex lens.

Theory : Let parallel rays of monochromatic light fall normally on the plane of the grating. The rays falling on the ruled lines are scattered in all directions while the rays falling on the clear spaces give rise to diffracted rays. Let A and C be one pair of corresponding points from the first and second transparent spaces of the grating. Let us consider as shown in Figure 99 a particular set of the diffracted rays diffracted from all the transparent spaces of the grating in a direction making angle θ with the normal to the grating. Draw AG perpendicular to the diffracted rays. Then light from each aperture or clear space, will travel from the line AG in the same phase and reach the point P at the same instant. But since light rays falling normally on the grating start from these apertures in the same phase, the light from the second aperture CD will be behind the light coming out of the first aperture AB and will cause interference at P. If the difference between the paths of the rays

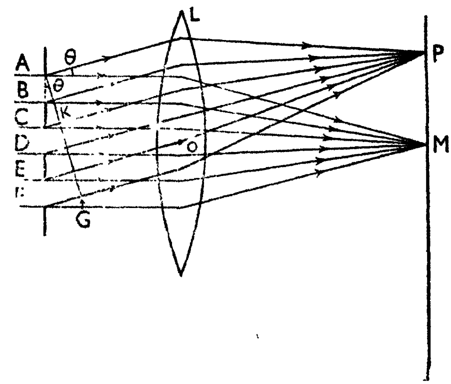


Fig. 99

from the first aperture and the corresponding rays from the second aperture be equal to CK (CK being the perpendicular distance of C from the line AG) and if CK be equal to an even

number of half wave-lengths, the rays from these two aperture will then reach P in the same phase and produce brightness The same remark will also apply to other consecutive pair c apertures. So when all the apertures are considered, the illuminatio at P will be very great. If the rays from each pair of aperture meet the point P in opposite phase, the illumination at P will b zero so that point P will be dark. As rays from all the aperture meet the central point M in the same phase, the point M will b very much bright.

Hence if CK be equal to an odd number of half wave-length the light from the first aperture will be destroyed by that fror the second and so on for the other consecutive pairs. Thus ther will be darkness at P.

Let a be the width of any aperture and b, that of each ruling $(a+b)$ gives the **grating space** or **grating constant.** The $CK=(a+b)\sin\theta$, where θ is the angle which diffracted rays mak with the normal to the grating.

Then point P will be bright, if $CK=(a+b)\sin\theta=2n\frac{\lambda}{2}$, *i.e*

$(a+b)\sin\theta=n\lambda$, and dark, if $CK=(a+b)\sin\theta=(2n+1)\frac{\lambda}{2}$, where n i any integer from zero upwards.

157(a). General Discussion : If θ gradually increases fro zero, n will have different values such as 0, ± 1, ± 2, ± 3 etc., wher 1, 2 etc. refer to order number.

When $n=0$, the corresponding value of $\sin\theta=0$ and light fro all the apertures arrives at M in the same phase and consequentl the point M will be the centre of a bright band.

So for the first bright band, when $n=1$
we have $(a+b)\sin\theta_1=\lambda$ (1)
For the second bright band when $n=2$, $(a+b)\sin\theta_2=2\lambda$
and so on.

Similarly for the first dark interval, when $n=1$, we hav

$(a+b)\sin\theta=3\frac{\lambda}{2}$ and so on for the second, third etc. order

Thus we have a succession of bright bands or regions separate by dark intervals on either side of the central bright band

If the total number of apertures or clear spaces in the gratin be even, then the rays diffracted at an angle θ from the pairs c consecutive apertures will produce brightness or darkness accordin to the conditions deduced above. If the total number be odd, then on

extreme clear space may be disregarded, since the rays from a single clear space will not produce any appreciable effect.

157(b). Formation of spectrum : With white light composed of rays of different wave-lengths, spectrum in all the bands except at the central one, will be obtained. In case of the central band n is zero, so that θ is zero for all the wave-lengths and no spectrum is therefore produced. Since θ is greater for longer wave-length, red will appear in the outermost region as λ_r is greater than λ_v. We find therefore that light of different wave-lengths are diffracted in different directions in the first and successive higher orders forming what is called a grating spectrum.

Mathematically also, as θ changes with the wave-length of light the positions for bright bands will be different for different colours when white light is used.

For different values of n, we have a series of spectra on both sides of the central image which is perfectly white, each pair being separated from the other by completely dark intervals.

We have, $\sin\theta = \dfrac{\lambda}{(a+b)}$ for the first order, the dispersion of the spectrum of the first order between blue and red rays is

$$= \sin\theta_r - \sin\theta_b = \frac{\lambda r - \lambda_b}{(a+b)}.$$

For the second order $\sin\theta'_r - \sin\theta'_b = \dfrac{2(\lambda r - \lambda_b)}{(a+b)}$, and so on for the third etc. orders.

Thus we see that the dispersion in the spectrum becomes greater and greater as the order of the spectrum is raised.

157(c). General character of illumination : The general character of illumination due to a diffraction grating is of two patterns. In the first pattern there are marked peaks called **principal maxima** and in the second pattern there is a set of much smaller maxima and minima between two consecutive principal maxima known as **secondary maxima and minima**.

Case I. If the number of lines (rulings) in the grating is very large *i.e.*, if the distance between the rulings is very small, the principal maxima are exceedingly bright and narrow when monochromatic light is used and the secondary maxima disappear entirely.

If the lines are very close so that the distance between them is less than the wave-length of light used, the grating will practically become opaque and no transmission of light is possible.

Case II. If, again, the number of lines in the grating is small *i.e.*, if the distance between them is large both the principal and secondary maxima will be observed.

158. Oblique incidence ; Minimum deviation : Let the incident parallel ray be inclined to the normal to the grating at an angle i. Then the equation for the path difference between corresponding points on successive slits becomes

$$(a+b)(\sin\theta+\sin i)$$

The condition for maximum, then, becomes

$$(a+b)(\sin\theta+\sin i)=n\lambda \quad \ldots \quad (1)$$

Then deviation D for light of wave-length λ in the nth spectrum is given by $D=i+\theta$

This will be minimum, when $dD=0$ or $di+d\theta=0$.

From equation (1) assuming λ and n constant, we get by differentiating $\cos\theta.\ d\theta+\cos i\ di=0$

If $di=-d\theta$, $\cos\theta=\cos i$ and therefore $\theta=i$.

The incident and diffracted rays must be equally inclined to the surface of the grating for obtaining minimum deviation.

This is the condition for **minimum deviation.**

The definition of a diffracted image is very much increased when the condition of minimum deviation is satisfied.

In this case the condition for the diffracted maxima *i.e.* the equation (1) becomes $2(a+b)\sin\frac{1}{2}D=n\lambda$.

159. Dispersive Power of a Grating : It is defined as the ratio of the angular interval to the corresponding variation of the wave-length. Let θ be the angle of diffraction for light rays of wave-length λ corresponding to nth order of bright band, then we have $(a+b)\sin\theta=n\lambda$ ...(1)

$$\text{or} \quad \sin\theta=\frac{n\lambda}{a+b} \quad \ldots \quad \ldots \quad \ldots(2)$$

If $\theta+d\theta$ be angle of diffraction in the nth order bright band corresponding to very neighbouring rays of wave-length $\lambda+d\lambda$, then dispersive power is expressed by $\frac{d\theta}{d\lambda}$; then differentiating (2)

$$\cos\theta.\ \frac{d\theta}{d\lambda}=\frac{n}{a+b}$$

or $\frac{d\theta}{d\lambda}=\frac{n}{(a+b)\cos\theta}=\frac{Nn}{\cos\theta}$, where $N=\frac{1}{a+b}$ where N is the number of rulings per cm.

The above expression shews that the dispersive power increases with the order of the spectrum and is inversely proportional to the width $(a+b)$ or directly to the number of rulings of the grating. Hence the finer the ruling of the grating, the higher is the dispersive power.

So doubling the number of lines to the inch means that the spectra produced are exactly doubled in length and the dispersion in each part of the spectrum is doubled.

In the grating spectrum the red end is most deviated while a prism refracts the violet end most.

The grating spectrum is called the **normal spectrum.**

Most gratings have about 14,000 lines to the inch. in which case d is $\frac{1}{14000}$ inch or about $1{\cdot}81 \times 10^{-4}$ cms.

If the wave-length λ for sodium light is $5{\cdot}8 \times 10^{-5}$ cms. the equation for the maximum becomes sin $\theta = 3897n$.

160. Experimental Measurement of Wave-length with Diffraction Grating: To determine the wave-length of any monochromatic light the grating is placed on the table of a spectrometer with its surface perpendicular to the rays of light coming out of the collimator adjusted previously for parallel rays and the rulings on it parallel to the slit in the collimator. The telescope which is also focussed for parallel rays is turned to view the bright bands on either side of the central band corresponding to the direct light and half the difference between the readings obtained gives the value of θ in the expression $(a+b)$ sin $\theta = n\lambda$.

If $a+b=d$ *i.e.* if the combined width of a space and a line be equal to d then, d $\sin\theta = n\lambda$

That is, in the above expression θ is obtained and if d, the distance apart of the lines is measured from centre to centre, λ may be determined.

The exp. d $\sin\theta = n\lambda$ may be written as $\lambda = \dfrac{\sin\theta}{n\text{N}}$, where N is the number of lines per centimetre in the grating. If n, N and θ be known λ is calculated. The value of λ for sodium light is 5890×10^{-8} cm. [Description and theory are given in Art. 157]

161. Grating Spectrum and Prism Spectrum:

(1) The grating spectrum is pure but this is not generally the case with the spectrum formed by a prism but with proper arrangement the refraction spectrum may be made pure.

(2) In a grating spectrum the relative dispersion of any two colours is invariable since the dispersion of colours depends on the

wave-length and on the distance $(a+b)$ *i.e.*, on the number of lines per centimeter in the grating. For this reason the spectrum formed by any grating is exactly similar to any other and it is called a normal spectrum.

(3) In the spectra formed by different prisms the relative dispersion of any two colours is different and in some cases even the order of the colours is altered. So the spectra produced by different prisms are not exactly similar to one another.

(4) As regards brightness, diffraction spectra are far inferior to those obtained by prisms. In the case of grating the whole energy of the incident beam is spread over a number of spectra of different orders, while in the case of prism it is distributed over a single spectrum.

(5) Resolving power of a prism is much less than that of a grating.

162. Resolving Power of Optical Instruments : The term is used in connection with (1) **Spectroscopic apparatus** *e.g.*, prisms, grating etc. (2) **lens system** *e.g.*, telescopes and microscopes.

For the first of these, the resolving power is a measure of the power of the apparatus to separate two neighbouring lines in the spectrum while for the second, it is the power of an optical instrument by which the overlapping images of two objects are rendered smaller so that each is distinct from the other.

We know that the propagation of waves is associated with interference. So when a divergent wave passes through a lens it becomes convergent due to the fact that the lens retards the central portion of the incident wave more than the peripheral portion. The convergent wave thus produced is propagated by reinforcement and interference.

The focus is the small space within which all of the secondary wavelets rainforce each other. This space will always possess certain magnitude so that the optical image of a geometrical point will never itself be a point but will possess certain dimension.

It can be proved mathematically that the *diameter of the image is inversely proportional to the aperture of the lens.* So a lens with a large aperture will form an image with a very small diameter.

Thus two very distinct stars or objects though considered as mere points when viewed with naked eyes will possess certain dimension when looked through the telescope and may overlap.

But by using a telescope of large aperture the image may be made smaller so that each is distinct from the other.

The resolving power of the instrument is considerably diminished if a stop is used to allow light to pass through the central portion of the lens for diminishing

the spherical aberration, so Rayleigh used a stop which allowed light to pass only through the peripheral portion of the lens.

By this means, spherical aberration is diminished without any loss of resolving power.

According to Airy, the resolving power of a telescope is given by $\frac{1{\cdot}22 f\lambda}{D}$ where f is the focal length and D, the diameter of the lens. The resolving power is said to be high when this expression is small.

It can also be proved mathematically that in a microscope of a high resolving power, the objective must have a wide aperture and the distance between the nearest points which can be resolved is proportional to the wave-length of light used. So it is impossible to see any object which is very much smaller than the wave-length of light; in particular we cannot hope to see atoms or molecules.

163. Resolving Power of a Telescope: We know that when the image of a distant object is formed by the objective of a telescope, each point of the object will give rise to a diffraction pattern, the brightest central image being surrounded by alternate maxima and minima on both sides.

The instrument is said to have resolved the two points when the first maximum of each falls on the first minimum of its neighbour.

Let AB be the aperture (Fig. 100) of the objective of the telescope of diameter D and let PF be its focal plane and F be its focus on which the brightest central image is formed.

Let P be the point on the first minimum of the diffraction pattern on one side of F and let PF $= x$ and OF $= f$, the focal length of the objective.

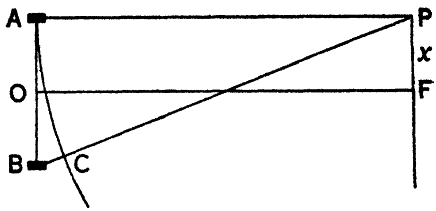


Fig. 100

With centre P and radius equal to PA describe an arc cutting PB in C. Thus BC is the path difference of light coming from A and B to the point P. So for P to be on the first minimum, BC must contain an ***even number*** of half-period elements for P and therefore BC $= \lambda$

Now $\frac{PF}{OF} = \frac{BC}{AB}$ or $\frac{\lambda}{D}$ $\therefore$ $x = \frac{f\lambda}{D}$

Thus two points on the image at a distance $\frac{f\lambda}{D}$ will be just distinguishable as separate.

The quantity $\frac{f\lambda}{D}$ is the **resolving power** of the telescope.

The smaller the quantity, the higher will be the resolving power.

According to Airy the resolving power is given by $1{\cdot}22\,\dfrac{f\lambda}{D}$.

The angular limit of resolution is $\dfrac{x}{f}$ or $\dfrac{\lambda}{D}$.

164. Resolving Power of a Grating : If the light incident on a grating consists of two wave-lengths λ and $\lambda+d\lambda$ very nearly equal in magnitude, two separate and distinct lines in the spectrum can not be obtained unless the first maximum of each falls on the first minimum of its neighbour. If the grating produces two distinct lines in the *n*th spectrum corresponding to these waves, it is said to have resolved the light of mean wave-length λ.

The ratio $\dfrac{\lambda}{d\lambda}$ is called the **Resolving Power** of the grating.

In a grating, the resolving power is expressed by $\dfrac{\lambda}{d\lambda}=\mathrm{N}n$, where N is the total number of lines in the grating per cm. and n the order of the spectrum.

We know that $n\lambda=(a+b)\sin\theta$ $\quad \dfrac{d\lambda}{d\theta}=\dfrac{a+b}{n}\cos\theta$

Path retardation D between the extreme ray and the central ray is given by $\mathrm{D}=(a+b)\sin\theta\dfrac{\mathrm{N}}{2}$, where N is the total number of rulings and spaces. Thus $d\mathrm{D}=(a+b)\cos\theta.\ d\theta.\ \dfrac{\mathrm{N}}{2}=nd\lambda.\dfrac{\mathrm{N}}{2}$

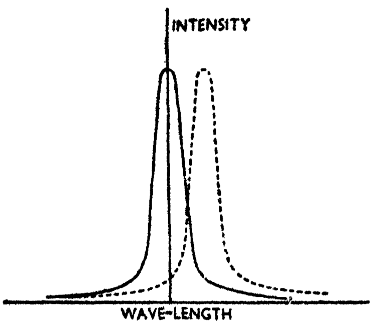


Fig. 101

The lines are said to be resolved when the maximum intensity of the line corresponding to the wave-length $\lambda+d\lambda$ begins at the point where the maximum brightness of the line corresponding to the wave-length λ ceases and this happens when the path retardation

$$d\mathrm{D}=\frac{\lambda}{2}.$$

The intensity-wave-length curve in Figure 101 explains this fact.

$\therefore$ **Thus we have** $d\mathrm{D}=\dfrac{\lambda}{2}=nd\lambda\,\dfrac{\mathrm{N}}{2}$ $\therefore$ the **resolving power** $\dfrac{\lambda}{d\lambda}=\mathrm{N}n$.

165. Resolving Power of a Prism Spectroscope : If light of two neighbouring wave-lengths λ and $\lambda + d\lambda$ be incident on the objective of the telescope the resolving power of the spectroscope is expressed by $\frac{\lambda}{d\lambda}$, where $d\lambda$ is the difference in the wave-lengths.

If the maximum of the second diffraction pattern falls on the first minimum of the first diffraction pattern, the objective will resolve the two wave-lengths as distinct if they subtend an angle at it equal to $d\theta = \frac{\lambda}{\mathrm{D}} = \frac{x}{f}$ So $\frac{}{d\lambda}$ $\mathrm{D}\frac{d\theta}{d\lambda}$

In the case of prism spectroscope observations are made when the telescope receives light refracted through the prism at an angle of minimum deviation θ.

Let A be the angle of the prism ABC and t, the path-difference BC between the extreme ray within the glass and the ray striking the edge A of the prism (Fig. 102).

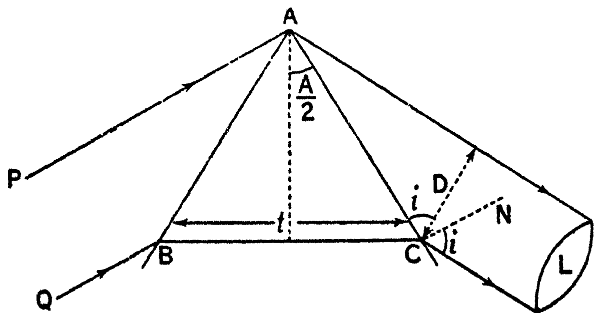


Fig. 102

When the ray undergoes minimum deviation, it cuts the prism symmetrically such that the angle of incidence is equal to the angle of emergence.

In this case the angle of incidence $i = \frac{\theta + \mathrm{A}}{2}$,

where θ is the angle of minimum deviation and A, the angle of the prism.

In Figure 102, D is the aperture of the objective and is equal to the width of the incident beam.

From Figure 102 $AC \sin \frac{A}{2} = \frac{t}{2}$, or $AC = \dfrac{\frac{t}{2}}{\sin A}$

Again, $AC = \dfrac{D}{\cos i} = \dfrac{D}{\cos\left(\frac{\theta + A}{2}\right)}$

$$\frac{\frac{t}{2}}{\sin \frac{A}{2}} = \frac{D}{\cos\left(\frac{\theta + A}{2}\right)}, \text{ or } D = \frac{t}{2} \cdot \frac{\cos\left(\frac{\theta + A}{2}\right)}{\cdot} \quad \ldots(1)$$

But refractive index, $\mu = \dfrac{\sin\left(\frac{\theta + A}{2}\right)}{\sin \frac{A}{2}}$ for glass

$$\therefore \qquad \frac{d\mu}{d\theta} = \frac{\cos\left(\frac{\theta + A}{2}\right)}{2 \sin \frac{A}{2}} \qquad \therefore \text{ from (1) } D = t\frac{d\mu}{d\theta}$$

$$\therefore \qquad \text{the resolving power } \frac{\lambda}{d\lambda} = D\frac{d\theta}{d\lambda} = t\frac{d\mu}{d\theta} \cdot \frac{d\theta}{d\lambda} = t\frac{d\mu}{d\lambda} \cdot \qquad \ldots \quad (2)$$

To determine the resolving power, two separate experiments are to be performed, one for measuring t and the other for measuring $\frac{d\mu}{d\lambda}$.

To measure t, the minimum deviation θ for the prism is first determined for the wave-length λ and from the knowledge of D, the aperture of the lens, t is calculated from Eqn. (1)

To measure $\frac{d\mu}{d\lambda}$, a $(\mu - \lambda)$ curve is plotted from the knowledge of the wave-lengths of different colours and the corresponding values of μ with the help of a spectrometer and a discharge tube.

From the curve, $\frac{d\mu}{d\lambda}$ is calculated for the wave-length λ.

Thus with the help of equation (2) the resolving power of the prism spectroscope is obtained.

165(a). Dispersion in a Prism Spectrograph : When rays of two different colours *i.e.* of different wave-lengths undergo different amounts of deviation, the quantity $\frac{dD}{d\lambda}$ is called dispersion where D is the minimum deviation and λ the wave-length of a particular colour. Now $\frac{dD}{d\lambda}=\frac{dD}{d\mu}\cdot\frac{d\mu}{d\lambda}$, where μ is the refractive index of the colour considered. For a ray suffering minimum deviation through a prism, we have

$$\mu=\frac{\operatorname{Sin}\frac{(D+A)}{2}}{\operatorname{Sin}\frac{A}{2}}\text{, where A is the angle of the prism}$$

$$\therefore\quad \frac{d\mu}{dD}=\tfrac{1}{2}\cdot\frac{\operatorname{Cos}\frac{(D+A)}{2}}{\operatorname{Sin}\frac{A}{2}}\ ;\ \text{or}\ \frac{dD}{d\mu}=\frac{2\operatorname{Sin}\frac{A}{2}}{\operatorname{Cos}\frac{(D+A)}{2}}=\frac{2\operatorname{Sin}\frac{A}{2}}{\operatorname{Cos} i}$$

where i the angle of incidence $=(D+A)/2$

$$\text{Hence}\ \frac{dD}{d\mu}=\frac{2\operatorname{Sin}\frac{A}{2}}{\sqrt{1-\operatorname{Sin}^2 i}}=\frac{2\operatorname{Sin}\frac{A}{2}}{\sqrt{1-\mu^2\operatorname{Sin}^2\frac{A}{2}}}$$

Thus $\frac{dD}{d\mu}$ is obtained from the knowledge of μ and A.

So for dispersion which depends on $\frac{dD}{d\mu}$ *i.e.* on A and μ, it is necessary that the angle of the prism should be properly altered to get the desired separation of colours.

166. Zone Plate : It is a practical application of the theory of Fresnel's half-period zones.

We have noticed before (Art. 134(a)) that if a plane or spherical wave-front be divided into a number of half-period zones the resultant effect at a point on the normal to the wave-front is equal to half the amplitude or rather the displacement of the first half-period zone. This is due to the fact that the consecutive zones of equal area send waves to the point in opposite phases.

If the alternate zones, say the 2nd., 4th., 6th. etc. are blackned or covered with some opaque substance the remaining zones 1, 3, 5, 7 etc. send waves to the point in the same phase. The result is just as if the source and the point were the conjugate foci for a convex lens.

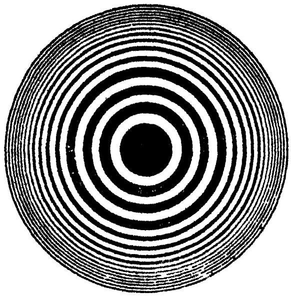


Fig. 103

To construct a zone plate a sketch is made on a sheet of paper by drawing on it a number of concentric circles with radii proportional to 1, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$ etc. for we know that, $r_n{}^2 = nd\lambda$ where r_n is the radius of the nth. circle, d the distance of the point from the pole of the wave-front and λ the wave-length of the light.

The alternate circles (rings) are blackned and photographed on a glass-plate. The plate thus prepared is called a **zone plate** and acts as a converging lens.

167. Mathematical Study of Zone Plate: Its similarity with a Convex Lens: Suppose O is a monochromatic point source of light and PQ a transparent plane sheet perpendicular to the plane of the paper. Draw OP perpendicular to PQ and produce it to I. It is required to compute the illumination at I due to secondary wavelets produced on PQ under the action of spherical waves emanating from O.

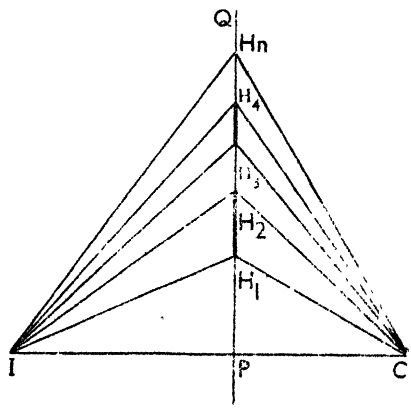


Fig. 104

Let OP $= u$ and PI $= v$. Let us consider points H_1 H_2, H_3, ... H_n on the plane PQ such that

$$OH_1 + H_1I = OP + PI + \frac{\lambda}{2}$$

$$OH_2 + H_2I = OP + PI + 2\frac{\lambda}{2}$$

$$OH_n + H_nI = OP + PI + \frac{n\lambda}{2} = u + v + \frac{n\lambda}{2} \quad (1)$$

The area of the circle of radius PH_1 on the plane is called the first half-period zone. The area of the annular space between the circles of radii PH_2 and PH_1 is second half-period zone and so on. Suppose $PH_n = r_n$; Then we have $OH_n^2 = u^2 + r_n^2$.

$$\text{or} \quad OH_n = (u^2 + r_n^2)^{\frac{1}{2}} \quad u\left(1 + \frac{r_n^2}{u^2}\right)^{\frac{1}{2}} = u\left(1 + \frac{r_n^2}{2u^2}\right) \text{ approx.}$$

$$\text{Similarly} \quad IH_n = v\left(1 + \frac{r_n^2}{2v^2}\right)$$

Putting these in (1)

$$u\left(1 + \frac{r_n^2}{2u^2}\right) + v\left(1 + \frac{r_n^2}{2v^2}\right) = u + v + \frac{n\lambda}{2}$$

$$\text{or} \quad \frac{r_n^2}{2}\left(\frac{1}{u} + \frac{1}{v}\right) = \frac{n\lambda}{2}$$

$$r_n^2 = \frac{uv}{u+v} . n\lambda \qquad \ldots(2)$$

Taking $n = 1, 2, 3$ etc. in the relation (2) the radii r_1, r_2, r_3 etc. of the 1st, 2nd, 3rd zones etc. are obtained and they are found to be proportional to the square roots of natural numbers.

$$\text{Area of the } n\text{th zone} = \pi r_n^2 - \pi r_{n-1}^2 = \pi\left(\frac{uv}{u+v} n\lambda - \frac{uv}{u+v}(n-1)\lambda\right)$$

$$= \frac{\pi uv}{u+v} . \lambda$$

Thus for given values of u and v, the areas of all the zones are the same.

The displacements $d_1, d_2, d_3 \ldots$ at I due to 1st, 2nd, etc. zones decrease only slightly with increase of the order of the zones. But as the displacements from the alternate zones have opposite phases, the resultant displacement d is expressed by

$d = d_1 - d_2 + d_3 - d_4 + \ldots$. Hence the wave-lets from the 2nd, 4th etc. zones have been intercepted and the resultant displacement becomes $d = d_1 + d_3 + d_5 + \ldots$

This is much greater than that due to the effects of all the zones, and I will be a point of maximum illumination and we can regard that light from O will be focussed at I. Thus the zone plate acts like as lens of focal length "f" given by

$$\frac{1}{v} + \frac{1}{u} = \frac{n\lambda}{r_n^2} \text{ from (2)}$$

$$= \frac{1}{f} \qquad \therefore f = \frac{r_n^2}{n\lambda}.$$

168. Distinction between a zone plate and a converging lens : (1) A converging lens gives only one image for a particular position of the object but a zone plate gives a series of images of which the nearest one is the brightest.

(2) With white light the lens converges the most refrangible violet to a point nearest the lens and the least refrangible red to a point furthest away from the lens. But with white light the zone plate converges the least refrangible red to the nearest point.

(3) In a lens all the waves converge to the focus after refraction in the same time and in the same phase. Whereas with a zone plate the consecutive waves converge to the focus in the same phase but a complete period later.

QUESTIONS

1. Explain the phenomenon of diffraction with illustrations. What are the two different classes of diffraction phenomenon. Explain them.

2. Explain the essential difference between interference and diffraction, illustrating your answer by diagrams. [C. U. 1943, '45, '52, '53]

3. What is a diffraction grating and what is meant by grating constant. Shew how the wave-length of light can be measured with a grating. [C. U. 1939, '45, '48, '50, '54, '58]

4. What do you understand by the spectrum of the second order ? [C. U. 1945]

5. What is a zone-plate ? Compare its function with that of a lens. [C. U. 1945]

6. The photograph of the shadow of a small circular disc formed by a point source of light is enlarged. What do you expect to find and why ? [C. U. 1943]

7. What do you understand by the resolving power of an instrument ? Deduce expressions for the resolving power of (*a*) Prism spectroscope (*b*) Diffraction Grating. [C. U. 1952, '57, '59]

8. Explain the action of a diffraction grating. [C. U. 1956]
What would be the effects, if the distance between the rulings were (*a*) very large (*b*) very small compared with the wave-length. [C. U. 1950]

9. Find an expression for the angular dispersion of a plane transmission grating. [C. U. 1958]

EXAMPLES

1. A parallel beam of light is allowed to be incident normally on a grating having 4250 lines per cm. and the second order spectral lines are observed to be deviated through 30°. Calculate the wave-length of sodium light. [C. U. 1937]

We have $\lambda = \frac{\sin\theta}{nN}$, where λ is the wave length of light, n the order of spectral lines, N the number of lines per cm. and θ the deflection. Here $n=2$, $N=4250$ and $\theta=30°$.

$$\therefore\ \lambda = \frac{\sin 30°}{2\times 4250} = \frac{1}{2\times 2\times 4250} = 5882{\cdot}35\times 10^{-8} \text{ cm.}$$

2. How many lines per centimeter there are in a grating which gives a deflection of 30° in the first order for light of wave-length 6×10^{-5} cm. ?

Ans. 8333·3 lines per cm. [C. U. 1946]

3. Calculate the wave-length of light which gives a deflection of 30° for the third order spectral lines with a grating having 3000 lines per cm. [C. U. 1956]

We know that $\lambda = \frac{\sin\theta}{nN}$, where λ is the wave length of light, n, the order of the spectral lines, N, the number of lines on the grating per cm. and θ the deflection.

Here $\theta = 30°$, $n = 3$ and $N = 3000$

$$\lambda = \frac{\sin 30}{3 \times 3000} = \frac{1}{2 \times 3 \times 3003} = \frac{1}{18000} = 5555{\cdot}5 \times 10^{-8} \text{ cm.}$$

$= 5555{\cdot}5$A
[A = Angstrom unit
$= 10^{-8}$ cm.]

CHAPTER XV

POLARISATION AND DOUBLE REFRACTION

169. Light vibrations are transverse: We know that the vibrations of air particles which constitute sound waves are longitudinal, *i.e.*, the particles of air vibrate in the direction in which sound is being propagated. We also know that a solid rod or a

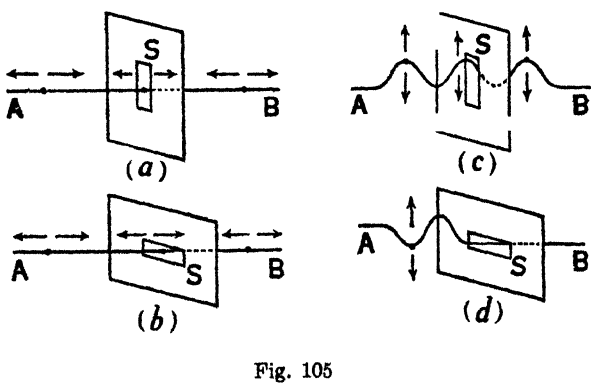


Fig. 105

cord can be vibrated longitudinally or transversely. When it vibrates longitudinally the vibration of the cord is the same on all sides of it. Again when the cord vibrates transversely *i.e.*, when the particles vibrate in a direction perpendicular to its length it looks like a flat ribbon and vibrates in a definite plane and is said to have acquired *sides* or to be **polarised.**

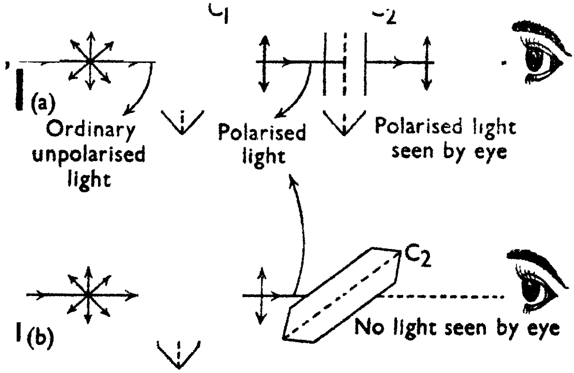
Now if the cord be made to pass freely through a narrow vertical slit the longitudinal vibration of the cord will not be affected

but pass through the slit unmodified. But if the chord vibrates transversely, so long as the vibrations are in a direction parallel to the length of the slit they will not be affected but when the slit is rotated so that the slit is at right angles to the plane of vibration, the vibration will be damped and the cord will not freely oscillate.

Longitudinal vibrations can be maintained in both positions of the slit (Fig. 105 *a* and *b*). Transverse vibration is possible in position (*c*), (Fig. 105) of the slit but it will stop when the slit is in position (*d*) (Fig. 105).

169(a). Experiment with tourmaline crystals: The same remark may be applied to the case when light is transmitted through two tourmaline crystals in succession and will thereby help us to investigate the nature of ether vibrations constituting light.

If a beam of ordinary light be allowed to fall perpendicularly on a plate of tourmaline crystal C_1 cut parallel to its axis the transmitted



(a) Axes of crystals parallel ; (b) Crystals Crossed.

Fig. 106

light is not diminished in intensity but appears a bit coloured due to the nature of the crystal but when the transmitted light is again allowed to pass through another similar plate of tourmaline C_2, the light will come out undiminished in intensity when the axes of the crystals are parallel to one another. But as the second plate rotates the light gradually diminishes in intensity and when the axes of the crystals are at right angles to one another i.e, the tourmalines are crossed, the light is completely cut off in just the same way as the transverse vibration of the cord is stopped when the slit is rotated rounde it

and placed at right angles to the plane of vibration of the cord. If the rotation of the second plate be continued, the light will reappear again when its axis is parallel to that of the first plate and vanish when the axes of the plates are at right angles. It is then clear that the light after transmission through the first plate has acquired sides or been polarised or have vibrations transverse to its direction of propagation.

The first crystal which polarises the ordinary light is called the **polariser** and the second crystal which detects whether the beam incident on it is polarised or not is called an **analyser.**

170. Polarisation : Ordinary light consists of transverse vibrations of ether particles and since there is no diminution in its intensity when a plate of tourmaline is rotated round the original beam the vibrations must take place in all planes at right angles to the direction of the beam. In the adjoining figure a ray of light is incident at O and made to pass through a tourmaline plate AB cut parallel to the axis of the crystal and after its passage through the plate the vibrations are confined to a single plane and the beam is said to be **plane-polarised** and the plane EFGH drawn through the ray perpendicular to the directions of vibrations of the ether particles is called the **plane of polarisation**. Thus a ray of light is said to be polarised when the transverse vibrations of ether particles take place parallel to some definite direction in a plane known as the **plane of vibration**. Here ABCD is the plane of vibration.

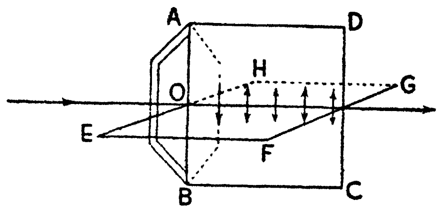


Fig. 107

The facts described in Article 169(*a*) and in Article 170 will be clear from Figures 106 and 107.

171. Plane of polarisation : It is a plane drawn through the direction of the ray and perpendicular to the direction in which the ether particles vibrate.

172. Polarisation by Reflection : When a beam of light is reflected from a mirror or a polished surface and then transmitted through a plate of tourmaline it will be found that on rotating the plate round the beam the reflected light varies in intensity and at a particular position of the plate the light is entirely cut off. The same phenomenon may be observed if the beam of light be first transmitted by a plate of tourmaline and then reflected from a

mirror. The intensity of the reflected light is seen to vary as the mirror is rotated round the beam transmitted by the tourmaline. Thus ordinary light may be polarised by reflection as observed by analysing the reflected beam by a plate of tourmaline. The degree of polarisation in a reflected beam depends on the angle of incidence. For a particular angle, the reflected beam is completely polarised and for all other angles of incidence the beam is partly polarised in the plane of incidence. The angle of incidence for which the reflected beam is completely plane-polarised is called the **polarising angle**.

We must not necessarily infer that for every substance there is an angle of complete polarisation.

It has been found by experiment that as the angle of incidence increases from O to $\pi/2$, the polarisation increases at first, then reaches a maximum value and finally decreases.

Thus for each substance there is an angle of incidence which gives **maximum polarisation** and this angle is called the polarising angle. For glass the polarising angle is about $57\frac{1}{2}°$ and for pure water it is about 53°.

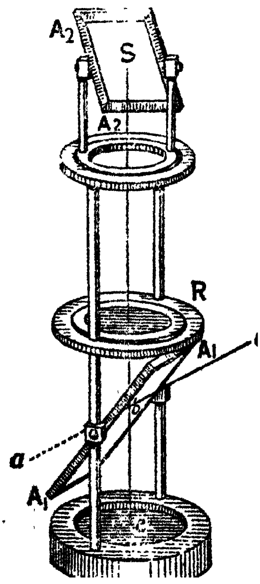


Fig. 10

Mr. Jamin has found that only a few substances of refractive index of about 1·46 polarise light completely by reflection.

Experimental Verification : The above phenomenon of polarisation by reflection can be demonstrated by an instrument known as **Biot's polariscope.** A plate of glass A_1 is supported (Fig. 108) by two vertical uprights in such a way that it can rotate about a hyrizontal axis and thereby vary the angle of incidence of a beam of light falling upon it. The light reflected from the plate of glass, called the polariser is examined through a tourmaline crystal fitted inside a ring and placed above the glass plate. The ring containing the crystal can be rotated about a vertical axis. The tourmaline crystal through which the light is examined is called the analyser.

Since a glass plate at the polarising angle acts as a polariser, a second sheet of glass A_2 may be used as a substitute for the tourmaline to analyse the reflected light, inclined to the horizon at the

same angle as the first plate. The second plate supported on two uprights is placed above the first plate and can rotate about a horizontal axis. The uprights are fixed to an annulus of metal which can rotate about a vertical axis over a circular scale graduated in degrees. When a ray is directed on to A_1 along a_1b at the polarising angle the reflected ray travels vertically along bS and falls on A_2. As A_2 is rotated through a certain angle, no light is reflected from A_2. The normals to A_1 and A_2 are now parallel. Then from geometry of the apparatus the polarising angle can be found out.

173. Brewster's Law : Sir David Brewster from a series of experiments on the angle of polarisation for different substances found that *the tangent of the angle of polarisation* i.e. the polarising angle, *is numerically equal to the index of refraction of the reflecting medium.*

That is, $\tan p = \mu$ where p is the angle of polarisation and μ, the refractive index.

It can be shown that the reflected and refracted rays are at right angles. Let an oblique ray PO incident on a transparent surface AB of a medium of refractive index μ be partly reflected and partly refracted along OR and OQ respectively. If the angle of incidence i be equal to the polarising angle p for the medium, then by Brewster's Law $\tan p = \mu$.

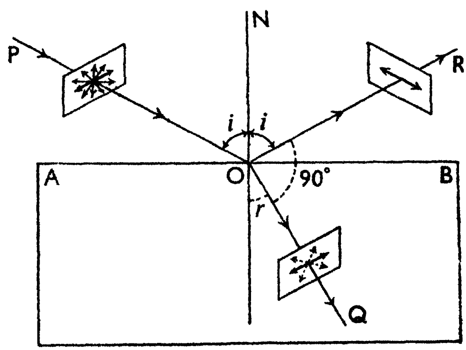


Fig. 109

We know that $\mu = \dfrac{\sin p}{\sin r}$

(by laws of refraction)

and $\tan p = \dfrac{\sin p}{\cos p}$

(by Brewster's Law)

$\therefore \quad \sin r = \cos p \quad$ or $\cos (90 - r) = \cos p \quad \therefore \quad p + r = 90°$.

Thus the angle between the reflected and the refracted ray is 90°.

If μ of a transparent medium be known the polarising anlge for the medium can be calculated.

In Figure 109 the reflected ray OR is completely polarised. the refracted ray OQ is partly polarised and PO, the incident ray is unpolarised. Mode of vibrations along three rays will be evident from the figure.

173(a). Pile of Plates : The strength of the light reflected from the surface of glass or any other similar transparent medium is very faint. This defect may be overcome by using a number of plane-parallel glass plates separated by air

films, piled on the top of one another and by reflecting ordinary light at eth polarising angle from both the back and fornt surfaces of each plate.

At each succeding plate the reflected light is enriched in waves polarised in the plane of incidence so that with a reasonable number of plates a strong beam of polarised light may be obtained.

174. Double Refraction: We know that when a beam of light falls on a transparent isotropic (having same properties in directions) body, refraction takes place and the refracted ray pursues a single definite direction. But when the beam falls on a aelotropic (having different physical properties in different directions) body such as **Iceland spar** it is refracted into two determinate directions, one obeying the ordinary laws of refraction known as the **ordinary ray** O and the other which does not obey the laws is known as the **extra-ordinary ray** E. The phenomenon that the ray incident on one of the faces of a crystal of Iceland spar is refracted in two determinate pencils is known as **Double Refraction.** In Figure 110 the ray PQ normally incident on a crystal ABCD undergoes two refractions, one along QO corresponds to ordinary ray, and the other along QE corresponds to extra-ordinary ray.

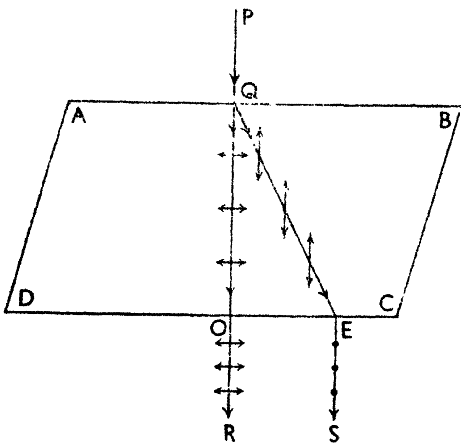


Fig. 110

175. Different types of doubly-refracting crystals: Calcite (Iceland spar), tourmaline, quartz, ice etc. are **Uniaxial Crystals.**

Borax, Mica, Sulphur, Selenite, etc. are **Biaxial Crystals.**

Calcite (Iceland spar) and Tourmaline are *negative crystals* and in these crystals the extraordinary ray travels faster than the ordinary ray and so μ_o is greater than μ_e where μ_o and μ_e are respectively the refractive indices of the ordinary and the extra-ordinary rays.

In *positive crystals* such as Quartz and Ice the ordinary ray travels faster than the extraordinary and so $\mu_e > \mu_o$.

If a crystal of calcite be placed over a small black dot on a sheet of white paper, two images of the dot are seen on looking through the crystal.

If the eye be vertically above the crystal, and the latter rotated one image remains stationary and is termed the **ordinary image** and obeys the ordinary laws of refraction.

The other image which rotates round the first image does not obey the laws of refraction and is termed the **extraordinary image.**

176. Optic Axis: Calcite (Iceland spar) is a transparent crystal. It crystallises in many forms, each of which may be reduced by cleavage to a rhombohedron.

The faces of the rhombohedron are similar parallelograms, each face having angles of $101°55'$ and $78°5'$.

There are two opposite solid angles which are contained by **three obtuse angles.** A line drawn through either of the corners formed by these obtuse angles so as to be equally inclined to the three edges meeting there, is called the **optic axis** of the **crystal** (Fig. 111).

Any line in the crystal parallel to its direction has important optical properties and is called the **Optic axis.**

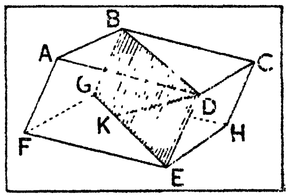


Fig. 111

If a rhombohedron be so cut that all of its edges are equal, the line drawn through the two obtuse solid angle corners will be parallel to the optic axis.

The optic axis is a direction and not a particular line. It is therefore clear that through every point in the crystal an optic axis can be drawn.

Definition: Optic axis of a crystal is that direction such that a ray travelling along it will suffer no double refraction and we shall get a single refracted ray for a single incident ray according to ordinary laws of refraction. Along the optic axis the ordinary and extraordinary rays behave exactly alike and merge into one.

176(a). Principal Plane: A plane drawn through the optic axis perpendicular to the opposite faces is called the Principal Plane.

The Figure 111 represents the calcite crystal ABCDEFGH and in which any plane parallel to the plane BDEG is a principal section of the crystal, the line DK being the optic axis.

177. Huyghens' Construction: According to Huygens, the wave propagated in a uniaxial crystal consists of two surfaces, one within the other.

The surface corresponding to the Ordinary ray AC (Fig. 112) is a sphere and that for the extraordinary ray AC′ an ellipsoid.

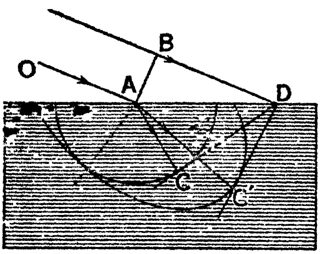


Fig. 112

The two surfaces touch each other at two points and the dotted line joining them is the optic axis.

In negative crystals the sphere is within the ellipsoid and in positive crystals the sphere is outside the ellipsoid.

Let a plane wave-front AC moving in air strike the surface of the crystal PB at an angle i. In the figure the optic axis DF is in the plane of the incidence (Fig. 113).

When the point C on the wave-front reaches the point B on the crystal, the ordinary wave-front generated at A expands to a sphere of radius AO and the extraordinary wave-front to an ellipsoid of revolution round the optic axis DAF. The sphere and the ellipsoid touch each other at points D and F lying on the optic axis.

The tangents BO and BE give the positions of the ordinary and extraordinary wave-fronts and the straight lines AO and AE represent the directions of the ordinary and extraordinary rays respectively.

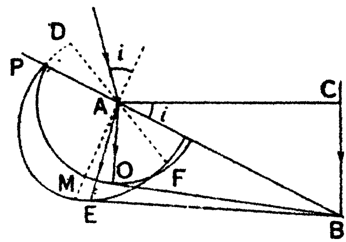


Fig. 113

This construction is for the *negative* crystals such as calcite etc. A similar construction may be made in the case of *positive* crystals such as quartz etc. in which the ellipsoid lies within the sphere.

177(a). To determine the directions of the ordinary and extraordinary rays in the crystal in some special cases.

(1) *Normal Incidence : the optic axis inclined to the surface and in the plane of incidence.*

The ordinary ray O and extraordinary ray E travel in different directions with different velocities and the extraordinary ray lies in the plane of incidence.

(2) *Normal Incidence : the optic axis perpendicular to the surface and in the plane of incidence.*

E and O rays coincide and the rays travel in the same direction nd at the same speed.

(3) *Normal Incidence : the optic axis is parallel to the surface n the plane of incidence.*

E and O rays travel in the same direction but with different elocities.

(4) *Normal incidence : the optic axis is parallel to the face of he crystal and perpendicular to the plane of incidence.*

The sections of the wave surfaces by the plane of incidence are wo concentric circles. In this case only both the ordinary and xtraordinary rays obey the laws of refraction and travel with two ifferent velocities in the same direction.

In all the cases stated above, the optic axis lies in or perpendicular to the face of the crystal and both the ordinary and extraordinary rays lie in the plane of incidence.

178. Polarisation by Double Refraction : If the rays transmitted through a doubly refracting crystal are received on a plate of tourmaline and if the plate is gradually rotated it is found that for some position of the plate the ordinary ray is extinguished and for position at right angles to the former position, the extraordinary ray is extinguished. The experiment proves that both the ordinary and the extraordinary rays are plane-polarised and that the planes of polarisations are at right angles to one another.

In every doubly refracting crystal there is at least one and in many, two optic axes, and if the incident beam passes along the axis, *no separation of the beam occurs after refraction*, but if the beam be perpendicular to the axis the separation between the refracted rays becomes maximum.

If a plane be drawn perpendicular to the refracting surface and through the optic axis, the plane is called the **principal plane**.

Now if the refracted beam from a doubly refracting crystal be allowed to pass through a second crystal of Iceland spar, each of the beams will be singly or doubly refracted according to the relative positions of the principal planes of the crytals.

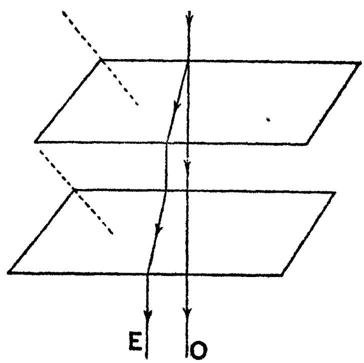


Fig. 114

If the principal planes are parallel, the ordinary ray through the first crystal will become ordinary ray through the second

crystal and the extraordinary ray through the first will become extraordinary ray through the second. But if the planes are at right angles to one another, the ordinary ray through the first crystal will be the extraordinary ray through the second and vice versa. But in any other position of the planes, the ordinary and extraordinary rays while passing through the second crystal will each give rise to an ordinary and an extraordinary, the intensity of the rays varying with angle between the principal planes of the crystals.

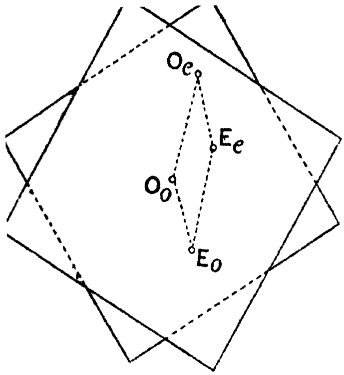


Fig. 115

That is, when the second crystal is rotated through an angle, each of the rays O and E is doubly refracted through it so that four images emerge from the second crystal.

The ordinary ray of the first crystal gives rise to O_o and O_e lying in a plane parallel to the principal section of the second crystal and similarly the extraordinary ray gives rise to to the pair E_o and E_e lying in a parallel plane.

Starting with the principal sections parallel, we get O_o and E_e and as the angle between the principal planes increases these two fade, and the other pair O_e and E_o increase in brightness until when the sections are at right angles the latter pair remain.

If the rotation be continued, the pair O_e and E_o will fade and the other pair, O_o and E_e will appear and increase in brightness until the crystal is rotated through 180° from the first position. Here we have two images O_o and E_e as at start.

The vibrations in the ordinary and extraordinary rays are executed at right angles to each other, and that the ordinary ray is polarised in the principal plane and the extraordinary ray is polarised perpendicular to the principal plane.

Let a beam of plane polarised light be incident normally on a crystal of calcite, cut parallel to the optic axis. Let the plane of incidence of the beam be horizontal so that the vibrations are vertical.

Now if the crystal be held so that its optic axis is vertical, the light is transmitted as an extraordinary ray, since the directions of vibrations are parallel to the optic axis.

But if the axis is horizontal, no light is transmitted through the crystal.

For any intermediate position, the direction of the vibrations in the incident polarised beam makes an angle with the principal plane

of the crystal and the light will be transmitted as ordinary and extraordinary rays. They are polarised at right angles.

The intensities of the transmitted beams are, according to Law of Malus the same as that of the incident beam.

179. Law of Malus : Let OP the amplitude of vibration in the incident plane-polarised ray make an angle θ with the principal plane OY of the calcite crystal. The incident vibration of amplitude a may be resolved into two directions, one $a \cos \theta$ parallel to the principal plane and the other $a \sin\theta$ perpendicular to it.

The vibration of amplitude $a \cos \theta$ is transmitted along the extraordinary ray and the intensity in the ray is proportional to $a^2 \cos^2\theta$.

Again the vibratlon of amplitude $a \sin \theta$ is transmitted along the ordinary ray and the intensity in this ray is proportional to $a^2 \sin^2\theta$.

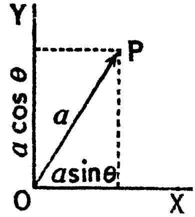


Fig. 116

Therefore the sum of intensities of the two transmitted rays is proportional to $a^2 \sin^2\theta + a^2 \cos^2\theta = a^2$ *i.e., the sum of the intensities of transmitted rays is equal to the intensity of the incident ray.*

180. Polarisation by Tourmaline : We know that polarised light is obtained by allowing a beam of ordinary light to pass through a thin plate of tourmaline. Since tourmaline is a doubly refracting crystal it divides the ordinary beam passing through it into two parts, one, the ordinary and the other, the extraordinary ray. But if the thickness of the plate is greater than 1 or 2 mm. the ordinary ray is absorbed leaving the extraordinary ray which emerges out as a plane-polarised beam. It is this property that renders the crystal to be used as a **polariser** or as an **analyser.**

As tourmaline is not a very transparent plate, strong beams of polarised light can not be obtained and also the light coming out through it is of pinkish colour.

181. Nicol's Prism : To get a strong beam of polarised light the most convenient method is to stop one of the plane-polarised beams into which the ordinary light is divided when transmitted through a crystal of Iceland spar.

There are various ways of stopping one of the beams but that adopted in Nicol's prism is to stop one of the beams by total reflection inside the crystal.

Nicol's prism consists of a long rhomb of calcspar (calcite) with the corners B and bounded by three obtuse angles and of

length equal to three times its breadth. The faces AB and CD of the natural crystal make an angle at 71° with the sides AD and BC respectively. The faces are ground and polished so that the new faces AE and CF make an angle of 68° with the longitudinal edges AF and CE respectively. The prism is cut into two parts obliquely by a plane EF perpendicular to the principal plane ABCD for the face AE. The cut faces are then polished and cemented together in their original position by a thin film of Canada balsam.

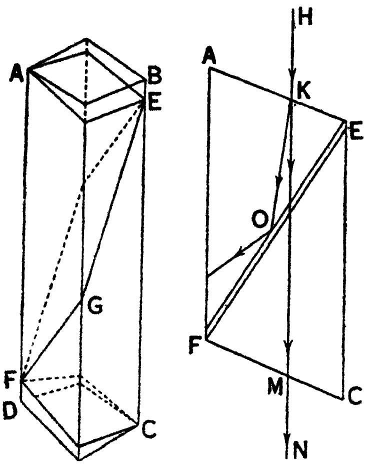


Fig. 117

The refractive index of Canada balsam (1·65) is intermediate between those of the ordinary (1·668) and the extraordinary (1·486) for the Iceland spar. So if the natural light be incident nearly normally on the end face AE *i.e.*, in a direction parallel to the long faces of the rhomb, it is split up into ordinary and extraordinary rays. The ordinary ray will strike the film EF of canada balsam at an angle greater than the critical angle (69°30′) for calcite to Canada balsam film, ordinary ray will be totally reflected at the surface of the Canada balsam and refused transmission through it for total reflection occurs in passing from a more to a less refracting medium, and the extraordinary ray will only come out as a plane-polarised beam polarised at right angles to the principal plane. It can be used both as a polariser and as an analyser.

When two Nicol's prisms are placed end to end so that their principal sections are parallel, light passing through them is plane-polarised but when they are crossed *i e.* when the principal sections are at right angles to one another no light is obtained.

To test whether a beam is partially or completely polarised, a Nicol's prism is held longitudinally along the beam of light. The Nicol is rotated round the beam and the intensity of transmitted beam varies but in no position of the Nicol complete darkness is obtained in the case of a partially polarised beam. But in the case of a completely polarised beam a position of the Nicol is found for which there will be complete darkness.

182. Methods of producing Plane Polarised light : Plane-Polarised light may be obtained by the following methods.

(1) By **reflection** from some transparent substance such as glass at the polarising angle.

It is a very convenient method but it is less perfect than the method of double refraction.

(2) **By double refraction** through a doubly refracting crystal such as Iceland spar.

The refracted pencils thus obtained are both plane-polarised and their planes of polarisation are at right angles to one another.

There are **three ways** of obtaining a single beam of polarised light by double refraction.

(*a*) By stopping one of the refracted polarised rays by an opaque diaphragm and allowing the other to pass for examination. This is possible in *Iceland spar* in which the separation between the two refracted rays is large.

(*b*) By causing one of the refracted pencils to be absorbed and the other transmitted.

This is done with a crystal known as *tourmaline.*

(*c*) By intercepting one of the refracted pencils by total reflection and the other transmitted.

This is done in Nicol's prism.

183. Methods of detecting Polarised Light : The instrument which acts as a polariser may be used also as an analyser.

In the case of light polarised by *reflection* from a glass plate a *second glass plate* placed above the first plate and equally inclined to the horizon as the first plate is used as an *analyser.*

A *Nicol's prism* or a *tourmaline plate* is used as an analyser, if the principal plane of an analyser is parallel to the plane of polarisation of the incident light.

184. Characteristics of a plane-polarised wave :

(1) It is not divided into two others by a doubly refracting crystal in two positions (parallel and perpendicular) of the principal section with respect to the ray while in the other positions it is divided into two pencils which vary in intensity and are complementary as the crystal is rotated.

(2) It is not reflected at the surface of a transparent substance when the plane of incidence is perpendicular to the plane of polarisation and when the angle of incidence has a certain value depending on the nature of the substance.

185. Ordinary light or Unpolarised light: It may be supposed to consist of vibrations, the direction of which changes many times a second being restricted in a plane perpendicular to the ray.

On transmission through a doubly refracting crystal such as Iceland spar it is divided into two pencils of equal intensity and polarised in planes at right angles to one another.

186. Circular and Elliptical Polarisation: If an unpolarised light be incident normally on a plate of uniaxial crystal cut parallel to the axis and perpendicular to the plane of incidence, both ordinary and extraordinary rays travel in the same direction with different velocities and both these rays obey the ordinary laws of refraction.

Now let us consider a plane-polarised ray incident normally on a thin plate of quartz, its plane of polarisation making an angle θ with its principal section at the point of incidence.

It will be split up into two components, one in the principal section parallel to OA of amplitude $a\cos\theta$ and the other at right angles to it of amplitude $a\sin\theta$, where a is the amplitude of the incident ray. Fig. 116.

As the two components travel with different velocities there will be a difference of phase between the two components when they come out from the plate.

In the general case the transmitted ray will be divided into two parts differing in amplitude and phase.

If $\theta = 45^\circ$, the two components have amplitude of values $a \cos 45^\circ$ and $a \sin 45^\circ$ and equal to $\frac{a}{\sqrt{2}}$.

Thus the two parts of the emergent ray are polarised in perpendicular planes and *have equal amplitude but differ in phase.*

(1) If the phase difference is any multiple of π, the resultant motion of the particle executing two S. H. Ms. of equal amplitudes at right angles is a *straight line* and the light will be **plane-polarised.** (See General Physics Art. 36 case II).

The plate which introduces this phase difference is called a **half-wave plate.**

(2) If the phase difference is $\frac{\pi}{2}$, or any odd multiple of it the light is said to be **circularly polarised.**

[See General Physics Art. 32 Case V]

The crystal plate which introduces this phase difference is called **a quarter-wave plate.**

(3) For any other phase difference the light is **elliptically polarised.**

187. Detection of circularly and elliptically polarised light : We know that a circular vibration is equivalent to two equal rectilinear vibrations at right angles to each other initially differing in phase by $\frac{\pi}{2}$.

So when a ray of circularly polarised light is analysed by a Nicol, the intensity of one component is diminished by rotating the Nicol, that of the other is increased by the same amount. Thus a rotation of the Nicol produces no alteration in the intensity in the transmitted ray. In this respect, circularly polarised light resembles unpolarised light.

To distinguish between the two, a quarter-wave plate placed in the path of the beam before the Nicol will introduce a phase change of $\frac{\pi}{2}$, so that on emergence, the phase difference between the component vibrations is π or 0 and the light is plane polarised and one position of the Nicol will extinguish it. On the other hand, unpolarised light after transmisson through a quarter-wave plate will not be extinguished.

An elliptic vibration is equivalent to two unequal rectilinear vibrations at right angles to each other. When elliptically polarised light is analysed by a Nicol it will change in intensity when viewed through a rotating Nicol. In this respect elliptically polarised light resembles a mixture of plane polarised and unpolarised light.

To distinguish between the two a quarter-wave plate is arranged so that its axis is parallel to one of the axes of the elliptic vibration, a phase change of $\pi/2$ is introduced as the light traverses the plate so that on emergence a plane polarised beam is the result and it can be extinguished by a Nicol.

Partially polarised light when transmitted through a quarter-wave plate will not be extinguished by a Nicol.

188. Rotation of the plane of polarisation : If a plate of calcite, cut perpendicular to the axis is inserted between two crossed Nicols, a parallel pencil of light is still refused transmission but when a plate of quartz having faces perpendicular to the axis of the crystal be interposed between the crossed Nicols the beam is no longer cut off by the second Nicol, but extinction of light is restored by rotating the second Nicol which is here the analyser. This shews that the quartz plate has the power of rotating the plane of polarisation of the light it transmits.

If white light is used, the field appears coloured, the colour depending on the rotation of the analysing Nicol, but if the light is monochromatic, the field appears dark.

Some crystals of quartz produce rotation in one direction and others in the opposite direction.

On looking towards the source of light the rotation may be *right-handed* if clockwise and *left-handed* if anti-clockwise.

Many liquids and vapours produce a similar rotation but to a smaller extent than quartz.

185(a). Specific Rotation : The specific rotation is defined as the amount of rotation produced by traversing a path of 10 cm. (1 decimeter) length in the substance when the density is unity. Therefore the specific rotation S is given by, $S=\frac{\theta}{ld}$, where θ is the observed rotation, l, the length of the substance traversed in decimetres and d, the density of the substance. For solution, $S=\frac{\theta}{lc}$, where c is the concentration of the solution in gram per c. c.

189. Laurent's Polarimeter or Saccharimeter: It is a special form of apparatus used to measure the amount of rotation in a given substance.

The apparatus consists of the parts shewn below in Figure 118.

Sodium light from a burner illuminates the slit *a* and made parallel by the lens *e* placed in the tube containing the polarising Nicol *d*. It then illuminates the half-shade *f*.

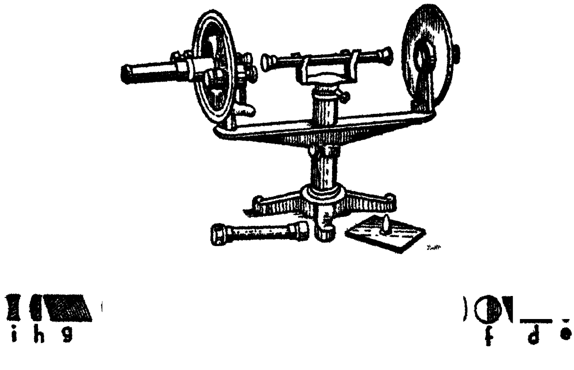


Fig. 118

The analysing Nicol *g* is fixed in a tube which carries the telescope with lenses *h* and *i*. The tube can be rotated on its axis and its position is observed by means of the vernier and the fixed circular scale shown in the upper figure.

By looking through the telescope and rotating it and the analysing Nicol, the position for extinction of light, *i.e.*, when the field is dark is observed from the position of the vernier on the circular scale.

Then a tube PP containing a rotatory solution, say sugar, having glass ends is placed between the polariser and the analyser and some light is found to be transmitted. Then again, by rotating the analyser a new position for extinction of light is found and the angle as observed in the vernier and the scale is the angle through which the substance has rotated the plane of polarisation of light passing through it.

As it is not possible to judge accurately when complete extinction of light takes place Laurent overcame this difficulty by using a **half-wave plate** of quartz or mica known as **half-shade** and his method of adjustment depends on the judgment of equal brightness.

In Figure 118 the **half-shade** *f* is placed behind the polarising Nicol *d*.

Adjustment with half-shade : The analyser is adjusted so that the half-shade appears uniformly bright. The tube PP containing the sugar solution is then placed between the half-shade and the analyser. The light from either portion of the half-shade has its plane of polarisation turned through the same angle and the amount of rotation of the analyser required to restore equality in the illumination of the two halves of the field is a measure of the rotation of the plane of polarisation produced by the solution.

190. Laurent's Analyser—The Half-shade: In Laurent's Analyser one half of the field is occupied by a plate of quartz or gypsum and the other half empty or covered by a glass plate of sufficient thickness to absorb waves of length λ to the same extent as the plate.

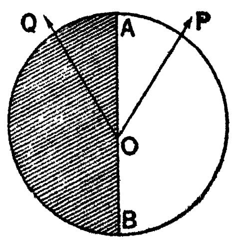


Fig. 119

The quartz plate is cut parallel to the axis and of such a thickness that it introduces a retardation of $\lambda/2$ between the ordinary and the extraordinary rays.

It is placed immediately after the polarising Nicol and cover half the field.

Let ABQ be the semicircular plate of quartz cut so that the optic axis is in the plane of the paper parallel to AB and let APB be the semicircular plate of glass.

If the plane of polarisation of the incident light be parallel to OP it will emerge out through it still plane-polarised parallel to OP.

But in the quartz plate it will be divided into two components one parallel to OA and the other perpendicular to it with a relative phase difference of $\frac{\lambda}{2}$.

The result is that the light emerges out of the crystal as plane-polarised beam with its plane of polarisation inclined to OA on the other side at an angle $AOQ = \angle AOP$.

Thus the planes of polarisation of the rays coming out of the two halves of the compound plate will therefore be inclined to each other at an angle equal to 2θ where $\theta = \angle AOP$.

Now if a Nicol be placed before the plate, two halves of the field will appear unequally illuminated ; but if the principal plane of the Nicol be parallel to AB, the illumination of the two halves will be the same.

The quartz plate AQB is called **a half-wave plate** and the combination of glass and quartz semicircles is termed a **half-shade.**

191. Fresnel's Explanation of the Rotation of the plane of polarisation : Fresnel assumed that a ray of plane polarised light incident normally on a plate of quartz cut perpendicular to the axis is decomposed into two circularly polarised rays which are transmitted along the axis with unequal velocities.

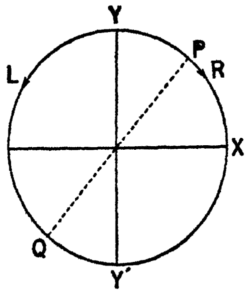


Fig. 120

On account of this inequality in velocity, one wave will be retarded over the other and consequently the plane of polarisation of the emergent resultant beam will be rotated with respect to the primitive plane.

In General Physics Art. 42 we have noticed that two circular motions in opposite senses are compounded into a simple harmonic motion of double the amplitude and also a simple harmonic motion is resolved into two equal circular motions in opposite senses (Art. 42b).

Let two particles execute two circular vibrations in opposite senses, one in the right-handed direction and the other in the left-handed direction (Fig. 120).

If the particles start at the same instant from Y they will cross each other at Y or Y′ as the circular motions have the same period. But if the left-handed motion be by any means relatively retarded *i.e.*, they do not start at the same instant

the two motions will not cross each other at Y or Y′ but at two diametrically opposite points say P and Q.

Thus the two circular motions in opposite senses and relatively retarded will be compounded into a simple harmonic motion of double the amplitude along PQ.

Thus, to explain the rotation of the plane of polarisation we are to consider the fact that a plane-polarised beam when incident normally on a plate of quartz cut perpendicularly to the axis is decomposed into two opposite circularly polarised rays which are transmitted along the axis with unequal velocities.

As they are transmitted through the crystal, a relative retardation is produced between the two circular motions and on emergence the two motions combine to form a plane-polarised beam, the plane being rotated through a certain angle from the original incident beam. Thus the rotation of the plane of polarisation is explained.

192. Mathematical Treatment :

Let the two opposite circular vibrations by represented by

$$x_1 = a \cos wt \qquad y_1 = a \sin wt \text{ (right-handed)}$$
$$x_2 = -a \cos wt \qquad y_2 = a \sin wt \text{ (left-handed)}$$

If the latter be retarded in phase by an amount δ the equations of transmitted vibrations may be written in the form

$$x_1 = a \cos wt \qquad y_1 = a \sin wt$$
$$x_2 = -a \cos (wt + \delta) \qquad y_2 = a \sin (wt + \delta)$$

So for the transmitted vibration we have

$$x = a \cos wt - a \cos (wt + \delta) = 2a \sin \tfrac{1}{2} \delta \sin (wt + \tfrac{1}{2}\delta)$$
$$y = a \sin wt + a \sin (wt + \propto) = 2a \cos \tfrac{1}{2} \delta \sin (wt + \tfrac{1}{2}\delta)$$

These are two perpendicular vibrations of the same phase, which when compounded gives the resultant rectilinear vibration, making an angle θ with the axis YY' such that

$$\tan \theta = \frac{x}{y} = \tan \tfrac{1}{2}\delta$$

193. Scattering of Light—Colour of the sky : In a previous Article 131, we have noticed that the effect of a wave is mainly confined to a few half-period elements into which the wave is divided in front of the point at which the influence is to be considered.

If the half-period elements are covered by an obstacle the point will be effectively screened.

Since the width of the half-period element increases with the wave-length of light a given obstacle will act most effectively as a barrier of the waves of shortest wave-length.

We may, therefore, say that when ordinary light passes through a medium such as our atmosphere in which very small particles such as of frozen moisture dust etc. are suspended the waves of greater wave-length will be freely transmitted and the shorter waves of higher refrangibilities are scattered in any direction.

Now the light which reaches us from the sky is that part of light which has been scattered by the particles which are very small compared with the wave-length of light.

Lord Raleigh has shown that the intensity of scattered light varies inversely as the fourth power of the wave-length.

$$\text{That is} \quad I \propto \frac{1}{\lambda^4}$$

where I is the intensity of the scattered light, λ, the wave length and so the intensity of the blue portion of the spectrum becomes very much greater and the scattered light appears blue. The scattered light is found to be polarised.

Thus the blue colour of a distant mist or of the smoke from a wood fire and of the intensely blue colour of some rivers in which divided chalk particles are suspended is due to the scattering of light.

If a ray of sunlight passes through a sufficient distance through the atmosphere, scattering will take place from successive layers of particles and it will therefore be wholly or partially robbed of the blue and the violet ray and consequently the remaining light will be of a yellow or red colour.

The red colour of the sky at sunrise or sunset is thus accounted for, as it is seen through a thick layer of the atmosphere.

In estimating the quality of light we are to consider both the action of transmission and that of scattering.

QUESTIONS

1. Light is supposed to be propagated as transverse waves through space. Give some evidence in support of this theory. [C. U. 1922, '23, '25, '31]

2. Define Polarisation, Polarised light, Plane of Polarisation. [C. U. 1943, '44, '47, '49, '50, '52, '53, '54, '55]

Describe any polarising apparatus you know. [C. U. 1924]

3. Describe the method of producing plane-polarised light by reflection. Explain Brewster's Law. [C. U. 1943]

Indicate how polarising angles of light crown glass and rock-salt are calculated with sodium light, using Brewster's law. [C. U. 1952]

4. Explain what is meant by double refraction, principal plane, optic axis, ordinary and extraordinary rays. [C. U. 1934, '42, '44, '46, '53, '54, '55, '57]

5. Distinguish clearly between polarised light and ordinary light. [C. U. 1940, '44, '50]

6. What are the different methods for producing and detecting polarised light waves. [C. U. 1941, '50]

7. Describe in detail the construction of Nicol's prism and explain how it produces plane-polarised light. [C. U. 1940, '43, '46, '49, '52, 53, '55, '57, '58]

Explain how the plane of polarisation of a beam of light can be tested with a Nicol's prism. [C. U. 1953]

8. Explain how a beam of plane polarised light can be regarded as composed of two circularly polarised beams and how you can explain with the aid of this conception, the rotation of the plane of polarisation of a plane-polarised beam on passage through a doubly refracting crystal. [C. U. 1941]

9. Describe the construction and use of a saccharimeter (Polarimeter). [C. U. 1941, '50, '59]

10. What is the specific rotation of an optically active substance? [C. U. 1958]

11. What do you understand by the 'scattering of light'?

EXAMPLES

1. If for a certain wave-length, the rotation of the plane of polarised light due to 1 *mm.* of quartz (cut perpendicular to the optic axis) is 20°, for what thickness of quartz will no light of this wave-length be transmitted when the piece is placed between two Nicol prisms with parallel principal planes? Explain your answer.

Let t be the thickness of the plate of the crystal which when placed between two Nicols with parallel principal planes rotates the plane of vibration through 90° and no transmission of light takes place.

Therefore $t = \frac{90}{20}$ *mm.* $= 4 \cdot 5$ *mm.*

2. For a given wave-length one millimetre of quartz cut perpendicular to the optic axis the plane of polarisation is rotated through a certain angle. If for a thickness of 27 cms. no light of this wave-length be transmitted when the quartz piece is interposed between a pair of parallel Nicols, find the angle of rotation of the plane of polarisation. Why is it necessary that the quartz plate be cut perpendicular to the optic axis? Explain your answer. [Ans. 20″]

3. Plane polarised light is incident on a piece of quartz cut with faces parallel to the axis. Find the least thickness for which the ordinary and extra-ordinary rays combine to form plane polarised light, given that,

$\mu_O = 1{\cdot}5442$, $\mu_E = 1{\cdot}5533$, $\lambda = 5 \times 10^{-5}$ cm. [Ans. $5{\cdot}49 \times 10^{-3}$ cm.]

4. Calculate the thickness of a quartz half-wave plate for sodium light ($\lambda = 5893 \times 10^{-8}$ cm.) given that the indices of refraction of quartz for the ordinary and extra-ordinary rays are 1·5442 and 1·553 respectively. [Ans. $6{\cdot}476 \times 10^{-3}$ cm]

HEAT

CHAPTER I

THERMOMETRY

1. Heat and Temperature : We are familiar with sensation of heat and cold from our very childhood. Heat is nothing like a material body of definite shape and size with distinguishing colour or smell. Heat is an invisible form of energy whose absorption makes a body hot and abstraction of which from the body makes it cold.

When we say, a body is hot or cold, we use the terms in a more or less relative sense. We ordinarily form a sort of judgment as to the greater or lesser sensation of warmth of a body on touching it as compared either to the state of warmth of our own body or to some other body we happen to touch just before.

The degree of hotness of a body is called *temperature*. Heat and temperature may be related to each other as cause and effect. The degree of hotness of a body is closely connected with how much of heat has been absorbed by the body. When a body seems to be very hot to touch, we say that it has attained a higher temperature. Again when a body seems to be very cold to touch, we say that the body is at a lower temperature.

Temperature in a more general sense is the thermal condition of a body which determines whether the body will communicate heat to or receive heat from, another body if put into contact with each other. If, of the two bodies A and B, A be hot and B cold, heat will flow from A to B, and A is said to be at higher temperature than B. Heat and temperature though closely associated are not the same thing. Two bodies may be at the same temperature although the heat content in each is different. Again heat content of a body at a higher temperature may be much less than the heat content of a body at a lower temperature.

2. Different effects of heat : When a body is sufficiently heated or cooled the following among other effects are observed : (1) Change of temperature, (2) Change of volume, area or length, (3) Change of state, (4) Change in electrical resistance of a conductor, (5) Production of electromotive force at the junctions of two dissimiliar metals, (6) Change of pressure of a gas or vapour.

3. Measurement of temperature : Sense of touch is the primitive way of measuring temperature. But this very often leads to uncertain results even from qualitative stand point. Scientific precision demands that every physical quantity must be

measurable in numerical terms. These measurements again should be easily reproducible and exact. The instrument devised for measurement of temperature is called a thermometer. We cannot, strictly speaking, measure temperature absolutely. What we can do by a thermometer is to compare temperature difference. Therefore, in order to construct a scale of temperature, a standard temperature difference or **fundamental interval** is selected. To define this fundamental interval, two fixed and easily reproducible temperatures are chosen. Two numerical values are then arbitrarily assigned to them. The one, called the lower fixed point or the ice point corresponds to the temperature at which pure ice would always melt at normal atmospheric pressure, and the other called the upper fixed point or the steam point corresponds to the temperature of the steam from water boiling under normal atmospheric pressure. These fixed points are by convention, called 0°C and 100°C respectively according to the Centigrade scale of graduation.

After defining the fixed temperatures, we can now make a scale of temperature which will give temperatures. For this, we consider any property of a substance which changes uniformly with temperature, say volume of a given mass of mercury or alcohol, pressure of a gas at constant pressure, resistance of a metal, say platinum wire etc.

Let the quantitative measures of the property chosen be x_0 and x_{100} at 0°C and 100°C, *i.e.*, at lower and upper fixed points respectively. Then, when its measure is x_t at any other temperature the numerical value t of that temperature is expressed by the linear relation

$$\frac{t}{100} = \frac{x_t - x_0}{x_{100} - x_0}$$

Fig. 1

In this relation t may be any temperature at which the property selected continues to exist.

4. Mercury-in-glass Thermometer : In mercury thermometers cubical expansion of mercury is utilised for the measurement of temperature and the principle of its construction is given below.

A capillary tube of uniform bore is taken and a bulb B is blown at one end of the tube and at the other end a thistle funnel E is attached. A small quantity of mercury is placed in the funnel after heating the bulb slightly so as to drive out some of the imprisoned air (Fig. 1).

As the bulb cools, the air will contract and some of the mercury will be drawn inside the bulb. This process of alternately heating and cooling the bulb is repeated for sometime and finally the bulb with a part of the stem will be filled with mercury.

The mercury in the bulb is then heated to its boiling point so as to drive out air from inside the tube and when the tube contains no air but mercury vapour it is sealed hermetically at C.

The graduation of the thermometer is then performed by determining two fixed points in it. The bulb of the thermometer is placed in powdered ice and the mercury column in the stem is seen to recede and remain stationary at a fixed point known as the **Ice Point.** This is the lower fixed point of the thermometer.

The bulb is again exposed to steam issuing from boiling water in a hypsometer. The mercury column will rise up and remain stationary at a point which is the upper fixed point or the **Boiling point** of the thermometer.

The two fixed points being determined, the interval is divided into a number of equal parts according to three different **scales of temperature,** namely Centigrade, Fahrenheit and Reaumur.

In centigrade scale, the interval is divided into 100 equal parts. The ice and steam points for Fahrenheit scale are respectively 32°F and 212°F and the fundamental interval is divided into 180 equal parts. The ice and steam points for Reaumur scale are respectively 0°R and 80°R, and the interval is divided into 80 equal parts.

If three thermometers in these different systems be placed in a given hot bath and if the readings in Centigrade, Fahrenheit and Reaumur scales be C, F and R respectively, then $C/5 = (F - 32)/9 = R/4$. This relation enables conversion of temperature reading in one scale into any one of the others.

4a. Absolute Scale : In this scale the zero is taken to be that temperature at which volume of a perfect gas would tend to vanish and the average kinetic energy of the molecules of the gas is also zero. This is found to happen at **−273°C** called the **absolute** 0°. In the absolute scale, the ice and steam points are respectively 273°A and 373°A, the interval and size of a degree in absolute scale being the same as in the centigrade scale.

4b. Range of Mercury Thermometers : Mercury freezes at **−39·8°C** and boils at **357°C.** Hence its range is limited between these two temperatures. But the higher limit can be raised to about 500°C by filling the space above mercury with nitrogen under high pressure. For this, the thermometric glass should be strong, stable and should return to its normal condition after being exposed to high temperatures.

4c. Errors of Mercury Thermometer: The following principal errors must be corrected for, if a mercury thermometer is to be used for accurate work :

(1) *Error for inequalities in the bore of the thermometer.*

If the bore of the tube be not uniform, the thread of mercury in the tube will occupy different lengths at different parts of the tube and the graduations will not be uniform

(2) *Error due to Capillarity.*

If the tube be of very fine bore, mercury does not move freely but may remain stationary for sometime when the temperature is altered, and then suddenly change its position.

(3) *Error due to exposed column.*

This error is due to the fact that the whole amount of enclosed mercury is not at the same temperature, the mercury in the stem outside the bath in which the thermometer is placed being at a lower temperature than that in the bulb.

(4) *Error due to alteration in the pressure to which the bulb is subjected.*

The bulb of the thermometer is generally made thin in order to enable mercury to quickly take up the temperature of the bath. So any alteration in pressure alters the volume of the bulb and consequently the mercury thread moves in the stem and introduces error.

(5) *Error due to softness of the glass.*

The error is due to the expansion of the glass when the thermometer is strongly heated.

(6) *Error due to heat capacity of the thermometer.*

This error is due to the fact that when the thermometer is placed in a hot liquid, the liquid is cooled as the thermometer is heated. So the temperature indicated will be somewhat lower than the actual temperature just before the thermometer was introduced into the liquid.

5. Advantages of Mercury Thermometer :

(1) The expansion of mercury is regular at ordinary temperatures.

(2) It can be obtained pure.

(3) Its specific heat is very low, so it takes up a very small quantity of heat to rise through a certain range of temperature.

(4) As it has a good conducting power, it can readily take up the temperature of the substance.

(5) It does not wet glass.

(6) It is opaque and can be easily seen.

Note : It remains liquid over a considerable range of temperature *viz.* from about −39°C to 357°C.

6. Other types of Thermometers : Besides mercury thermometer already described, other types such as *Alcohol thermometer, Clinical thermometer, Maximum and Minimum thermometers and Sensitive thermometers* are used for various purposes.

6a. In **Alcohol Thermometers** alcohol is used as the thermometric substance for registering much lower temperature than is possible with mercury thermometer for alcohol solidifies at **−130°C** and mercury at about −39°C.

It is more sensitive than a mercury thermometer for **alcohol** expands more than mercury for a given rise of temperature.

This thermometer is used to record the temperature in the polar regions. But it is not suitable for temperatures above about 70°C as alcohol boils at 78°C.

6b. Maximum and Minimum Thermometer : The Maximum thermometer registers the highest and the minimum thermometer the lowest temperature occurring during a certain time.

These thermometers are usually used to record the daily variation of temperatures.

In a minimum thermometer, alcohol and in maximum thermometer, mercury is used as thermometric substances.

In **Rutherford's type** the minimum and maximum thermometers are used as two separate themometers on a single frame.

6c. Six's Thermometer : This is a combined maximum and minimum thermometer (Fig. 2). It is a self-registering thermometer. The cylindrical bulb A and part of the tube contains alcohol up to B. Portion BC of the tube is full of mercury; above the level C there is alcohol which also partially fills the bulb D. Two light steel indices S_1, S_2 with spring are placed respectively above the levels B and C. When the temperature of the surrounding air rises, the alcohol in A expands, the level B goes down leaving the index S_1 undisturbed. At the same time mercury level C goes up pushing the index S_2 to its maximum limit corresponding to maximum temperature of air. When temperature of air falls again, alcohol contracts, mercury in BC is drawn back leaving S_2 in position. At the same time the receding mercury pushes the index S_1 above B, to a certain position. The index S_1 remains here, when temperature rises again. To reset for new observations, each index is drawn down to the level B and C by a magnet. The spring attached to each index prevents its slipping down under its own weight.

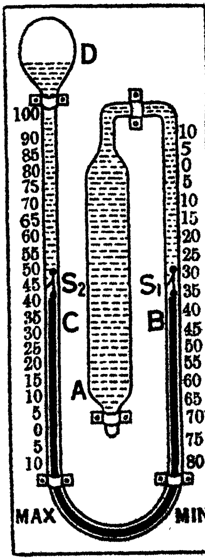


Fig. 2

6d. The **clinical thermometer** used to measure the temperature of human body is a mercury thermometer of short range and of maximum type. There is, in the stem just above the bulb, a constriction, through which mercury can pass when the bulb becomes heated. On cooling, mercury in the stem cannot quickly force its way back and remains detatched from the mercury in the bulb. The stem is graduated from about 95°F to 110°F.

Fig. 3

6e. Sensitive Thermometers are constructed for measuring small difference in temperature in cases where the change in temperature is important and a knowledge of actual temperature is not required. The Beckmann's thermometer (Fig. 3) is employed to measure small difference or changes of temperature with fairly high degree of accuracy.

As intended for indicating only small temperature difference, this thermometer is made with a large bulb, and the scale which extends only over 5 or 6°C is divided into hundredths of a degree. To make the thermometer suitable for different ranges there is a bulb at the top into which some of the mercury can be driven from the lower bulb by appropriate shaking and heating.

7. Sensitivity of mercury thermometer : A thermometer is said to be sensitive if, for a small change of temperature there is a large movement of the mercury column.

The sensitivity of a thermometer depends on

(1) the bore of the tube The finer the bore the greater is the sensitivity.

(2) the volume of the bulb. The larger the volume, the greater is the sensitivity.

Note . If the bore of the tube is *very fine*, the sensitivity will increase as the expanded mercury will occupy a longer length in the stem.

Again if the *size of the bulb is increased* the volume of mercury in the bulb will be greater and due to a rise of temperature the expansion of mercury will be greater and so the graduations will be widely spaced and the sensitivity will increase.

8. Measurement of high temperatures : Under normal pressure, mercury boils at 357°C. So ordinary mercury thermometer can not be used for temperatures higher than 357°C.

For measuring higher temperatures several methods have been employed. Instruments used for measuring very high temperatures are known as **Pyrometers.**

8a. Platinum Resistance Thermometers : The electrical resistance of platinum varies regularly with temperature according to the formula $R_1 = R_0 (1 + \alpha t + \beta t^2)$.

This fact enables high temperatures to be measured by determining the resistance of platinum wire at various temperatures by a method described in *Current Electricity*. The thermometer used for this purpose is a *Platinum Resistance thermometer.*

It has a wide range from $-200°C$ to about $1000°C$.

8b. Besides this, **Thermo-Couple** thermometers have been constructed with two dissimilar metals such as Plantinum and an alloy of Platinum and Iridium forming a couple in which E.M.F. is generated producing an electric current of varying strength when the junctions of the couple are heated to different temperatures.

From the knowledge of the E.M.F. at different temperatures the unknown high temperature is determined by a method described in *Current Electricity*. The usual range of thermo-electric thermometer is about $-200°C$ to $1600°C$. Suitable couples of different metals may be used to give temperatures up to $2100°C$.

8c. Optical Pyrometers are used for measuring higher temperatures in metallurgical operations. Radiation pyrometers are used to measure exceedingly high temperatures up to about $6000°C$ which is the temperature of the outer envelope of the sun.

8d. Gas Thermometers : The high expansion of gases is utilised for adopting gases as suitable thermometric substances for constructing high-temperature gas thermometers.

Constant Volume Hydrogen Thermometer with a Platinum Iridium bulb gives temperatures up to $500°C$. (See Chapter on Expansion of gases).

For higher temperatures Nitrogen is employed instead of Hydrogen.

9. Measurement of Low Temperatures :

(1) Alcohol thermometers may be used for temperatures down to about $-130°C$ as alcohol remains in the liquid state up to this temperature.

In polar regions this thermometer is suitably employed for registering low temperatures.

(2) Platinum Resistance Thermometer may also be used for temperatures from $0°C$ to $-200°C$.

(3) For temperatures below $-200°C$.

Copper-Constantan and Iron-Constantan Couples may be used to give temperatures up to $-255°C$.

(4) Constant volume hydrogen and helium thermometers can be used to register temperature down to about $-250°C$.

(5) To measure temperatures below the temperature of boiling helium ($-268°C$), the vapour-pressure thermometer of helium can be used.

The pressure in this case is measured by a suitable manometer at very low temperatures and the temperature is obtained by the formula connecting vapour pressure and temperature. The temperature down to $\cdot 75°A$ i.e. $-272°\cdot 25C$ may be obtained.

(6) For measuring still lower temperatures the method of demagnetising samples of suitable materials has been used.

Using a mixture of chrome-potassium alum and aluminium-potassium alum de Haass and others reached a temperature $0\cdot 0034°A$.

Note: Scales of temperature based on various properties of matter always disagree with one another to a greater or less extent.

The constant volume hydrogen scale with the gas at a pressure of 1 metre of mercury at 0°C is considered as a standard scale of temperature.

All other thermometers are calibrated with reference to the constant volume hydrogen scale.

QUESTIONS

1. Explain the construction of an ordinary mercury thermometer, its scale, and show how the fixed points are determined. [C. U. 1928]

2. Discuss fully the various sources of errors in a mercury-in-glass thermometer and the corrections necessary. [C. U. 1928]

3. What factors must be taken into account in the design of a sensitive thermometer?

4. What are the advantages and disadvantages of mercury in a thermometer as compared to other thermometric substances?

5. What is the principle on which construction of different thermometers depend? Mention a temperature scale which is independent of the nature of the thermometric substance.

6. What do you mean by (a) the perfect gas scale of temperature, (b) the platinum resistance scale of temperature? Why do the scales give different numbers for the same temperature above or below ice point and boiling point respectively?

7. Mention three physical properties which have been utilised for measurement of temperature. Compare relative advantages, disadvantages and ranges of temperature of thermometers based on each of these principles. [C. U. 1957]

CHAPTER II

EXPANSION OF SOLIDS AND LIQUIDS

10. Expansion of Solids : All solids, as a rule expand when heated and contract when cooled. A stretched rubber band and leather however contract on being heated. Iron expands continuously up to about 890°C ; it then contracts slighty on further heating, and then expands again at higher temperatures. Alloys exhibit an anomaly near the melting point of each of the components. For isotropic bodies, *i.e.*, bodies whose properties are the same in all directions, the expansion is also the same in all directions. Metals, glass, rock-salt belong to this type. In anisotropic bodies which include crystals, the properties are different in different directions and thermal expansions are also different in different directions. Certain crystals even contract along particular direction on being heated.

11. Coefficient of Linear Expansion : The ratio of the increase in length of a solid rod produced by a rise of temperature of 1°C, to the original length is called the coefficient of linear expansion of the material of the rod.

If l_0 denote the length of a bar of any substance at 0°C and l_t its length at t°C then $l_t = l_0\ (1 + \alpha t)$ ... ... (1)

where α is the mean coefficient of linear expansion of the substance between 0°C and t°C.

It is the mean increase in the length of a bar per unit length when heated through unit difference of temperature.

Since α is a very small quantity the equation (1) may be written in general case as $l_2 = l_1\ \{1 + \alpha(t_2 - t_1)\}$

where l_2 and l_1 are the lengths of the bar at temperatures t_2 and t_1 respectively.

The value of the coefficient of linear expansion does not depend on the unit of length used. But as one Fah. degree is $\frac{5}{9}$ Cent. degree, the Fah. coefficients are obtained by multiplying the Cent. coefficients by $\frac{5}{9}$. Thus coefficient of linear expansion of brass is ·0000189/°C. Its value in Fah. degree = ·0000189 × $\frac{5}{9}$ or ·0000105/°F.

The mean coefficient α is found to change with temperature and hence, relation between length and temperature does not in general remain a linear one. The general form of equation (1) is given by $l_t = l_0(1 + \alpha t + \alpha_1 t^2 + \alpha_2 t^3 + \ldots)$, where α, α_1, α_2 etc. are coefficients of rapidly decreasing magnitude. It is enough to consider for general purpose terms up to second power of t. In that case the length-temperature relation becomes parabolic.

In ordinary work, the linear equation (1) is used since the variation of α with temperature is almost negligible.

12. Measurement of Linear Expansion :

Comparator method : It is a standard method for solid in the form of a long bar, rod or tube. If the specimen be in the form of a tube, its ends are closed and it is fitted with two side tubes near two ends for entrance and exit of steam. There is another side tube near the middle for insertion of a thermometer The tube is held horizontally and it can expand freely at two ends. Two fine marks are made near the ends of the tube. Two microscopes used for observing the marks are each either provided with an eye-piece micrometer or can be moved in the direction of the expansion by means of a micrometer screw. The microscopes attached to the micrometer screw are mounted on heavy slabs independently.

In actual experiment, the length (l) of the tube between two marks is measured accurately by a standard metre scale at room temperature (t_1). It is then placed horizontally below the microscopes. The microscopes are focussed on the two marks by means of the micrometer arrangement. The micrometer readings at two ends, (x_1, y_1) are noted. The tube is then raised to some high steady temperature t_2 by passing steam. The microscopes are again focussed on the marks. The micrometer readings at two ends, (x_2, y_2) are noted. Then total expansion $e=(x_1 \sim x_2)+(y_1 \sim y_2)$ is obtained. The value of α can be found from the relation $\alpha = e/l(t_2 - t_1)$, where l is length of the tube at t_1°C.

13. Expansion of crystal :

Fizeau devised an optical method based upon the formation of interference fringes, to find coefficients of linear expansion of crystal along different axes. The crystal is cut into a thin parallel plate P. (Fig. 4). It is placed on a plane metal platform AB through which project three levelling screws (two only visible in the figure). On the upper ends of the screws rests a glass plate CD, which is so adjusted that it lies almost parallel to and at a very short distance from the upper face of the crystal P. If the air film between P and CD be illuminated by monochromatic light, say sodium flame, interference fringes are produced. When the above arrangement is placed in a heated chamber, the thickness of the air film changes owing to the differential expansion of the

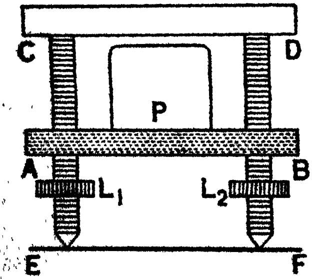


Fig. 4

crystal and the screws, and the fringes are displaced from original position at room temperature (t_1). By measuring the shift of the fringes the differential expansion can be found and from that the expansion of the crystal (e) can be determined. Then α of the crystal along the axis perpendicular to the parallel faces can be calculated from the relation $\alpha = e/d(t_2 - t_1)$, where d is the length of the crystal (thickness) at t_1°C, and t_1°C, t_2°C the initial and final temperatures of the crystal.

14. Coefficients of Linear Expansion of some Common Solids : In the following table are given the mean coefficients of expansion (α) of some solids between 0 and 100°C.

Soild	α-per°C	Solid	α-per°C
Copper	0'0000167	Nickel	0'000013
Brass	0'0000189	Lead	0'000028
Iron (cast)	0'0000102	Glass	0'000008

15. Coefficient of Superficial Expansion : It is defined as the increase in area which a plate of unit area undergoes when its temperature is raised through one degree centigrade.

Then, by definition $S_2 = S_1 \{1 + \beta (t_2 - t_1)\}$

where S_2 and S_1 are respectively the areas of the plate at temperatures t_2 and t_1 and β, the coefficient of superficial exapansion.

It can be easily proved that $\beta = 2\alpha$.

16. Coefficient of Cubical Expansion : It is defined as the increase in volume which a unit volume undergoes when the temperature is raised through one degree centigrade.

Then, by definition $V_2 = V_1 \{1 + \gamma(t_2 - t_1)\}$

where V_2 and V_1 are respectively the volumes of the substance at temperatures t_2 and t_1, and γ the coefficient of cubical expansion.

It can be proved that $\gamma = 3\alpha$. Then, $\beta/\gamma = 2\alpha/3\alpha = 2/3$.

The coefficient of cubical expansion of a solid for all practical purposes is obtained by multiplying its coefficient of linear expansion by three.

17. Compensated Pendulum : The period of a simple pendulum depends on its length, *i.e.*, on the distance of the point of support from the centre of gravity of the bob. If the pendulum consists of a single metal rod attached to a bob, the increase or diminution in the length of the pendulum due to a change of temperature, increases or diminishes the time of oscillation. Conse-

quently when the length of the pendulum increases the clock having such a pendulum loses, and when it contracts the clock gains. Thus to get correct time under all variations of temperatures, variation of the length of the pendulum due to the change of temperature is to be compensated.

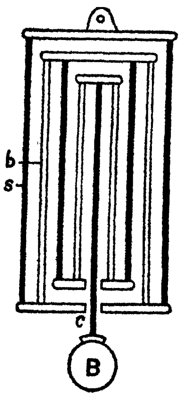


Fig. 5

In Harrison's Grid-iron Pendulum the bob B is supported from a frame work consisting of alternate rods of steel and brass. The steel rods (s) are fitted in the frame work in such a way that they expand in the downward direction and carry the bob with it, but the brass rods (b) on the other hand expanding in the upward direction compensate the downward expansion of the steel rods. So the position of the bob remains unaffected due to changes of temperature and hence, the period of the pendulum is not altered. (Fig. 5).

Let l_1, be the total length of steel rods in one half of the frame including that of the central rod and l_2 be the total length of brass rods in the same half. The length in one half is considered, since all the bars are in pairs except the central rod. The downward expansion of the steel rods for a rise of temperature t°C is $l_1 \alpha_1 t$ and the upward expansion of the brass rods is $l_2 \alpha_2 t$ for the same rise of temperature, where α_1 and α_2 are their respective coefficients of expansion.

Then to keep the effective length of the pendulum always constant,

$$l_1 \alpha_1 t = l_2 \alpha_2 t \quad \text{or} \quad \frac{l_1}{l_2} = \frac{\alpha_2}{\alpha_1} = \frac{\cdot 000019}{\cdot 000012} = \frac{3}{2} \text{ (approx.)}$$

Here α_1 for iron = ·000012 per°C ; α_2 for brass = ·000019 per°C

Thus for compensation, the effective lengths for iron and brass rods should be approximately proportional to 3 : 2.

17a. Compensated Balance Wheel: The balance wheel is usually made of three radial spokes S (Fig. 6) each supporting a separate portion of the rim of the wheel and carrying a weight W near its free end. Each portion of the rim is composed of two strips of metal, one lying outside the other, the outer strip being more expansible than the inner one.

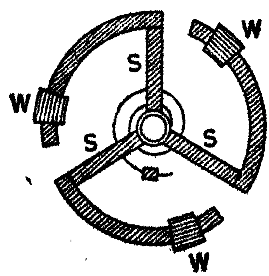


Fig. 6

As the temperature rises the expansion of the spokes will cause the rim and the weight to recede from the centre of rotation and the rate of oscillation of the wheel will thereby change. But as at the same time the strips become more

curved due to the greater expansibility of the outer strip, the free end of the rim or the weight is brought near to the centre of rotation. By this means the moment of inertia of the wheel is kept unchanged and consequently the rate of oscillation of the wheel is not altered by change of temperature.

18. Expansion and Elasticity of solids : A rod can be extended in length by a pull in the direction of the length. The rod can be lengthened as well by increasing its temperature. In the first case, if the rod returns to its original length on the removal of the pull, the stretching is perfectly elastic. The force is experimentally found to be proportional to the elongation it produces, if it is within elastic limit. For a given material, the stress (*i.e.* force per unit area) is proportional to the strain (*i.e.* elongation per unit length) and the ratio stress/strain which is called Young's modulus Y for the material is given by $Y = \frac{\text{stress}}{\text{strain}}$

or Stress = Y × strain. The above law of stress and strain known as Hooke's law holds good also for compression *i.e.* negative extension.

Let us now consider a rod of length l_0 and cross-section A heated through t°C. The expansion or elongation is $l_0 \alpha t$, where α is the coefficient of linear expansion of the rod.

The strain is therefore equal to $l\alpha t/l = \alpha t$. Suppose the rod is now fixed at the ends by clamps and allowed to cool down to initial temperature. The process is just the same as if it were not heated but pulled out mechanically, so that the force required to keep it extended will be equal to that required to produce the elongation.

Now, strain $= \alpha t$ $\therefore$ Stress $= Y.\alpha t$

Hence, total force = stress × area $= Y\alpha t.$ A

Force required to stop extension can be found as follows. Suppose a bar whose length $l(1+\alpha t)$ at t°C is compressed by an amount $l\alpha t$ such that the length is decreased to l.

The force required = stress × area

$$= \frac{Y\alpha t \times A}{1+\alpha t} \left[\because \text{ stress} = Y.\frac{l\alpha t}{l(1+\alpha t)} = \frac{Y\alpha t}{1+\alpha t} \right]$$

19. Relation between Density and Coefficient of Cubical Expansion of a solid :

A body of volume V_0 and density ρ_0 at 0°C is heated to t°C and consequently its volume and density changes to V_t and ρ_t respectively.

Since the mass of the body remains constant, $V_0\rho_0 = V_t\rho_t$ = Mass. But $V_t = V_0\,(1+\gamma t)$ where γ = coefficient of cubical exp. of the solid.

$\therefore\ \rho_t = \frac{V_0\rho_0}{V_t} = \frac{\rho_0}{1+\gamma t} = \rho_0\,(1-\gamma t)$ neglecting higher powers of γ

or $\rho_0 = \rho_t(1-\gamma t)^{-1} = \rho_t(1+\gamma t)$

$\therefore$ density decreases with the increase of temperature.

20. Expansion of hollow vessel : Consider a hollow sphere. Let abc be a circular section (Fig. 7) passing through the centre of the sphere. Let the radius of the sphere and hence of the circular section be r at 0°C.

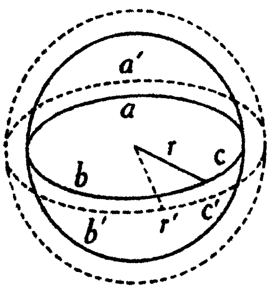


Fig. 7

Length of the *circumference* of this circular section at $0°C = 2\pi r$

Length of the *circumference* of the circular section at $t°C = 2\pi r\,(1+\alpha t) = 2\pi r'$

where r' = radius at t°C and α = Coefficient of linear expn.

$\therefore$ Radius of circular section at $t°C$ = radius of the sphere at $t°C = r' = r(1+\alpha t)$

Now vol. enclosed by the sphere at $0°C = V_0 = \frac{4}{3}\pi r^3$

" " " " $t°C = V_t = \frac{4}{3}\pi r'^3 = \frac{4}{3}\pi r^3\,(1+\alpha t)^3 = \frac{4}{3}\pi r^3\,(1+3\alpha t)$

neglecting higher powers of αt

$\therefore\ V_t = \frac{4}{3}\pi r^3\,(1+3\alpha t)$

$= V_0\,(1+gt)$ since $3\alpha = g$, the coefficient of cubical expansion of the walls of the vessel. Thus the hollow vessel expands in the same manner and to the same extent as a solid body of the same size, shape and material.

21. Expansion of Liquids : We are to consider only cubical or volume expansions in case of liquids. If V_0 be the volume of a given mass of liquid at 0°C, V_t its volume at t°C, and γ the coefficient of cubical expansion of liquid, then we have, $V_t = V_0(1+\gamma t)$.

Liquids expand to a much greater extent than solids. But its measurement becomes rather difficult owing to the expansion of the containing vessel. The expansion considered ignoring the expansion of the vessel, is called apparent expansion. Absolute expansion of a liquid is its actual or real expansion.

22. Coefficient of Apparent and Absolute expansion of a liquid : The coefficient of apparent expansion of a liquid is the coefficient of the expansion of liquid relative to the containing vessel, while the coefficient of absolute expansion refers to the expansion of the liquid which would be observed if the liquid were contained in a vessel incapable of expansion.

The coefficient of absolute expansion of a liquid in any **vessel is the actual** increase in volume of unit volume for a rise in **temperature of 1°C.**

It is denoted by the symbol γ. Its mean value between 0°C and 100°C, is constant for a given liquid and independent of the material of the containing vessel.

The **coefficient of apparent expansion** of a liquid in any vessel is the apparent increase in volume of unit volume for a rise of temperature of 1°C.

It is denoted by the symbol γ'. Its value for a given liquid may be different, depending on the material of the vessel containing it.

23. Relation between two coefficients : To determine the relation between the coefficients of real and apparent expansion of a liquid, let us consider the case of a liquid filling completely a flask at 0°C, the volume of the flask and liquid being V. Let v be the volume of liquid expelled when heated to 1°C. Then neglecting the expansion the vessel v denotes the apparent expansion of the liquid. If γ, γ' be coefficients of real and apparent expansion of the liquid, and g the coefficient of cubical expansion of the material of the flask (glass),

v = Actual increase in volume of liquid at 1°C − increase in volume of flask at 1°C

$$= V.\gamma - Vg = V(\gamma - g)$$

$$\text{or,}\quad \gamma - g = \frac{v}{V} = \frac{\text{apparent increase in volume}}{\text{original vol.} \times \text{rise of temp. (1°C)}} = \gamma'$$

$$\therefore\quad \gamma = \gamma' + g.$$

That is, the coefficient of absolute exp. of the liquid is equal to the sum of the coefficient of app. exp. of the liquid and the coefficient of cubical expansion of the vessel.

This relation is not strictly accurate. The error in using the relation increases for a large temperature rise. It is, however, true for all practical purposes for ordinary high temperature.

N. B. Relation between density and coefficient of cubical expansion (γ) of a liquid :—(Deduction same as in Art. 19). If ρ_1, V_1 be the density and volume of a given mass of liquid at t_1°C, and ρ_2, V_2 corresponding values of same mass at t_2°C, then $V_2 = V_1\,\{1+\gamma(t_2-t_1)\}$ and hence, $\rho_1 = \rho_2\{1+\gamma(t_2-t_1)\}$.

24. Correction due to exposed column in a thermometer : For measuring temperatures by a thermometer it is observed that the upper part of the stem is at a temperature somewhat lower than that of the bulb.

Hence, the observed temperature requires correction.

Let t_1 be the observed temperature and let t be the true temperature of the bulb *i.e.*, of the body.

Let t_2 be the average temperature of the stem containing n divisions and exposed to air.

Then n divisions would become $n\{1+\gamma'(t-t_2)\}$, if the temperature of the thermometer were uniform throughout.

Here γ' is the coefficient of apparent expansion of mercury in |

Then the increase in the number of divisions is $n.\gamma'(t-t_2)$

This is very nearly equal to $n.\gamma'(t_1-t_2)$

∴ the true temperature of the body is given by $t=t_1+n\gamma'(t_1-t_2)$.

25. Measurement of coefficient of expansion of a liquid : Dilatometer Method.

Dilatometer Method. A glass bulb provided with a long graduated stem is filled with the liquid at initial temperature t_1°C up to a certain mark in the stem and then heated to t_2°C. The column of the liquid inside the stem rises up and stands at a particular mark at the temperature t_2. Let v_1 be the original volume of the liquid up to the mark to which it stands at the temperature t_1 and v_2 the volume up to the mark at t_2. If the instrument be calibrated at t_1, then v_1 is the true volume of the liquid at t_1 and the true volume of the liquid at t_2 is not v_2 but $v_2\{1+g(t_2-t_1)\}$ where g is the coefficient of cubical expansion of glass.

Thus the coefficient of absolute expansion of the liquid is given by

$$\gamma = \frac{v_2\{1+g(t_2-t_1)\}-v_1}{v_1(t_2-t_1)} = \frac{v_2-v_1}{v_1(t_2-t_1)} + \frac{v_2}{v_1}\,.\,g$$

In our experiment we do not actually get the change in volume *e.g.* v_2-v_1 for a rise of temperature (t_2-t_1), but we get the length l cms. through which the column rises through this rise of temperature. If s be the area of the cross-section of the tube, then the increase in volume for a rise of temperature (t_2-t_1) is $l\times s$ c.c. Thus if the original volume v_1 be known, the coefficient is easily determined by the formula given above.

Again, if we neglect the expansion of the vessel, the apparent coefficient of expansion is determined by the expression

$$\gamma' = \frac{v_2-v_1}{v_1(t_2-t_1)}$$

where γ' is the coefficient of apparent expansion.

26. The Weight Thermometer Method :

A weight thermometer consists of a glass bulb with a bent stem of narrow bore as shown in figure 8. It is first washed, cleaned, dried and weighed empty. Let its weight be W. The bulb is then filled with the liquid to be experimented upon, by dipping the open end of the stem into the liquid and by alternately heating and cooling the bulb. The process is repeated several times and the bulb together with the stem is completely filled.

The bulb is then placed in a water bath with the stem under the liquid and allowed to stand for sometime until it attains a constant temperature t_1°C, of the bath. The bulb is then taken out of the bath, its outer wall carefully dried and then weighed. Let this weight be W_1. The bulb is then placed in the water bath and its temperature is raised to t_2°C, and some of the liquid is expelled. The bulb is then taken out of the hot bath and allowed to cool down to the room temperature. It is then dried and weighed again. Let this weight be W_2.

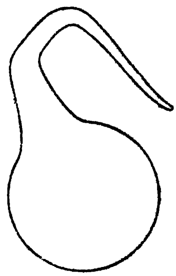


Fig. 8

Mass of the liquid completely filling the weight thermometer at $t_1°C = W_1 - W = m_1$ (say).

Mass of the liquid completely filling the weight thermometer at $t_2°C = W_2 - W = m_2$ (say).

Let the density of the liquid at $t_1°C = \rho$.

Now, volume of the liquid filling weight thermometer at $t_1°C = \dfrac{m_1}{\rho}$

= Internal volume of weight thermometer at t_1°C = Internal volume of weight thermometer also at t_2°C, since expansion of the bulb is to be ignored when we consider apparent expansion.

$\therefore$ Mass m_2 of liquid occupies at t_1°C a volume $= \dfrac{m_2}{\rho}$.

and „ „ „ „ „ „ t_2°C a volume $= \dfrac{m_1}{\rho}$.

Thus a vol. of liquid, $\dfrac{m_2}{\rho}$ of mass m_2 at t_1°C appears to occupy a vol. $\dfrac{m_1}{\rho}$ at t_2°C. $\therefore$ Apparent increase in vol. of liquid $= \dfrac{m_1}{\rho} - \dfrac{m_2}{\rho}$.

If γ' be coefficient of apparent expansion of the liquid

$$\gamma' = \frac{\text{apparent increase in vol.}}{\text{original vol.} \times \text{rise of temp.}} = \frac{\dfrac{m_1}{\rho} - \dfrac{m_2}{\rho}}{\dfrac{m_2}{\rho}(t_2 - t_1)} = \frac{m_1 - m_2}{m_2(t_2 - t_1)}$$

$$= \frac{\text{mass expelled}}{\text{mass left} \times \text{rise of temp.}} \quad \ldots\ldots(1)$$

Knowing m_1, m_2, t_1 and t_2, γ' can be found out.

If expansion of the bulb be considered, then a volume $\frac{m_2}{\rho}$ of liquid of mass m_2 at t_1°C will actually occupy a volume $\frac{m_1}{\rho}\{1+g(t_2-t_1)\}$ at t_2°C, where g is the coefficient of cubical expansion of glass material of the bulb.

Then coefficient of absolute expansion γ of the liquid is given by

$$\gamma = \frac{\frac{m_1}{\rho}\{1+g(t_2-t_1)\} - \frac{m_2}{\rho}}{\frac{m_2}{\rho}(t_2-t_1)} = \frac{m_1-m_2}{m_2(t_2-t_1)} + \frac{m_1}{m_2}g \qquad \ldots (2)$$

Knowing m_1, m_2, t_1 and t_2 and using value of g, γ can be found out. In (2) m_1g/m_2 may be taken as g, since m_1/m_2 is nearly equal to unity, *i. e.*, we can write $\gamma = \frac{(m_1-m_2)}{m_2(t_2-t_1)} + g$.

26a. Alternative method of Calculation :

Let V_1 and V_2 be volumes of the bulb and hence of the liquid at t_1°C and t_2°C respectively in the above experiment and let ρ_1 and ρ_2 be corresponding densities of the liquid,

Then $m_1 = V_1.\rho_1$; $m_2 = V_2.\rho_2$. so that, $\frac{m_1}{m_2} = \frac{V_1}{V_2} . \frac{\rho_1}{\rho_2}$.

Now, $V_2 = V_1\{1+g(t_2-t_1)\}$, where g = coefficient of expansion of the bulb, and $\rho_1 = \rho_2\{1+\gamma(t_2-t_1)\}$, where γ = coefficient of real expansion of liquid

$\therefore \frac{m_1}{m_2} = \frac{1+\gamma(t_2-t_1)}{1+g(t_2-t_1)} = 1+(\gamma-g)(t_2-t_1) = 1+\gamma'(t_2-t_1)$. Since $\gamma-g=\gamma'$ the coefficient of apparent expansion of the liquid.

Then $1+\gamma'(t_2-t_1) = \frac{m_1}{m_2}$ or $\gamma' = \frac{m_1-m_2}{m_2(t_2-t_1)}$. Using the relation $\frac{m_1}{m_2} = 1 + (\gamma-g)(t_2-t_1)$, the value of γ also can be found out assuming the known value of g

Note 1. The weight thermometer can be used to find the cubical expansion of a solid indirectly by enclosing a specimen of the solid inside the bulb.

Note 2. An apparatus called Pyknometer is also used to find the coefficient of apparent expansion of a liquid the principle being the same as in weight thermometer.

27. Hydrostatic method :

The method consists in the determination of the loss of weight of a solid when weighed in the liquid at two different temperatures the coefficient of cubical expansion of the solid being known.

A solid sinker of known weight W in air is weighed in the liquid at t_1°C and t_2°C ($t_1 < t_2$) by a hydrostatic balance and the apparent mass of the sinker determined in each case.

Let W_1, W_2 be the apparent weights of the sinker at t_1 and t_2°C. Then weight of the liquid displaced at t_1°C, by Archimedes' principle $= W - W_1 = m_1$ (say) $\therefore$ its volume at t_1°C, $= \frac{m_1}{d_1}$, where d_1 = density of the liquid at t_1°C. Evidently $\frac{m_1}{d_1} = V_1$ = the volume of the sinker at t_1°C. Similarly the volume of the liquid displaced at t_2°C $= \frac{W - W_2}{d_2} = \frac{m_2}{d_2}$ where $W - W_2 = m_2$ and d_2 = density of the liquid at t_2°C,

Let $\frac{m_2}{d_2} = V_2$ = volume of the sinker at t_2°C.

$$\text{Now } \frac{m_1}{d_1} = V_1\,; \quad \frac{m_2}{d_2} = V_2, \quad \therefore \frac{m_1}{m_2} = \frac{V_1 d_1}{V_2 d_2}$$

But $V_2 = V_1\{1 + g(t_2 - t_1)\}$ where g = coefficient of cubical expansion of the material of the sinker.

And $d_1 = d_2\{1 + \gamma(t_2 - t_1)\}$, where γ = coefficient of real expansion of the liquid.

$$\text{Hence } \frac{m_1}{m_2} = \frac{1 + \gamma(t_2 - t_1)}{1 + g(t_2 - t_1)} \text{ or } \gamma = \frac{m_1 - m_2}{m_2(t_2 - t_1)} + \frac{m_1}{m_2} g = \frac{m_1 - m_2}{m_2(t_2 - t_1)} + g$$

$$\therefore \quad \gamma = \gamma' + g \quad \because \frac{m_1}{m_2} \text{ is nearly equal to 1.}$$

If g be assumed, γ can be found out. Neglecting the value of g, $\gamma' = \frac{m_1 - m_2}{m_2(t_2 - t_1)}$, whence γ' can be obtained.

28. Hydrostatic Balancing-Columns method for γ:

Dulong and Petit's Experiment: A direct method in which the cubical expansion of the vessel (g) need not be assumed was first given by Dulong and Petit for the determination of the absolute expansion of a liquid. It depends on the hydrostatic principle that the heights of two liquid columns in an U-tube which produce equal pressures are inversely proportional to their densities, a principle independent of the diameters of the tubes and therefore of their expansion when heated.

The apparatus used for this purpose consists of a glass tube BACD bent twice at right angles with its limbs AB, CD surrounded by two wide glass jackets J_1, J_2 fitted with corks and having side tubes for entrance and exit of water and steam respectively. The whole thing is fitted vertically on a vertical stand. The tube is filled with the experimental liquid, (say mercury) so that the levels of the liquid in the two limbs are visible above the jackets. (Fig. 9).

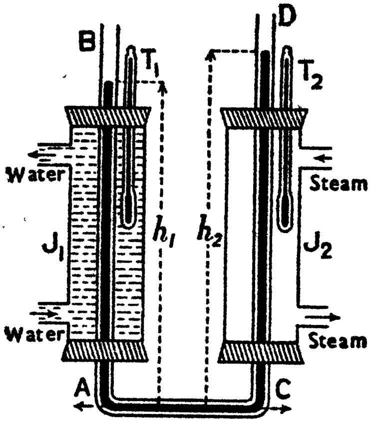


Fig. 9

The temperature of the liquid in one of the limbs is kept constant by filling the jacket J_1 round it with water, and that in the other limb is raised by passing steam through the jacket J_2 from a boiler. The temperatures of the liquid t_1, t_2 in the cold and hot limbs are noted by thermometers T_1 and T_2 suitably placed and the heights h_1, h_2 of the liquid in them are carefully observed by a cathetometer when a steady state has been attained. Care should always be taken to keep the horizontal portion of the tube cool by wrapping it with pieces of cloth constantly soaked in water, otherwise flow of heat may take place through it from the hot to the cold limb and vitiate the result.

If P be the atmospheric pressure, then since the pressure at the ends of the horizontal portion AC of the tube are equal, we have,

$$P + h_1\rho_1 g = P + h_2\rho_2 g \quad \dots \quad (1)$$

where h_1 and h_2 are the heights of the liquid columns at temperatures t_1 and t_2, and ρ_1, and ρ_2 are the densities of the liquid at the corresponding temperatures.

From $\dfrac{h_2}{h_1} = \dfrac{\rho_1}{\rho_2} = 1 + \gamma(t_2 - t_1)$, where γ is the coefficient of absolute expansion of mercury. Therefore $\dfrac{h_2 - h_1}{h_1(t_2 - t_1)}$

The coefficient of expansion thus determined is independent of the expansion of glass, since the relation of the heights of liquid columns to the densities of the liquid does not depend on the cross-section of the glass tube and hence on its expansion.

Note :—A cathetometer is a telescope attached with a vernier such that the combination can be slided along a scale graduated on a vertical metal stand.

28a. Errors and precautions :

(1) The horizontal portion of the tube should be made narrow otherwise ransference of heat from the hot to the cold mercury will take place.

(2) The heights of the mercury columns should be measured from the axis of the horizontal portion of the tube.

(3) The two free mercury surfaces being at different temperatures have different surface tensions. The error is avoided to some extent by using wide-mouthed glass tubes.

(4) The thermometer may not give the correct mean temperature of the hot column since there is no efficient stirring arrangement. As portion of hot column is outside the jacket an error in its temperature occurs.

29. Regnault's method : Two wide vertical tubes AB and CD are connected at the top by a horizontal tube of fine bore and at the bottom by a tube BEFGHD in the form of an inverted U which is again connected at the top to a pump by the side tube K for forcing air into FG. The horizontal tube at the top is provided with a small opening at L so that the excess of mercury will flow out and its upper surfaces in the vertical tubes will be at the same level. Mercury is poured into the limbs AB and CD and sufficient pressure is produced by the pump until the levels of mercury occupy the positions somewhere in the middle of EF and HG. If the temperature of mercury in the limbs be the same, levels of mercury in the central tubes would be also the same.

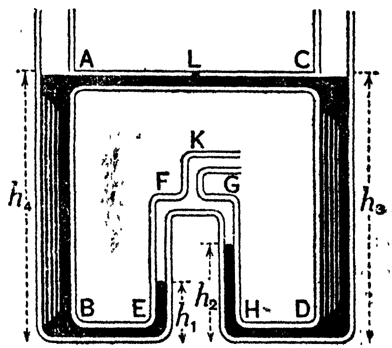


Fig. 10

Let the temperature of AB be raised from t_1°C to a certain higher steady temperature t_2°C, the mercury in the bulbs EF, GH and DC being kept at constant lower temperature t_1. In this condition, level of mercury in EF will be lower than that of mercury in HG. Since other conditions being same, cold column h_1 in EF balances a hot column of liquid h_4 in AB, whereas cold column h_2 in GH balances a cold column of liquid h_3 in CD of same height as h_4.

Let P = atmospheric pressure ; P_1 = pressure of compressed air above E and H ; ρ_1, ρ_2 = densities of mercury at t_1°C and t_2°C respectively.

Since the portion BE and HD are coaxial the pressures at B, E, H and D are measured from the same axis.

Now, pressure at H = pressure at D or $P_1 + h_2.\rho_1.g = P + h_3.\rho_1.g$

or $P_1 - P = (h_3 - h_2)\rho_1.g$ ... ... (1)

Again, pressure at E = pressure at B.

or $P_1 + h_1\rho_1 g = P + h_4\rho_2 g$. or $P_1 - P = (h_4\rho_2 - h_1\rho_1)g$ ....(2)

From (1) and (2) above, $(h_3 - h_2)\rho_1 g = (h_4\rho_2 - h_1.\rho_1)g$

or $h_3\rho_1 - h_2.\rho_1 = h_4.\rho_2 - h_1\rho_1$ or $(h_3 - h_2 + h_1).\rho_1 = h_4.\rho_2$

$$\text{or} \quad \frac{\rho_1}{\rho_2} = \frac{h_4}{h_3 - h_2 + h_1}$$

But $\dfrac{\rho_1}{\rho_2} = 1 + \gamma(t_2 - t_1)$; $\therefore$ $1 + \gamma(t_2 - t_1) = \dfrac{h_4}{h_3 - h_2 + h_1}$

$$\therefore \gamma = \frac{h_4 - h_3 + h_2 - h_1}{(h_3 - h_2 + h_1)(t_2 - t_1)}$$

Since $h_3 = h_4$ in actual experiment, $\gamma = \dfrac{h_2 - h_1}{(h_3 - h_2 + h_1)(t_2 - t_1)}$.

Knowing all quantities of the right hand side, the value of γ, the coefficient of absolute expansion of the liquid can be calculated The heights of various columns of liquid from the common axis of BE and HD were measured by a cathetometer. In Ragnault's method, error due to unequal surface tension was eliminated. Efficient stirring arrangement was provided. Heights of liquid columns were accurately measured. Temperature of hot jacket was taken by a mercury thermometer applying necessary corrections.

30. Expansion of Water :

The behaviour of water is anomalous in the region from 0°C to 4°C If water at 0°C be heated gradually, a contraction of water goes on up to 4°C. Above 4°C, water goes on expanding systematically with rise in temperature. This peculiarity is explained by the assumption of some complex structure of water molecules near 4°C. The assumption however seems to have no great justification.

The behaviour of water can be qualitatively studied by **Hope's apparatus**, and by **constant volume dilatometer** quantitatively. The latter consists of a bulb with a long graduated stem. Part of the bulb is filled with mercury. The rest of the bulb and a part of the stem is filled with air-free pure water. The volume of mercury is about 1/7th part of the volume of the bulb, so that expansion of mercury causing inside space to decrease is equal to the expansion of the bulb causing inside space to increase for a given rise of temperature. As the effects of two expansions cancel each other, the volume of water can change with temperature independently of the expansion of the bulb.

30a. Mathematical Proof: Let V = volume of the bulb of the dilatometer up to the junction of the bulb and the stem.

v = volume of the bulb filled with mercury.
γ = coefficient of real expansion of mercury.
g = coefficient of cubical expansion of glass.

Suppose the bulb with mercury is heated through t°C.

Then exp. of the bulb = $V.gt$
,, ,, mercury = $v\gamma t$.

But $v\gamma t = Vgt$ or $v = \frac{g}{\gamma}V = \frac{\cdot0000240}{\cdot000188}V = \frac{1}{7}V.$

30b. Experiment with Water: The dilatometer being constructed as above, the rest of the bulb and a part of the graduated stem is filled with pure water. The bulb is then placed in a bath of water at 10°C. Temperature is gradually lowered by adding ice, by a step of about 1° and for each temperature, volume V of water is noted from the stem. Plotting V against t a parabolic curve is obtained, the apex of which corresponds to the minimum volume of water taken for the experiment. In the curve, this minimum volume again is found to be at 4°C. Hence, the fact that water has maximum density at 4°C is proved.

QUESTIONS

1. Distinguish between the real and the apparent expansion of a liquid contained in a glass vessel. [C. U. 1928, '32, '45]

2. Prove that $\gamma = \gamma' + g$

where γ is the coeff. of absolute expansion of a liquid, γ' its coefficient of apparent expansion, and g, the coeff. of cubical expansion of the material of the vessel. [C. U. 1922, '33, '38.]

3. Describe the weight thermometer method of determining the apparent coeff. of expansion of a liquid. [C. U. 1933, '45.]

4. Describe a method of determining directly the coeff. of absolute expansion of a liquid. Deduce the working formula. [C. U. 1927, '49.]

5. Describe Regnault's method of finding the absolute expansion of mercury and deduce the formula. [C. U. 1936.]

EXAMPLES

1. Given that the coefficient of expansion of mercury is ·00018 and that the coefficient of apparent expansion of mercury in glass is $\frac{1}{6480}$, calculate the difference in length of a glass-tube 10 ft. long at 0°C and 20°C. [Ans. ·0016 ft]
[C. U. 1910.]

2. A glass bulb contains some mercury. If the coefficients of cubical expansion of glass and mercury are 0·000025 and 0·00018 respectively, what fraction of the whole volume of its empty part should remain constant when the glass and mercury are heated to the same temperature? [C. U. 1928.]

Let V c.c. be the whole volume of the glass vessel and let v c.c. be the volume of the vessel occupied by mercury.

Then, in order that the volume of the empty part of the vessel should remain constant, the expansion of the glass vessel and that of mercury when heated through the same range of temperature should be equal.

Then, we have

$$V \times \cdot 000025t = v \times \cdot 00018t\ ; \quad \text{Or} \quad v = \frac{\cdot 000025}{\cdot 00018} = \cdot 139\ V \text{ or } \frac{1}{7}\ V \text{ nearly.}$$

$\therefore$ Constant fraction of whole volume $= \frac{6}{7}$.

3. A cylindrical tube of glass is divided into 300 equal parts. is loaded with mercury and sinks into the 50th division from the top in water at 10°C. To what division will it sink in water at 50°C ? The ratio of the volumes of a given mass of water at these temperatures is as 1·000268 to 1·012 and the coefficient of cubical expansion of glass 0·000024. [C. U. 1926]

The weight of water displaced by the cylindrical tube at 10°C $= 250 \times V \times d_1 \times g$. where V = volume of each part ; and the weight of water displaced by the tube at 50°C $= x \times V(1 + \cdot 000024 \times 40) \times d_2 \times g$

where x is the number of divisons of the tube below water at 50°C ; and d_1, d_2, the densities of water at 10°C and 50°C respectively and g the acceleration due to gravity.

Since the tube floats in water at these temperatures the weight of the displaced water at each of these temperatures is equal to the weight of the body.

Therefore, we have

$$250 \times V \times d_1 \times g = x \times V(1 + \cdot 00096) \times d_2 \times g$$

$$= 250 \times \frac{1 \cdot 01205}{1 \cdot 000268} \times \frac{1}{1 \cdot 00096} = \frac{250 \times 1 \cdot 01}{1 \cdot 00096} = 252 \cdot 2$$

4. 6·2 grams of mercury overflows when a glass weight thermometer completely filled at 0°C with 400 grams of mercury is heated to 100°C. Calculate the coefficient of absolute expansion of mercury if the coefficient of linear expansion of the material of the bulb be 0·000008. [C. U. 1938.]

In a weight thermometer, the apparent expansion of a liquid γ' is given by

$$\gamma' = \frac{m_1 - m_2}{m_2(t_2 - t_1)} = \frac{6 \cdot 2}{(400 - 6 \cdot 2)100} = \frac{6 \cdot 2}{393 \cdot 8 \times 100} = \cdot 000157$$

But $\gamma = \gamma' + g$, where γ is the absolute expansion and g, the expansion of glass.

Or $\gamma = \gamma' + 3\alpha$, where α is the coefficient of linear expansion of glass.

$= \cdot 000157 + 3 \times \cdot 000008 = \cdot 000157 + \cdot 000024 = \cdot 000181.$

5. A weight thermometer weighs 40 gm. when empty, and 490 gm. when filled with mercury at 40°C. On heating it to 100°C., 6·85 gms. of mercury escape. Calculate the coefficeint of expansion of the glass. The coeff. of real expansion of mercury is to be taken as 0·000182. [C. U. 1945.]

Proceed as in Ex. 4. [Ans. ·0000274]

6. A glass bulb whose volume at 0°C equals to 10 c.c. is joined to a stem of diameter 2 mm. and filled with mercury at 0°C. What length of stem will the liquid occupy at 60°C ?

Coeff. of expansion of mercury is ·00018 and that of glass is ·000027.

Let the required length of the stem $= l$.

Then the volume of mercury at 60° = the volume of the bulb and the stem occupied at 60°C.

That is $\quad 10\,(1 + \cdot 00018 \times 60) = 10(1 + \cdot 000027 \times 60) + l.\pi(\cdot 1)^2(1 + \cdot 000027 \times 60)$

or $\quad 10 \cdot 108 = 10 \cdot 0162 + \cdot 03142l \quad$ or $\quad l = \dfrac{\cdot 0918}{\cdot 03142} = 2 \cdot 9$ cms.

7. Find the reading of a thermometer whose bulb is immersed in boiling water at 100°C and the stem is in the air at an average temperature of 10°C. The coefficient of apparent expansion of mercury = ·000155.

From Art. 24
we have $t = t_1 + n(t_1 - t_2)\gamma' = t_1 + n(t - t_2)\gamma'$ (approx.)
Here $t = 100$, $n = (t_1 - 0)$ since the bulb only is immersed in water and $t_2 = 10°$
$100 = t_1 + t_1 \times (100 - 10) \times \cdot000155 = t_1 + 90 \times \cdot000155 \times t_1$ $\therefore$ $t_1 = 98°6C$.

8. A solid weighs 42·78 gm. in air. When immersed in a liquid successively at 30° and 70°C its weights are 37·68 and 37·765 gms. respectively. Find the coefficient of real expansion of the liquid taking the coefficient of linear expansion of the solid as 0·00002.

$$\gamma = \frac{W_1 - W_2}{(W - W_2)(t_2 - t_1)} + 3\alpha = \frac{\cdot085}{5\cdot015 \times 40} + \cdot00006 = \cdot00042.$$

[Consult Art. 27.]

CHAPTER III

EXPANSION OF GASES

31. Introduction: While studying about expansion of solids and liquids we have not considered the pressure to which the substance is subjected for we find here no change of volume even for a great change of pressure.

But in the case of a gas a very small change of pressure produces an appreciable change in volume.

So to state the condition of a gas, its *volume*, *pressure* and *temperature* must be considered. A change in any one of these affects the others, and the relationships between the three variables are called the **Gas Laws**. The laws are given below :

31a. Boyle's Law : The law states that at constant temperature, the volume of a given mass of a gas is inversely proportional to the pressure to which it is subjected.

If p and v denote the pressure and volume of a given mass of a gas, then $v \propto \frac{1}{p}$ when the temperature t is constant.

or $pv = k$ where k is a constant, depending on the mass of the gas taken, *i. e.* according to Boyle's law, for a fixed mass of gas at constant temperature the product pressure × volume is constant.

The law is true for any gas at low pressures, but the law breaks down at very high pressures.

A curve showing relation between p and v of a gas at a constant temperature "t" is a hyperbola and is called an *isothermal* curve corresponding to a temperature t.

31b. Charles' Law : The law states that at constant pressure the volume of a given mass of any gas increases by a constant fraction of its volume at 0°C for each rise of temperature of 1°C.

If v_0 be the volume of the gas at 0°C, then the increase in the volume is $v_0\gamma_p t$ for a rise of temperature t°C, where γ_p is the constant fraction or the coefficient of expansion of any gas at constant pressure. Hence, the volume v_t of the gas at t°C is given by

$$v_t = v_0 + v_0\gamma_p t = v_0(1+\gamma_p t)$$

$$\therefore \quad \gamma_p = \frac{v_t - v_0}{v_0 t} \quad \ldots \quad (1)$$

This coefficient γ_p has been found to be $\frac{1}{273}$ or about ·003665.

From above we have $v_t = v_0(1+\gamma_p t)$.

The constant fraction γ_p is called the **volume coefficient** of gas. It is defined as the increase in volume of unit volume at 0°C for one degree centigrade rise in temperature, when a definite mass of that gas is heated at constant pressure.

If V_1 be the volume of a given mass of gas at t_1°C, and V_2 the volume at t_2°C at constant pressure, then to find a relation between them we must refer to volume V_0 of the gas at 0°C, as follows.

We have $V_1 = V_0(1+\gamma_p t_1)$ and $V_2 = V_0(1+\gamma_p t_2)$

$$\therefore \quad \frac{V_1}{V_2} = \frac{1+\gamma_p.t_1}{1+\gamma_p.t_2} = \frac{1+\frac{t_1}{273}}{1+\frac{t_2}{273}} = \frac{273+t_1}{273+t_2}.$$

It will be wrong to use the relation $V_2 = V_1\{1+\gamma_p(t_2-t_1)\}$ in deducing which higher powers of γ_p are to be neglected. This can not be done as the expansion of gas is very large, compared with that of solids and liquids.

31c. Law of Pressure : The law states that at constant volume, the pressure of a given mass of gas increases by a constant fraction of its value at 0°C for each rise of 1°C.

Hence, if p_0 be the pressure at 0°C and p_t, the pressure at t°C and γ_v the constant fraction called the **pressure coefficient**,

we have, $p_t = p_0 + \gamma_v.p_0.t = p_0(1+\gamma_v t)$ whence $\gamma_v = \frac{p_t - p_0}{p_0.t}$.

The value of pressure coefficient γ_v has been also found to be 1/273 or about ·003665.

32. The Absolute Scale of Temperature : By Charles' Law, we have, $V_t = V_0\left(1+\frac{t}{273}\right)$. If the gas be gradually cooled to $-273°C$, so that $t = -273°C$, then

$$V_{-273} = V_0\left(1-\frac{273}{273}\right) = 0$$

Thus at this temperature of $-273°C$ the volume of any gas is reduced to zero, which is practically impossible.

The temperature $-273°C$ is considered as the lowest temperature conceivable. It is called the **absolute zero** of the gas thermometer.

If a scale of temperature be constructed with $-273°C$ as its zero and having each degree equal to that on a centigrade scale, the temperature t on the Centigrade Scale becomes equal to $t+273°$ *i. e.* $T°$ on the Absolute Scale. The Scale of temperature is called the **Absolute Scale of Temperature.**

The value of the absolute zero in Fahrenheit scale is $-459°·4F$.

The temperature of the absolute zero on the constant valume Hydrogen Scale is $-273°·03C$ and that on the absolute thermo-dynamic Centigrade Scale is $-273°·2C$.

The symbol $T°A$ or $T°$ Abs. denotes the temperature on the constant volume Hydrogen Scale measured from absolute zero, approximately equal to $-273°C$.

The symbol $T°K$ (degrees Kelvin) denotes the temperature on the absolute thermo-dynamic Scale.

33. Other forms of Charles' Law and Pressure Law :

We have by Charles' Law

$\frac{V_1}{V_2} = \frac{237+t_1}{273+t_2} = \frac{T_1}{T_2}$, where T_1 and T_2 denote absolute temperatures corresponding to centigrade temperatures t_1 and t_2.

From above, $\frac{V_1}{T_1} = \frac{V_2}{T_2} =$ a constant when P is constant.

$\therefore$ $V \propto T$ *i.e.*, volume is proportional to the absolute temperature.

Similarly from the law of Pressure we have $\frac{P_1}{T_1} = \frac{P_2}{T_2} =$ a constant when V is constant ; or $P \propto T$. *i. e.*, pressure is proportional to the absolute temperature.

34. The Gas Laws Combined : Let P, V and T denote the pressure, volume and absolute temperature of a given mass of gas.

Then by Boyle's Law, $V \propto \frac{1}{P}$ when T is constant

and by Charles' Law, $V \propto T$ when P is constant

$\therefore \quad V \propto \frac{T}{P}$ when both P and T vary *i. e.* $V = R\frac{T}{P}$ where R is a constant or $PV = RT$. This is the *Equation of State* of a gas

If for a given mass of gas, pressure be P_1 and volume V_1 at T_1°A, and corresponding values be P_2 and V_2 at T_2°A.

then $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = \frac{P_0V_0}{T_0}$ where P_0, V_0 and T_0 are pressure, volume and temperature in absolute scale, of the given mass of a gas, at N. T. P.

Since the density of a given mass of gas at a given pressure and temperature varies inversely as its volume, the relation $P_1V_1/T_1 = P_2V_2/T_2 = P_0V_0/T_0$ may be written in the form $P_1/\rho_1T_1 = P_2/\rho_2T_2 = P_0/\rho_0T_0 =$ a constant, where ρ_0, ρ_1, and ρ_2 are densities at T_0, $T_1{}^{\circ}$ and $T_2{}^{\circ}$A.

If pressure remains constant *i.e.*, if $P_1 = P_2$, $\rho_1T_1 = \rho_2T_2$.

Thus density of a gas at constant pressure varie inversely as its absolute temperature.

35. Value of the Gas Constant : The constant R in the relation $PV = RT$ is called the **Gas constant**. Its value is different for different masses of a given gas. It is also different as a rule for a given mass of different gases. But R is constant and possesses the same value for **gram-molecular mass** of all gases. It is then called **Molar gas constant.**

35a. Calculation of molar gas Constant : We know that the gram-molecular volume of any gas at N. T. P. = 22·4 litres. If P_0 V_0 and T_0 be pressure, volume and temperature, of 1 gm. mol. of any gas at N. T. P.

then for a gram-molecule we have

$$R = \frac{P_0V_0}{T_0} = \frac{76 \times 22400 \times 13{\cdot}6}{273} = 84740 \text{ gm-cm.} = 84740 \times 981$$

$= 8{\cdot}31 \times 10^7$ *ergs per gram. mole per degree cent.* $= 1{\cdot}99$ *cals.*

Thus for most gases, the value of R is very nearly equal to double the mechanical equivalent of heat, the value of which being equal to $4{\cdot}2 \times 10^7$ ergs per calorie.

Again if P=1 atmosphere. V=22·4 liters.

$$R=\frac{1\times 22{\cdot}4}{273}={\cdot}082 \text{ litre-atmos. per degree.}$$

$$1 \text{ litre-atmosphere}=1000\times 76\times 13{\cdot}6\times 981 \text{ ergs}$$
$$=1000\times 1{\cdot}013\times 10^{6} \text{ ergs}=1{\cdot}013\times 10^{9} \text{ ergs}$$

Thus the constant R for the gram-molecule (mole) of any gas is the same, can be proved directly from theoretical reasoning that by Avogadro's hypothesis, the volume occupied by the gm.-mol. of all gases is the same at N.T.P., being 22·4 litres.

36. Calculation of R for one gram of any given gas :

The constant R in this case may be denoted by R_1.

Value of R_1 *for one gram of hydrogen* $=\frac{P_0V_0}{T_0}$.

Here $P_0=76\times 13{\cdot}6\times 981$ dynes per sq. cm.

$V_0=1/{\cdot}00009$ c.c. at N.T.P. for one gram of hydrogen, since density of hydrogen at N. T. P. is ·00009 gm/c.c.

$$\therefore\quad R_1=\frac{76\times 13{\cdot}6\times 981}{273\times {\cdot}00009}=4{\cdot}13\times 10^{7} \text{ erg/gm/}^\circ\text{C}$$

$$R \text{ mol.}=4{\cdot}13\times 10^{7}\times 2=8{\cdot}26\times 10^{7} \text{ erg/mol./}^\circ\text{C}$$

Value of R_1 for one gram of Oxygen.

In this case $P=76\times 13{\cdot}6\times 981$ dynes per sq. cm.

$V=699{\cdot}9$ c.c. at N.T.P.

$$\therefore\quad R_1=\frac{P_0V_0}{T_0}=\frac{76\times 13{\cdot}6\times 981\times 699{\cdot}9}{273}$$
$$=2{\cdot}60\times 10^{6} \text{ ergs/gm/}^\circ\text{c}\;;\; R \text{ mol}=2{\cdot}6\times 32\times 10^{6}$$
$$=8{\cdot}32\times 10^{7} \text{ ergs/mol/}^\circ\text{c}$$

Thus we see that the **gas constant for unit mass** is different for different gases. But molar gas constants are the same.

37. To find the Coefficient of increase of pressure at constant volume due to a rise of temperature, or Pressure Coefficient of a gas by Jolly's constant volume air thermometer :

A glass bulb B (Fig. 11) filled with dry air is connected by a glass capillary tube bent twice at right angles to a wider tube A. The tube A is again connected by rubber pressure tubing with the wide reservoir tube D open at the top. The whole thing is then mounted on a vertical board with scales fitted on it and is so arranged that the reservoir tube can slide along a vertical rod fixed by the side of the vertical board.

The rubber tubing, part of the tube D and portion of A are filled with clean dry mercury so that the level of mercury in A comes up to a fixed mark C near the junction of A and the capillary tube. The bulb B and the space up to C thus contain a definite volume of air. The bulb is then placed in melting ice, the volume of the air in it diminishes so that level of mercury in the wider tube falls due to the diminution of pressure of air in the bulb. The difference of level in the two limbs of the tubes is noted by adjusting the level of mercury to the fixed point C by raising D, so that the volume of air in the bulb and the space up to C remains constant. The pressure corresponding to this difference of level together with the atmospheric pressure gives the pressure p_0 of the air in the bulb at 0°C, the atmospheric pressure being found by a Barometer before starting the experiment.

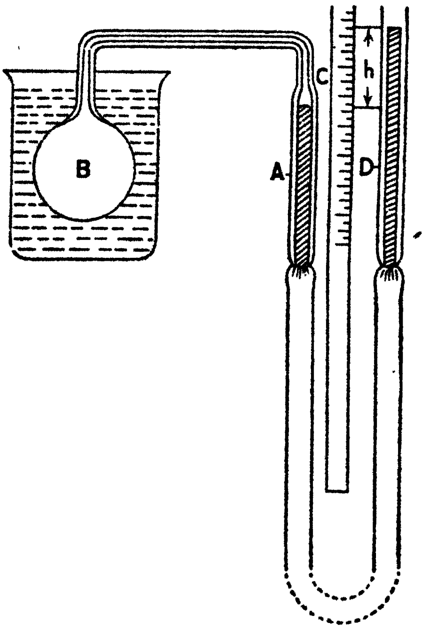


Fig. 11

The bulb is then placed inside a beaker of water whose temperature is raised in steps of 5°C or 10C° and is recorded by a thermometer held in the bath. The pressure increases due to the expansion of air inside the bulb and so the mercury level in the tube A falls and that in D rises. The levels of mercury in the tubes are then similarly adjusted until the volume of air in the bulb and the space up to C remains the same as before, and the total pressure p of air in the bulb is obtained at a certain steady high temperature t from the knowledge of the atmospheric pressure and the difference of mercury levels h in the tubes. The barometer is read again as a check, at the end of the experiment.

The coefficient of increase of pressue γ_v at constant volume is therefore determined by the formula,

$$\gamma_v = \frac{p_t - p_0}{p_0(t-0)} = \frac{p_t - p_0}{p_0 t} \tag{1}$$

For accurate results, starting with the water bath at room temperatue, the temperature of the bath is increased and recorded at nearly regular intervals and the corresponding pressures are obtained as described above.

A graph is then drawn with temperatures as abscissa and pressures as ordinates, taking 0°C and a suitable pressure at the origin. The graph is a **straight line**. Extrapolating *i.e.*, producing, the straight line either way it is found to cut the pressure axis at a certain point. The value of the pressure at this point which is pressure at 0°C is noted. Then taking from the line the value of pressure 'p_t' at some high temperature (say 100°C or 90°C) γ_v can be determined from the formula (1).

The increase of pressure per 1°C rise of temperature at constant volume, i.e., pressure coefficient is found to be $\frac{1}{273}$ or ·00367. It is also found to be same for all gases.

In the above experiment the "dead space" of the capillary tube causes an error, as the temperature of air in this space in much less than that of the air in the bulb. As the volume of air in the dead space is extremely small this error can be ignored. The expansion of the bulb introduces an error in γ_v ; this can be corrected by adding the value of "g" the coefficient of cubical expansion of glass to the observed value of γ_v.

Note : When the straight line showing $p-t$ relation is extrapolated, it is found to meet the temperature axis at -273°C, *the absolute zero* temperature. At this temperature pressure of a gas becomes zero.

38. Coefficient of increase of volume at constant pressure due to a rise of temperature or Volume Coefficient of a gas : Regnault's Constant Pressure Air Thermometer : The apparatus used for this purpose consists of a U-tube ADB (Fig. 12) with one limb graduated and ending in a bulb B, while the other limb A is open to atmosphere. At the bend of a U-tube a short glass tube with a stop-cock C is attached so as to withdraw required quantity of liquid when necessary.

The U-tube is placed inside a wide glass jacket fitted with a cork at the bottom through which the short glass tube with the stop-cock C passes.

The jacket is filled up with water so as to keep the bulb B inside water.

The temperature of the bulb *i.e.* of the air contained in the bulb is altered by passing steam through the copper tube E in the shape of an inverted U placed inside the jacket in such a way that the two limbs of the tube pass out through the cork.

The lower part of the U-tube is filled with Sulphuric acid while the bulb B and the upper part of B contains air.

To keep the air in the bulb at atmospheric pressure the liquid in both the limbs is adjusted to the same level by pouring in more of the liquid or letting out some by the stop-cock C.

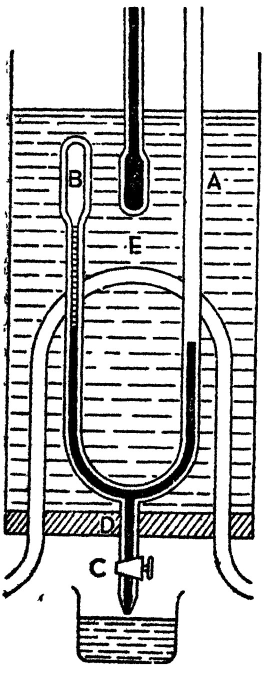


Fig. 12

Steam is passed through the copper tube E and the water in the jacket stirred well. As the temperature rises the air in the bulb expands and forces down the liquid which therefore rises in the limb A.

By adjusting the flow of steam the temperature is kept constant and the liquid in the limbs is adjusted to the same level by letting out the desired quantity of the liquid by the stop cock C. The air in the bulb is then at atmospheric pressure.

The volume of air enclosed in the bulb and a portion of the limb is read off from the graduations at regular intervals of temperatures at constant pressure, until six or seven sets of readings between room temperature and 100°C are obtained.

A graph is then plotted with temperatures as abscissa and volumes as ordinates, taking 0°C and a suitable volume at the origin. The graph is a **straight line.** By extrapolating *i. e.*, extending the line either way, the volume V_0 at 0°C and the volume V_t at some high temperature t (100°C) are obtained.

Then γ_p is determined from the formula $\gamma_p = \dfrac{V_t - V_0}{V_0 \cdot t}$

The value of γ_p is found to be nearly $\frac{1}{273}$ or ·00365 nearly.

Note : When the straight line showing $V-t$ relation is farther extrapolated backward it is found to meet the temperature axis at -273°C, called the absolute zero temperature. If a gas could continue to remain a gas, its volume would be zero at this temperature.

39. Equality of Pressure Coefficient and Volume Coefficient of a gas (that is $\gamma_v = \gamma_p$) : Consider a certain mass of gas to have volume v_0 and pressure p_0 at 0°C. Let the gas be heated to t°C at constant pressure p_0, so that volume is v_t ; then by Charle's law.

$v_t = v_0\ (1+\gamma_p t)$, where γ_p = volume coefficient of the gas. ... (1)

After this operation the gas has a volume v_t and pressure p_0 at t°C...(1a)

Let us suppose now, that the gas is heated from the initial condition at 0°C to t°C at constant volume v_0 so that pressure becomes p_t ; then by the law of pressure, $p_t = p_0\ (1+\gamma_v t)$···(2) where γ_v = Pressure Coefficient of the gas.

After this operation the gas has a volume v_0 and pressure p_t at t°C...(2a).

From (1a) and (2a), by Boyle's law, $p_0 v_t = p_t v_0$: substituting for v_t and p_t from (1) and (2)

$$p_0 v_0\ (1+\gamma_p.t) = p_0 v_0\ (1+\gamma_v t).\ or\ 1+\gamma_p t = 1+\gamma_v t$$

$$\therefore\ \gamma_p = \gamma_v.$$

40. General discussions about different gases : In a perfect gas it has been assumed that the diameters of the molecules are very small in comparison with the distance traversed between successive encounters *i.e.*, the molecules are considered as mere points and that the effects of mutual attraction or repulsion between the molecules may be neglected.

But, since ordinary gases such as CO_2, O_2, H_2 etc. do not strictly follow the conditions of the perfect gas, the laws which have been found to be applicable to perfect gases do not apply to the ordinary gases. The more a gas approaches a perfect gas the more it will obey the laws relating to it. The gases such as H_2, O_2, N_2 etc. are called permanent gases since they approach very nearly the conditions of a perfect gas, while CO_2 gas at ordinary temperatures differs widely from a perfect gas and therefore the conditions for a perfect gas are not strictly obeyed.

If we study these coefficients at constant volume and at constant pressure for different gases, we see that in ordinary gases such as CO_2 etc. the coefficients increase much more rapidly with the increase of pressure than in the case of air, N_2 etc. In the case of H_2 which very closely resembles a perfect gas, this change is not noticable until a very high pressure is reached.

It has also been found that in most cases the coefficient of expansion at constant pressure is greater than that at constant volume and at lower pressure these coefficients of expansion for different gases are almost exactly the same.

From all these observations we conclude that in a perfect gas these coefficients will be numerically equal to one another and in gases which deviate from a perfect gas they will differ considerably.

40a. Equation of the Isothermal of perfect gas in terms of its absolute temperature : The relation between the pressure and the volume of a given mass of gas at constant temperature as enunciated by Boyle is expressed as $pv = k$ ... ... (1)

The above equation represents a curve which is as already stated, called an *isothermal* curve of the gas. Let the equation (1) be the equation to the isothermal curve of a perfect gas at a certain temperature.

At different temperatures the isothermal curves will be different and the equation to the isothermals can be expressed in terms of the absolute temperature of the gas.

Let us now suppose that a particular volume v_1 of gas at 0°C and pressure p_1 is heated to t°C, the pressure meanwhile remaining constant and let the increase in volume be v_2.

Then, as long as the temperature of the gas remains constant at t°C, the product of the pressure and the volume of the gas will remain constant and equal to $p_1(v_1+v_2)$.

Again if p and v denote the pressure and volume of this quantity of gas at t°C we have, $pv = p_1(v_1+v_2) = p_1v_1\left(1+\dfrac{v_2}{v_1}\right)$

$$= p_1v_1(1+\gamma_p t) \quad \left[\because \frac{v_2}{v_1} = \gamma_p t\right]$$

$$or,\ pv = k(1+\gamma_p t) \qquad [\because p_1v_1 = k \text{ a constant}].$$

This is the equation to the isothermal curve at t°C.

Again, since $\gamma_p = 1/273$, the above equation reduces to

$$pv = k(1+t/273) = k.\frac{273+t}{273} = \frac{k}{273}.\mathrm{T}$$

Or, $pv = \mathrm{RT}$ *where* T *is the absolute temperature and* R *is the constant equal to* $k/273$.

41. Characteristics of a perfect gas :

(1) In a perfect gas the molecules are considered as mere points and that the effects of mutual attraction and repulsion between them may be neglected.

(2) It should obey Boyle's Law at all pressures and temperatures and also the relation $pv = \mathrm{RT}$.

(3) It should not be liquefied at any temperature other than the Absolute Zero

(4) It should have no viscosity.

(5) It should have no Joule-Thomson effect.

42. Gas Thermometers : A permanent or perfect gas obeying Boyle's law can be conveniently used as a thermometric substance. Such a thermometer in which the thermometric substance is a gas is called a gas thermometer. The construction of a gas thermometer depends on the fundamental principle of thermometry—utilising any effect of heat on matter which changes uniformly with temperature. This is possible in two ways in case of gases—(*a*) change of volume with temperature at constant pressure, (*b*) change of pressure with

temperature at constant volume. According to (*b*) we have constant volume gas thermometer and temperature is given by the relation $\frac{t}{100}=\frac{p_t-p_0}{p_{100}-p_0}$, where p_0, p_{100} are pressures at 0° and 100°C respectively and t is the temperature corresponding to pressure p_t. This relation fits well with the general equation of thermometers (Art. 3).

According to (*a*) above, we have constant pressure gas thermometer and temperature is known from the relation,

$$\frac{t}{100}=\frac{v_t-v_0}{v_{100}-v_0}$$ where v_0, v_{100} are volumes at 0° and 100°C respectively, and v_t the volume at unknown temperature t°C.

This equation also fits with the general equation for thermometers. (Art. 3).

43. Constant Volume Hydrogen Thermometer : Jolly's constant volume air thermometer has many defects. Besides these defects, it is a tedious job to have for accurate result to read barometer every time an observation is made.

Harkar and Chappius overcame many of these defects and difficulties by devising a constant volume thermometer using hydrogen as the thermometric substance. It consists of a cylindrical bulb A of platinum-iridium alloy (Fig. 13) of volume about 1 litre containing pure and dry hydrogen gas. The bulb is connected by a capillary tube to the manometer MLN. The constant volume mark is the tip of a pointer P_2 inside the limb M of the manometer. The manometer communicates with a reservoir of

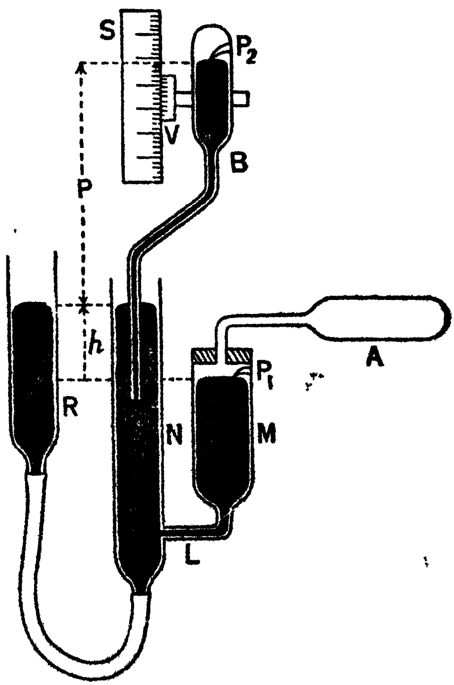


Fig. 13.

mercury R which can be raised or lowered, thus enabling mercury in M to touch the tip of the pointer P_1. Inverted over the mercury in the limb N is a barometer B which is so bent that the level of mercury in the wider upper part of tho barometer is vertically above the level of mercury in M. The barometer which is also free to move vertically carries a vernier which can be moved over a vertical scale (Portion shown in fig. 13). There is a second pointer P_2 in the vacuum space of the barometer and during work, the level of mercury in B is made to touch the tip of the pointer P_2. As the tubes N and R are open at the top, levels of mercury in them must be always at the same height

To measure pressure of gas at a certain temperature of the bath in which the bulb A is placed, the reservoir R is first adjusted to make mercury in M touch P_1. Then the barometer is raised or lowered so that mercury in B touches P_2. This affects the first adjustment slightly, and there is a fine-adjustment device (not shown in the figure) to make necessary allowance for it. The vertical distance between the tips of the pointers P_1 and P_2 gives the total pressure $(P+h)$ of the gas, where P is the atmospheric pressure and h is the difference of levels of mercurry in M and N. The levels of mercury in M and B may also be observed by a cathetometer. The difference of two levels at once gives the pressure of hydrogen at the temperature of the bath.

The obesrved pressure readings are converted into corresponding temperatures as follows. Let p_o, p_{100} denote the pressures indicated by the manometer at the ice point (T_0A) and the steam point $(T_o+100)A$ on the perfect gas scale having fundamental interval 100. Then since volume is constant,

$$\frac{p_{100}}{p_o}=\frac{T_o+100}{T_o} \quad \text{or} \quad \frac{p_{100}-p_o}{p_o}=\frac{100}{T_o}$$

$$\therefore \quad \frac{1}{T_o}=\frac{p_{100}-p_o}{100.p_o}=\gamma_v \text{ (pressure coefficient)}$$

Thus we can find γ_v from a measurement of p_{100} and p_o. To find then the temaperature T, in absolute scale corresponding to any observed pressure p, we have by pressure law $\frac{p}{p_o}=\frac{T}{T_o}$ or $T=\frac{p}{p_o}\cdot\frac{1}{\gamma_v}$

$[\because \; T_o=1/\gamma_v]$.

Knowing p_o, p and γ_v T can be found out.

The constant volume hydrogen thermometer as described above is suitable for measurement of temperatures between $-200°C$ and 500°C. The range can be extended with proper modifications from

$-260°C$ to $1600°C$. The inclusion of a barometer in this thermometer has largely increased its efficiency.

44. Callender's compensated constant pressure Air Thermometer : It consists of a bulb T, known as the thermometer bulb, which is connected to a bulb M almost filled with mercury by the connecting tube C. The tube M is provided with a tap at the bottom for drawing out mercury when required (Fig. 14).

There is another bulb S similar to T which is filled with dry air and connected to C through a compensating tube C$'$, and a sulphuric acid pressure gauge or manometer G.

The bulb M is graduated in volume scale (*i.e.* in c.c.) and mercury may be drawn out from it to preserve the equality of pressure of the air in T and S by keeping mercury levels in the two limbs of G same. In this thermometer the mass of air contained by T, C and M is kept the same as the mass of air contained by S and C$'$.

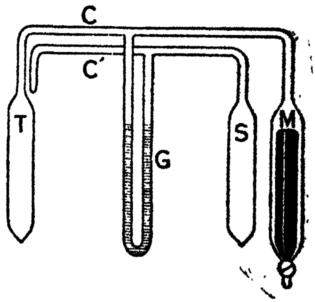


Fig. 14

To measure a temperature, the bulbs S and M are kept at a constant temperature $0°C$, and the thermometer blub T is immersed in a bath whose temperature is to be determined.

The temperature of the air in the bulb T rises and consequently its pressure increases thereby depressing the sulphuric acid column in the pressure gauge G.

Mercury is then drawn out from the bulb M until the levels in the pressure gauge G become the same as before and the pressure of confined air is kept at the original value.

Let v, v_1, v_2 be the volume of air in the bulb T, in the tube C joining T and M (including part of G), air above mercury in M respectively ; T, T_1, T_0 their respective temperatures, m gms. the total mass of air in T, C and M and p the common pressure. From the gas equation we have.

$$\frac{pv}{T}+\frac{pv_1}{T_1}+\frac{pv_2}{T_0}=mR \quad \ldots \quad (1)$$ where R = gas constant for 1 gm.

Again let v_3, v_4 be the volumes of air in the tube C$'$ joining G and S, and in the bulb S, T_3, T_0 their respective temperatures. m_1 the total mass of air in C$'$ and S, and p' the common pressure ; then as in the above case we have the relation,

$$\frac{p'v_3}{T_3} + \frac{p'v_4}{T_0} = m'R \qquad (2)$$

In actual experiment $m = m'$ $\quad$ $mR = m'R$

$$\left(\frac{v}{T} + \frac{v_1}{T_1} + \frac{v_2}{T_0}\right) = p'\left(\frac{v_3}{T_3} + \frac{v_4}{T_0}\right)$$

$$\text{or} \quad \frac{v}{T} + \frac{v_1}{T_1} + \frac{v_2}{T_0} = \frac{v_3}{T_3} + \frac{v_4}{T_0} \quad \ldots \quad \ldots \qquad (3)$$

since in actual experiment $p = p'$

Again, since the tubes C and C′ have equal volumes and are placed side by side at the same temperature, we have, $v_1 = v_3$, $T_1 = T_3$

Hence, relation (3) becomes

$$\frac{v}{T} + \frac{v_2}{T_0} = \frac{v_4}{T_0} \quad \text{or} \quad \frac{v}{T} = \frac{v_4}{T_0} - \frac{v_2}{T_0} = \frac{v_4 - v_2}{T_0}$$

Again, since the bulbs T and S are exactly similar, we have

$$v = v_4 \quad \therefore \quad \frac{v}{T} = \frac{v - v_2}{T_0} \quad \text{or} \quad T = \frac{v}{v - v_2} T_0$$

Knowing v, v_2 and T_0 the unknown temperature T of the bath can be found out.

45. Advantages of a gas thermometer :

(1) A gas thermometer has a much larger range of temperature than a mercury thermometer, for the properties of a gas remain unchanged within wide limits of temperature.

A thermometer containing permanent gases such as Hydrogen, Nitrogen, etc., is used for registering very high as well as very low temperatures.

(2) Since the expansion of a gas is much greater than that of a liquid, a gas thermometer is much more sensitive than a liquid thermometer and the readings of a gas thermometer would practically remain unaffected by the expansion of the vessel containing the gas.

(3) Since all the permanent gases expand very nearly equally, the thermometer containing these gases will give similar readings, but readings of thermometers containing different liquids will never agree as the expansions of the different liquids are always different.

(4) Gases can be obtained in pure state.

46. Disadvantages :

It is bulky and can not be easily moved about.

It is too large to be used in barometric work.

Note : The constant volume hydrogen gas thermometer is preferred to constant pressure air thermometer since it permits easier manipulation and fixed scale may be provided in it.

46. Discussion on Gas Scales : The constant volume hydrogen scale with the gas at a pressure of 1 metre of mercury at 0°C is considered as a standard scale of temperature.

All gas scales, whether constant volume or constant pressure type agree, at very low pressure, at temperatures between the fixed points, for the reason that all gases obey Boyle's law strictly under these conditions and so they are called ideal gases.

Ideal gas scale can not be realised in practice and temperature can not be measured with this scale but hydrogen gas scale is preferred to all others for hydrogen agrees closely with the ideal gas scale.

Lord Kelvin conceived an absolute scale of temperature from thermo-dynamic considerations and the scale is independent of the properties of any particular material substance. Lord Kelvin's absolute scale agrees with the ideal gas scale. If the two fixed points are the same in the two scales, any temperature observed in any of the two scales will have the same value.

The correction to constant volume hydrogen scale can be calculated from *Joule-Kelvin effects.*

47. Barometer : Errors and their corrections :

It consists of a glass tube (Fig. 15) closed at one end and open at the other. The tube is filled with mercury and made to stand vertically in a trough filled with mercury. In Fortin's Barometer the trough is a leather bag closed by a small button resting against a screw S. The level of mercury in the bag is maintained constant by turning the screw and allowing the surface to touch a small ivory pin I fixed to the lid of the brass chamber in which the leather bag is fixed.

The pin point is the zero of the barometer reading and the height of the mercury column from this point up to the top of the mercury meniscus is the true barometric height. The top reading is measured by a brass scale U and vernier V fitted to a brass jacket having an opening covered with glass plate for viewing the mercury column. The middle portion of the tube is also enclosed in a brass jacket C of smaller diameter having a thermometer T on it for registering the temperature of the air.

The observed barometric height is subject to several corrections of which the following corrections are considered.

(1) Temperature corrections for mercury.

(2) Correction for the expansion of the scale.

Case I. Let us suppose that the scale and the tube containing mercury are kept at 0°C whilst the temperature of mercury is raised from 0° to t°C.

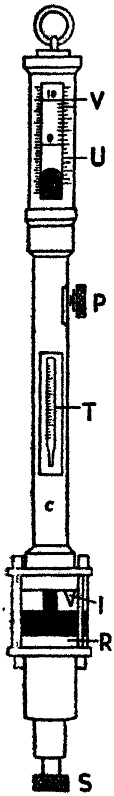


Fig. 15

As the temperature rises, the density of mercury decreases and consequently the observed height appears to be greater than the true height.

Let h = the observed height at t°C.

h_0 = the height of the mercury column hen the temperature of mercury is 0°C.

d = the density of mercury at t°C

d_0 = the ,, ,, ,, at 0°C

γ = coefficient of cubical expansion of mercury

therefore $hd = h_0 d_0$; But $d_0 = d(1+\gamma t)$

$$\therefore \quad h_0 = h(1-\gamma t) \quad \dots \quad \dots \quad (1)$$

Case II. The scale is supposed to be correct at 0°C. As the temperature is raised, the true distance between two marks on the brass scale increases and consequently the height shewn by the scale will be too small at all temperatures higher than 0°C.

Let the distance or height h cms. measured on the brass scale at t°C be equal to a true distance $h(1+\alpha t)$ cms. ... ... (2)

Then combining the corrections (1) and (2) we have the corrected height of barometer for temperature t

$$\mathrm{H} = h(1+\alpha t)(1-\gamma t) = h\{1-(\gamma-\alpha)t\}$$
$$= h\{1-(\cdot 000182 - \cdot 000020)t\} = (1 - \cdot 000162t)$$

Besides the above two corrections there are other minor corrections which are given below.

(a) Correction for the variation of gravitational pull on the mercury.

The value of 'g' *i.e.*, acceleration due to gravity depends on the latitude and elevation of the place above the sea-level. The variation of g due to these causes produces a change in the observed height of the mercury column. Hence, corrections are necessary.

(b) Correction for the vapour pressure of mercury.

The pressure of mercury vapour in the tube depresses the column. Hence, a correction is necessary.

(c) Correction for Capillarity.

Surface tension tends to depress the mercury column. Hence, a correction is necessary.

These corrections are very small and may be neglected for practical purposes.

48. Normal or Standard Pressure :

Standard (or normal) temperature and pressure, S. T. P. or N. T. P. A temperature of 0°C or 273°A and a pressure of 76 cm. of mercury under specified conditions are called S. T. P. or N. T. P. ; normal or standard pressure is defined as such that will support a column of mercury 76 cm. high at a latitude 45° and at the sea-level, the temperature of mercury being 0°C. The absolute value of Normal or standard pressure $P = 76 \times 13{\cdot}595 \times 980{\cdot}6$ dynes/sq. cm. $= 1013250 = 10^6$ dynes (nearly), where 13·596 is the density of mercury, and 980·6 cm./sec^2 is value of g *i.e.* acceleration due to gravity at the sea-level at latitude 45°.

49. Reduction of gas volume to N. T. P. Let V be the volume of a given mass of gas at pressure P and temperature t°C, and V_0 the required volume at 76 cm. pressure and 0°C.

We have, $0°C = 273°A = T_0 A$; $t°C = (273 + t)°A = T$; $P_0 = 76$ cm.

By gas equation $\dfrac{P_0.V_0}{T_0} = \dfrac{P.V}{T}$ or $V_0 = \dfrac{P.V.T_0}{P_0.T} = \dfrac{P \times V \times 273}{76 \times (273 + t)}$

if P, V and t are given, V_0 can be obtained.

QUESTIONS

1. What is meant by the absolute scale of temperature ? Does it differ from the perfect gas scale ? [C. U. 1953]

2. Describe some form of constant-volume gas thermometer and explain how it is graduated. [C. U. 1928, '50, '53]

3. Discuss the advantages of using one of the permanent gases as a thermometric substance for constructing a primary standard thermometer. [C. U. 1951]

Why are gas thermometers preferred as primary standards in thermometry. [C. U. 1953]

4. Explain accurately what you mean by a perfect gas. Is there any gas that is perfect ?

Prove the relation $pv = RT$ for a perfect gas. Is R an absolute constant for different masses of different gases ? Give reasons. [C. U. 1940, '57]

Explain the physical significance of the gas constant R. [C. U. 1957]

5. Deduce from the first principles the equation of a perfect gas ? [C. U. 1951]

6. Describe Callender's compensated air-thermometer and explain how it is used to measure temperature. [C. U. 1931, '51]

7. Describe exactly how you would construct a mercury barometer, detailing the precautions you would take to ensure (a) the purity of the mercury, (b) the exclusion of air from the tube. [C. U. 1930]

8. State and explain the necessity for corrections that are applied to the observed barometric height. Deduce an expression for the temperature correction. [C. U. 1937]

Explain fully what you understand by the Normal atmospheric pressure. [C. U. 1937]

9. What are the advantages of using a permanent gas as a thermometric substance?

Describe, with a neat sketch, the constant volume hydrogen thermometer and explain how it is used in practice to measure temperature. [C. U. 1955, '58]

EXAMPLES

1. The pressures indicated by a constant-volume hydrogen thermometer are 23·5 cm., 75·0 cm. and 102·4 cm. in a certain scale, when the bulb is immersed in liquid air, ice and steam respectively. What is the temperature of the liquid air on the constant volume hydrogen scale? [C. U. 1950]

We know that $\gamma_v = \dfrac{p_{100}-p_0}{p_0(100-0)} = \dfrac{102{\cdot}4-75}{75\times 100} = \dfrac{27{\cdot}4}{7500} = {\cdot}00365$

Again temperature of liquid air, $t = \dfrac{p_t - p_0}{p_0 \gamma_v} = \dfrac{23{\cdot}5-75}{75\times{\cdot}00365} = -188{\cdot}12°C.$

2. A cylindrical test tube 10 cms. long is plunged mouth downwards into mercury. How deep must it be plunged so that the volume of the enclosed air may be diminished by one half? [C. U. 1924]

Let h be the depth of the open end of the test tube below the free surface of mercury when the volume of the enclosed air in the tube is halved.

Since the length of the tube is 10 cms., the volume of the enclosed air is therefore 5 cms. and its pressure is equal to $(h-5)+76$ where the pressure of atmosphere is 76 cms.

Then by Boyle's Law, we have $76\times 10 = \{(h-5)+76\}\times 5$ or $760 = 5\times 76 + 5(h-5)$ or $760 = 380 + 5h - 25$ or $5h = 405$ $\therefore$ $h = 81$ cms.

3. If the value of R for 2·016 grams of hydrogen be 0·082 litre-atmosphere per degree. What is its value for 8 grams of Oxygen. [C. U. 1940]

Note: Gram, $\pm$mol. wt. of $O_2 = 32$ gms.; 8 grams of $O_2 = \frac{1}{4}$ mol.; R for one mol. = ·082 *litre-atoms.* $\therefore$ R for $\frac{1}{4}$th mol. = ·0205 *litre-atmos.*]

4. A barometer provided with a brass scale, which is correct at 50°F., reads 754 mm. at 40°F., what will be its true height at 32°F. (Coeff. of linear expansion of brass = ·000018 per 1°C; Coeff. of cubical expansion of mercury = ·00018 per 1°C.)

α for brass per 1°F $= \frac{5}{9}\times{\cdot}000018 = {\cdot}00001.$
γ for mercury per 1°F $= \frac{5}{9}\times{\cdot}00018 = {\cdot}0001.$

We have $H_{32} \cdot \rho_{32} = H_{40}\cdot\rho_{40}$; $H_{32} = H_{40}\cdot\dfrac{\rho_{40}}{\rho_{32}} = H_{40}\{1-\gamma\ (40-32)\}$

$= H_{40}\ \{1 - {\cdot}0001\times 8\}$

For Scale Correction, H_{40} when corrected becomes $H_{40}\{1+\alpha(40-50)\}$
$= H_{40}\ (1-\alpha.10) = 754(1-{\cdot}00001\times 10)$
The corrected heigh H_{32} at 32°F $= 754(1-{\cdot}0001)\ (1-{\cdot}0008) = 753{\cdot}4$mm.

5. A mercury barometer is provided with a brass scale, whose divisions are correct at 68°F. The reading of the barometer is 29·5 inches of mercury when the temperature is 20°C. Calculate the barometric height corrected at 0°C.

(Coefficient of linear expansion of brass = ·000018 per degree centigrade. Coefficient of cubical expansion of mercury γ = ·00081 per degree centigrade.) [C. U. 1949]

The scale is correct at 68°F *i.e.*, $(68-32)\times\frac{5}{9}$C = 20°C

H_0 be the correct height at 0°C and ρ_0 the density of mercury at 0°C.

$$H_0\rho_0 g = H_{20}\rho_{20} g = 29{\cdot}5 \times \rho_{20}\ g$$

$$\text{or}\quad H_0 = \frac{29{\cdot}5 \times \rho_{20}}{\rho_0} = 29{\cdot}5\ (1-20\gamma)$$

$$= 29{\cdot}5(1 - 20 \times {\cdot}00018) = 29{\cdot}4 \text{ inches.}$$

Scale correction is not required as at 20°C *i.e.* 68°F, the scale is correct.

6. A certain volume of gas enclosed in a glass bulb gives a pressure of 760 mm. of mercury at 20°C. To what temperature must it be raised so as to change the pressure to 800 mm. of mercury (assume the coeff. of linear expansion of glass to be $8{\cdot}08 \times 10^{-6}$ per degree cent.) [D. U. 1944]

Use the formula $v_2 = v_1(1+3\alpha t)$ and $\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$ and proceed.

CHAPTER IV

CALORIMETRY

50. Quantity and unit of heat: The subject of *calorimetry* deals with the measurement of quantities of heat. The vessel in which the measurement is carried out is known as a *calorimeter*.

In the *metric system* the unit of heat, **calorie** or *gram-degree centigrade* is defined as the quantity of heat required to raise 1 gram of water through 1°C—from 14°·5C to 15°·5C for accurate work. This is called 15°C Calorie.

The other unit known as **Kilo-calorie** or the *large calorie* is also used and it is defined as the quantity of heat required to raise 1 Kgm. of water through 1°C.

In the *British system* the **British Thermal Unit** (B. Th. U.) is the quantity of heat required to raise the temperature of 1 lb. of water through 1°F.

The **therm** is a larger unit and is equal to 100,000 B. Th. U. and is defined as the quantity of heat required to raise 1000 lbs of water through 100°F.

There is another unit known as **pound Centigrade** *unit* and it is defined as the quantity of heat required to raise 1 lb of water through 1°C. This unit is also known as *lb.deg. c* unit.

Relation between the units.

$$1 \text{ B. Th. U.} = \left(453{\cdot}6 \times \frac{5}{9}\right) \text{ calories} = 252 \text{ calories}$$

$$1 \text{ lb deg. C.} = \frac{9}{5} \text{ or } 1{\cdot}8 \text{ B. Th. U.} = 453{\cdot}6 \text{ calories}$$

51. Specific Heat, Water Equivalent and Thermal Capacity of a body :

The *specific heat* of a substance is defined as the quantity of heat which it absorbs or gives out when the temperature rises or falls through 1°C compared with the quantity of heat which would be absorbed or given out under the same condition by the same weight of water.

The *water equivalent* of a body is defined as the quantity of water in grams which will be raised through 1°C by the amount of heat required to raise the temperature of the given body through 1°C.

It is expressed as *Ms grams* where M is the mass of the body and s its sp. heat.

The *Thermal Capacity* of a body is the amount of heat required to raise the temperature of the body through 1°C.

It is expressed as Ms Cal., where M and s are mass and sp. ht. of the body respectively.

Thermal Capacity per unit volume is the amount of heat required to raise unit volume of the body through 1°C.

It is expressed as ρs. where ρ is the density of the body, and s the specific heat.

The specific heat of a body is sometimes defined as the thermal capacity per unit mass.

Note : If m gms. of a body of specific heat s fall or rise through a temperature t°c, the heat given up or absorbed by the body $= m.\ s.t$ calories.

It is to be noted that for a given amount of heat absorbed or given up by a body, greater is the mass of the body lesser will be its rise or fall of temperature and vice versa. Again for a given mass of a body, lesser the sp. heat of it greater rise of temperature will be caused by a given amount of heat applied to it.

52. Newton's Law of Cooling : The amount of heat radiated per sec. by a particular vessel when filled with any liquid is proportional to the difference of temperature between the vessel and its surroundings.

It has been found that this law is true for small difference in temperature and at low temperatures, but fails when the temperature of the radiating body is high.

Newton's Law of Cooling may be treated mathematically in the following way.

The average rate of loss of heat during an extremely small interval of time dt is expressed as $-\frac{dQ}{dt} = -ms.\frac{d\theta}{dt}$ ... (1)

where $d\theta$ is the small change in temperature during the interval

dt, dQ the corresponding heat change, m the mass of the body and s its specific heat.

According to Newton's Law of Cooling, the rate of loss of heat of a body is proportional to the temperature excess of the body over the surroundings.

$$\text{Therefore, } -\frac{dQ}{dt}=\mathrm{K}(\theta-\theta_0) \quad \ldots \quad (2)$$

where θ is the temperature of the body θ_0 the temperature of the surroundings and K is a constant.

From (1) and (2) we have

$$\frac{d\theta}{dt}=\frac{K}{ms}(\theta-\theta_0)=C(\theta-\theta_0), \text{ where C is another constant.}$$

So from Newton's Law of Cooling it may be said that rate of fall of temperature is proportional to the temperature excess.

53. Different methods in Calorimetry: (1) Method of Mixtures, (2) Method based on Newton's law of Cooling, (3) Method based on change of state, (4) Electrical method.

54. To determine the specific heat of a substance by method of mixtures: The body is weighed, and then heated in a double-walled steam jacket or steam-heater until its temperature T is constant and then quickly dropped into a calorimeter containing a known weight m_2 of water (or some oil) at a temperature t_1°C. The temperature of the water (or oil) gradually rises and attains a maximum value t_2°C, after which it begins to fall.

Let m_1, and w be the masses of the body and the calorimeter respectively, and s_1 and s be the sp. heats of the body and the calorimeter respectively.

Then the heat lost by the body in cooling from T°C, the temperature of the solid in the steam jacket, to t_2°C is gained by the water and the calorimeter in rising from t_1°C to t_2°C.

Thus, we have $m_1s_1(\mathrm{T}-t_2)=(m_2s_2+ws)(t_2-t_1)$

$$s_1=\frac{(m_2s_2+ws)(t_2-t_1)}{m_1(\mathrm{T}-t_2)}$$

Here s_2 is the sp. heat of water or any other liquid which is used in the calorimeter.

54a. Sources of Errors: In arriving at the above expression for determining the specific heat of solid by method of mixtures, it has been assumed that all the heat given out by the hot body is taken up by the calorimeter and its contents. But this is not really so, due to the following reasons.

(1) Some heat might have been lost during the transfer of the hot body from the heater to the calorimeter. This loss is minimised by placing the heater just above the calorimeter and dropping the hot body straight down into the calorimeter. This loss is also reduced by increasing the mass of the body to such an extent that it may be conveniently suspended inside the heater.

(2) The stirrer and the thermometer, take up a certain amount of heat which is not generally taken into account but this can be easily calculated if the water equivalents of the stirrer and the thermometer are known.

The loss of heat due to conduction may be made as small as possible if the calorimeter is supported on non-conducting legs such as pieces of cork inside a surrounding jacket.

(3) Since after the introduction of the hot body into the water (or oil) in the calorimeter, the temperature of the water becomes higher than that of the surroundings some amount of its heat is lost by *radiation*.

55. To minimise the loss of heat due to radiation : The loss of heat due to radiation may be minimised in the ways described below.

(a) The outside of the calorimeter should be highly *polished* since a polished surface is a bad radiator of heat.

(b) Radiating *surface* should be as *small* as possible. If the calorimeter be cylindrical in form its height should be equal to the diameter of the base for in that case for a given volume the ratio of the volume to the height is the greatest.

(c) To make the loss from the outside of the calorimeter regular it is necessary to enclose the calorimeter in a double-walled jacket containing water at a constant temperature.

(d) The calorimeter should be made of a *good conducting material* and it should be nearly filled with water, otherwise the portion of the calorimeter above water will be at a different temperatures from the rest and the change in temperature of the water will not be the same as that of the whole of the calorimeter.

56. Radiation Correction : The loss of heat due to radiation depends on (1) the area of the radiating surface, (2) the nature of the radiating surface, (3) the excess of temperature of the radiating surface over the temperature of the room and the interval during which radiation takes place.

The losses due to the first cause may be minimised by making the area of the radiating surface as small as possible and by polishing the radiating surface.

The loss due *to the third cause* may be calculated in the way described below.

When the temperature of the radiating surface *i.e.*, of the calorimeter is the same as the temperature of the room, no heat is lost or gained by the calorimeter but if the temperature of the calorimeter gradually rises either by heating the water or by dropping the hot body into the water contained in the calorimeter, there will be a loss of heat due to radiation which will be greater as the excess of temperature of the calorimeter over the room temperature gradually increases. This loss will be maximum when the temperature of the calorimeter i.e., of the water in the calorimeter becomes maximum.

56a. Calculation of loss of heat by radiation: To calculate the loss of heat due to radiation when the common temperature of the calorimeter and its contents is maximum we are to know the different losses of heat at intermediate temperatures and by adding all these losses we can calculate the loss of heat at the maximum common temperature.

Since it is very tedious to calculate these different losses at different temperatures for different excesses of temperature, we generally take a mean of all the rising temperatures and determine the excess of this mean temperature over the room temperature.

In our experiment the loss of heat is represented by the fall in temperature.

To calculate the loss of heat *i.e.* the fall in temperature during any interval we must first known the fall in temperature in one minute when the excess of temperature of the radiating surface over the room temperature is only 1°C. To determine this a preliminary experiment is arranged. A certain quantity of hot water is taken in a calorimeter and its falling temperatures at an interval of one minute are noted during five minntes. Let θ_0, θ_1, θ_2, θ_3, θ_4, θ_5, be the minute-interval readings during five minutes.

Let θM be the mean of these temperatures and let θA be the temperature of the room. Then the mean excess of temperature over that of the room is $\theta M - \theta A$.

With this excess the fall of temperature is $\theta_0 - \theta_5$ in 5 minutes. Therefore the rate of fall *i.e.*, the fall of temperature in one minute when the excess of temp. is 1°C is equal to K where

$$K = \frac{\theta_0 - \theta_5}{5(\theta M - \theta A)}$$

Having determined the rate of fall in temperature we note the temperature of the water in the calorimeter when the hot body is dropped into it and since the temperature rises very quickly it is advisable to take *half-minute readings* of temperatures from the moment at which the body is dropped to the time when the temperature of the calorimeter and its contents is maximum.

Since the maximum temperature as recorded is less by a certain amount than what it ought to be, had there been no loss due to radiation, a correction is to be applied to the recorded maximum to get the true temperature.

Let $\theta_1 M$ be the mean of all these half-minute readings during interval of t minutes.

Then $(\theta_1 M - \theta A)$ is the mean excess of temperature during this interval of time.

Since K is the rate of fall in temperature *i.e.*, the fall in temperature in one minute when the excess of temperature is one degree, the fall in temperature when the excess is $\theta_1 M - \theta A$ in time t is $(\theta_1 M - \theta A)t.K^\circ$.

Therefore the corrected maximum temperature is $\theta + (\theta_1 M - \theta A)t \times K$ where θ is the observed maximum temperature.

57. Specific heat of a liquid by Method of Cooling :

A certain quantity of warm water is taken in a wide test-tube fitted with a cork and a thermometer and suspended inside a vessel, the walls of which are surrounded by melting ice to maintain a constant temperature. Let t_1 be the time taken by water to fall from a temperature θ_2 to θ_1.

The water is then replaced by an equal volume of a hot liquid of which the specific heat is required and the temperature of it is allowed to fall through the same range *i. e.* from θ_2 to θ_1 under similar conditions.

Then, according to Newton's law of cooling the amount of heat lost by the liquid is proportional to the time t_2 taken by the liquid in cooling from θ_2 to θ_1.

If m_1 be the mass of water and m_2, that of the liquid of which the specific heat is S, we have

$$\frac{\text{Heat lost by liquid}}{\text{Heat lost by water}} : : \frac{m_2 S(\theta_2 - \theta_1)}{m_1(\theta_2 - \theta_1)} = \frac{t_2}{t_1} \quad \text{or} \quad \frac{m_2 S}{m_1} = \frac{t_2}{t_1}$$

Therefore S, the sp. heat of the liquid $= m_1 t_2 / m_2 t_1$

Knowing m, m_2, t_1 and t_2 S can be found out.

58. Specific heat under various conditions : The specific heat of a substance is not the same in its three states. It is usually the greatest in the liquid state.

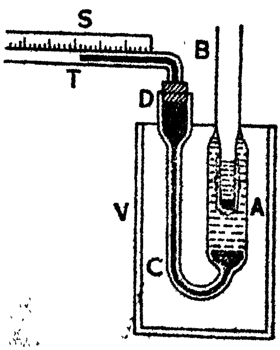


Fig. 16

The specific heat of an elementary substance depends on the condition in which it exists. Its value in the amorphous condition differs from its value in the crystalline condition. Carbon in different states has different specific heats.

The specific heat of a substance varies with the temperature of the body and is greatly altered by the presence of impurities.

59. Specific heat by Change-of-State method : Bunsen's Ice Calorimeter : We know that a gram of ice at 0°C in melting to water at 0°C decreases in volume by about ·091 c.c. This change in volume has been utilised by Bunsen to determine the specific heat *of a solid when available in a small quantity.*

The apparatus (Fig. 16) used by Bunsen consists of a test tube B fused into a wider tube A. A tube C leads from the bottom of the tube A and is fashioned into a U-shaped tube whose open end is fitted with a cork through which passes a bent, fine capillary tube T placed horizontally by the side of a millimeter scale S.

The lower part of the tube A, the tube CD and a part of the capillary tube T is filled with pure dry mercury and the upper part of A is filled with distilled water, free from air.

The whole apparatus is first immersed in pure melting ice in a box V and then a stream of alcohol or ether is passed through B till a shell of ice is formed round the lower part of B. With continued evaporation of alcohol or ether the shell of ice will increase and the mercury in the capillary tube T will be seen to advance.

Now the evaporation is stopped, the cooling agent is removed and some amount of water at 0°C is pourea into the tube B. The instrument is then allowed to stand for a long time till it comes to 0°C and the position of mercury in T remains steady. This position is then read off by the scale S.

To determine the *Specific heat* of a metal, a small quantity of it, say m gram, is heated to a constant high temperature t and then put into water in the tube B which is at once corked. The heat given out by the solid melts some of the ice round B and causes a contraction in volume and the mercury meniscus in T is found to recede.

When the meniscus is steady its position is read off.

From the knowledge of the cross-section a of the tube T, and the distance d through which the meniscus moves, the contraction in volume of ice, v *c.c.* *i.e.*, $a \times d$ is determined and from this the quantity of ice melted and the heat supplied can be calculated.

Heat lost by the metal in cooling from t°C to 0°C $= mst$.

This quantity of heat is absorbed by ice to melt at 0°C, only.

We know that the contraction of 1 gm. of ice on melting at 0°C is ·091 c. c. In other words a contraction of 1 c. c. is due to melting of 1/·091 gm. of ice ; ∴ a contraction of v c. c. is due to melting of v/·091 gm. of ice.

Hence, the quantity of ice melted $= \dfrac{v}{\cdot 091}$ gm.

Heat absorbed by ice in melting $= \dfrac{vL}{\cdot 091}$ cal.

where L, the latent heat of fusion of ice $= 80$

$$\therefore \; mst = \frac{vL}{\cdot 091} = \frac{80v}{\cdot 091} \quad [\because \text{ Heat lost} = \text{Heat gained}]$$

That is, $s = \dfrac{80v}{\cdot 091 \times mt} = \dfrac{80\, a.d}{\cdot 091 \times mt}$

Knowing all quantities of the right hand side s can be found out.

60. Specific heat by Electrical method ; Nernst's Vacuum Calorimeter :

It is a specially designed calorimeter used by Nernst and Lindemann for measuring the specific heat of a substance at very low temperatures. If the substance is a metal such as copper or silver it is shaped into the form shown by the shaded parts in figure 17, both the outer cylinder and the central plug being made of the same materials. The central plug is wound over with a platinum coil (shown dotted in the figure and well insulated by paraffined paper) which serves both as an electric heater and resistance thermometer. The platinum coil is connected in series with a battery, a rheostat, an ammeter A and a voltmeter V indicating respectively the current in it and the potential difference across it. The calorimeter is suspended by connecting leads in a vessel which can be filled with any gas or evacuated and the whole is surrounded by low temperature baths such as liquid air or liquid hydrogen.

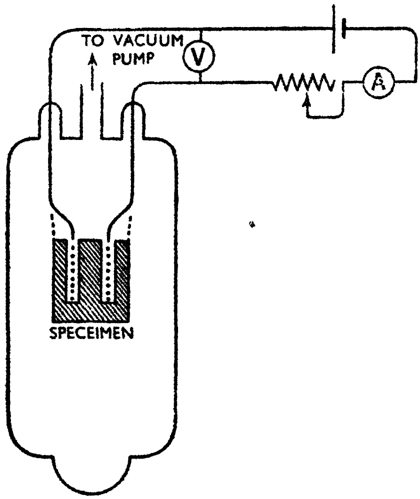


Fig. 17

Let a current be passed through the heater for t seconds and the potential difference maintained constant by varying the rheostat.

If I_1, I_2 and R_1, R_2, denote the initial and final values of the current and the resistance of the heater (platinum coil) respectively we have

$R_1 = V/I_1$ and $R_2 = V/I_2$ where V is the constant potential difference.

From the average value of the current the heat supplied is equal to $\dfrac{VIt}{J}$ calories.

If m is the mass of the substance forming the calorimeter, s its specific heat and θ, a small rise in temperature we have

$$\frac{VIt}{J} = m.s.\theta + h$$

where h is the heat lost by radiation which is very small in this case.

The rise in temperature θ is determined by the change in resistance of the platinum coil.

In determining the specific heat at low temperatures, cooling correction which is appreciable, is eliminated by suspending the calorimeter in a vacuum and the small rise in temperature is obtained by controlling the current.

For non-conducting substance they are placed inside a hollow silver vessel on which the platinum coil is wound and which is closed with a lid.

The silver vessel with the platinum coil is surrounded by a piece of silver foil to prevent loss of heat and also to assist in the uniform distribution of heat in the silver vessel.

For other electrical methods *see* "Electricity and Magnetism".

61. The two Specific Heats of Gases : In determining the specific heat of a solid or liquid we have assumed that the change in volume of the heated body is so small that the internal work done and the heat necessary to do this work are negligible but when a gas is heated its change in volume is considerable and therefore the heat required by a certain mass of the gas to raise its temperature through 1°C at constant pressure differs considerably from the amount of heat required to raise the same mass of gas through the same range of temperature at constant volume.

For, when a gas expands and does some external work a certain amount of its internal energy is used up for the external work and consequently the gas is cooled and its temperature lowered. So heat must be supplied from external source to raise its temperature to its initial value.

Thus, if one gram of a gas be heated through 1°C at constant volume a certain amount of heat known as the **Specific Heat at constant volume** would be required or rather absorbed. If the gas be now allowed to expand until its pressure becomes the same as it originally possessed, it will be lowered in temperature due to the external work performed by the gas and so an additional amount of heat h over and above that required for heating the gas through 1°C will be required to raise the gas through 1°C. Thus the quantity of heat required to raise one gram of the gas through 1°C at constant pressure which is called the **Specific Heat of gas at constant pressure**, is greater than the specific heat at constant volume.

Hence, $C_p = C_v + h$, where C_p and C_v are the Sp. Heats of gas at constant pressure and at constant volume respectively, and h is the heat equivalent of work done.

62. Relation between C_p and C_v : Consider unit mass o a gas at atmospheric pressure enclosed within a cylinder ACBL (Fig. 18) by means of an air-tight, weightless and frictionless piston EF If the gas be heated the gas expands and the piston moves outwarc through a distance dx until it pressure becomes same as origina atmospheric pressure. If A be th area of the cross-section of th cylinder, the expansion of the gas when heated through 1°C is Ad or dv.

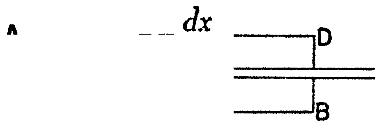


Fig. 18

Now, the work done = Force × displacement = $p.A.dx = pdv$. Th heat equivalent for this work is pdv/J calories, where J is the Joule' equivalent. Thus we have $C_p = C_v + h$, where Cp, Cv = Sp. heats o a gas at constant pressure and at constant volume respectively h = heat equivalent of work done.

$$\text{or} \quad C_p = C_v + \frac{pdv}{J} \text{ Cal.} \quad \text{or} \quad C_p - C_v = \frac{pdv}{J} \text{ Cal.} \quad \dots \quad (1)$$

If the gas obeys Boyle's Law *i.e.*, if it is a perfect gas,

we have $pv = \text{RT}$, where p, v and T denote respectively the pressure, volume and temperature (absolute) of the gas and R is a constant quantity, called the gas constant.

Then when the gas expands at constant pressure by the applicatio of heat, we have $p(v + dv) = \text{R}(\text{T} + d\text{T})$ or $pv + pdv = \text{RT} + \text{R}d\text{T}$ or $pdv = \text{R}d\text{T}$; [Alternately, $pv = \text{RT}$, taking p constant and differentiat ing it, we have $pdv = \text{R}d\text{T}$].

If dT the change in temperature be 1 $pdv = \text{R}$. Then the equation (1) becomes $C_p - C_v = \frac{\text{R}}{J}$ or $C_p = C_v + \frac{\text{R}}{J}$ where C_p, C_v and R/J are expressed in calories. Expressing all in ergs we can write $C_p = C_v + \text{R}$.

In the above expressions, R is the gas constant for the gram-molecules of gas and C_p and C_v are really the molecular heats (specific heat × gram-molecular weight) of the gas at constant pressure and at constant volume respectively.

But if unit mass of the gas is considered the expression $C_p - C_v = \frac{\text{R}}{\text{J}}$ becomes

$$C_p - C_v = \frac{\text{R}_1}{J} \quad \dots \quad \dots \quad \dots \quad (2)$$

Here R_1 is the gas constant for unit mass and C_p and C_v are the specific heats of the gas at constant pressure and at constant volume.

The expressions (2) may be written in the form $C_p - C_v = R_1$.

Here C_p and C_v are measured in calories per gram per °C and R_1 is calculated n ergs per gram per °C.

For *hydrogen* $C_p = 3{\cdot}4$ cal. per gram per °C
$C_v = 2{\cdot}4$ cal. per gram per °C

$\therefore \quad C_p - C_v = 3{\cdot}4 - 2{\cdot}4 = 1$ calorie $= R_1$.

But R_1 for hydrogen $= 4{\cdot}13 \times 10^7$ ergs per gram per °c.
1 calorie $= 4{\cdot}13 \times 10^7$ ergs = Joule's equivalent.

The ratio $\frac{Cp}{Cv}$ is denoted by γ and for air it has been found be 1·41.

63. Difference of the Sp. Heats of Air :

$$C_p - C_v = \frac{R}{J} = \frac{P.V}{273J} = \frac{76 \times 13{\cdot}6 \times 981}{{\cdot}001293 \times 273 \times 4{\cdot}2 \times 10^7} = {\cdot}068 \text{ at N.T.P.}$$

The values of C_p and C_v for air have been determined and found to be as follows.

$C_p = 0{\cdot}2375$ cal. per gm. per 1°C
$C_v = 0{\cdot}1684$ cal. ,, ,, ,, ,,

64. Specific Heat of a gas at constant volume by Joly's Differential steam calorimeter : Two hollow copper spheres A and B, having equal thermal capacities or water equivalents (Fig. 19) are suspended from the opposite arms of a balance by means of fine platinum wires p, p, and enclosed in a chamber EC which can be filled with dry steam through O. One of the spheres, say B, is exhausted and the other A is filled with the gas under pressure and the mass of the gas is determined by adding weights on the pan from which the exhausted sphere is suspended. Two catch-pans C_1, C_2 are attached at the bottom of the spheres to catch the water which may condense on their surfaces. Steam is passed into the chamber surrounding the spheres and the excess of steam condensed on the sphere containing the gas is accurately determined. It is to be noted that the volume of the gas during the course of the experiment is maintained constant.

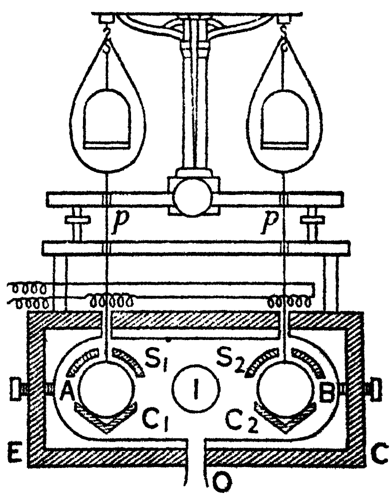


Fig. 19

The specific heat of the gas at constant volume is calculated in the following way.

Let m_1 = mass of the gas in sphere A at temperature t_1°C
m_2 = mass of excess of steam condensed on sphere
t_2 = the temperature of steam
L = Latent heat of steam
C_v = the Sp. heat of the gas at constant volume

Then, heat gained by the gas in being heated from t_1 to t_2°C = heat lost by steam in condensing at t_2°C.

$\therefore$ we have $m_1 C_v(t_2 - t_1) = Lm_2$ $\quad\therefore\quad C_v = \dfrac{Lm_2}{m_1(t_2 - t_1)}$

Knowing all quantities of the right hand side, C_v can be found out.

Corrections :

(1) Correction for the thermal expansion of the sphere.
(2) Correction for any unequal thermal capacities of the spheres.
(3) Buoyancy correction due to increased volume at higher temperatures.

The sphere with the gas will increase more in volume than the exhausted one. So the effect of buoyancy will be different at the higher temperatures.

(4) Reduction of weight of steam condensed due to the gas, in vacuum.

Precaution :

A difficulty arises in the weighing due to the condensation of steam on the suspending platinum wires.

This is overcome by passing a current through coils of fine platinum wires surrounding the suspending wires.

Note 1 : Correction for unequal thermal capacities can be done as follows. Let the value of C_v found above be C_v'. The experiment is next repeated under identical conditions keeping sphere A exhausted and filling the sphere B with the gas. If the value of C_v be C_v'', the correct value of C_v will be $\frac{1}{2}(C_v' + C_v'')$.

Note 2 : To test equality of thermal capacity of A and B the two spheres are exhausted and counterpoised. Steam is then passed. If equal mass of steam condense on A and B their thermal capacities must be equal and equilibrium of the balance will not be disturbed.

65. Specific heat of solid by steam calorimeter : Joly's calorimeter is modified to determine the *specific heat of a solid*. One of the pans of the balance is removed and in its place a small platinum pan is suspended by a wire in the steam chamber. The solid whose specific heat is to be determined is placed on this pan and counterpoised by putting weights on the other pan which is outside the chamber. Steam is admitted into the chamber, which

condenses on the solid and the pan. When condensation has stopped, additional weight is required to counterpoise the balance.

Then, if M be the mass of the solid and m the additional weight, we have heat gained by solid and pan = heat lost by steam of mass m.

$$\therefore \text{MS}(100-t)+\text{W}(100-t)=m\text{L}$$

where t is the initial temperature of the chamber, 100°C the temperature of steam, W the thermal capacity of the pan, S, the specific heat of the solid and L, the latent heat of steam

$$\therefore \quad \text{S}=\frac{m\text{L}}{\text{M}(100-t)}-\text{W}.$$

66. Specific heat of a gas at constant pressure by Regnault's method : The gas whose specific heat is to be determined is stored under pressure in a large metal reservoir R (Fig. 20) which is surrounded by a bath, the constant temperature of which is observed by a thermometer T. It is then allowed to pass through a bent tube provided with a valve V and a manometer M into a long spiral tube made of copper immersed in an oil bath S_1 whose temperature is raised by placing a burner underneath and determined by a thermometer T_1.

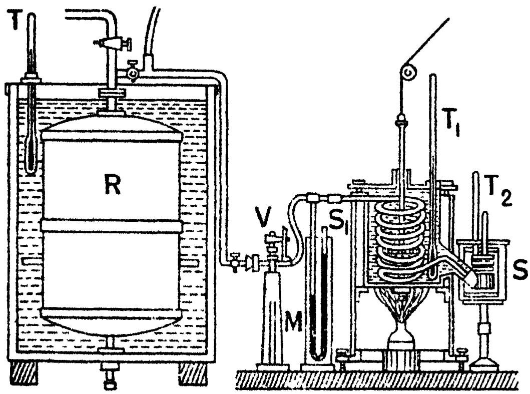


Fig. 20

As the gas escapes from the reservoir, its pressure is reduced and so by continuously adjusting the valve V and observing the same difference of level of the liquid in the manometer, a steady flow of the gas at constant pressure is maintained. The gas while passing through the spiral tube acquires the temperature of the bath S_1 and is then directly passed into another spiral copper tube immersed in a calorimeter S_2 containing water whose temperature is determined by a thermometer T_2.

The mass of the gas which passes through the calorimeter out into air is obtained by noting the pressure of the gas in the reservoir before and after the experiment by another manometer (not shown in the figure).

Let M = mass of the gas of sp. heat C_p (at constant pressure)
T = temperature of the oil bath
t_1 = original temperature of the water in the calorimeter
t_2 = final temperature of the water in the calorimeter
m = mass of water in the calorimeter
w = water equivalent of the calorimeter

As the hot gas has not passed through the calorimeter all at once, it has cooled from T°C to $(t_1+t_2)/2$°C which is the average of initial and final temperature of the calorimeter. Then heat lost by the gas = Heat gained by calorimeter and contents.

$$\therefore \quad M.\ C_p\left(T-\frac{t_2+t_1}{2}\right)=(m+w)(t_2-t_1)$$

$$C_p=\frac{(m+w)(t_2-t_1)}{M(T-T')} \quad \text{where } T'=\frac{t_2+t_1}{2}$$

whence Cp can be found out knowing all quantities of right hand side.

Note: To calculate the mass M of the gas, the pressure of the gas before and after the experiment is noted and then the densities at these two stages are calculated. Let d_1 and d_2 be respectively the densities of the gas when the pressures are p_1 and p_2 and let d be the density at N. T. P.

Then $d_1=d\times\frac{p_1}{760}\times\frac{273}{273+t}$ where t is constant temp. of the gas in R.

and $d_2=d\times\frac{p_2}{760}\times\frac{273}{273+t}$

Hence, $d_1-d_2=\frac{d\times 273}{760\times(273+t)}\times(p_1-p_2)$; if V = volume of the reservoir,

then $M=V(d_1-d_2)=V.\ \frac{d.273(p_1-p_2)}{760(273+t)}$ whence M can be found out.

To determine the final temperature of the water in the calorimeter heat lost due to radiation and conduction should be taken into consideration.

Errors: (1) Heat may be conducted from the bath to the calorimeter, (2) Heat may be received or lost by radiation.

67. Atomic Heat ; Dulong and Petit's Law :

According to Dulong and Petit's Law masses of elementary substances equal to the atomic weights of the respective elements will comprise equal number of atoms. The atomic heat of a substance is the amount of heat required to raise a mass of the substance equal to its atomic weight through 1°C.

The atomic heat of a substance is obtained by multiplying its specific heat by its atomic weight.

Dulong and Petit's Law states that the atomic heats of the elementary substances are approximately constant at ordinary

temperatures in the solid state. The mean value of the constant is 6·38 with extremes of 6·76 and 5·7.

It has been found by experiments that atomic heat diminishes with temperature and becomes zero at the absolute zero temperature.

The product of the atomic weight and the specific heat of all elementary substances is constant. This law is known as Dulong and Petit's law of specific heat.

The atomic heats of the elementary gases possess a different mean value from those of the elementary solids. The atomic heats for gases vary from 3·94 to 3·40.

According to the kinetic theory of matter a very simple explanation may be given of the approximate agreement of the atomic heats of the elements existing in the same state.

We know from the kinetic theory (*see* Kinetic Theory) that $pv=\frac{1}{3}MV^2$, where V is the velocity of gas molecules.

We also know that $pv=RT$ and the kinetic energy of translation of a gram-molecule is $\frac{1}{2}MV^2=\frac{3}{2}pv=\frac{3}{2}RT$.

Since the velocities of molecules are different and have components in three perpendicular directions, the molecules have **three degrees of freedom.**

Then, by **the law equipartition of energy**, if the energy is equally distributed between the three degrees of freedom, the energy associated with each degree of freedom per gram-molecule is $\frac{1}{2}RT$.

67a. Solids: A molecule of a solid which is considered as an elastic sphere held in position by the attracting force of the other molecules vibrates with a simple harmonic motion about a position of rest.

The molecule will have three component velocities and three degrees of freedom. The kinetic energy per gram-molecule is $\frac{3}{2}RT$. On the average the harmonic vibration will have equal kinetic and potential energies.

Therefore the total energy is $2\times\frac{3}{2}RT=3RT$. So the molecular specific heat is equal to $3R=3\times1{\cdot}99$ Cal. $=5{\cdot}97$ Cal.

67b. Gas: In a monatomic gas the molecules possess only kinetic energy and no potential energy.

As before the total kinetic energy associated with a gram-molecule $=\frac{3}{2}RT$.

The molecular specific heat at constant volume = 2·98 Cal.

For all perfect gases we have

$$C_p - C_v = R \quad \text{or} \quad C_p = C_v + R = \tfrac{3}{2}R + R = \tfrac{5}{2}R = 3{\cdot}95 \text{ cal.}$$

and $\gamma = \dfrac{C_p}{C_v} = \tfrac{5}{2}R \cdots = 1{\cdot}66$

This agrees with the experimental value of γ for monatomic gases such as *argon*, *helium*, etc.

68. The knowledge of γ is useful for the following purposes:

(1) It throws valuable light on the constitution of the molecules of a gas with the help of Kinetic theory of gases.

(2) It is important for the determination of the velocity of sound from the expression $V=\sqrt{\frac{\gamma P}{D}}$

(3) In adiabatic change of volume of a gas the knowledge of γ is important for the determination of volume, pressure or temperature from the expressions $pv^{\gamma}=$ constant, $\frac{P^{\gamma-1}}{T^{\gamma}}=$ constant and $TV^{\gamma-1}=$ constant.

(4) It is important for the calculation of the work done for an adiabatic change of the volume of a gas from the expression $W=\frac{p_1v_1-p_2v_2}{\gamma-1}$

QUESTIONS

1. Give a critical account of a method of measuring the specific heat of a solid accurately in the laboratory—state the nature of the results obtained from the variation of the atomic heat with temperature and indicate how theory accounts for such results. [C. U. 1948]

2. Why is the specific heat of a gas at constant pressure greater than that at constant volume? [C. U. 1940, '50, '56]

3. Explain clearly what you mean by the specific heats at constant volume and at constant pressure of a gas. [C. U. 1945]

4. Describe a method of determining accurately the specific heat of gases at constant volume and mention the corrections that are to be made. [C. U. 1940, '43, '45, '47, '53, '55, '57]

5. Describe the significance of γ and point out in what way a knowledge of its value is important. [C. U. 1943, '47]

6. Explain why a gas has two kinds of specific heats. Show that the difference between the two specific heats is equal to the gas constant. [C. U. 1953, '55]

7. Show how the difference in the specific heats of a gas has been utilised to estimate the value of the mechanical equivalent of heat. [C. U. 1950]

8. What is Dulong and Petit's law of specific heat for solids and gases? How has it been explained from the standpoint of the Kinetic theory of gases? [C. U. 1941]

9. Describe briefly Regnault's method of finding the specific heat of a gas at constant pressure. [C. U. 1926, '54]

10. State Newton's Law of cooling. [C. U. 1928]

EXAMPLES

1. A body cools from 50°C to 40°C in 5 minutes when its surroundings are maintained at 20°C. What will be its temperature after a further 5 minutes?

According to Newton's law of cooling

$\frac{d\theta}{dt}=-c(\theta_2-\theta_1)$, where c is a constant; $\frac{d\theta}{dt}=-c(\theta-20)$.

Here $\theta_2=\theta$ and $\theta_1=20°C$ or $\dfrac{d\theta}{(\theta-20)}=-cdt$

By integration $\log_e(\theta-20)=-c.t+k$, where k=another constant. Now for $t=0$, $\theta=50°C$; $t=5$ min., $\theta=40°C$; $t=10$ min., $\theta=\theta_x°C$

Then, $\log_e(50-20)=-c\times 0+k$ $\therefore$ $k=\log_e 30$.

Again, $\log_e(40-20)=-c\times 5+\log_e 30$ $\therefore$ $c=-\frac{1}{5}\log_e\frac{20}{30}=-\frac{1}{5}\log_e\frac{2}{3}$

Finally, $\log_e(\theta_x-20)=\frac{1}{5}\log_e\frac{2}{3}\times 10+\log_e 30$

$$\text{or } \log_e\frac{\theta_x-20}{30}=2\log_e\frac{2}{3}=\log_e\left(\frac{2}{3}\right)^2 \quad \therefore \quad \frac{\theta_x-20}{30}=\frac{4}{9},$$

$$\text{or } 9\theta_x-180=120 \quad \text{or } 9\theta_x=300, \quad \therefore \quad \theta_x=33°\cdot3C.$$

2. It was observed that a heated piece of iron was cooled from 100°C to 90°C in 1 m. 38 secs. and from 50°C to 45°C in 2 m 24 sec. Assuming Newton's law of cooling to hold through out the range, estimate the temperature of the surrounding atmosphere to which heat was being radiated. [D. U. 1944]

According to Newton's law of Cooling,

Average rate of fall $=c\times$ *average temperature excess,* where c is a constant.

$$\therefore \frac{100-90}{98}=c(95-t_0) \quad \left[\because \; t_m=\frac{100°+90°}{2}=95°C\right]$$

Here t_m is the average temperature and t_0 the temperature of air.

$$\text{Again} \quad \frac{50-45}{144}=c(47\cdot5-t_0) \quad \left[\because \; t_m=\frac{50+45}{2}=47°\cdot5C\right]$$

$$\therefore \frac{10}{98}=c(95-t_0) \quad \dots \quad (1) \qquad \frac{5}{144}=c(47\cdot5-t_0) \quad \dots \quad (2)$$

From (1) by (2) t_0 is found out to be equal to 23°C.

3. A copper calorimeter weighing 15 gms. is first filled with water and then with a liquid. The times taken in the two cases to cool from 65°C to 60°C are 170 sec. and 150 sec. respectively. The weight of the water is 11 gm. and that of the liquid is 13 gms. Calculate the specific heat of the liquid. The specific heat of coper is 0·1 [C. U. 1928]

According to Newton's law of cooling the total quantity of heat leaving the vessel is proportional to the time taken by the liquid in cooling from one temperature to the other.

$$\text{Then, } \frac{\textit{Heat lost by liquid and calorimeter}}{\textit{Heat lost by water and calorimeter}}=\frac{150}{170}$$

$$\text{or } \frac{(15\times\cdot1+13.\,s)\times5}{(15\times\cdot1+11)\times5}=\frac{150}{170} \quad \text{Where } s \text{ is the sp. heat of the liquid.}$$

$$\therefore \quad s=\cdot733$$

4. The inner vessel of a calorimeter of copper weighs 130 grammes and the specific heat of copper is ·095. 50 grammes of iron whose specific heat is ·11 are thrown into 500 grammes of water at 15°C. Calculate the temperature of water after the immersion of iron, the temperature of iron before immersion having been 100°C. Ans. 15°·9C. [C. U. 1910]

5. 7 grammes of ice float in water in a calorimeter of thermal capacity 5 calories. When 4·6 grammes of steam (at 100°C) are passed into the calorimeter the final temperature becomes 60°C, how much water was there in the calorimeter ? Latent heat of steam at 100°C=540. [C. U. 1916]

Ans. 29·9 grammes.

6. A volume of air originally one litre, expands under a constant pressure of 750 *mm.* as it is raised from 0°C to 10°C. Calculate the work done during expansion in terms of c.g s. units. [C. U. 1912]

Let a given mass of gas be enclosed in a cylindrical vessel fitted with an air-tight and frictionless piston. Let the gas expand under a constant pressure p through a distance x of the cylinder. Then if A be the area of the piston, force on the piston $=p.A$. Since the piston has moved through a distance x during expansion, work done = force × distance $=p.Ax$. But Ax represents the increase of volume.

∴ Work done during expansion = pressure × increase of volume.

Pressure of gas $=75\times 13{\cdot}6\times 980$ dynes.

Increase of volume $=1000\times\dfrac{1}{273}\times 10$ c.c.

$$\text{Work done}=\frac{75\times 13{\cdot}6\times 980\times 10^4}{273}\ ergs=36{\cdot}6\times 10^6\ ergs.$$

7. 25 grams of water at 15°C are put into the tube of a Bunsen ice calorimeter and it is observed that the mercury moves through 29 cm. 15 grams. of a metal at 100°C are then placed in the water and the mercury moves through 12 cm. Find the specific heat of the metal.

Since at 0°C the volume of ice is greater than the volume of water at 0°C by ·09 c.c , a reduction of this volume of ice when condensed into water means an absorption of 80 calories of heat. Consequently a decrease in volume of v c.c. corresponds to $\dfrac{80\times v}{{\cdot}09}$ calories.

Here the heat given out by water melts a certain amount of ice producing a reduction in volume as indicated by the backward movement of mercury and is equal to 25×15 calories.

Again the heat given out by the metal of specific heat s melts ice producing the backward movement of mercury and is equal to $15\times s\times 100$ calories.

In the first case $\quad 25\times 15=\dfrac{80\times 29\times\alpha}{{\cdot}09}$ where α is the cross-section of the capillary tube.

In the second case $\quad 15\times s\times 100=\dfrac{80\times 12\times\alpha}{{\cdot}09}$

Therefore $\quad \dfrac{15\times s\times 100}{25\times 15}=\dfrac{12}{29}\quad$ or $\quad s={\cdot}1034$

8. A solid sphere weighing 100 grm. and of specific gravity 10 is cooled to −180°C and immersed in water at 0°C. The apparent weight of the sphere and the ice formed on it is 88 grms. Calculate the specific heat of the solid. Density of ice = ·92 gm/c.c.

Volume of the sphere $=\dfrac{100}{10}=10$ c.c. Let the mass of ice formed $=m$

Then, $100+m=88+10+\dfrac{m}{{\cdot}92}\qquad 2=m\left(\dfrac{1}{{\cdot}92}+1\right)=\dfrac{{\cdot}08}{{\cdot}92}m$

∴ $m=\dfrac{1{\cdot}84}{{\cdot}08}=23$ grm. ; $\quad 100\times s\times 180=23\times 80 \qquad s={\cdot}102.$

9. The diameter of the capillary tube of a Bunsen Calorimeter is 1·4 mm. On dropping into the instrument a piece of metal whose temperature is 100°C

and mass 11·09 grms., the mercury thread is observed to move 10 cms. Calculate the Sp. heat of the metal. (Given the latent heat and density of ice to be 80 and ·9 gms per c. c. respectively.) [D. U. 1948]

Volume change per gram of ice $=\left(\frac{1}{\cdot 9}-1\right)=\frac{1}{9}$ c.c.

$\therefore\ 11{\cdot}09\times S\times 100=80\times\pi({\cdot}07)^2\times 10\times 9 \quad \therefore\ S=\frac{80\times\pi({\cdot}07)^2\times 10\times 9}{11{\cdot}09\times 100}={\cdot}1$

10. The temperature of a body falls from 30°C to 20°C in 5 minutes. The air temperature is 13°C. Find the temperature of the body after a further 5 minutes. [Ordinary non-calculus method is followed. For calculus method *see* Problem 1, Page 59.]

Let x = final temperature. For first 5 minutes, average temp. $=(30+20)/2=25$°C

Average temperature excess $=25-13=12$°C

Average rate of fall of temperature $=\frac{30-20}{5}=2$°C/min.

Then, since the average rate of fall of temp. = c × average temp. excess, where c is a constant. $\therefore\ 2=c\times 12$ or $c=\frac{1}{6}$

For the second 5 minutes :

Average temp. $=\frac{20+x}{2}$ °C ;

Average temp. excess $=\left(\frac{20+x}{2}-13\right)=\left(\frac{x}{2}-3\right)$°C degrees

Average rate of fall of temp. $=\frac{20-x}{5}$ °C degrees/min.

But $\frac{20-x}{5}=c\left(\frac{x}{2}-3\right)=\frac{1}{6}\left(\frac{x}{2}-3\right)$ or $4-\frac{x}{5}=\frac{x}{12}-\frac{1}{2}$ or $\frac{x}{12}+\frac{x}{5}=4+\frac{1}{2}=\frac{9}{2}$

or $5x+12x=270$ or $17x=270$ $\therefore\ x=15{\cdot}88$°C.

11. **Specific heats of air at constant pressure and constant volume are respectively 0·2375 and 0·1684 cal./gm./°C. If density of air be ·001293 find mechanical equivalent of heat.**

We have $J=\frac{R}{C_p-C_v}=\frac{P_0V_0}{T_0(C_p-C_v)}=\frac{70\times 13{\cdot}6\times 981}{{\cdot}001293\times 273\times{\cdot}0691}$

$=4{\cdot}16\times 10^7$ ergs per calorie.

12. **Given that the specific heat of a gas at constant volume enclosed in one sphere of the Joly's differential steam calorimeter is 0·3. Find the excess mass of water which will condense on this sphere, the volume is 10^3 c.c.; its initial and final temperature 15°C and 100°C and the gas is at a density 8×10^{-3} gm/c.c. (L = 540 cal/gm.)** [C. U. 1954]

We know that $C_v=\frac{Lm_2}{m_1(t_2-t_1)}$ [Art. 64]

where m is the mass of the gas at a temp. t_1°C, m_2, the mass of excess of steam condensed, t_2, the temp. of steam, L the latent heat of steam and C_v the sp. heat of the gas at constant volume.

Therefore $m_2=\frac{C_v\times m_1(t_2-t_1)}{L}=\frac{{\cdot}3\times 10^3\times 8\times 10^{-3}\times(100-15)}{540}$

$=\frac{{\cdot}3\times 8\times 85}{540}={\cdot}378$ gm.

13. Calculate the value of J, given that the gram-molecular specific heat of hydrogen at constant pressure $=6{\cdot}865$ calories, that at constant volume $=4{\cdot}880$ calories, atmospheric pressure $=1{\cdot}013\times10^6$ dynes/cm.^{2}, gram-molecular volume of hydrogen at N.T.P. $=22{\cdot}4$ litres and the coefficient of expansion of hydrogen at constant pressure $=1/273$ per °C. [C. U. 1956]

We know that $C_p - C_v = \dfrac{R}{J}$ or $J = \dfrac{R}{C_p - C_v} = \dfrac{P.V}{273(C_p - C_v)}$

$$\therefore\quad J = \frac{1{\cdot}013\times10^6\times22400}{273(6{\cdot}865-4{\cdot}880)} = 4{\cdot}18\times10^7 \text{ ergs per calorie.}$$

14. If the volume of each sphere of Joly's differential steam calorimeter is 500 c.c. and the excess water condensed is 0·1 gm., find the specific heat of the gas at constant volume. The initial temperature was 15°C and density 6×10^{-3} gm/c.c.

The latent heat of condensation is 540 calories per gm. [C. U. 1957]

We have $Lm_2 = m_1\ C_v(t_2 - t_1)$

where L = Latent heat of steam, m_1 = mass of the gas at temp. t_1, m_2 = mass of excess steam condensed and C_v, the sp. heat of the gas at constant volume

$$\text{or}\quad 540\times{\cdot}1 = 500\times6\times10^{-3}\times C_v\times(100-15)$$

$$\therefore\quad C_v = \frac{540\times{\cdot}1}{500\times6\times10^{-3}\times85} = {\cdot}2117$$

CHAPTER V

CHANGE OF STATE

69. Fusion : Fusion or melting is the change of a solid substance on heating from the solid to the liquid state.

If a solid body is heated, its temperature rises gradually until at a certain temperature the solid begins to melt. This temperature remains constant throughout the entire process of melting and is called the *melting point* of the substance. This is different for different substances.

70. Laws of fusion :

(*a*) A solid melts at a definite temperature which generally coincides with that at which the corresponding liquid solidifies.

(*b*) Since the temperature of the solid remains the same while it is melting it follows that a certain amount of heat, known as *latent heat*, is absorbed. An equal amount of heat is evolved when the solid in the liquid state solidifies.

71. Solidification : Solidification or freezing is the change of a liquid on cooling from the liquid to the solid state. The temperature at which the liquid solidifies remains constant during the process of solidification and is known as the *freezing* or solidification *point*. This is different for different substances.

Many substances occupy a larger volume in the liquid than in the solid state, so that contraction takes place on solidification. But there are exceptions. Some substances such as ice, cast iron, bismuth, etc. contract on melting and expand during solidification.

In the case of water which expands on solidification, 1 c.c. of water at 0°C becomes 1·09 c.c. of ice at 0°C.

72. Super-cooling or Surfusion : Under certain conditions it is possible to cool water to a temperature considerably below zero degree without solidification. If a test tube filled with distilled water is cooled without stirring the water or disturbing it in any way, the temperature will be observed to fall below zero degree without any ice being formed.

If the surface be covered with a layer of oil to keep it free from dust the water may be cooled to − 12°C. The phenomenon is called *Super-cooling* or *Surfusion*. This condition is unstable. If a small piece of ice is dropped into the super-cooled water, solidification at once begins, the temperature quickly rising to 0°C.

73. Latent Heat : We know that when heat is applied to a body its temperature begins to rise but after some time it will remain stationary although heat is being continually supplied to the body. This takes place when the body melts *i.e.* passes from the solid to the liquid state. At this stage of the substance, a thermometer when introduced into it will show no change in temperature and heat supplied to the body from the beginning to the end of fusion is used up in overcoming the mutual attraction between the molecules and changing the body from the solid to the liquid state. Thus when heat is applied to ice at 0°C it will melt and the temperature will remain stationary until the whole of the ice has been melted and after that it will rise again.

Since the heat applied to ice at its melting point is not indicated by a thermometer, and as it goes with the water formed and remains hidden in water, it is called **latent heat**.

Again, when heat is supplied to a liquid its temperature will gradually rise until a stage is reached when the liquid will show signs of boiling. As soon as the liquid begins to boil the temperature will cease to rise and will remain stationary until the whole of the liquid is vaporised. The heat supplied during the process of boiling is used up only in changing the liquid into its vaporous condition.

Since the heat applied to water at its boiling point is not indicated by a thermometer and as it goes with the steam formed and remains hidden in the steam, it is also called **latent heat. Latent**

heat of fusion of a solid is the amount of heat required to change unit mass of the solid at its melting point to the liquid condition without change of temperature. Again, **latent heat of vaporisation** of a liquid is the amount of heat required to change unit mass of the liquid at its boiling point to the vapour state without any change of temperature.

The latent heat of fusion of ice is 80 cal/gm. Its value in lb-degree centigrade is also 80, but in lb-degree Fah. the value is $80 \times 9/5 = 144$.

Similarly, the latent heat of vaporisation of water is 537 cal/gm. Its value in lb.-degree centigrade is also 537, but in lb-degree Fah. the value is $537 \times 9/5 = 967$ (nearly).

74. Latent heat of fusion of ice by Method of mixtures : The calorimeter with stirrer is weighed. It is placed inside an enclosure on a non-conducting wooden stand, and the initial temperature taken. Small pieces of ice carefully dried, are added gradually and the water is stirred until each piece has melted before the next piece of ice is added. When the temperature has fallen to a steady value, the temperature is noted. The calorimeter is then weighed to obtain the mass of ice melted.

Let M be mass of ice melted, m the mass of calorimeter, S the specific heat of material of the calorimeter, w the mass of water. Let t_1°C and t_2°C be the initial and final temperatures of the calorimeter and its contents, and L cal/gm. be the latent heat of fusion of ice.

Then, heat gained by ice first in melting at 0°C + heat gained by melted ice (ice-water) in rising from 0°C to t_2°C = heat lost by calorimeter and water in cooling from t_1°C to t_2°C.

$\therefore$ $ML + Mt_2 = (mS + w)(t_2 - t_1)$. Whence L is determined.

Discussion. The chief error is due to the fact that the ice pieces added are not completely dry. A single piece of ice of suitable mass being held below surface of water by the stirrer with wire-gauze cage is preferable as it can be more easily freed of moisture, and it will have no chance of melting due to heat of air. The mass of ice should be such that no dew deposits on the walls of the calorimeter.

The cooling correction can be eliminated as follows. The calorimeter with water is heated previously to temperature t_1 which is as much above the air temperature as t_2 is below, the average rate of loss in the first part is then equal to average rate of gain in the later stages.

75. Conversion of ice below 0°C to steam : If M gms. be mass of ice at $-t$°C, S be specific heat of ice, L_f the latent heat of

fusion of ice and L_v the latent heat of vaporisation of water, then heat required to change the ice into steam is given by

$$Q = M.S\{0-(-t)\} + ML_f + M.(100-0) + ML_v$$
$$= (MSt + ML_f + M.100 + M.L_v) \text{ calories.}$$

76. Latent heat of Vaporisation of water : A calorimeter with a stirrer is taken and weighed. It is filled two-thirds with water and weighed again. It is placed inside its enclosure on a nonconducting stand and the temperature t_1 noted. Steam from a boiler is passed into the water-trap (Fig. 21) until only dry steam will issue out from the long vertical tube. The tube is immersed in water in the calorimeter and some dry steam is passed into it until temperature of water rises by about 5°C. The final temperature of the calorimeter is noted after stirring, and when cooled, the calorimeter and contents are weighed again. The difference of second weight from third weight is the mass of the steam condensed. Let it be M. Let m be mass of calorimeter, S its specific heat and w mass of water, taken ; let the initial and final temperatures be t_1 and t_2 ; let the boiling point of water after correcting for pressure be T and L be latent heat of steam at this temperature.

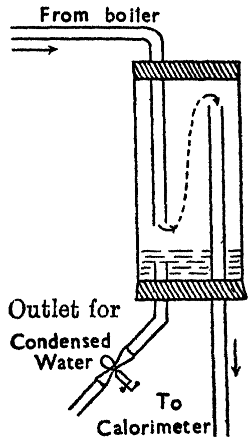


Fig. 21

The, total heat lost by steam = heat gained by calorimeter and contents or $ML + M(T - t_2) = (mS + w)(t_2 - t_1)$ whence the value of L can be found out.

77. Effect of pressure on the melting point of ice and boiling point of water : It is a fact that substances such as brass, cast iron, bismuth, type metal and ice etc. which contract on melting have all their melting points **lowered** by the *increase of pressure* and substances such as wax, lead etc., which expand on melting have their melting points **raised** by the *increase of pressure.*

This is obvious from the general reasoning. For, in case of ice any increase of pressure tends to diminish the volume and thus it helps melting and so the melting point is lowered under increased pressure. In case of wax, increase of pressure which tends to diminish the volume, will oppose melting and so the melting point is raised by increased pressure.

The fact that the melting point of ice is lowered by the increase of pressure is illustrated in the phenomenon of **Regelation :**

A single turn of a thin copper wire supporting a heavy weight is made to rest on a block of ice standing on two legs. It will be seen that the wire slowly cuts its way through the ice without dividing the block of ice into two pieces. The explanation is that the portion of ice just under the wire is subjected to a considerable pressure so that its melting point is lowered and some ice melts. The wire then easily passes through the water which again solidifies as the pressure is removed. The process goes on until the wire comes out finally leaving the ice block intact.

A liquid, say water, boils when its vapour pressure equals that of the pressure to which the liquid is subjected. If the pressure be greater, the boiling point will be raised.

A substance in the vaporous condition occupies a much greater volume than in the liquid state. So an increased pressure will tend to hinder expansion during vaporisation of the liquid into vapour. Hence, when the pressure is high the liquid will boil at a higher temperature than when the pressure is low.

We know that ice melts at 0°C when the pressure of air is one atmosphere, and this temperature remains constant so long as the pressure remains the same. If the pressure of air be increased the melting point would be lowered *i.e.* would be below 0°C.

Again it has been found that for a change of pressure of one atmosphere the change in the melting point of ice is ·0375°C and for a change of pressure of 1 cm. of mercury the change in temperature of boiling point of water is ·035°C.

78. Total heat of Steam : The total heat of steam at any temperature t°C is defined as the quantity of heat which must be communicated to one gram of water at 0°C in order to convert it to saturated vapour t°C.

If Q be the total heat of steam, or the amount of heat required to convert one gram of water at 0°C to steam at t°C and L_t the latent heat of vaporisation at t°C, then $Q = t + L_t$.

Regnault found that $Q = 606{\cdot}5 + {\cdot}305t$. $\therefore$ $L_t = 606{\cdot}5 + {\cdot}305t - t = 606{\cdot}5 - {\cdot}695t$ or $L_t = a - bt$ where $a = 606{\cdot}5$, $b = {\cdot}695$.

The latent heat of steam at $100°C = L_{100} = 606{\cdot}5 - {\cdot}695 \times 100$
$= 537$ calories per gram.

QUESTIONS

1. What is regelation? Explain the phenomenon by illustrations.

2. How does the change of pressure affect the temperature of freezing? Explain this change by the mechanical theory of heat.

3. State briefly the effect of change of pressure on the melting point of ice and boiling point of water. [C. U. 1926]

EXAMPLES

1. Lead melts at 326°C. Its specific heat is ·0314 in the solid and ·0402 in the liquid state. Find what mass of water at 0°C will be raised one-tenth of a degree centigrade by dropping into it 100 grams of melted lead at 350°C.

[C. U. 1921]

Let m = mass of water required and L = the latent heat of fusion of lead.

Heat given out by 100 *gms.* of lead in falling from 350°C to 326°C, in being solidified and finally in falling again from 326°C to ·1°C.

$= 100(350-326)\times{\cdot}0402+100\text{L}+100\times{\cdot}0314(326-{\cdot}1)$ cal.
$= 100(24\times{\cdot}0402+\text{L}+325{\cdot}9\times{\cdot}0314)$ cal.

Heat required to raise m grammes of water through ·1°C $= m\times{\cdot}1$ cal.

$m\times{\cdot}1 = 100(24\times{\cdot}0402+\text{L}+325{\cdot}9\times{\cdot}0314) = 100(\text{L}+11{\cdot}198)$
$m = 1000(\text{L}+11{\cdot}198)$ grammes nearly,

If L = 5·38 which is not given in the problem, then

$m = 1000(5{\cdot}38+11{\cdot}198) = 1000\times16{\cdot}578 = 16578$ grams. nearly.

2. Find the result of mixing equal masses of, ice at -10°C and water at 60°C. Latent heat of water is 80 and specific heat of ice is ·504. [C. U. 1922]

Let m be the mass of ice and water taken.

Heat required in melting m grammes of ice from -10°C to 0°C $= m\times{\cdot}504\times10$ cal. $= 5{\cdot}04\,m$ cal.

Heat required in melting m grammes of ice at 0°C to water at 0°C $= 80m$ cal.

And heat given out by m grammes of water in cooling from 60°C to 0°C $= 60m$ cal.

Now $60m > 5{\cdot}04m$ but $< 85{\cdot}04m$. It follows that of $60m$ calories of heat given out by m grams of water in cooling from 60°C to 0°C, 5·04 m calories will go to heat m grams of ice from -10°C to 0°C and the rest *i.e.* $(60-5{\cdot}04)m$ calories will melt a part of ice. Therefore the water is reduced to 0°C before the whole of the ice melts.

Again, mass of ice melted $= \dfrac{(60-5{\cdot}04)m}{80}$ $gms = {\cdot}686m = {\cdot}69m$ gm. nearly.

∴ About ·69 part of the total mass of ice taken is melted.

3. Water is boiled at 120°C and the vapour formed is passed into a vessel containing 100 grammes of ice. If the whole of the ice is melted and the resulting liquid (due to ice and vapour) is at 0°C, find the amount of water vapour passed into the vessel.

Latent heat of water at 100°C	= 537
,, ,, ,, at 90°C	= 544
,, ,, fusion of ice	= 80

[C. U. 1911]

Change of latent heat of water the temperature changes from 90°C to 100°C = 7

Change for a difference of 20°C $= 2\times7 = 14$

Hence, latent heat of water at 120° $= 537-14 = 523$

Heat lost by m grams of water vapour at 120°C in condensing to water at 0°C $= m\times523+m(120-0)$.

Heat gained by 100 grammes of ice in melting to water at 0°C $= 100\times80$ cal.

∴ $m(523+120) = 100\times80$ ∴ $m = 12{\cdot}44$ grams.

4. A piece of ice weighing 22·1 gms. at initial temperature $-3°C$ is dropped into a copper calorimeter weighing 97·9 gms. and containing 129·1 gms. of petroleum at 28°C, the specific heat of which is ·52. Find the resulting temperature neglecting the loss by radiation. [C. U. 1915]

Specific heat of ice = ·5
Specific heat of copper = ·094
Let t°C be the resulting temperature.
Heat lost by calorimeter and petroleum in cooling from 28°C to t°C
$= 97{\cdot}9 \times {\cdot}094 \times (28-t) + 129{\cdot}1 \times {\cdot}52(28-t)$ cal.
$= (97{\cdot}9 \times {\cdot}094 + 129{\cdot}1 \times {\cdot}52)(28-t)$ cal.

Heat required by 22·1 gms. of ice in being heated from $-3°C$ to 0°C, then in being melted and finally in being again heated (in the liquid state) from 0°C to t°C $= 22{\cdot}1 \times {\cdot}5 \times 3 + 22{\cdot}1 \times 80 + 22{\cdot}1 \times t = 22{\cdot}1 \times (t + 81{\cdot}5)$ cal.

Since, heat lost = heat gained.

$$(97{\cdot}9 \times {\cdot}094 + 129{\cdot}1 \times {\cdot}52)(28-t) = 22{\cdot}1(t + 81{\cdot}5)$$

or $76{\cdot}3(28-t) = 22{\cdot}1(t+81{\cdot}5)$ or $2136{\cdot}4 - 76{\cdot}3t = 22{\cdot}1t + 1801{\cdot}15$

or $98{\cdot}4t = 335{\cdot}25$ $\therefore$ $t = 3{\cdot}4°C$ nearly.

5. Assuming that the specific volume of water and saturated steam at 100°C are 1 c.c. and 1·601 c.c. respectively and the latent heat of evaporation is 536 calories per gram, find the change in temperature of the boiling point due to a change of pressure of 1 cm. of mercury. [C. U. 1948]

From Clapeyron's equation $dT = \dfrac{T.dp.(v_2 - v_1)}{L.J}$

Here dT is the increase in temperature, dp the increase in pressure, v_2 and v_1 denote the specific volumes of vapour and liquid respectively at 100°C, L, the latent heat of evaporation and J, the Joule's heat equivalent.

$$\therefore \quad dT = \frac{373 \times 1 \times 13{\cdot}6 \times 981 \times (1601 - 1)}{536 \times 4{\cdot}2 \times 10^7} = {\cdot}3537°C$$

In the question 1·601 c.c. should be 1601 c.c.

6. Find the change in the freezing point of water at 0°C by a change of pressure of one atmosphere.

Latent heat of fusion $= 79{\cdot}6 \times 4{\cdot}18 \times 10^7$ ergs per gm.

Sp. vol. of water at 0°C = 1·000 c.c.
" " of ice " " = 1·091 c.c.

$$dT = \frac{273 \times (1 - 1{\cdot}091) \times 76 \times 13{\cdot}6 \times 981}{79{\cdot}6 \times 4{\cdot}18 \times 10^7} = -0{\cdot}0075°C.$$

CHAPTER VI

KINETIC THEORY OF GASES

79. Evolution of Kinetic Theory : The kinetic theory of gases or more generally kinetic theory of matter is based upon two fundamental hypothesis :—the molecular structure of matter and the identification of heat energy with molecular motion.

The evidence of molecular agitation in matter is provided by many phenomena a few of which are stated as follows: (1) The phenomena of diffusion and solution, (2) Tendency of a gas to expand at a given temperature, (3) Evaporation and vapour pressure and (4) Brownian Movement.

The phenomenon of diffusion in which a heavy gas such as CO_2 moves upwards and mixes with the lighter gas H_2 placed above, clearly proves that although the molecules of a gas appear at rest yet there must be some kind of movement going on in the whole volume of the gas.

80. Pressure exerted by a Perfect gas: All gases consist of molecules in motion. Hence, they must exert pressure on the walls of the containing vessel or their enclosures. The following assumptions are made for the calculation of this pressure, to simplify matters.

(1) The molecules of a gas are all alike, and are perfectly elastic spheres, there being no force of the attraction or repulsion between them, or between them and the walls of the containing vessel.

In other words all the energy of the gas molecules is kinetic and they do not suffer any loss of momentum or kinetic energy after collision with the walls of the enclosure, their direction of motion being only reversed.

(2) The smallest volume of a gas contains a very large number of molecules, and the size of the molecules is so small that the actual volume of the molecules is negligibly small compared with the volume of the enclosure.

(3) Though the molecules are ceaselessly colliding against one another in all directions with different velocities and having their velocities altered in direction and magnitude at each collision, yet the molecules do not in the steady state collect more at one place than at another; in other words the collisions do not affect the molecular density of the gas.

(4) The molecules being material bodies move, according to the laws of motion, in straight lines with uniform velocity between two collisions.

(5) The time for which a collision lasts is negligible compared with the time interval between two collisions or the time required to cover the distance traversed between two collisions. This distance is called the **mean free path** of the molecules and depends on the pressure and temperature of the gas. The dimension of the molecules is negligible in comparision with this mean free path.

81. Deduction of Pressure of a gas: Let a hollow cube (Fig. 22) of volume one cubic centimetre contain a single molecule only and let its mass be m.

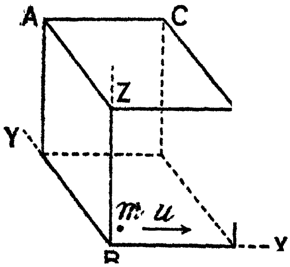


Fig. 22

Let the molecule be moving along a line perpendicular to the opposite faces AB, CD, striking the wall CD rebounding and striking the wall AB and so on. The velocity is reversed in each impact. If u be the mean velocity of the molecule, the momentum before impact on wall $CD = mu$. After impact on CD the molecule rebounds with velocity reversed, to strike upon wall AB again.

$\therefore$ Momentum after impact on $CD = -mu$

$\therefore$ Change of momentum due to 1 impact on $CD = 2mu$.

Now time taken to travel from wall CD to wall AB and back to CD, *i.e.* time between two successive impacts on $CD = \frac{2}{u}$ sec

Therefore the number of impacts per second is $\frac{u}{2}$, and hence the total change of momentum per second $= \frac{u}{2} \times 2mu = mu^2$

By Newton's second law of motion rate of change of momentum is equal to the force acting upon wall CD of unit area. But force per unit area is pressure. Therefore pressure on the wall $CD = mu^2$.

If the cube contains n molecules, the pressure of the whole gas on the wall $CD = mnu^2$, where u^2 is square of mean molecular velocity previously assumed.

Since the molecules have different velocities in different directions it is proper to take into consideration the mean of the square of molecular velocities instead of the square of the mean of molecular velocities.

So the pressure $p = mn\bar{C}^2$, when $\bar{C}$ is the root of mean of the square of the molecular velocities.

Instead of supposing all the molecules to be travelling at a given instant backwards and forwards along a line perpendicular to the two opposite walls, it is more reasonable to consider the molecules to be moving along three perpendicular lines drawn parallel to the

adjoining edges of the centimeter cube, and as many molecules will at any instant, be moving along one of these lines as along any other.

Of the total number of molecules n in the cube, $\frac{n}{3}$ molecules may therefore be considered to be moving in any direction at any instant.

Then expression for pressure on the wall CD becomes

$p=\frac{n}{3}m\bar{C}^2=\frac{1}{3}mn\bar{C}^2$; but $mn=\rho$ the density of the gas, since mn is the mass per unit volume.

$$\therefore\ p=\frac{1}{3}\rho\bar{C}^2=\frac{2}{3}\cdot\frac{1}{2}\bar{C}\,\rho^2=\frac{2}{3}E$$

where E is the kinetic energy of translation per unit volume.

Thus we see that **the pressure of a gas is numerically equal to two-third of the kinetic energy of translation per unit volume.**

81a. Root-mean-Square Velocity : The root-mean-square velocity or R. M. S. value $\bar{C}$ of the velocity of a molecule of a gas is the quantity obtained by taking out the square root of the mean square velocity $\bar{C}^2$, *i.e.*, $C=\sqrt{\bar{C}^2}$. It is the velocity with which if each molecule of the gas were supposed to move, the total kinetic energy of the gas moles. will be same as with their actual variable velocities.

82. Boyle's Law : We have, $p=\frac{1}{3}mn\bar{C}^2=\frac{1}{3}\rho\bar{C}^2$; if v be the volume of the cube and hence, that of the gas under the same condition of temperature, and M be the mass of the gas, then

$$pv=\tfrac{1}{3}M\bar{C}^2.$$

Since M is constant, pv will be constant if $\bar{C}^2$ remains constant. Again, we know that the increase of kinetic energy of the molecules of the gas is the cause of a rise of temperature. Hence, $\bar{C}^2$ is constant at constant temperature.

$$\therefore\ pv=\frac{1}{3}M\bar{C}^2=\text{constant, when temperature is constant.}$$

Thus Boyle's law is proved.

Note : If M = the molecular weight of the gas and v = the volume occupied by a gram-molecule of the gas

$$pv=\tfrac{1}{3}.M\bar{C}^2$$

Comparing this equation with the ideal gas equation $pv=RT$, R being the gas constant for 1 gram-molecule of the gas, we have

$$\tfrac{1}{3}M\bar{C}^2=RT \qquad (\because\ pv=RT)$$

or $\quad \tfrac{1}{2}M\bar{C}^2=\tfrac{3}{2}RT \quad \ldots \quad \ldots \quad (1)$

Dividing both sides by N, the number of actual molecules in 1 gram-molecule of the gas, called **Avogadro's number**, we have,

$$\tfrac{1}{2}\frac{M}{N}\ \bar{C}^2=\tfrac{3}{2}\frac{R}{N}\ .\ T$$

But $\frac{M}{N}=m$ (mass of a molecule) or $\frac{1}{2}m\bar{C}^2 = \frac{3}{2}KT$ (2)

where $K=\frac{R}{N}$ and is called **Boltzmann's Constant.**

The equation (1) states **that the mean kinetic energy of translation of one molecule is $\frac{3}{2}$ KT.**

83. Charles's Law : If in the equation $pv = \frac{M}{3}\bar{C}^2$, p is constant. $v \propto \bar{C}^2$ (1)

According to Kinetic theory of gases, the mean square velocity $\bar{C}^2$ is proportional to the absolute temperature of the gas.

$\therefore$ $\bar{C}^2 \propto T$. (abs. temp.) ... (2)

From (1) and (2) we have $v \propto T$.

Thus in a given mass of gas, the volume is proportional to the absolute temperature provided the pressure is constant.

84. Avogadro's Law : It is known that the temperature of a gas molecules is proportional to the kinetic energy possessed by them due to their linear velocity and that at a given temperature kinetic energies possessed by molecules of different gases have equal values.

Let us consider two vessels of equal capacities and containing two gases A and B at the same temperature and pressure.

Let n_1 and n_2 be the numbers of molecules of the gases in the two vessels.

Let m_1 and m_2 be the masses of the molecules of gases A and B respectively and V_1 and V_2 be the square roots of the mean of the squares of the volocities at the absolute temperature T, the kinetic energy of the molecules of gas A is $\frac{1}{2}m_1V_1^2$ and that of the molecules of gas B is $\frac{1}{2}m_2V_2^2$.

But according to our assumption we have $\frac{1}{2}m_1V_1^2=\frac{1}{2}m_2V_2^2$ or $m_1V_1^2=m_2V_2^2$. ... ... (1)

Since the two vessels are filled with different gases A and B at the same temperature and pressure, then pressure p is given by $p=\frac{1}{3}m_1n_1V_1^2=\frac{1}{3}m_2n_2V_2^2$ or $m_1n_1V_1^2=m_2n_2V_2^2$ ... (2)
Dividing (2) by (1) $n_1=n_2$.

Here n_1 and n_2 are respectively the numbers of molecules of gases A and B in the vessels.

Thus *at the same temperature and pressure equal volumes of all perfect gases contain equal number of molecules.* This is Avogadro's Law.

85. Graham's Law of diffusion: Graham's law of diffusion states that the rates of efflux of different gases through a porous membrane under given conditions of temperature and pressure are inversely proportional to the square root of their densities. For two gases A and B at the same pressure, the rate of diffusion is again proportional to the root-mean-square (R.M.S.) of the velocity of each gas molecule.

Let ρ_1, ρ_2 be density of gases A and B, and $\bar{C}_A$, $\bar{C}_B$ the root-mean-square velocities of A and B, respectively.

Then for gas A, pressure $p = \frac{1}{3}\rho_1 \bar{C}_A^{\,2}$

" " " B, " $p = \frac{1}{3}\rho_2 \bar{C}_B^{\,2}$

$$\therefore \quad \tfrac{1}{3}\rho_1 \bar{C}_A^{\,2} = \tfrac{1}{3}\rho_2 \bar{C}_B^{\,2}$$

$$\text{or} \quad \frac{\bar{C}_A}{\bar{C}_B} = \sqrt{\frac{\rho_2}{\rho_1}}$$

$$\text{Hence,} \quad \frac{\text{Rate of diffusion of A}}{\text{Rate of diffusion of B}} = \sqrt{\frac{\text{density of B}}{\text{density of A}}}$$

Thus Graham's law is proved.

86. Viscosity from Kinetic Theory of Gas: We know that when two adjacent layers of a gas move relatively to each other with different velocities, forces are called into play between them tending to destroy the relative motion. The property of fluid (gas or liquid) which opposes the relative motion of the adjacent layers is called *viscosity*.

To explain viscosity of gas from the kinetic theory of gases let us consider three parallel planes A, B and C (Fig. 23) in a gas such that the distances between A and B and B and C are each equal to the *mean free path* of the molecules of the gas.

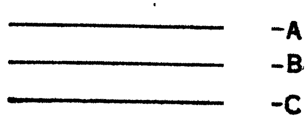


Fig. 23

Let us suppose that the gas in the immediate neighbourhood of the plane A is moving bodily form left to right and that the rate of flow of gas decreases as we move from A through B to C.

Some of the molecules of the gas in the plane A moves towards B and reaches it without suffering an encounter. So they become part of the gas below B and have the same velocity as those of the molecules in the plane A. Similarly, those of the molecules from the plane C move up and pass through B and become part of the gas above B retaining the same velocity as those of the molecules of the gas in the plane C.

Thus the result of this interchange of molecules having different velocities between the different layers of the gas decreases the velocity of the movement of the gas above the plane B and increases it below and therefore tends to oppose the relative motion of the adjacent layers and endow the medium with a property known as the *viscosity* of the gas.

87. Heat of Compression of a gas; Cooling of gas by expansion.

Consider a gas enclosed in a cylinder having a piston P. Let the velocity of impact of a gas molecule on the piston be u. (Fig. 24, left hand side). To compress the gas the piston is to be moved inward with some velocity v. The rebounding gas molecule will have total velocity $u+v$. This gain of velocity causes a rise of temperature.

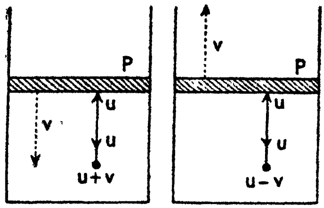


Fig. 24

Again, for expansion of the gas the piston (Fig. 24, right hand side) will move outward with some velocity say v. The rebounding gas molecule will have total velocity $u-v$ where u is the velocity before impact. This loss of velocity causes a fall of temperature.

88. Determination of the value of $\bar{C}$: We know that $p=\frac{1}{3}\rho\bar{C}^2$ where ρ = density of the gas.

$$\text{or } \bar{C}^2=\frac{3p}{\rho} \quad \therefore \bar{C}=\sqrt{\frac{3p}{\rho}}$$

This gives the square root of mean-square-velocity (R. M. S. value of velocity) which is not the same as the average velocity.

For hydrogen at N. T. P.

$p=76\times13{\cdot}6\times981$ dynes/sq. cm.; $\rho=0{\cdot}00009$ gm/c.c.

$$\bar{C}=\sqrt{\frac{3\times76\times13{\cdot}6\times981}{0{\cdot}00009}}=1{\cdot}86\times10^5 \text{ cm./sec.}$$

Note: Comparison of value of $\bar{c}$ with velocity of Sound in the same gas :—

Velocity of sound $V_s=\sqrt{\frac{\gamma p}{\rho}}$ where $\gamma=1{\cdot}4$ for diatomic gases. (ratio of two sp. hts. of a gas)

From Kinetic theory $\bar{c}=\sqrt{\frac{3p}{\rho}}$

From above $V_s=\sqrt{\frac{1{\cdot}4p}{\rho}}$ $\quad \bar{c}/V_s=\sqrt{3/1{\cdot}4}=\sqrt{1{\cdot}4}$ nearly.

QUESTIONS

1. What is the kinetic theory view-point of gaseous systems? How would you relate the pressure, volume and temperature of an ideal gas from this view point? [C. U. 1952, '57]

2. Deduce Boyle's law, Charles' law and Avogadro's hypothesis for a perfect gas from the principles of kinetic theory. [C. U. 1947, '49]

3. How are pressure and viscosity of a gas explained on the kinetic theory? [C. U. 1948]

4. Obtain an expression for the pressure of a gas in terms of its density and the root-mean-square velocity of the molecules from the stand-point of kinetic theory. [C. U. 1949, '55, '58]

5. Shew that the pressure of a gas is equal to two-thirds of the kinetic energy of translation per unit volume. [C. U. 1951]

Calculate the kinetic energy of hydrogen per gram-molecule at 0°C.

6. Explain what is meant by the root-mean-square velocity of a molecule of a gas. Deduce the relation between the root-mean-square velocity, pressure and density of the gas molecules. [C. U. 1958]

EXAMPLES

1. Find the sq. root of the mean square velocity of the molecules (of oxygen) at 0°C. The density of oxygen at N. T. P. = 1·43 gm. per litre. [C. U. 1948]

$$\bar{c}=\sqrt{\frac{3p}{\rho}}=\sqrt{\frac{3\times 76\times 13{\cdot}6\times 981}{{\cdot}00143}}=4{\cdot}6\times 10^4 \text{ cms/sec.}$$

2. Calculate the kinetic energy of hydrogen per gram-molecule at 0°C. [C. U. 1951]

We know that Kinetic energy $\frac{1}{2}M\bar{c}^2=\frac{3}{2}RT=\frac{3}{2}\times 8{\cdot}3\times 10^7\times 273$

$[\because R=8{\cdot}3\times 10^7$ ergs per degree]

$=3{\cdot}4\times 10^{10}$ ergs.

3. Calculate the molecular Kinetic energy of 1 gm. of helium at N. T. P. what will be the energy at 100°C?

We have $\frac{1}{2}M\bar{c}^2=\frac{3}{2}RT$

$\therefore$ Energy for 1 gm. $\frac{1}{2}1.\bar{c}^2$ at N.T.P. $=\frac{3}{2}\ \frac{RT}{M}=\frac{3}{2}\ \frac{8{\cdot}3\times 10^7\times 273}{4}=8{\cdot}5\times 10^9$ ergs.

Energy at 100°C $=\frac{8{\cdot}5\times 10^9\times 373}{273}=1{\cdot}16\times 10^{10}$ ergs.

4. Calculate the root-mean-square velocity of a hydrogen molecule at N.T.P. The density of hydrogen = ·00009 gm/c.c. [C. U. 1956]

We know that $\bar{c}=\sqrt{\frac{3p}{\rho}}$

For hydrogen at N. T. P.

$p=76\times 13{\cdot}6\times 981$ dynes/cm², $\rho={\cdot}00009$ gm/c.c.

$$\therefore\ c=\sqrt{\frac{3\times 76\times 13{\cdot}6\times 981}{{\cdot}00009}}=1{\cdot}84\times 10^5 \text{ cms/sec.}$$

5. If the density of a gas at N.T.P. (temperature 0°C, pressure 760 m. m. of mercury) is 9×10^{-4} gm/c.c., find the root-mean-square velocity of the gas molecules. [C. U. 1958]

The expression for pressure is $p=\frac{1}{3}\rho\bar{C}^{-2}$, where ρ is the mass of gas per unit volume.

But $$\bar{C}=\sqrt{\frac{3p}{\rho}}=\sqrt{\frac{3\times 76\times 13{\cdot}6\times 981}{{\cdot}0009}}=5{\cdot}8\times 10^{-4} \text{ cms.}$$

CHAPTER VII

VAPOURS AND THEIR PROPERTIES

89. Vapour tension or Saturation Vapour pressure : Like gases, vapours of different liquids exert pressure on the walls of the vessels in which they are contained. When a small quantity of any volatile liquid is introduced into a space devoid of air it will at once evaporate and fill up this space with vapour which will exert a definite pressure. If the space be saturated with vapour, the pressure in the staturated space is called the **saturation pressure or maximum pressure of vapour.** The saturation pressure is independent of the volume occupied by the vapour but entirely depends on the *temperature* of the vapour.

When the space is unsaturated, the vapour in this unsaturated space obeys Boyle's Law and the pressure of vapour will increase or diminish according as the volume is diminished or increased. But for changes of temperature the unsaturated vapour will very nearly obey Charles's Law.

If an enclosure contains vapour of a liquid in presence of the liquid the vapour is definitely saturated. But in this condition of saturation, evaporation does not stop; it continues all the while, but is balanced by condensation which takes place exactly at the same rate. A phenomenon such as this, in which no change is manifest since two continuously opposing processes are taking place at the same rate is known as dynamic equilibrium. Thus a saturated vapour at a given temperature is a vapour which is in dynamic equilibrium with its liquid at that temperature.

Note : When an enclosed space contains maximum quantity of vapour it can hold at a given temperature the vapour is said to be saturated at that temperature. If the said space contains a lesser amount of vapour, the vapour is called unsaturated at the same given temperature.

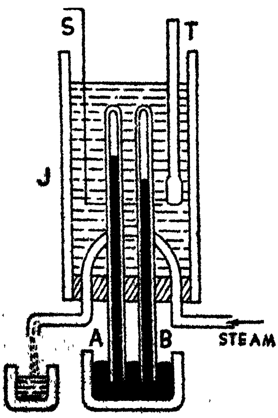


Fig. 25

90. Measurement of Saturation Vapour pressure of water : Dalton's Experiment (Between 0° and 50°C): Two barometer tubes A and B (Fig. 25) surrounded by a long jacket were arranged side by side, fed from the same trough of mercury. The barometer A is kept as such with its space above mercury a vacuum, while water is gradually introduced into the vacuous space in B., where the water evaporates. More water is introduced till a small layer remains over mercury in B. At this stage, vapour in the tube B is saturated and the pressure of this vapour is maximum and remains

constant as long as the temperature is not altered. The difference in heights of two mercury columns in A and B which are observed with a cathetometer, gives the saturated vapour pressure of water at the temperature of water in the jacket. Similar measurements were repeated by raising temperature gradually to 50°C, care being taken all the while to introduce more and more water in B to make the vapour saturated at a particular temperature.

As the bath was long, efficient stirring of water was not possible. Hence, readings of temperature were not correct. Again as observation was made by cathetometer through the curved wall of jacket, some error due to refraction was introduced.

91. Regnault's experiment : To eliminate several errors in Dalton's method, Regnault surrounded only the upper portion of the barometer tubes B and E by a shorter vessel, (Fig. 26) the front face of which was a plane glass plate. Thus uniform stirring of water was made possible, and error due to refraction was reduced. The measurements of saturated vapour pressure of water were made at different temperatures between 0° and 50°C by cathetometer, in the same way as in Dalton's experiments.

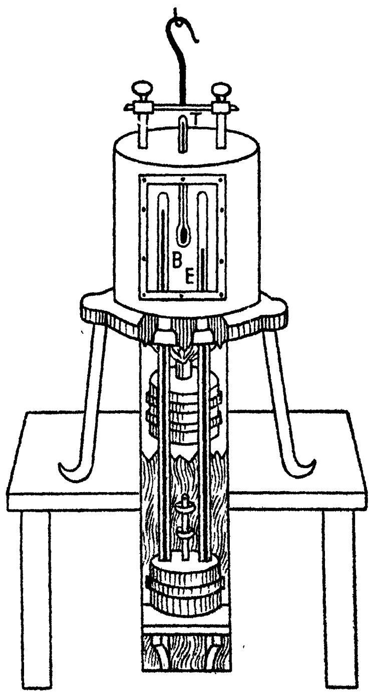


Fig. 26

92. Regnault's Dynamical method : From 50°C (upwards). To determine the *vapour pressure at temperatures above* 50°C, Regnault made use of the fact **that the vapour pressure of any liquid at its boiling point is equal to the superincumbent air pressure.** The apparatus used by Regnault consists of a strong copper vessel B (Fig. 27) containing water which can be raised to different temperatures indicated by a sensitive thermometer T fitted with the copper vessel. The upper part of the copper vessel is connected by means of a tube inclined upwards, with a large metal sphere E immersed in a water-bath W at a constant temperature. The metal sphere contains air whose pressure is altered at will by means of a pump and determined by a manometer M connected to E. The vapour

produced from the liquid passes up the inclined tube leading to the metal sphere, gets condensed by a cold water jacket C placed round the tube and finally flows back into the boiler. With the help of this arrangement Regnault determined the vapour pressure of any liquid at temperatures above 50°C. For temperature above 100°C a compression pump, and for temperature below 100°C an exhaust pump are to be connected to E.

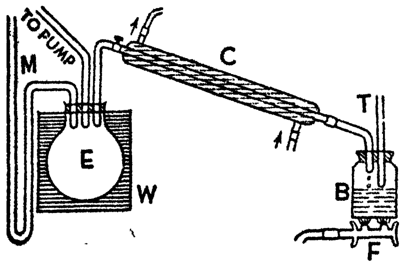


Fig. 27

93. Vapour Pressure at Low temperatures (below 0°C) by Gay Lussac's method : The method depends on the fact that if a number of enclosures of a system in communication with one another, contain vapours or gases at different temperatures, then in the steady state, the pressure everywhere in the system corresponds to that of the coldest part of the system.

Gay Lussac determined the vapour pressure of substances at temperatures below zero degree by using two barometer tubes B and E, (Fig. 28) but the top of the experimental tube E was bent round and it terminated in a spherical bulb which contained water. The bulb was immersed in freezing mixture. The difference in the heights of the mercury in the tubes determines the vapour pressure of the liquid at the temperature of the bulb in the freezing mixture. Lowering the temperatures more and more down to the possible limit, the corresponding vapour pressures of water were similarly obtained.

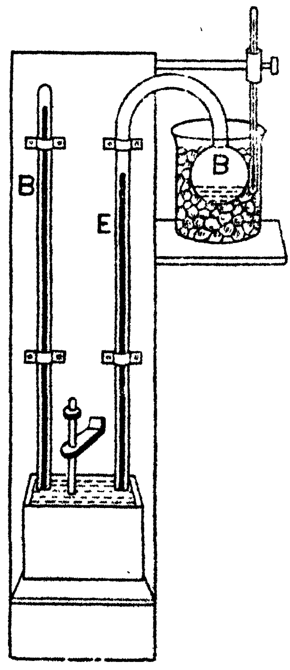


Fig. 28

94. Determination of the boiling point of a very small quantity of liquid : A bent tube in the form of an U with a longer open arm and a shorter closed arm is filled with mercury all but a

small space containing a very small amount of a liquid, say water, but in the longer arm the mercury stands just near the bend. The bent tube is then plunged into a beaker-full of water so that the shorter arm is inside and a part of the longer arm stands outside the water.

The water in the beaker is boiled and the water occupying the upper part of the shorter arm is partially converted into steam which exerts a pressure on the mercury column causing it to descend until the levels of mercury in both the arms are the same. We know that at the boiling point of any liquid the vapour pressure is equal to the atmospheric pressure but since in this experiment the vapour pressure of water at the temperature of boiling of the water in the beaker has become equal to the atmospheric pressure, the boiling point of the small quantity of water contained in the shorter arm is equal to that of the water in the beaker, which is found by a thermometer to be equal to about 100°C.

Note : Saturated vapour pressure increases with temperature, the rate of increase being greater, the higher the temperature. Unsaturated vapours however behave almost like gases.

94a. Datton's law of partial pressure : It states that when a mixture of vapours (or gases) which do not chemically react with each other, is kept in an enclosure, the pressure exerted by each constituent is the same as that which it would exert if it alone occupied the whole volume. The total pressure of the mixture is evidently equal to the sum of the pressures of the constituent vapours, which are called partial pressures. The law applies both for unsaturated and saturated vapours, but with the difference, that when the volume of the enclosure is changed, the pressure of the former changes according to Boyle's law, while that of the latter remains constant.

95. Height of a mountain from the boiling point of a liquid : Since the tension of aqueous vapour at the boiling point of water or any other liquid is always equal to the atmospheric pressure, the height of a mountain can easily be determined from the knowledge of the difference of the pressures between the mouatain top and the surface of the earth. This difference of pressure is obtained from Regnault's table of vapour pressures from the knowledge of the boiling points of water at these two stations as determined by a hypsometer. This difference of pressure is again equal to the weight of a column of air of 1 sq. cm. cross-section and of height equal to the height of the mountain. Since the two stations are further apart the densities of air at these stations are different and so the average density of air should be taken into consideration.

If H be the height of the mountain, the weight of the column of air extending between the two stations of average density $D = H.D.g$.

If $P_2 - P_1$ be the difference of pressure between the two stations expressed in centimetres, then $P_2 - P_1$ expressed in dynes is equal to $(P_2 - P_1) \times 13.59 \times g$.

Then $H.D.g = (P_2 - P_1) \times 13.59 \times g$. or $H = (P_2 - P_1) \times 13.59/D$

. To calculate the average density, let the mean pressure be P and let t be the mean temperature of air between the two stations. Then the density of air at temp. t°C and pressure P

$$= \frac{.001293 \times P \times 273}{760 \times (273 + t)} \quad \left\{ \begin{array}{l} \text{where } .001293 \text{ is the density} \\ \text{of dry air at N. T. P.} \end{array} \right.$$

$$H = \frac{(P_2 - P_1) \times 13.59 \times 760 \times (273 + t)}{.001293 \times P \times 273}$$

Another formula is generally used to determine approximately the height H, *i.e.* the difference in height in centimetres between the two stations where the pressures are P_1 and P_2 respectively. The formula is given as $H = 1830300 (\log P_1 - \log P_2)$.

Note: We know that the pressure p at a height h is expressed by the relation $p = \rho g h$. For a small change δh of the height h the change of pressure is δp, *i.e.* $\delta p = \rho g \delta h$ ... (1)

But $pv = RT$, we have, therefore $p/RT = 1/v = \rho$ for unit mass ... (2)

Therefore from (1) and (2) we have $\delta p/p = g\delta h/RT$

If the temperature is constant, we have by integration the change of altitude between the barometric heights p_0 and p.

$$\log \frac{p}{p_0} = -\frac{gh}{RT} \text{ or } \log p - \log p_0 = -\frac{gh}{RT} \text{ or } h = \frac{RT}{g} (\log p_0 - \log p)$$

It may be written as $H = 1830300 (\log P_1 - \log P_2)$ ... (3)

where $h = H$, $\frac{RT}{g} = 1830300$, $p_0 = P_1$ and $p = P_2$

The formula (3) needs correction for the temperature is not constant but varies with the height.

The height of a mountain can also be determined from the fact that an ascent of 900 ft. produces a depression of an inch in the height of the barometer and an ascent of 1080 ft. produces the lowering of 1°C in the boiling point of water.

QUESTIONS

1. What is meant by the saturation vapour pressure of a liquid at a given temperature and how it may be measured? [C. U. 1944]

2. Describe the method adopted by Regnault to obtain the pressure of water vapour below and above 100°C. Clearly explain the principle on which the experiment is based. [C. U. 1929]

3. How do you determine the height of a mountain from the boiling point of a liquid?

CHAPTER VIII

HYGROMETRY

96. Dew point : When a mixture of air and vapour is cooled at constant pressure, the temperature at which saturation occurs is called the *dew point* of the original mass ; if the mixture be cooled below the dew point, some of the vapour will be condensed. Since the process of cooling does not alter the vapour pressure the actual vapour pressure in any portion of the air at its initial temperature at a certain time is equal to the maximum vapour pressure at the dew point, at the same time.

97. Relative Humidity : It may be defined as the fractional saturation of the air and is expressed as the ratio of the pressure of the aqueous vapour actually present at the existing temperature to the saturation pressure of the vapour at the same temperature.

Again if m be the mass of aqueous vapour actually present in a given volume of air at t°C and M, the mass necessary to saturate it at that temperature, then the *Hygrometric State* or *Relative Humidity* (H) is the ratio m/M, *i.e.* $\mathrm{H}=m/\mathrm{M}=f/\mathrm{F}$ where f is the maximum vapour pressure at the dew point and F, the maximum vapour pressure at the actual temperature of the air. Since pressure is proportional to mass, $m/\mathrm{M}=f/\mathrm{F}$. The second ratio ($f/\mathrm{F}$) refers to the first definition of humidity given above.

98. Determination of Relative Humidity :

(A) Regnault's (Dew point) Hygrometer :

A highly polished silver tube closed at the bottom is filled with ether. Air is forced into the liquid through a tube causing its rapid evaporation and thereby producing cooling of surrounding air. The cooling goes on and the temperature of the liquid is noted as soon as dew appears on the outer side of the tube. The air current is then stopped and the temperature of the disappearance of dew is noted. The mean of these two temperatures is the *dew-point temperature.*

The maximum vapour pressures F and f at the temperature of the air and at the dew point respectively are obtained from Regnault's table of vapour pressures of water and the relative humidity is calculated from the ratio of f and F.

$\therefore$ **Expressing in percentage, relative Humidity = $100f/\mathrm{F}$ per cent.**

Note : For the above experiment two tubes, one the experimental and the other the comparison tube, fitted with silver parts at E and F are arranged (Fig. 29) on a hollow brass vertical tube connected to an aspirator D. The aspirator is worked and air is sucked through the ether in the tube E *via* the side tube A and the vertical tube, causing rapid cooling by evaporation and consequent formation of dew on the silver part of E. The thermometer T gives the dew point temperature and the thermometer t, the temperature of air.

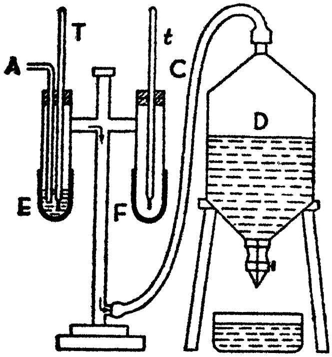


Fig. 29

(B) Dry and Wet Bulb Thermometers :

A wooden frame carries two thermometers, the bulb of one (Fig. 30) of them being covered with muslin and kept moist by a lamp wick dipped into an evaporating basin containing water. The Dry Bulb thermometer registers the temperature of the atmosphere and the Wet Bulb thermometer always indicates a lower temperature unless the air is saturated. These two temperatures with the help of suitable tabular readings for masses of saturated vapour per c.m. will determine the relative humidity.

The relative humidity may also be obtained as in the case of Regnault's experiment from the knowledge of the dew point as determined by Glaisher's formula which is given by

$$t - t_{dp} = A(t - t_w)$$

where t = temp. of dry bulb, t_{dp} = temp. of the dew point. A = Glaisher's Factor ; t_w = temp. of wet bulb thermometer.

Then knowing A and hence, dew point t_{dp}, the values of f and F at t_{dp} and t respectively can be ascertained from Regnault's table and H, the relative humidity can be found out.

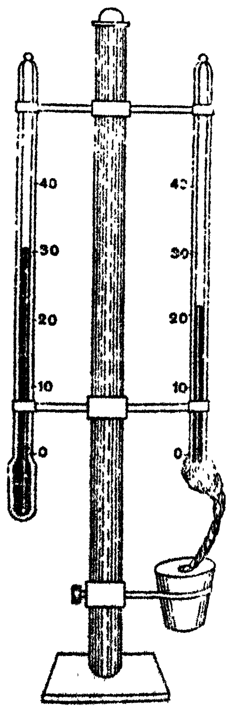


Fig. 30

99. Shew that the actual vapour-pressure in any portion of air at a certain time is equal to the maximum vapour-pressure at the dew point at the same time:

Let us consider a vessel surrounded by air containing moisture in it and let it be gradually cooled down until dew point is reached. Let f be the pressure of vapour present in the air, and when the vessel is cooled the vapour just surrounding the vessel should diminish in pressure, but it has always the same value of pressure f as there is free communication between this vapour and that in the outside atmosphere. The pressure of vapour in the air just outside the vessel will remain the same and will be of the same value until the vessel is cooled to the dew point at which the maximum pressure of vapour is equal to f, which is the actual vapour-pressure at the temperature of the experiment.

100. To determine the pressure of vapour in the laboratory with the help of a chemical hygrometer : We know that with the help of a chemical hygrometer the mass of aqueous vapour in a known volume of air can be determined.

Let m be the mass of aqueous vapour in a volume V at temperature t and let f be its pressure.

$$\text{Then } m = \frac{f \times V}{273 \times t} \times \frac{273}{760} \times {}^{\cdot}001293 \times \tfrac{5}{8}$$

where ·001293 is the mass of 1 c.c., of dry air at N. T. P., and $\frac{5}{8}$ is the density of aqueous vapour with respect to dry air.

From the above expression, the pressure f is easily determined since m, V and t are known.

QUESTIONS

1. Write a short note on Relative humidity and its measurement. [C. U. 1952, '56]

Write a short note on Wet and Dry bulb hygrometer. [C. U. 1945]

2. Define dew point.

The actual vapour-pressure in any portion of air is equal to the maximum vapour-pressure at the dew point. Why so? [C. U. 1926]

EXAMPLES

1. Calculate the mass of aqueous vapour in V volume of moist air, the hygrometric state of which is H, the temperature t and the pressure P, the density of vapour being $\frac{5}{8}$th of that of air.

We have the formula $H = \frac{f}{F}$ or $f = H.F.$

$\therefore$ the tension f of the vapour in the air is obtained since H is known and F is obtained from the table of maximum vapour-pressures at different temperatures.

Then at a pressure f and temperature t the volume V of dry air becomes equal to $\frac{V.f.273}{760\times(273+t)}$ *c.c.* at N. T. P.

Again since the mass of 1 *c.c.* of the dry air at N. T. P. is equal to $\cdot 001293$ gm., the mass of $\frac{V.f.273}{760\times(273+t)}$ *c.c.* of dry air becomes equal to $\frac{\cdot 001293\times V.f.\times 273}{760\times(273+t)}$

So the mass of V *c.c.*, of moisture present in V *c.c.*, of moist air at $t^\circ c$ and pressure f *mm.* $=\frac{5}{8}\times\cdot 001293\times\frac{V.f.\times 273}{760\times(273+t)}$

since the density of aqueous vapour is $\frac{5}{8}$th of that of air.

$\therefore$ mass of vapour $=\frac{5}{8}\times\cdot 001293\times\frac{V.F.H\times 273}{760\times(273+t)}$ gms.

2. Calculate the quantity of water vapour in a room of which the capacity is 10,000 litres when the temperature is 30°C and the humidity is 80 per cent. Maximum pressure of water vapour at 30°C is 31·5 mm.

10,000 litres $=10000\times 1000$ c. c. $=10^7$ c. c.

Let f = pressure of aqueous vapour present in the atmosphere.

We know that humidity $H=\frac{f}{F}=\frac{f}{31\cdot 5}=\frac{80}{100}$ $\qquad f=\cdot 8\times 31\cdot 5=25\cdot 2$ mm.

Now since the density of aq. vapour $=\cdot 622$, the mass of 10^7 c.c. of aq. vapour at 30°C and 25·2 mm. pressure is determined by multiplying the mass of 10^7 c.c., of dry air at 30°C and 25·2 mm. pressure by ·622

Let V be the volume of this much of dry air at N. T. P.

then by gas equation, $\frac{10^7\times 25\cdot 2}{273+30}=\frac{V\times 760}{273}$ $\quad\therefore\quad V=\frac{10^7\times 25\cdot 2\times 273}{303\times 760}$

Again the density of dry air at N.T.P. $=\cdot 001293$ gm./c. c.

$\therefore$ mass of 10^7 c.c. of aq. vapour at 30°C and 25·2 mm., pressure $=\cdot 622\times$ mass of 10^7 c.c., of dry air at 30°C and 25·2 mm. pressure

$$=\cdot 622\times\frac{10^7\times 25\cdot 2\times 273}{303\times 760}\times\cdot 001293=240\cdot 3 \text{ grammes nearly.}$$

3. Calculate the weight of 15 litres of air saturated with aqueous vapour at 20°C and 750 mm. The maximum pressure of aqueous vapour at 20°C is 17·39 mm. The weight of 1 c.c., of dry air at N.T.P., is ·0012932 gm. The density of aqueous vapour is ·622. [C. U. 1923.]

Mass of 1 litre of dry air at N.T.P. $=1000\times\cdot 0012932=1\cdot 2932$ gms.

Pressure of dry air $=750-17\cdot 39=732\cdot 61$ mm.

$\therefore$ **Mass of 15 litres of dry air at 20°C and 732·61 mm.**

$$=\frac{15\times 732\cdot 61}{273+20}\times\frac{273}{760}\times 1\cdot 2932 \text{ gms.}=17\cdot 42 \text{ grammes nearly.}$$

Again,

$$\cdot 622=\frac{\text{Mass of 15 litres of aq. vapour at 20°C and 17·39 mm.}}{\text{Mass of 15 litres of dry air at 20°C and 17·39 mm.}}$$

∴ Mass of 15 litres of aq. vapour at 20°C and 17·39 mm.

$= \cdot 622 \times$ mass of 15 litres of dry air at 20°C and 17·39 mm.

$$= \cdot 622 \times \frac{15 \times 17 \cdot 39}{273+20} \times \frac{273}{760} \times 1 \cdot 2932 \text{ gms.} = \cdot 257 \text{ gms. nearly.}$$

the total mass $= 17 \cdot 42 + \cdot 257 = 17 \cdot 677$ gms. nearly.

(Since 15 litres of air saturated with aq. vapour = 15 lit. of dry air + 15 lit. of aq. vapour.)

3. On a certain day the temperature was 25°C, pressure 755 mm. of mercury and relative humidity 70%. Assuming density of dry air at N.T.P. $= 1 \cdot 29 \times 10^{-3}$ gm/cm³ and density of water vapour relative to dry air $= \cdot 62$, find the density of moist air of the atmosphere on that day.

(Saturation pressure of water vapour at 25°C = 23·67 mm. of Hg.

[D. U. 1943]

We know that $\frac{f}{F} = \frac{70}{100} = \cdot 7$ $\qquad f = F \times \cdot 7 = 23 \cdot 67 \times \cdot 7 = 16 \cdot 569$ mm.

∴ pressure of dry air $= 755 - 16 \cdot 569 = 738 \cdot 431$ mm.

Then proceed for determining D_{mt} as before. [Ans. ·001164]

CHAPTER IX

VAPOUR DENSITY

101. Vapour density : Strictly speaking, the density of a substance means the mass of unit volume of the substance. In the case of gases or vapours the density may be defined as the mass in grams per litre.

By the term vapour density we generally mean *the ratio of the mass of a given volume of the vapour to the mass of an equal volume of the standard substance, air, under the same condition of temperature and pressure.* Sometimes dry Hydrogen or Oxygen is taken as the standard substance.

102. Determination of Vapour density (Unsaturated Vapour) :

The density of an unsaturated vapour can be accurately determined by any of the standard methods used by Hofmann, Victor Meyer, Dumas and others. The methods of Victor Meyer and Dumas being more important are described below.

(1) Victor Meyer's Method : This method is used in the case of a liquid (low boiling point) which at ordinary atmospheric pressure does not require a very high temperature to vaporise off.

The apparatus mainly used for this purpose consists of cylindrical glass bulb A (Fig. 31) connected to a long straight glass stem B having a bent delivery tube near its open end which is closed by a stopper D. The bulb and the greater part of the stem is surrounded by a larger glass vessel G containing water into which glass fragments have been dropped to prevent bumping. The open end of the delivery tube CE is inserted into a graduated glass tube V filled with water and placed in an inverted position in a trough containing water.

The liquid whose vapour density is to be determined is then taken in a small glass bottle called Hofmann's bottle, having a stopper, and its weight M obtained from the weight of the bottle filled minus its weight when empty, by method of successive weighings. The bottle is then dropped on the asbestos placed on the closed end of the cylindrical bulb to prevent breakage, by taking out the stopper D from the open end which is then immediately closed, the water in the outer vessel in the mean time boiling. The liquid at once vaporises, pushes the air in front of it, which comes out of the delivery tube and then collects in the graduated tube V by displacing water from inside it.

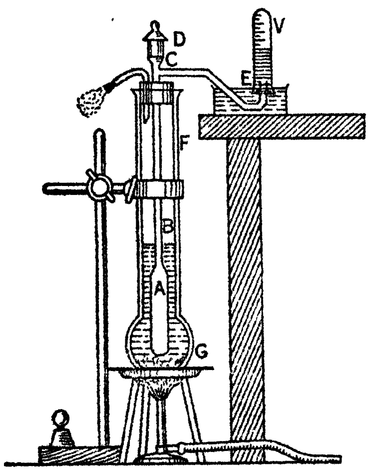


Fig. 31

The volume of the air collected in the graduated tube when corrected for temperature and pressure will be equal to the volume of the liquid vapour at 100°C and at the barometric pressure.

Let V be the volume of air inside the graduated tube at temperature t of the water in the trough, and at pressure P.

Let H be the barometric height and let h and f denote respectively the height of water in the tube above the free surface in the trough and the pressure of saturated aqueous vapour at t°C.

The pressure P is therefore equal to $\left(H-\frac{h}{13{\cdot}6}-f\right)$ where 13·6 is the density of mercury.

Therefore the volume V of air at N. T. P.

$$=\frac{273}{760\times(273+t)}\times\left(H-\frac{h}{13{\cdot}6}-f\right)V \text{ c.c.}$$

But since 1 c. c., of dry air at N. T. P. weighs ·001293 gm. the mass of $\frac{273}{760\times(273+t)}\times\left(H-\frac{h}{13{\cdot}6}-f\right)$ V c. c. of air $=\frac{273}{760\times(273+t)}\times\left(H-\frac{h}{13{\cdot}6}-f\right)\times V\times$ ·001293

Therefore, the required vapour density

$$=\frac{M}{\frac{273\ V}{760\times(273+t)}\times\left(H-\frac{h}{13{\cdot}6}-f\right)\times{\cdot}001293}$$

whence the value of vapour density can be obtained.

(2) Dumas' Method : This method can be adopted for both low and high boiling point liquids, using suitable liquids for bath. The apparatus used in this method consists (Fig. 32) of a glass or porcelain globe having a neck drawn out into a fine jet. The globe is at first washed, dried and then carefully weighed. A certain quantity of liquid is then introduced into the globe by alternately heating and cooling it. The globe is then immersed by a frame carrying thermometers, in a bath of water, (or oil or molten metal) at a temperature considerably above the boiling point of the liquid. As soon as the liquid is vaporised, a jet of vapour issues out expelling the inside air. When the whole amount of liquid has been converted into vapour and no more vapour is issuing out, the globe has then become full of vapour at the atmospheric pressure at the time of experiment and at the temperature of the bath. The end of the drawn-out neck is then sealed up and the globe is allowed to cool to the temperature of the room. It is then weighed and the difference between the weights of the globe when filled with vapour and when empty gives the weight of the vapour minus the weight of a certain volume of the air displaced by the external volume of the globe.

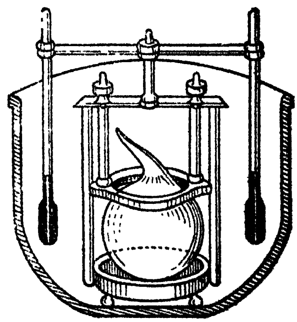


Fig. 32

Let W be the weight of the globe when empty. Then denoting the wt. of the globe in vacuo by w_g, weight of air inside the globe by w_a and the weight of air displaced by outer volume of the globe by w_1, we have

$$W=w_g+w_a-w_1 \quad \ldots \quad \ldots \quad (1)$$

Again let W_1 be the weight of the globe full of vapour alone. Then denoting the weight of the vapour by w_v, we have

$$W_1=w_g+w_v-w_1 \quad \ldots \quad \ldots \quad (2)$$

From (1) and (2) $W_1 - W = w_v - w_a$ ... (3)

In (3) w_a is the weight of air of same volume V as the vapour enclosed at the atmospheric pressure p at the time of experiment and at the temperature t of the bath.

Now density of air at N. T. P. = ·001293 gm/c.c. Then, volume of w_a gms. of air at N. T. P. = w_a/·001293 c.c. By gas equations

$$\frac{760 \times w_a}{\cdot 001293 \times 273} = \frac{p.V}{273+t} = \frac{p.V_0(1+gt)}{273+t} \qquad (4)$$

Since $V = V_0\ (1+gt)$, where V_0 = inside volume of the globe at 0°C and g = coefficient of cubical expansion of glass.

From (4) $w_a = \dfrac{p.V_0(1+gt) \times 273 \times \cdot 001293}{760 \times (273+t)}$

Knowing all qualities of the right hand side w_a can be found out. Again from (3)

$$w_v = W_1 - W + w_a$$

Substituing for w_a and using experimental data for W_1 and W, the value of w_v is also found out. Then knowing both w_v and w_a, from their ratio w_v/w_a we can easily calculate the vapour density.

Note : For comparatively high boiling point liquid the bulb is made of zena glass while for very high boiling point liquids the bulb is to be made of porcelain, the corresponding liquids for the hot bath being oil and molten metal respectively.

103. Density of Saturated Vapour : In determining the density of saturated vapour Fairbairn and Tate utilised the fact that the pressure of a saturated vapour increases more rapidly with temperature than the pressure of an unsaturated superheated vapour. The principle of the method used is illustrated by Fig 33. Let A and B be two glass bulbs connected together by means of a tube bent twice at right angles. The whole apparatus is exhausted, mercury being previously introduced into the tube which stands nearly at the same level in the two limbs.

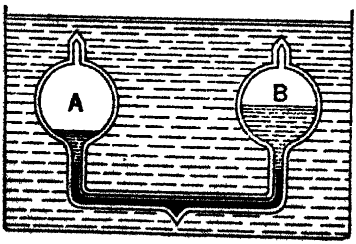


Fig. 33

Different quantities of any one liquid are also introduced into the two bulbs A and B (say m_1, in A and m_2 in B, where $m_2 > m_1$) and the level of mercury in the two limbs will remain

practicaly undisturbed since the pressures in both the bulbs are equal being due to saturated vapour at a given temperature.

The whole apparatus is then immersed in a water-bath whose temperature is gradually altered until the positions of mercury levels in the two limbs are just disturbed, mercury rising in A and that in B being depressed. This happens only when the whole amount of liquid in one bulb A is just completely vaporised keeping A saturated and some liquid is still left in B. Before this for all temperatures, as vapour pressure of water in both A and B acquire saturation value, the initial mercury levels in A and B remain fixed. The initial level of course is slightly lower in B as greater mass of water is contained in it.

Hence, if the temperature at which mercury just begins to rise in A, is noted, then it is known that at this temperature the liquid in A is all vaporised and just going to be superheated ; but more water in B being vaporised will keep the vapour in B saturated at the said temperature. The volume of the bulb A being known and the temperature noted, as stated already, the pressure may be found from tables of saturated vapour pressures, the density of the vapour at the saturation point can be obtained in the usual way.

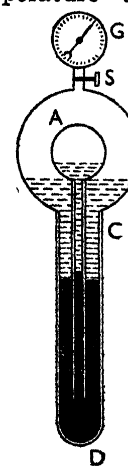


Fig. 34

The apparatus used for actual experiment is shown in Fig. 34. The bulb A containing smaller quantity of liquid is placed inside the large bulb B containing larger quantity of liquid, the bulb B being connected with a presure gauge G. The stem of A is immersed in mercury contained in the closed stem CD of the bulb B. The initial level of mercury in A is slightly higher than that in B, due to larger mass of liquid in the latter. The mode of working is identically same as in the illustrative experiment described above.

QUESTIONS

1. Define the term vapour density and describe Victor Meyer's method of determining it. [C. U. 1922, '24, '27, '33]

2. Describe a method of determining the vapour density of a substance which boils at a high temperature.

3. How would you determine the density of saturated vapour of water ? [C. U. 1925]

EXAMPLES

1. Calculate the vapour density of the liquid.

Mass of the liquid = 0·112 gm ; Volume of air driven off and collected over mercury = 38 c. c. ; Temperature of the air = 27°C ; Height of the mercury level inside the measuring tube above the free surface = 5 cms. Barometric height = 75 cms. [D. U. 1948]

2. Calculate the vapour density of a substance from the following data.

The bulb used weighed 14·94 gms., when full of air. 15·401 gms. when filled with the vapour and 175·103 gms. when filled with water after the experiment. Temperature of the laboratory = 20°C ; temperature of the bath at the time of sealing the bulb = 100°·16C ; barometer reading at the time of the experiment = 764·3 mm. The density of water may be taken as unity, and the expansion of glass may be neglected. [C. U. 1917]

From equation (1) in Duma's expt., we have $w_v = W_1 - W + w_a$

$\therefore$ the mass of vapour $= (15·401 - 14·94) + w_a = ·461 + w_a$

But w_a = mass of air filling the bulb at the temperature of the room and at atmospheric pressure. Now mass of water filling the bulb = 175·103 − 14·94 = 160·163 grammes.

$\therefore$ mass of air filling the bulb at 20°C and 764·3 mm.

$$= \frac{160·163 \times 764·3}{273+20} \times \frac{273}{760} \times ·001293 = ·194 \text{ gram nearly}$$

[Density of dry air at N. T. P. = ·001293]

$\therefore$ $w_v = ·461 + ·194 = ·655$ grames. Also mass of 160·163 c. c., of dry air at 100·16c. and 764·4 mm.

$$= \frac{160·163 \times 764·3}{273+100·16} \times \frac{273}{760} \times ·001293 = ·152 \text{ gram nearly}$$

$$\text{Vapour density} = \frac{·655}{·152} = 4·32 \text{ nearly}$$

CHAPTER X

ISOTHERMAL AND ADIABATIC CHANGES

104. Isothermal Changes : While studying Boyle's law we have noticed that at constant temperature the volume of a gas varies inversely as its pressure and the law is written in the mathematical form $pv = k$.

The change produced in the volume of a gas due to a change of pressure, temperature being kept constant, is called an *isothermal change.*

When a gas is compressed heat is generated, and when it expands cooling occurs and in order that the change should be

isothermal, the gas must be placed in thermal communication with surrounding bodies so that heat produced during compression is allowed to flow out by radiation or otherwise and cold produced during expansion is compensatod by heat flowing from the surrounding bodies.

In isothermal operations the temperature of the gas remains constant ; but in adiabatic operations temperature changes.

105. Adiabatic Change : The change produced in the volume of a gas due to a change of pressure, subject to the condition that heat generated during compression is not allowed to pass out and cold produced during expansion is not compensated by surrounding bodies, is what is known as *adiabatic change.*

106. Illustrations of Isothermal and Adiabatic Operations : To effect these changes let us suppose that a gas is contained within a cylinder C fitted with a frictionless and heat-insulated piston, the walls of which are non-conducting and that the bottom of the cylinder made of a perfect conductor of heat.

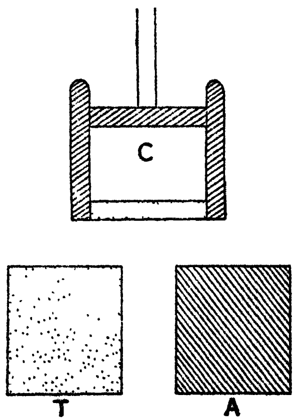


Fig. 35

Let us further suppose that the cylinder with the gas in it be placed on a stand fitted with a perfectly conducting top at a constant temperature say T. If now the gas be compressed very gradually some work will be done on the gas and certain amount of heat will be produced. Now since the bottom of the cylinder and the top of the stand are perfectly conducting, heat will flow from the heated gas to the stand and the temperature of the gas will be kept the same as before. This change in volume and pressure of a gas is known as an *isothermal change*. If we proceed in the reverse direction *i.e.*, if the gas be allowed to expand doing external work, temperature of the gas will fall which again will be kept the same by the supply of heat from the stand at the temperature T.

If, again the cylinder be placed on a stand A fitted with a perfectly non-conducting top and the gas compressed either gradually or suddenly, heat generated during compression will not be transferred to the stand as it is fitted with a non-conducting top and so there will be a rise of temperature. Similarly, during expansion of the gas there will be a fall of temperature.

Thus *adiabatic changes* in the pressure and volume of a gas are effected.

If the compression or expansion is very sudden, the condition at every step may not be retraced, yet the condition is *adiabatic*. Thus adiabatic changes are not necessarily *reversible*.

In an isothermal operation the temperature of the system remains constant but in an adiabatic operation the heat content of the system remains constant and the change in this case is always accompanied by a temperature change of the system.

107. Relation between the pressure and volume of a gas for an adiabatic change *i.e.*, pv^γ = constant.

Let an amount of heat dQ be supplied to unit mass of a gas so that volume changes by dV and temperature by dT. The heat supplied is spent in (1) raising temperature of the gas and (2) in doing external work. The first part is equal to $C_v dT$ and the second part equal to pdV/J, both expressed in calories. Hence, from principle of conservation of energy, we have,

$$dQ = C_v.dT + pdV/J \quad \dots \quad \dots \quad (1)$$

But, as in adiabatic change no heat is supplied or removed $dQ = 0$

$$\text{Then, } C_v.dT + \frac{pdV}{J} = 0 \quad \dots \quad \dots \quad \dots \quad (2)$$

By the gas equation, we have $pV = RT$.

Differentiating the above, $pdV + Vdp = RdT$ or $dT = \frac{pdV + Vdp}{R}$

Substituting for dT in (2), we get

$$Cv.\frac{pdV + Vdp}{R} + \frac{pdV}{J} = 0 \quad \text{or} \quad Cv(pdV + Vdp) + \frac{R}{J}.pdV = 0$$

$$\text{or} \quad Cv(pdV + Vdp) + (Cp - Cv)pdV = 0 \quad [\because R/J = Cp - Cv]$$

$$\text{or} \quad Cv.Vdp + Cp.pdV = 0$$

$$\text{Dividing by } Cv.pV, \ \frac{dp}{p} + \frac{Cp}{Cv}.\frac{dV}{V} = 0 \quad \text{or} \quad \frac{dp}{p} + \gamma.\frac{dV}{V} = 0 \quad \dots(3)$$

$$\left[\text{denoting } \frac{Cp}{Cv} \text{ the ratio of two specific heats of a gas by } \gamma\right]$$

Integrating (3), $\log p + \gamma \log V$ = a constant $= \log K$ (say)

$$\text{or } \log (pV^\gamma) = \log K \quad \therefore \quad pV^\gamma = K$$

Note: If p_1, v_1 be pressure and volume respectively of a quantity of gas, and p_2, v_2 their corresponding values after an adiabatic change, then $p_1V_1^\gamma = p_2V_2^\gamma$

108. Relation between volume V and temperature T in an adiabatic change. (Direct method):

Proceeding from the beginning exactly as in the deduction of (p-V) relation above, we have $Cv.dT + \frac{pdV}{J} = 0$ ... (2)

By gas equation, $pV = RT \quad \therefore \; p = RT/V$

Substituting for p in (2), we have $Cv.dT + \frac{RT}{J} \cdot \frac{dV}{V} = 0$

or $Cv.\frac{dT}{T} + \frac{R}{J} \cdot \frac{dV}{V} = 0$; or $Cv.\frac{dT}{T} + (Cp - Cv)\frac{dV}{V} = 0$ $\left[\because \frac{R}{J} = Cp - Cv \right]$

Dividing by Cv, $\frac{dT}{T} + \left(\frac{Cp}{Cv} - 1\right)\frac{dV}{V} = 0$ or $\frac{dT}{T} + (\gamma - 1)\frac{dV}{V} = 0$

$\left[\text{denoting } \frac{Cp}{Cv} \text{ (ratio of two sp. hts. of gas) by } \gamma \right]$

Integrating, $\log T + (\gamma - 1) \log V = \text{a constant} = \log C$ (say)

or $\log\left(TV^{\gamma - 1}\right) = \log C \quad \therefore \; TV^{\gamma - 1} = C$ a constant. ... (3)

Note: If a volume V_1 of a gas at temperature T_1 (abs.) be adiabatically compressed to a volume V_2 and if the final temperature be T_2, then $T_2V_2^{\gamma-1} = C$; $T_1V_1^{\gamma-1} = C \quad \therefore \; T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1}$

108a. Relation between density (ρ) and temperature in an adiabatic change:

Cor: We have, as deduced above, $TV^{\gamma - 1} = C$.

If m be a definite mass of the gas and ρ its density which changes with temperature, $V = m/\rho$; substituting this in above relation, we have, $T\left(\frac{m}{\rho}\right)^{\gamma - 1} = C$ or $T = \frac{C}{m^{\gamma - 1}} . \rho^{\gamma - 1}$

or $T = \text{a constant} \times \rho^{\gamma - 1} \quad \therefore \; T \propto \rho^{\gamma - 1}$.

Note: Deducing any of the relations $pV^{\gamma} = K$ or $TV^{\gamma - 1} = C$ and using $pV = RT$, we can deduce the other relations as follows:

(i) $TV^{\gamma-1} = C$; $pV = RT$ or $T = pV/R$

$\therefore \left(\frac{pV}{R}\right) . V^{\gamma-1} = C$; or $pV^{\gamma} = C.R = \text{a constant}$.

(ii) $pV^{\gamma} = K$; $pV = RT$ or $p = RT/V$

$\therefore \left(\frac{RT}{V}\right) . V^{\gamma} = K$ or $TV^{\gamma-1} = K/R = \text{a constant}$.

109. Relation between p (pressure) and T (temperature) in an adiabatic change :

Deducing the relation between p and V as before we have
$pV^{\gamma}=K$ ..(1)

By gas equation, $pV=RT$ or $V=\frac{RT}{p}$ subsitituting for V in (1)

we get, $p\left[\frac{RT}{p}\right]^{\gamma}=K$ or $\frac{T^{\gamma}}{p^{\gamma-1}}=\frac{K}{R^{\gamma}}=K'$ or $T^{\gamma}\propto p^{\gamma-1}$

Direct Method : Proceeding as in "$pv^{\gamma}=K$" case, we have in adiabatic change,

$$C_v.dT+\frac{pdv}{J}=0 \qquad (1)$$

By gas equation $pv=RT$ ... ... (2)

Differentiating (2) $pdv+vdp=RdT$.

$\therefore$ $pdv=RdT-vdp$, and from (2) $v=RT/p$

Then relation (1) becomes, $C_vdT+\left(RdT-\frac{RTdp}{p}\right)/J=0$

or $C_vdT+\frac{R}{J}\left(dT-\frac{Tdp}{p}\right)=0$; or $C_vdT+(C_p-C_v)\left(dT-\frac{Tdp}{p}\right)=0$

or $C_vdT-(C_p-C_v)\frac{Tdp}{p}=0$; or $\frac{dT}{T}-\left(1-\frac{1}{\gamma}\right)\frac{dp}{p}=0$, Integrating this

$\log T-\log p^{\frac{\gamma-1}{\gamma}}=\log K=$ a constant $\therefore$ $T/p^{\frac{\gamma-1}{\gamma}}=K$ *i.e.*, $Tp^{\frac{1-\gamma}{\gamma}}=K$

or $T^{\gamma}/p^{\gamma-1}=K^{\gamma}=$ a constant. $\therefore$ $T^{\gamma}\propto p^{\gamma-1}$.

110. An adiabatic curve is steeper than an isothermal curve :

Let DA and OD (Fig. 36) represent the pressure and volume of a gas. If the gas be compressed isothermally so as to reduce its volume by DE the condition of the gas is represented by B on the isothermal curve AB. If the gas instead of being isothermally compressed, be compressed adiabatically through the same volume DE, heat produced during compression does not pass into the surrounding bodies as in isothermal compression but remains in the gas and raises its pressure so that the condition of the gas is now represented by the point C on the adiabatic curve AC.

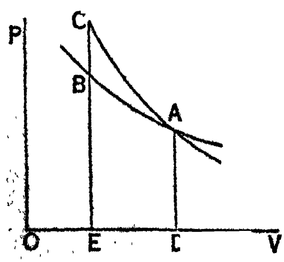


Fig. 36

Hence, the adiabatic curve AC is steeper than the isothermal curve AB.

In the P-V curve the slope at a given point is measured by the tangent at the point and expressed as $\frac{dp}{dv}$. For any isothermal change we have pv = constant. Differentiating this,

we have $v\frac{dp}{dv}+p=0$ or $\frac{dp}{dv}=-\frac{p}{v}$

Again for adiabatic changes, we have pv^{γ} = constant. Differentiating this, $v^{\gamma}\frac{dp}{dv}+\gamma pv^{\gamma-1}=0$ or $\frac{dp}{dv}=-\gamma\frac{p}{v}=\gamma\times\left(-\frac{p}{v}\right)$

Thus the slope of the adiabatic curve is γ times that of the isothermal curve through the same point, *i.e.* an adiabatic curve is γ times steeper than the isothermal curve.

111. Adiabatic elasticity is γ times the isosthermal elasticity :

Let a change of pressure dp produces a change in volume dv when original volume of the gas is v.

Then Bulk modulus of elasticity $E=\frac{\text{stress}}{\text{strain}}=\frac{dp}{\frac{dv}{v}}=\frac{vdp}{dv}$.

For an isothermal change, $pv=K$. $\therefore$ $pdv+vdp=0$

or $p=\frac{-vdp}{dv}$ [Negative sign indicates that when pressure increases volume decreases.]

But $\frac{vdp}{dv}=E_i$ where E_i, denotes isothermal elasticity.

Therefore $E_i=p$, *i.e.* isothermal elasticity of a gas is numerically equal to its pressure.

For an adiabatic change, $pv^{\gamma}=C$.

Differentiating, $\gamma pv^{\gamma-1}.dv+v^{\gamma}dp=0$

or $\gamma p=\frac{-v^{\gamma}dp}{v^{\gamma-1}dv}=\frac{-vdp}{dv}$.

But $\frac{vdp}{dv}=E_a$, where E_a denotes adiabatic elasticity

$$E_a=\gamma p=\gamma E_i.$$

Thus adiabatic elasticity of a gas is γ times the pressure of the gas and hence, adiabatic elasticity is γ times isothermal elasticity.

112. External Work done by a gas :

We know that the external work done by a gas against an external pressure p when it expands by an amount dv is equal to pdv ergs.

If the volume of the gas is reduced by an amount dv work done on the gas $=pdv$ ergs and work done by the gas $=-pdv$ ergs.

For a finite expansion from initial conditions p_1, v_1 to final conditions p_2, v_2, in respect of pressure and volume, the external work W done by the gas is given by

$$W=\int_{v_1}^{v_2} pdv \text{ ergs.}$$

(A) Work for an isothermal change : For an isothermal change, Boyle's law is obeyed. So for all values of p and v, at constant temperature for a given mass of gas, $pv=k$ where k is a constant.

$$\text{Hence, work done}=\int_{v_1}^{v_2} pdv=k\int_{v_1}^{v_2}\frac{dv}{v}=k\Big[\log v\Big]_{v_1}^{v_2} \quad \left[\because p=\frac{k}{v}\right]$$

$$=k\Big[\log v_2-\log v_1\Big]=k\log\frac{v_2}{v_1}=p_1v_1\log\frac{v_2}{v_1} \text{ ergs}$$

$$=p_2v_2\log\frac{v_2}{v_1}=\text{RT}\log\frac{v_2}{v_1}\text{ergs.} \quad [\because K=pv=p_1v_1=p_2v_2=\text{RT}]$$

(B) Work for an adiabatic change : Work done when its volume changes from v_1 to $v_2=\int_{v_1}^{v_2} pdv$, where p is the applied pressure.

Since in adiabatic change $pv^{\gamma}=C \quad \therefore \quad p=C/v^{\gamma} \quad \ldots \quad (1)$

$$\text{Work} = C\int_{v_1}^{v_2}\frac{dv}{v^\gamma} = C\int_{v_1}^{v_2} v^{-\gamma}.dv = C\left[\frac{v^{-\gamma+1}}{-\gamma+1}\right]_{v_1}^{v_2}$$

$$= \frac{C}{1-\gamma}\left[v_2^{-\gamma+1} - v_1^{-\gamma+1}\right]$$

$$= \frac{C}{\gamma-1}\left[v_1^{-\gamma+1} - v_2^{-\gamma+1}\right] = \frac{Cv_1^{-\gamma+1} - Cv_2^{-\gamma+1}}{\gamma-1}$$

But $C = pv^\gamma = p_1v_1^\gamma = p_2v_2^\gamma$.

$$\therefore\quad \text{work} = \frac{p_1v_1^\gamma v_1^{-\gamma+1} - p_2v_2^\gamma v_2^{-\gamma+1}}{\gamma-1} = \frac{p_1v_1 - p_2v_2}{\gamma-1}\text{ ergs.}$$

Note : To express work in terms of pressure for an adiabatic change

W (work) $= \frac{p_1v_1 - p_2v_2}{\gamma-1}$; we have $pv^\gamma = C \therefore v^\gamma = \frac{C}{p}$ or $v = \left(\frac{C}{p}\right)^{\frac{1}{\gamma}}$

$$= \frac{1}{\gamma-1}\left[p_1\left(\frac{C}{p_1}\right)^{\frac{1}{\gamma}} - p_2\left(\frac{C}{p_2}\right)^{\frac{1}{\gamma}}\right] = \frac{C^{1/\gamma}}{\gamma-1}\left[p_1^{\frac{\gamma-1}{\gamma}} - p_2^{\frac{\gamma-1}{\gamma}}\right]$$

$$= \frac{C^{1/\gamma}}{\gamma-1}\left[p_1^{(\gamma-1)/\gamma} - p_2^{(\gamma-1)/\gamma}\right].$$

113. Direct determination of γ by Clement and Desormes' method :

Clement and Desormes measured the value of γ for air by allowing it to undergo a single adiabatic expansion. The apparatus consists of a large (Fig. 37) vessel A, fitted with a stop-cock S through which the vessel can be connected to an air compression pump. There is also a valve V communicating with outer air and a manometer M containing some oil.

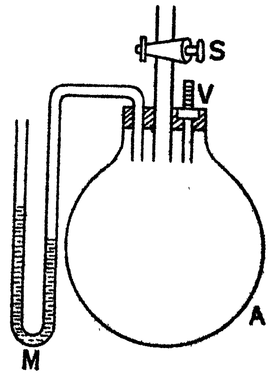


Fig. 37

Air is first pumped into A by means of the compression pump and the stop-cock is closed. The pressure p_1 of the enclosed air is obtained from a knowledge of the atmospheric pressure p and the head of the oil h_1 *i.e.*, $p_1 = p + h_1$. The valve V which is originally closed is opened for a second or so, allowing the enclosed compressed air to expand adiabatically and to attain atmos-

pheric pressure p. In this operation the temperature of the inside air falls. The valve is then closed and the enclosed air is allowed to come to initial room temperature. The manometer M now indicates a different head of oil h_2, so that the pressure of enclosed air is $p_2 = p + h_2$

If T_1 be the initial temperature corresponding to pressure p_1 and T the temperature corresponding to pressure p, then in the first process, which is adiabatic, we have,

$$\left(\frac{p_1}{p}\right)^{\gamma-1} = \left(\frac{T_1}{T}\right)^{\gamma} \quad \ldots \quad (1)$$

In the second process, pressure at constant volume changes to p_2 at the original temperature T_1

$$\text{Thus, } \frac{p_2}{p} = \frac{T_1}{T} \quad \ldots \quad \ldots \quad (2)$$

$$\text{From (1) and (2) } \left(\frac{p_1}{p}\right)^{\gamma-1} = \left(\frac{p_2}{p}\right)^{\gamma}$$

Taking logarithms of both sides of the equation we have.

$$(\gamma-1)(\log p_1 - \log p) = \gamma(\log p_2 - \log p).$$

or $\gamma(\log p_1 - \log p - \log p_2 + \log p) = \log p_1 - \log p$

or $\gamma(\log p_1 - \log p_2) = \log p_1 - \log p$

$$\therefore \gamma = \frac{\log p_1 - \log p}{\log p_1 - \log p_2}.$$

Thus knowing p_1, p and p_2, the value of γ can be calculated.

Note: When the valve is momentarily opened, the pressure of inside air does not fall to that of atmosphere immediately. In fact, the pressure oscillates above and below p for some time until it becomes equal to it. This is a source of error in the method.

114. Internal work of expanding gas: If appreciable attractions are exerted between neighbouring parts of a real gas, work will be performed in separating these molecules during expansion and a fall in temperature will result unless heat is communicated to it from surrounding bodies. In 1845 Joule carried an experiment to determine whether any temperature change occurs when a gas expands without doing any external work.

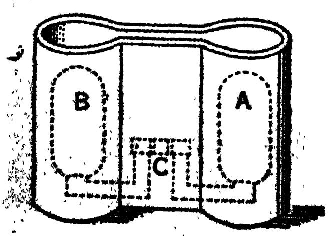


Fig. 38

The apparatus used (Fig. 38) by Joule consisted of two copper vessels A and B connected by a bent pipe having a stop-cock C. The vessel B contained dry air at very high pressure, and A was exhausted. Both A and B were immersed in a water-bath the temperature of which

could be read with great accuracy. The temperature of water being noted, the stop-cock was opened. Air flowed from B to A until pressures in A and B were equal. The water was stirred, but no fall in temperature of water was observed. Joule performed another experiment with the same two vessels immersed in separate water-baths and got same result, no cooling of the whole gas due to internal work being detected.

The above experiment was not a delicate one since the thermal capacity of the calorimeter and water were many times greater than that of the air and hence Joule failed to detect any internal work effect.

The experiment really shows, not that no internal work is done when a gas expands, but any internal work done in the process must be very small.

115. The Porous Plug Experiment ; Joule-Kelvin (or Joule-Thomson) Effect : Lord Kelvin and Joule investigated the phenomena attending the free expansion of gases using a method in which the temperature of the gases could be directly measured.

The principle of their method was to drive the gas by a compressor so as to allow it to expand freely through an orifice which is a porous plug of cotton wool or silk fibres. This expansion, of course, does external work but if Boyle's law is obeyed, no external work is done by the gas at the expense of the internal energy of the gas, since it does not use its own expansive force to get through the porous plug but is driven through by the compressor.

Experiment : The gas is passed from the compressor slowly and uniformly through a long copper spiral immersed in a water-bath and its temperature just before entering the porous plug is taken by a platinum resistance thermometer.

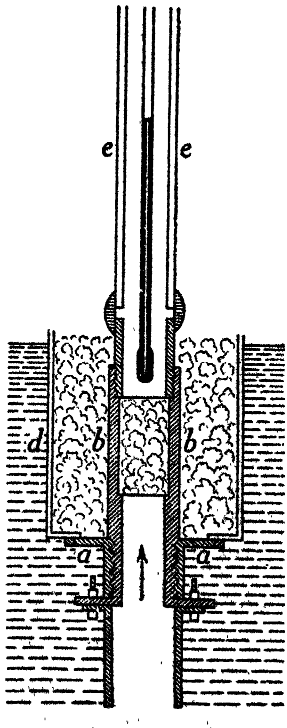


Fig. 39

The porous plug (Fig. 39) made of cotton wool is contained in a box-wood tube bb between two perforated

brass plates a, a. The box-wood tube is fixed to the bottom of a copper tube ee and is surrounded by a brass case d full of cotton wool, which is immersed in water-bath at constant temperature.

Another platinum resistance thermometer is placed just above the plug to measure the temperature of the gas after its passage through the plug. The plug of cotton is used for the orifice instead of a fine hole in a plate so as to keep down the velocity of flow of the gas on emergence. The thermometer above the plug is surrounded by a glass tube.

Joule and Kelvin's experiment has been repeated for a number of gases over a wide range of temperatures and the results may be summarised as follows :—

(*a*) All gases show a change in temperature on passing through the plug.

This effect is called the *Joule-Kelvin* (or *Joule-Thomson*) *effect.*

(*b*) This change in temperature is proportional to the difference of pressure on the two sides of the plug.

(*c*) At ordinary temperatures all gases, except Hydrogen, show a cooling. Hydrogen shows a heating effect.

(*d*) For every gas there is a temperature known as the **temperature of inversion**. If the gas be initially above this temperature, it shows a heating effect on passing through the plug, If it be below this temperature it shows a cooling effect.

For gases such as O_2, N_2 etc., the inversion temperatures are above the ordinary temperatures but for H_2 it is $-80°C$ and for Helium it is $-243°C$.

(*e*) The drop in temperature per atmosphere decreases with the rise in temperature, becomes zero at a certain temperature, which is different for each gas and becomes a rise above that temperature.

The cooling or heating effects are called **Joule-Kelvin effects** and the temperature at which it changes sign is called the **Inversion temperature.**

For air, a *cooling* of 0·275°C per atmos.-diff. for CO_2 a *cooling* of 1·39°C and for hydrogen above $-80°C$ a *heating* of 0·03°C per atmos.-diff. have been noticed.

116. Explanation of Results : The Joule-Kelvin effect is made up of two parts, one due to intermolecular force and the other due to the small amount of external work done by or on the gas.

If the gas obeys Boyle's Law, no external work is done by a free expansion. But if the gases, both the liquefiable and

the so-called permanent gases, do not obey Boyle's Law, there will be internal work as well as external work done.

If there is intermolecular attraction in the gas, cooling will be produced during the expansion of the gas. Again if the gas deviates from Boyle's Law heating will be produced due to external work done on the gas.

Let us suppose that the temperature on both sides of the plug is maintained constant.

Let p_1, v_1 be the pressure and volume of a unit mass of gas on the high-pressure side and let p_2, v_2 be the corresponding pressure and volume on the low-pressure side.

Let us consider the following cases :

(1) If the gas obeys Boyle's law, $p_1v_1 = p_2v_2$; Hence no external work is done.

(2) If the temperature of the gas be above its Boyle temperature at which a real gas obeys Boyle's Law closely, p_1v_1 is greater than p_2v_2, for according to Holborn and others the product of pressure and volume is greater at high pressure than at low pressure. So $p_1v_1 > p_2v_2$ and a little external work is done on the gas by the compressor which will produce **a rise in temperature.**

(3) If the temperature of the gas be below its Boyle temperature, then $p_1v_1 < p_2v_2$, so that the gas does a little external work in passing through the plug and this of itself will produce **cooling**.

At ordinary temperatures the cooling is the sum of the external work and internal work coolings for most gases. But for hydrogen and helium the heating is the difference between external work heating and internal work cooling.

QUESTIONS

1. Distinguish clearly between isothermal and adiabatic expansion of a [C. U. 1931, '38, '40, '42, '49, '58]

2. Obtain an expression for the work done by a perfect gas in expanding (a) isothermally, (b) adiabatically, from volume v_1 to v_2. [C. U. 1931, '42, '47]

3. Establish the equation $pv^{\gamma} =$ constant in the case of a perfect gas, (γ = ratio of the two specific heats) and find its adiabatic elasticity also. [C. U. 1949, '51, '54, '57]

4. Prove that for adiabatic expansion of a gas $T \propto \rho^{\gamma-1}$ where T is the absolute temperature, ρ the density and γ the ratio between the sp. heats of a gas at constant pressure and at constant volume. [C. U. 1941, '51, '55]

5. Deduce the relation between the volume and temperature of a mass of a perfect gas undergoing adiabatic compression. [C. U. 1938, '58]

6. Describe some adiabatic effects, giving wherever possible the quantitative relations involved. [C. U. 1931]

7. Show that the work done by an ideal gas undergoing adiabatic change is $\dfrac{p_1v_1 - p_2v_2}{\gamma - 1}$ [C. U. 1954]

8. What do you understand by temperature of inversion ?

9. Give a description of the porous plug experiment of Joule and Thomson. Discuss the results and indicate their significance for the problem of liquefaction of gases. [C. U. 1948]

EXAMPLES

1. V vol. of dry air at t°C is adiabatically compressed to occupy v vol. Find an expression for the rise of temperature. [C. U. 1938]

We have $pv^\gamma = k$ a constant ; we have also $pv = \mathrm{RT}$, then $pv^\gamma / pv = \mathrm{K/RT}$;

or $\mathrm{T}v^{\gamma-1} = \mathrm{K/R} = \mathrm{C}$, a constant.

From this question we have $\mathrm{T}V^{\gamma-1} = \mathrm{T}'v^{\gamma-1}$

$$\frac{\mathrm{T}'}{\mathrm{T}} = \frac{V^{\gamma-1}}{v^{\gamma-1}} = \left(\frac{V}{v}\right)^{\gamma-1} \quad \text{or} \quad \frac{273+t'}{273+t} = \left(\frac{V}{v}\right)^{\gamma-1}$$

$$\text{or} \quad \frac{t'-t}{273+t} = \left(\frac{V}{v}\right)^{\gamma-1} - 1 \quad \text{or} \quad t'-t = (273+t)\left\{\left(\frac{V}{v}\right)^{\gamma-1} - 1\right\}$$

2. A quantity of dry air at 15°C is adiabatically compressed to one-fourth its volume. Calculate the final temperature. [C. U. 1938]

$[\gamma = 1{\cdot}4$ and $4^{0\cdot4} = 1{\cdot}741]$

We have $Tv^{\gamma-1} = \text{constant}$ $\quad \therefore \quad T_1 v_1^{\gamma-1} = T_2 v_2^{\gamma-1}$

But $T_1 = 15° + 273° = 288°\mathrm{A}$; $v_2 = \frac{v_1}{4}$; $\gamma = 1{\cdot}4$

$$\therefore \quad T_2 = \frac{T_1 v_1^{\gamma-1}}{v_2^{\gamma-1}} = \frac{288 \times v_1^{(1\cdot4-1)}}{\left(\frac{v_1}{4}\right)^{(1\cdot4-1)}} = \frac{288 \times v_1^{\cdot4}}{v_1^{\cdot4} \times 4^{-\cdot4}}$$

$$= 288 \times 4^{\cdot4} = 288 \times 1{\cdot}74 = 501°{\cdot}4\mathrm{A}.$$

The required temperature $= 501{\cdot}4 - 273 = 228°{\cdot}4$C.

3. A given mass of dry air at t°C is adiabatically compressed from volume v_1 to volume v_2. Find an expression for the final temperature. [C. U. 1940]

We have $pv^\gamma = \text{constant}$, and $pv = \mathrm{RT}$, where R is gas constant.

$$\therefore \quad \frac{v^\gamma}{v} = \frac{\text{Constant}}{\mathrm{T}} \quad \text{or} \quad \mathrm{T}v^{\gamma-1} = \mathrm{C} \text{ (a constant)}$$

From the question we have $\mathrm{T}_1 v_1^{\gamma-1} = \mathrm{T}_2 v_2^{\gamma-1}$ where T_1, T_2 and v_1 and v_2 are the initial and final temperatures and volumes respectively.

$$\therefore \quad \mathrm{T}_2 = \mathrm{T}_1 \cdot \frac{v_1^{\gamma-1}}{v_2^{\gamma-1}} = \mathrm{T}_1 \left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

4. Dry air at 27°C is compressed to one-third its volume. Find the resulting rise in temperature. $\gamma = 1{\cdot}40$ [*Ans.* 165°68.] [C. U. 1941, '52, '55]

5. A mass of dry air at 15°C is expanded adiabatically to double its volume. Calculate approximately its new temperature, given $\gamma=1\cdot40$. [C. U. 1957]

We know that $Tv^{\gamma-1}=\text{const.}$

$$\therefore \quad T_1v_1^{\gamma-1}=T_2v_2^{\gamma-1} \qquad T_2=T_1\left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

Again $T_1=15°+273°=288°\text{A}$ and $v_2=2v_1$

$$T_2=T_1\left(\frac{v_1}{2v_1}\right)^{\gamma-1}=T_1\left(\tfrac{1}{2}\right)^{\gamma-1}=288\times(\cdot5)^{\cdot4}$$

$$=218°\cdot3\text{A}=218\cdot3-273=-54°\cdot7\text{C}$$

6. Calculate the change of temperature of helium initially at 15°C when it is suddenly expanded to 8 times its bulk. (Ratio of specific heats $=\frac{5}{3}$).

[C. U. 1958]

We know that $Tv^{\gamma-1}=\text{const.}$

$$\therefore \quad T_1v_1^{\gamma-1}=T_2v_2^{\gamma-1} \quad \text{or} \quad T_2=T_1\left(\frac{v_1}{v_2}\right)^{\gamma-1}$$

Again $T=15°+273°=288°$ and $\frac{v_1}{v_2}=\frac{1}{8}$

$$\therefore \quad T_2=288\times\left(\tfrac{1}{8}\right)^{\frac{5}{3}-1}=288\times\left(\tfrac{1}{8}\right)^{\frac{2}{3}}$$

$$=288\times\tfrac{1}{4}=72°\text{A}$$

$\therefore$ Change of temperature $=288°\text{A}-72°\text{A}=216°\text{A}$

7. At the beginning of the compression stroke of a gas engine, the gas is at a pressure of 1 atmos. At the end of the stroke when the volume of the gas is compressed to $\frac{1}{7}$ of the original volume, the pressure is found to be 15 atmos. Calculate γ from the above data. [D. U. 1947] [*Ans.* $\gamma=1\cdot4$].

8. Find the changes of volume and temperature of 76 c.c. of a gas at 20°C when the pressure is suddenly doubled. [D. U. 1944]
(Assume $\gamma=1\cdot4$)

Use the formula $PV^{\gamma}=P_1V_1^{\gamma}$ [*Ans.* $V_1=46\cdot31$ c.c.]

$$\therefore \quad TV^{\gamma-1}=T_1V_1^{\gamma-1}$$ [*Ans.* $T_1=84°\cdot1$C]

9. Find approximately the energy of isothermal compression, stored in a cylinder of compressed air, the internal volume being five litres, and the pressure one hundred atmospheres. [C. U. 1942]

Work for an isothermal compression

$$W=-\int_{v_1}^{v_2}p\,dv=C-\int_{v_1}^{v_2}\frac{dv}{v} \qquad \because \; pv=C$$

Here $v_2=5$ litres, $p_2=100$ atmos., then $v_1=500$ litres when $p_1=1$ atmos.

$$\therefore \quad W = -C\int_{500}^{5} \frac{dv}{v} = -500 \log \frac{5}{500} \quad (\because \quad pv = p_1v_1 = p_2v_2 = C = 500)$$

$$= -500 \log_e 10^{-2} = 1000 \log_e 10 \text{ Lt. atmos.}$$

$$= \frac{1000}{\log_{10} e} = \frac{1000}{\cdot 4343} \text{ Lt. atmos.} = 2302 \text{ Lt. atmos.}$$

Since 1 litre-atmosphere $= 1000 \times 76 \times 981 \times 13 \cdot 6$ *ergs*

$= 1000 \times 1 \cdot 013 \times 10^6$ *ergs*

$W = 2302 \times 1000 \times 1 \cdot 083 \times 10^6$ *ergs* $= 2332 \cdot 9 \times 10^9$ *ergs* $= 23 \cdot 32 \times 10^{11}$ *ergs*.

10. A motor tyre is pumped to a pressure of two atmospheres at 15°C, when it suddenly bursts. Calculate the resulting drop in temperature. [C. U. 1951]

Use the formula $\dfrac{P^{\gamma-1}}{T^{\gamma}} = C$

$$\therefore \quad \frac{p_1^{\gamma-1}}{T_1^{\gamma}} = \frac{p_2^{\gamma-1}}{T_2^{\gamma}} \qquad \text{Let } p_1 = 2 \text{ atmos., } p_2 = 1 \text{ atmos.}$$

$$T_1 = 273 + 15 = 288, \; T_2 = ?$$

$$T_2^{\gamma} = T_1^{\gamma} \cdot \left(\frac{p_2}{p_1}\right)^{\gamma-1} = 288^{\gamma}\left(\tfrac{1}{2}\right)^{\gamma-1}$$

$$\text{or} \quad T_2 = 288 \times \left(\tfrac{1}{2}\right)^{\frac{\gamma-1}{\gamma}} = 288 \times \left(\cdot 5\right)^{\frac{\gamma-1}{\gamma}} = 288 \times (\cdot 5)^{\cdot 4/1 \cdot 4} \qquad \because \quad \gamma = 1 \cdot 4$$

$$\log T_2 = \log 288 + \frac{\cdot 4}{1 \cdot 4} \log \cdot 5$$

$$= \log 288 + \tfrac{2}{7} \log \cdot 5 = 2 \cdot 4594 + \tfrac{2}{7} \times (\bar{1} \cdot 6990).$$

$T_2 = 236 \cdot 2°A = 236 \cdot 2 - 273 = -36 \cdot 8°C$; drop in temp. $= 15 - (-36 \cdot 8) = 51 \cdot 8°C$.

CHAPTER XI

CONTINUITY OF STATE

117. Boyle's Law : Its accuracy at high pressures :

Boyle's law states that at constant temperature the volume of a given mass of gas is inversely proportional to its pressure. This law holds accurately in the case of a perfect gas but in nature we do not get any gas obeying Boyle's law truly. The so-called permanent gases such as Hydrogen, Oxygen, Nitrogen *etc.*, very closely obey Boyle's law at ordinary temperatures under moderate changes of pressure but in the case of a gas such as CO_2, a certain divergence is noticed under similar conditions.

It has also been noticed that the higher the temperature of a gas the more nearly does it obey Boyle's law.

The apparatus used by Boyle to verify his law was not sufficiently accurate to detect small deviations from the law. So Regnault used a special form of apparatus and avoided to a great extent the following errors introduced into Boyle's experiment.

(1) The range of pressure was small.

(2) As the pressure increases the volume of the gas becomes so small that the incidental errors made in measuring this volume bear a larger ratio to the volume to be measured.

117a. Regnault's Experiments: The main principle of Regnault's experiment was to use the same volume of gas which being initially at different pressures was always reduced to one-half. If Boyle's law were true the pressure would be doubled when the volume is halved for a given mass of gas, but Regnault in his experiment showed that no gas accurately obeyed Boyle's law.

The apparatus (Fig. 40) used by Regnault for this experiment mainly consists of an iron reservoir M containing mercury and provided with a mercury force pump. The lower part of the reservoir communicates with a strong iron cylinder E in which there are two openings for the admission of two vertical tubes, one very long and the other short. The longer tube CD is open at the top and acts as a manometer while the shorter one BT, having a stop-cock T at the top for opening communication with an air pump is finely graduated and contains the gas to be operated. The temperature of the gas is maintained constant by surrounding the tube with a water jacket.

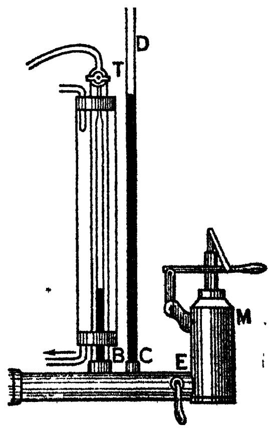


Fig. 40

At first opening the stop cock T, levels in both the manometer tube, CD and the experimental tube BT containing the gas, are kept the same and the gas is now at the atmospheric pressure. Now, closing the stop-cock, mercury is pumped through the tubes until the gas in the experimental tube is reduced to half its initial volume. The pressure of the gas is now determined by adding the barometric height to the difference in the heights of the mercury surfaces in the tubes.

The stop-cock is now opened and by means of the air pump the gas is forced into the tube until the gas attains its initial volume under a higher pressure (2 atmos.) noted by the manometer. Mercury is again pumped until the volume of gas is reduced to half and the increased pressure is noted as before. The same operation is repeated with higher pressures and from his observation Regnault concluded that for most gases, if curves were plotted with p as abscissae and pv as ordinate the product pv decreased as the pressure p was increassed shewing that a given increase of pressure produced a greater diminution in volume when the initial pressure was high than when it was low. These gases are therefore, less elastic and more compressible.

In the case of H_2 gas, the product pv increased as the pressure was increased and the gas is more elastic and less compressible.

*More recent experiments by Natterer and others have shown that for all gases including H_2 at very low temperatures pv at first decreases with pressure and then increases after reaching a certain lower limit.

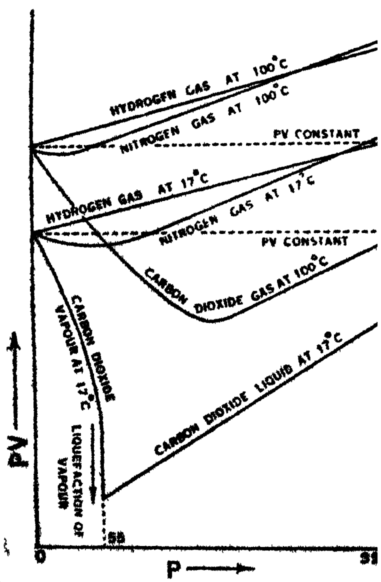


Fig. 41

For each gas there is a certain temperature called the **Boyle temperature** at which pv is approximately independent of p and Boyle's law is obeyed closely over a wide range of pressure at this temperature.

Amagat also performed a similar set of experiments subjecting different gases to very high pressures and plotted curves of pv against p (Fig. 41). He found that for Nitrogen, the product pv at first decreased. The behaviour of N_2 was different from that of H_2 for which the product pv instead of decreasing with pressure increased throughout with the increase of pressure.

In case of CO_2 gas at low temperatures the product pv decreased rapidly with pressure, reached a minimum value and then increased with the increase of pressure.

As the temperature of CO_2 gas is gradually raised and the change of pv with the change of p is noted, it will be seen that the curves shewing this change resembles the curves for N_2. It is also seen that at high temperatures the lower part of the curves either for N_2 or CO_2 are very nearly parallel to the axis of p *i. e.* the product pv is constant and at this point of the curve Boyle's law is applicable.

Fig. 41, shews the behaviour of H_2, N_2 and CO_2 by PV-P curves at different pressures and temperatures.

Andrews performed a series of experiments on CO_2 gas and explained the peculiar behaviour of the gas under increasing pressures by plotting isothermals for CO_2 gas at different temperatures.

118. Andrew's Experiment on CO_2 gas : The apparatus used by Andrews in his experiments (Fig. 42) consists of two similar capillary tubes *c, c* fused to two wide tubes and carefully graduated so that the volume per c. c. was accurately known. The whole of the lower portions together with the wider portion and a part of the upper portions of the tubes is filled with mercury. One of the tubes contains the gas (CO_2) to be operated and the other contains air and acts as a manometer. The lower portions of the two tubes are filled with clean mercury and then firmly fixed into two strong copper cylinders *d, d* containing water and communicating with one another by a cross-tube. The copper cylinders are fitted at the bottom by two screw plungers S, S by means of which pressure can be transmitted to the enclosed gases. The volumes of the enclosed gases are determined from observations of the mercury surfaces in the tubes and the corresponding pressures determined. The capillary tubes projecting from copper vessel are surrounded by bath (not shown in the diagram) whose temperature can be changed and kept constant.

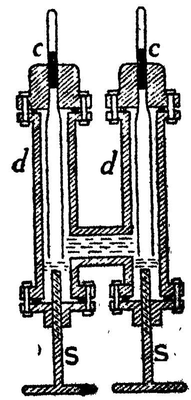


Fig. 42

A series of observations of pressure P and volume V at a number of temperatures was made. The curves obtained by plotting pressure against volume for a fixed mass of gas at a constant temperature is called an isothermal for that temperature. The isothermal curves obtained by Andrews for CO_2 gas, at different temperatures are shown in fig. 43.

118a. Study of the curves and discussion of results : Let us start with the isothermal for temperature 13°.1C. The

portion ab of the curve shows that the volume of the gas decreases with increase in pressure according to Boyle's law. At the point b liquefaction of the gas begins and as long as the liquefaction goes on, the pressure remains constant, and the volume continually decreases, more and more of the gas being changed into liquid state. This state of affairs is shown by the horizontal portion bc. At the point c the whole of the gas has changed into liquid form and the practically vertical rise of the curve represented by portion cd implies that liquids are not altogether incompressible, but are only slightly compressible.

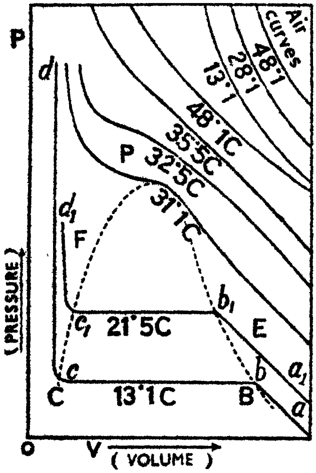


Fig. 43

The isothermal for 21°·5C is practically of same form but horizontal part b_1c_1 is much shorter. The specific volume of the vapour for this curve at the commencement of liquefaction is smaller while that of the liquid at the end of liquefaction is greater than the corresponding volumes for the isothermal curve for 13°·1C. For curves at gradually higher temperatures, these changes go on in the same direction as above till at 31°·1C isothermal, the horizontal part just disappears and the two volumes, volume at the start of liquefaction and volume at the end of liquefaction, become identically the same. The **curve for 31°·1C** is called the **critical isothermal** of CO_2. All the isothermal curves above this temperature show no horizontal part, and as the pressure is increased no liquid is found to be formed; the volume however decreases quickly till it equalises the volume of the liquid at some lower temperature. This characteristic of the isothermal also vanishes at still higher temperatures which will be evident from the 48°·1C isothermal. The isothermal for 48°·1C is similar to the isothermals for air, a permanent gas, which are shown at the right-hand top of the figure.

The temperature 31°·1C is called the **critical temperature** (T_c) for carbon dioxide. At this temperature it is just possible to liquefy the gas by pressure, and at the point when this happens the volumes of the gas and the liquid are the same. Above this temperature it is not possible to change the substance into or obtain it in the liquid state.

118b. Certain Terms :

(1) The *Critical temperature* (t_c°C or T_c°A) of a substance may be defined as that temperature at or above which a gas cannot be liquefied however great the pressure may be. In other words it is the highest temperature at which a gas can be just liquefied by pressure alone. Critical temperature of CO_2, O_2 and H_2 are equal to 31°·1C, −140°C and −240°C respectively.

(2) The *Critical pressure* (p_c) is the pressure required to liquefy a gas at the critical temperature. It is the pressure of the saturated vapour of the substance at its critical temperature. The critical pressure for CO_2 gas is 73 atmospheres or 1074 lbs. wt. per sq. inch.

(3) The *Critical volume* (V_c) is the volume of unit mass of gas or its liquid at the critical temperature and pressure.

(4) The critical point on the isothermal curve for the critical temperature at which a gas can just pass to liquid state and at which specific volumes of gas and its liquid are the same.

119. Continuity of State : A gas can be liquefied even without its entering into the region enclosed by the dotted curve $Cc_1 Pb_1B$ in the Andrew's diagram for CO_2 gas, the highest point P of which is the critical point, lying on the critical isothermal. In the region just referred to liquid and gaseous states co-exist. On the left of the line PC is a liquid region, and on the right of PB we have a gaseous region. Now if we want to convert gaseous CO_2 at 21°·5C into its liquid state at the same temperature without any discontinuity occuring so that the substance does not separate into a liquid and a gaseous part having a layer between them we must not reach the inside of the curve Cc_1Pb_1B. This is done as follows. Let us raise the temperature of the gas from 21°·5C to some value above 31°·1C and then compress is to a volume equal to the volume of the liquid at that temperature. If then we cool the gas to 21°·5C and reduce the pressure to the appropriate value, we will get liquid CO_2 at 21°·5C. At no stage of these processes any heterogenous discontinuity or meniscus appears in whole substance. This result is expressed by saying that ***there exists a continuity of the liquid and gaseous states.***

120. Determination of Critical Constants of a Gas : An approximate value of T_c can be found by sealing up a quantity of liquid and vapour in a strong glass tube from which all air is removed before. The tube is heated in a suitable bath. At the critical temperature, the surface layer between liquid and its vapour flickers and disappears. As the tube is then cooled again to the critical

temperature a turbidity is visible in the gas which condition quickly vanishes and a liquid meniscus appears at the bottom of the tube.

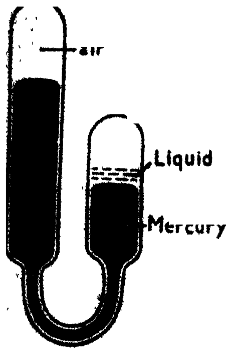


Fig. 44

To find p_c Cagniard de la Tour enclosed some liquid in one limb of an U-tube of the shape shown in fig. 44. The longer limb contains some air separated from the liquid in the shorter limb by a column of mercury. Observing the volume of air in the limb when the critical temperature, at which liquid meniscus just disappears, is reached, the pressure of vapour in shorter limb *i.e.*, the critical pressure p_c can be calculated.

Amagat determined the critical density by plotting the densities of the liquid and vapour at a number of temperatures near the critical temperature (Fig. 45). When curves were drawn joining the points which represent the density of liquid and vapour at the same temperature, he found that the middle points lay on a straight line. By extrapolating the line to the critical temperature he got the value of the critical density at a point where the curves for the vapour and the liquid meet indicating the value of common density.

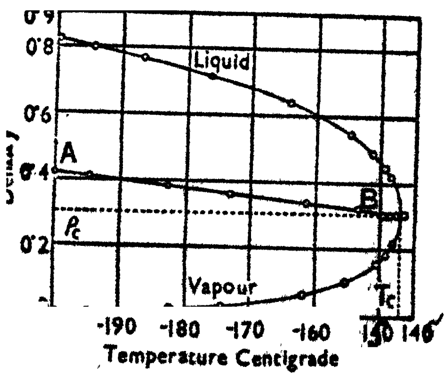


Fig. 45

121. Vapour and Gas : A gaseous substance at temperatures below its critical temperature is called a vapour and at temperatures higher than the critical temperature the substance is called a gas. A vapour can be liquefied by pressure alone while a gas requires cooling as well as pressure to be converted into the liquid state.

122. Van der Waals' Equation : We have seen before, that experiments over a wide range of pressure show that actual gases deviate from Boyle's law and to account for the difference between the compressibility of an actual gas and that of an ideal gas Van der Waal allowed for *intermolecular attractions* and for the *size of the molecules* and deduced an equation of state of gas relating to pressure volume and temperature.

The equation of state for a perfect gas is given by $pV = RT$

Van der Waal modified this equation to fit the observed results for a real gas, introducing pressure correction for inter-molecular attraction and volume correction for finite size of gas molecules.

(A) Correction for the inter-molecular attraction :

A gas molecule situated within a gas in an enclosure is attracted equally from all directions by the neighbouring molecules and remains unaffected as there is no unbalanced resultant force, while a molecule impinging on the walls of the containing vessel is pulled back by the molecules behind it. So the pressure inside the gas exceeds the observed pressure p at the walls by an amount proportional to $\frac{1}{V^2}$, or equal to $\frac{a}{V^2}$ where a is a constant and V the volume of the enclosure.

Hence, we may write the corrected pressure as $\left(p + \frac{a}{V^2}\right)$

(B) Correction for the finite size of gas molecules :

The space available for the gas molocules to move about inside an enclosure containing the gas is less than the observed volume V which is the volume of the enclosure, since it is diminished by the space occupied by the volumes of the molecules themselves.

The effect of this practically amounts to a diminution of the volume V. Hence, we may write $(V - b)$ instead of V in the equation $pV = RT$. Here b is a constant which is some function of the actual volume of the gas molecules.

Hence, applying the corrections for molecular attraction and size of molecules, the equation of state of a real gas may be written as

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT.$$

This equation is known as **Van der Waals' Equation.**

Van der Waals' equation was obtained by making certain assumptions which are only approximately true. So it may be regarded as an approximate equation over wide ranges of pressure and temperature, for a wide range of substances.

If this equation be plotted graphically at a temperature below the critical temperature of the gas a curve of the form ABCD (Fig. 47) is obtained. The whole of the isothermal is continuous and nowhere exhibits an abrupt change of direction as was observed in Andrew's experimental curves for CO_2 gas.

The continuous nature of the isothermal curve is corroborated by the suggestion put forward by Prof. James Thomson.

For temperatures above the critical temperature all the isothermals are continuous and agree in their general form obtained experimentally by Andrews.

123. Van der Waals' Constants and Critical Constants :

Van der Waals' equation of state for a real gas is expressed as

$$\left(p+\frac{a}{V^2}\right)(V-b)=RT$$

This equation may be written in the form

$$-\left(b+\frac{RT}{p}\right)V^2+\frac{a}{p}V-\frac{ab}{p}=0.. \qquad (1)$$

Since a, b and R are constants, for constant values of p and T, this becomes a cubic equation in V. From the theory of equations, we know that a cubic equation must necessarily have either one or three real roots.

If we study the isothermals of carbon dioxide for different temperatures we notice that the isothermal for 31°·1C shows no sudden discontinuity and at this temperature there is no marked passage from the gaseous to be liquid state. This isothermal is called the critical isothermal and at this temperature the roots of the cubic equation are all real and equal.

Let v_c, p_c and T_c be the critical volume, critical pressure and critical temperature respectively.

A cubic equation in V possessing three equal roots may be written in the form $(V-v_c)^3=0$ ... ... (2)

where v_c is the value of each of the equal roots.

Expanding (2) we have $V^3-3v_cV^2+3v_c{}^2V-v_c{}^3=0$ ... (3)

Again from (1) we have, $V^3-\left(b+\frac{RT_c}{p_c}\right)V^2+\frac{a}{p_c}V-\frac{ab}{p_c}=0$ ...(4)

Equating the coefficients of equal powers of V in these two equations (3) and (4), we find that

$$3v_c=b+\frac{RT_c}{p_c}...(5); \quad 3v_c{}^2=\frac{a}{p_c}...(6); \quad v_c{}^3=\frac{ab}{p_c} \qquad (7)$$

Dividing (7) by (6) $v_c=3b$ (8)

From (6) and (8) $p_c=\frac{a}{3v_c{}^2}=\frac{a}{27b^2}$ (9)

From (5), (8) and (9) $T_c=\frac{3v_c-b}{R}\times p_c=\frac{8b}{R}\times\frac{a}{27b^2}=\frac{8a}{27Rb}$ ......(10)

Alternative method.

At the critical point C on the critical isothermal for a gas (Fig. 46), the tangent of slope is horizontal *i.e.* $dp/dV=0$. As dp/dV has a turning value from C to B,

$$\frac{d}{dV}\left(\frac{dp}{dV}\right)=0 \quad \text{or} \quad \frac{d^2p}{dV^2}=0$$

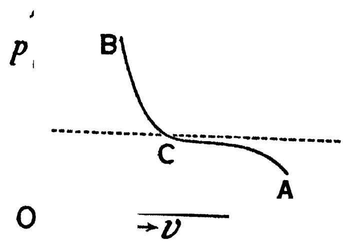


Fig. 46

According to Van der Waals' Equation,

$$\left(p+\frac{a}{V^2}\right)\left(V-b\right)=RT \quad \text{or} \quad p=\frac{RT}{V-b}-\frac{a}{V^2}$$

Differentiating $\dfrac{dp}{dV}=-\dfrac{RT}{(V-b)^2}+\dfrac{2a}{V^3}$

again $\dfrac{d^2p}{dV^2}=\dfrac{2RT}{(V-b)^3}-\dfrac{6a}{V^4}$

For the critical point, putting dp/dV and d^2p/dV^2 equal to O, and replacing all the above equations p, V and T by p_c, V_c and T_c respectively, we have

$$p_c=\frac{RT_c}{V_c-b}-\frac{a}{V_c^2} \qquad (1)$$

$$-\frac{RT_c}{(V_c-b)^2}+\frac{2a}{V_c^3}=0 \qquad (2)$$

$$\frac{2RT_c}{(V_c-b)^3}-\frac{6a}{V_c^4}=0 \qquad (3)$$

From (2) and (3) $\dfrac{RT_c}{(V_c-b)^2}=\dfrac{2a}{V_c^3}$ ...(2a)

and $\dfrac{2RT_c}{(V_c-b)^3}=\dfrac{6a}{V_c^4}$ ...(3a)

Dividing (2a) by (3a) $\dfrac{V_c-b}{2}=\dfrac{V_c}{3}$ or $3V_c-3b=2V_c$ $\therefore$ $V_c=3b$

Substituting for V_c in (2a) $\dfrac{RT_c}{4b^2}=\dfrac{2a}{27b^3}$ or $T_c=\dfrac{2a\times 4b^2}{27Rb^3}=\dfrac{8a}{27Rb}$; finally substituting for V_c and T_c in (1).

$$p_c=\frac{R.8a}{27Rb}\times\frac{1}{2b}-\frac{a}{9b^2}=\frac{4a}{27b^2}-\frac{a}{9b^2}=\frac{4a-3a}{27b^2}=\frac{a}{27b^2}.$$

124. Prof. James Thomson's Hypothesis : According to his hypothesis James Thomson suggested that all the isothermals above

and below the critical temperature of a gas are continuous shewing no abrupt discontinuity and that the dotted part CD of the curve CDEFB corresponds to the condition of *superheating*, and the portion DEF corresponds to the phenomenon of *bumping* and the portion FB corresponds to the *supersaturation* of vapour (Fig. 47).

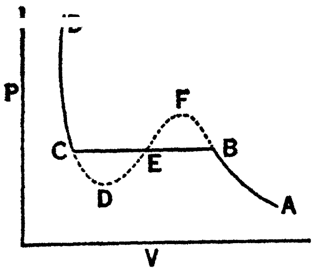


Fig. 47

125. Superheating and Bumping : A liquid may, under certain conditions, be heated to a temperature considerably above its boiling point, without ebullition occuring. This phenemenon is called *superheating*.

Defour prepared a mixture of oils of high boiling point whose density was the same as that of water and small drops of water suspended in the mixture were found to be heated to 178°C without any boiling taking place. Water enclosed in a vessel which has been previously cleaned to remove any dust, grease or occluded gas, may be heated to 137°C before it boils.

Thus the phenomenon of superheating is due to the absence of any nucleus, such as **bubbles of gas** or particles of dust to promote evaporation and subsequent formation of bubbles of vapour.

When a liquid has no facilities for producing bubbles of vapour, its temperature may rise far above the boiling pcint without the occurrence of boiling and ultimately a large bubble of vapour may be formed which throws the greater part of the liquid from its containing vessel with a bumping noise. This phenomenon is known as **bumping**.

It can be suppressed by putting a few pieces of broken glass or some sand into the water. The boiling at once becomes regular and quiet and the temperature resumes its normal value.

QUESTIONS

1. Describe experiments of Andrews on carbon dioxide and shew with the help of suitable curves the general nature of the isothermals above and below the critical temperature. [C. U. 1931, '34, '37, '44, '53.]

2. Explain the behaviour of carbon dioxide, nitrogen and hydrogen under high pressures. Account for the phenomenon and discuss the validity of Boyle's law for high pressure. [C. U. 1953.]

3. Why do real gases not obey Boyle's law ? [C. U. 1931, '43, '48, '51.]

Obtain relations between the critical constants of a gas and the constants of Van der Waal's equation. [C. U. 1948]

4. Give an account of Regnault's and Amagat's experiments on Boyle's law at high pressures. Mention the conclusions drawn from the experiments. [C. U. 1928, '39]

CHAPTER XII

LIQUEFACTION OF GASES

126. Introductory : A given mass of gas or vapour occupies much less volume when changed into liquid state. Hence the process of liquefaction of a gas or vapour involves the process of reduction of its volume. This again can be effected by (1) cooling and by (2) application of high pressure.

126a. Different methods of cooling a gas : There are three principal methods of cooling a gas :—

(1) By passing a gas through a tube immersed in a cold liquid or surrounded by a stream of cold gas.

(2) By allowing it to expand adiabatically.

(3) By allowing it to expand freely through a valve from a high to a low pressure (the Joule-Kelvin effect) which causes cooling if the intial temperature of the gas be below the Joule-Kelvin inversion temperature.

127. History of Liquefaction of Gases : The history of liquefaction of gases may be divided into **three stages.**

In the **first stage** when there was no clear idea of critical temperature, the gas was cooled as much as possible and then subjected to a high pressure. Faraday utilised this method to liquefy chlorine.

The **second stage** began with the liquefaction of carbon dioxide gas. Andrews experimenting with CO_2 gas showed that no amount of pressure was enough to liquefy a gas if it is above or at its critical temperature. So in the second stage the gas was at first cooled below its critical temperature and then subjected to a high pressure. The cooling was produced by freezing mixture as well as by the external work done by sudden expansion.

Pictet liquefied Oxygen by the **cascade process** in which the low temperature is reached in stages. Melhyl chloride was first liquefied by a compressor, which was then allowed to circulate

round a condenser through which ethylene was being passed. The ethylene was liquefied in a second condenser and boiled at a reduced pressure and a temperature of $-169°C$ was obtained which was sufficient to liquefy oxygen whose critical temperature is $-118°C$.

The difficulty arose with the liquefaction of Hydrogen whose critical temperature is $-240°C$. At this stage the method of liquefying a gas by cascade process broke down as it became very difficult to cool hydrogen below $-240°C$.

In the **third stage**, *Joule-Kelvin effect* was utilised to liquefy hydrogen. This method has a greater advantage over the cascade process.

In using the Joule-Kelvin effect it is often necessary to cool the gas below the *temperature of inversion* which is always higher than the *critical temperature* below which the gas is to be cooled in the cascade process.

A list of the important temperatures for the chief permanent gases is given in the following table :

Name of Gas	Normal Boiling point	Freezing point	Critical temp.	Joule-Thomson inversion temp.
Oxygen	$-183°C$	$-227°C$	$-118°C$	above ordinary temps.
Nitrogen	$-195°C$	$-211°C$	$-146°C$	
Hydrogen	$-252°C$	$-259°C$	$-241°C$	$-80°C$
Helium	$-268°C$	$-271°C$	$-268°C$	$-243°C$

128. Liquefaction of Gases : We know that a vapour either in the saturated or in the unsaturated state may be liquefied either by the increase of pressure or by the fall of temperature or by the joint action of both these causes.

A saturated vapour at a constant temperature may be reduced to the liquid state by compression alone but if its pressure remains constant, it may be liquefied by the diminution of temperature. Thus we see that there are two distinct means of liquefying a vapour, (1) by an increase of pressure and (2) by a fall of temperature. These two means may be employed separately and sometimes in conjunction.

A gas may be regarded as an unsaturated vapour and as unsaturated vapour is liquefied by first bringing it to the point of saturation and then diminishing the temperature or increasing the pressure, so a gas may be liquefied by the above means. But as gases are removed from their states of saturation they are subjected to the joint action of cold and pressure and then liquefied. Thus in liquefying a gas it is first to be cooled down below its critical temperature and then liquefied under a high pressure of appropriate value.

Gases such as CO_2, Cl_2, SO_2, NH_3, etc. are generally classed with vapours, their critical temperatures being higher than the

ordinary temperature and consequently they are liquefied by subjecting them to a high pressure at the ordinary temperature or to a fall of temperature at the ordinary pressure,

Faraday and Thirlorier first experimented on CO_2 and other gases and successfully liquefied them. Cailletet and Pictet liquefied the so-called permanent gases by first cooling them below their critical temperatures and then subjecting them to higher pressures.

Their extensive researches have effaced the distinction between vapours and gases by effecting the condensation of gases such as O_2, H_2, N_2 etc. which were supposed to be non-liquefiable.

Recent researches have shewn that vapours and gases can be liquefied by subjecting them to high pressures and lowering the temperature by causing them to expand through a narrow orifice in a vessel open to atmosphere *i.e.* by producing a sudden expansion in the compressed gases. The cold thus produced is sufficient to cause liquefaction.

The investigation of Joule and Kelvin regarding the change in temperature of a gas by forcing it through a narrow orifice or through a plug of cotton wool has opened up a new method of liquefying the so-called permanent gases. The gas, on being forced out through the orifice suddenly expands without doing any external work and is thereby lowered in temperature. This cooling effect is observed in all gases except H_2 which gets heated by free expansion. If the self-cooled gas be made to sweep over the pipe from which it is escaping, it will cool the pipe and thereby lower the temperature of the gas before it issues. In this way the gas is gradually cooled and finally reduced to the liquid state. This process of cooling a gas to a very low temperature is known as **Self-cooling method.** Linde used this method for liquefying air.

129. Faraday's Method (Chlorine): Faraday took some crystals of chloral hydrate in a stout, bent glass tube (Fig. 48) which was then sealed off. The end of the tube containing the crystals was placed in a water-bath, the other end being immersed in a bath of freezing mixture of ice and salt.

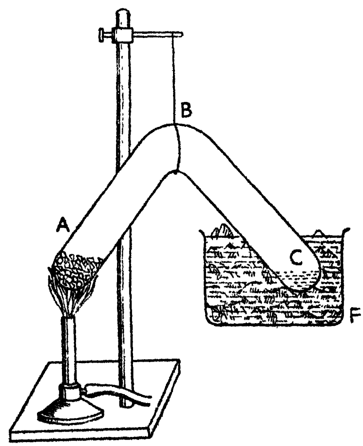


Fig. 48

The water in the bath was heated and chlorine hydrate was decomposed giving off chlorine, which by more and more accumulation sets up requisite high pressure in the closed tube. The gas was then found to be liquefied to a yellow oily substance.

Faraday liquefied carbon dioxide, cyanogen and nitrous oxide by similar method.

130. Liquefaction of Oxygen : Pictet's Method (Cascade Process) : The liquefaction of Oxygen is generally effected by compressing the gas in a closed vessel until a very high pressure is reached and then allowing it to pass through a tube surrounded by a jacket containing a suitable refrigerant *i.e.*, cooling agent. The gas is thereby cooled and a further reduction in temperature is produced by suddenly releasing the pressure and thereby causing a sudden expansion.

In Pictet's method Oxygen is generated by heating Potassium Chlorate, $KClO_3$ in a copper vessel V (Fig. 49) and the gas is passed

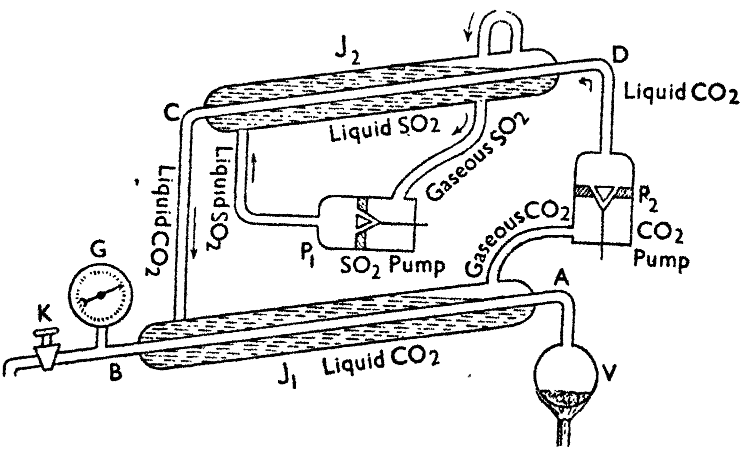


Fig. 49

through a tube AB fitted at its open end with a stop-cock and a pressure gauge to indicate the pressure of the gas. The tube through which the gas is passed is surrounded by a jacket J_1 provided with two tubes of which one is connected to a pump P_2 and the other forms a bent delivery tube and opens communication with the same pump P_2. The jacket J_1 contains liquid CO_2 which on evaporating cools the O_2 gas and is itself transformed into CO_2 vapour which is pumped by P_2 under reduced pressure, compressed and liquefied. The liquid CO_2 enters in the delivery tube, passes through the tube DC inside a second jacket J_2 containing liquid SO_2 and returns to jacket J_1. The supply of liquid SO_2 is maintained in the jacket J_2 by means of a second pump P_1 which compresses the SO_2 gas produced by rapid evaporation from the liquid SO_2 and delivers it in the liquid form, into the jacket J_2 again.

The liquefaction sets in when the pressure gauge indicates a pressure of 500 atmospheres and the gas is lowered down

to a temperature of about $-120°C$ by the cooling action of the refrigerant.

At this time the stop-cock is opened and the liquid O_2 is seen to flow out in a white jet. The liquid O_2 has the milky-white appearance but on being carefully filtered it presents a beautiful blue colour.

131. Principle of Regenerative Cooling : For most of the gases Joule-Thomson cooling is extremely small, and as such the cooling effect could not be utilised at first for liquefaction of gases. It was however discovered later on that the cooling effect can be made cumulative, *i.e.*, magnified by adopting what is called the regenerative process. In this process a portion of the gas which first undergoes Joule-Thomson expansion and becomes cooled thereby, is used to cool other portions of the incoming gas before the latter comes to the throttle valve. The incoming gas on issuing out through the valve becomes further cooled due to expansion. This process goes on and the cooling effect is generated again and again. The regenerative method has another advantage in the fact that the lower the temperature of a gas greater is the cooling by Joule-Thomson expansion. The regeneration principle is illustrated by figure 50.

Gas at high pressure goes from compressor C into spiral tube S surrounded by water-cooling jacket W. The gas then enters the regenerator coils at A, traverses the central tube, comes to the valve V and by expansion becomes cooled slightly. This cooled gas traverses the outer concentric tube B, cools the high pressure gas in the central tube and reaches D practically at the same temperature asthe incoming gas at A. The gas then goes into C to be compressed. This compressed gas cooled by water in W on passing through S, comes to A to re-enter the regenerative coil. As the process goes

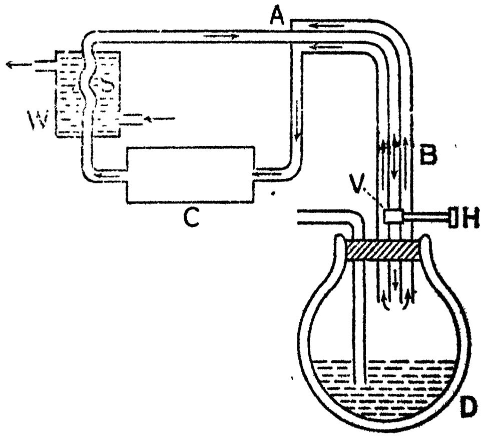


Fig. 50

on, the gas coming to the valve V becomes more and more cooled

till the Joule-Thompson expansion produces further cooling sufficient for liquefaction of the gas.

132. Linde's method of liquefying air : The arrangement adopted originally by Linde for the liquefaction of air is shown in fig. 51 and the principle is explained below.

The air is compressed in the pump P and delivered through a pipe to the cooler V where its temperature raised during compression is lowered. The compressed air then passes along the tube in the direction of the arrow, thence through the central tube D of the interchanger and finally comes out through the small orifice of the throttle valve R into the vessel T. The interchanger consists of three tubes arranged concentrically and coiled into a spiral. The interchanger and other parts of the liquefying apparatus are well 'lagged' with cotton wool to prevent the inflow of heat from the surrounding bodies to the air cooled by Joule-Thomson's effects.

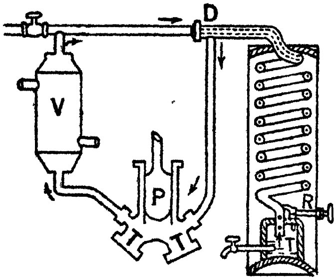


Fig. 51

The air thus cooled somewhat, passes back to the pump through the space between the central and the outer tubes of the interchanger and abstracts heat from the air approaching the throttle valve, which then attains lower and lower temperatures as the pump is worked. At a certain stage of cooling the temperature of air falls just below its critical temperature and liquid air is formed and is collected in the vessel T which is simply a Dewer's flask.

The arrangement is modified by using a two-stage air compressor and by surrounding the cooler V by a mixture of ice and salt. With this modified arrangement and by cooling the air preliminarily by carbonic acid snow, liquid air may be obtained in very short time.

133. Uses of liquid air :

Liquid air is usually used for the purposes mentioned below :

1. For production of high vacuum.
2. For drying and purifying gases.
3. For preparing gases from liquid air by fractional distillation.
4. It is used in scientific research for producing a very low temperature for investigating important properties of matter.

5. It is used in the manufacture of commercial O_2 and N_2 from liquid air.

134. Liquefaction of Hydrogen : We have noted that the so-called permanent gases can be liquefied by first cooling them below their critical temperatures and then subjecting them to high pressures.

Since the critical temperature of hydrogen is $-241°C$, which cannot be attained by any independent means, the Joule-Kelvin effect is the only means by which hydrogen can be liquefied.

The inversion temperature of hydrogen for moderate pressure is $-80°C$.

It is therefore necessary to cool the gas below $-80°C$ and allow it to expand freely through a valve.

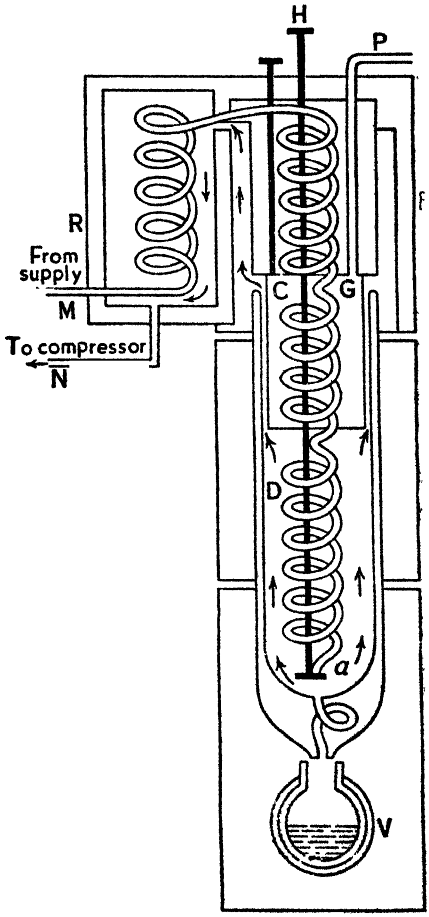


Fig. 52

Hydrogen prepared from zinc and sulphuric acid, first carefully purified, is compressed to about 150 atmos. and then passed through a coil connected to a supply tube M (Fig. 52) immersed in water in order to deprive the gas of the heat of compression. The gas is then passed through a series of refrigerating coils C in a bath of liquid air boiling under a reduced pressure of 100 mm.. This is a adjusted by allowing liquid air from uppermost chamber F to trickle into central chamber G by opening a valve near C and by evacuating G through a pump connected to P which communicates with central chamber G. The cooled gas $(-200°C)$ then passes through the coil D and suffer Joule-Thomson expansion at the valve *a* which

is operated by H. The cooled gas then passes round the chambers G and F thereby cooling the incoming gas in the coils D and C, to the chamber R and therefrom through the pipe N to the compressor. Then after a few cycles, the incoming gas at *a* falls to $-250°C$ and on suffering further Joule-Thomson expansion, it liquefies and is collected in the Dewar vessel V. Dewar solidified hydrogen by boiling the liquid under reduced pressure, reaching in this way a temperature of $-259°C$.

Both the liquid and solid Hydrogen are colourless and transparent.

135. Liquefaction of Helium : The temperature of inversion of Helium is about $-243°C$.

Method 1 : The gas is pre-cooled down to $-243°C$ by means of a bath of liquid hydrogen boiling under reduced pressure and then allowed to expand freely through a valve from 36 atmos. to one atmosphere. Cooling is produced by Joule-Thomson effect and the cold gas traverses through spirals surrounded by cold helium vapour and finally comes back to the compressor. After a few cycles of operation the gas becomes liquefied.

Prof. Kessom first succeeded in obtaining solid helium by subjecting the liquid helium to high pressures in 1926. The solid helium is obtained at a pressure of 250 atmos. and temperature 4·2°A.

Method 2 : The method is based on adiabatic expansion. The vessel E (Fig. 53) contains the helium to be liquified, under high pressure (about 150 atmospheres). The vessel E is surrounded by outer chamber F which is again inside a bath of liquid H_2 kept boiling under reduced pressure by means of exhaust pump connected to A. When filled with helium gas F maintains thermal contact between E and hydrogen bath. If F be evacuated through pipe C the vessel E becomes completely insulated from the hydrogen bath. The gaseous helium in E is then made to expand suddenly and the cooling produced is sufficient to liquefy the gas. The adiabatic condition is realised by perfect thermal insulation. The whole apparatus certain essential parts of which are not shown in the diagram, is made of metal to bear the high pressure difference.

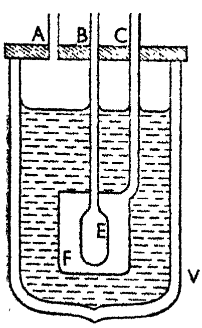


Fig. 53

136. Dewar's Thermos flask : Dewar's flask consists of double-walled vessel made either of glass or other material, the space

between the walls being thoroughly exhausted of air through a fine tube at the bottom of the outer wall which is then sealed off. The surfaces of the walls next the vacuum are silvered in the case of glass and polished in the case of metallic vessels.

For household purposes the ordinary form is used but for scientific purposes for storing hot and cold liquids at high and low temperatures various forms are used. In some forms there are tapping arrangements for withdrawing liquid without opening the mouth.

The usefulness of this instrument depends on the fact that vacuum is a perfect non-conductor of heat. Since the space between the walls of the vessel is thoroughly exhausted, transference of heat either from the outer to the inner or from the inner to the outer walls of the vessel by conduction or convention is materially reduced.

Again since the polished surfaces radiate much less heat than the blackened or dull surfaces, loss or gain of heat is prevented to a large extent due to reflection of radiant heat between the polished inner walls of the vessel.

QUESTIONS

1. Trace historically as far as is practicable, the development of methods of liquefaction of gases. [C. U. 1927, '43, '46, '47]

2. Explain the self-cooling method.

Describe the general construction of apparatus for liquefying air.

[C. U. 1943, '47, '56]

3. Describe the general construction of an apparatus for liquefying hydrogen and explain its action. [C. U. 1946]

4. Explain briefly the principles underlying some method of liquefying gases such as air. [C. U. 1950]

CHAPTER XIII

PRODUCTION OF LOW TEMPERATURES

137. Refrigeration: The term *refrigeration* means the production of a temperature lower than that of the ordinary air. There are various ways of producing a low temperature of which the following may be mentioned.

(1) By dissolving a salt in a liquid.
(2) By boiling a liquid under reduced pressure.

(3) By the adiabatic expansion of a gas doing external work.
(4) By utilising the heat of absorption.
(5) By the application of Joule-Thomson effect.
(6) By utilising the cooling due to Peltier effect.
(7) By the process of adiabatic demagnetisation.

137a. Refrigerating Machine: The method usually adopted in refrigerating machines is to alternately allow the working fluid such as NH_3, CO_2 and SO_2 to evaporate and condense into the liquid form by means of some mechanical device.

In a refrigerating machine (Fig. 54) the working fluid, say liquid ammonia is allowed to evaporate inside a pipe coil A which is surrounded by a substance to be chilled. Usually brine is placed inside the vessel containing the pipe coil A.

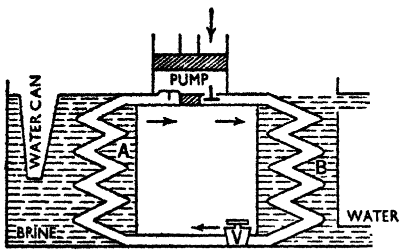


Fig. 54

During evaporation the liquid ammonia extracts the latent heat of vaporisation at the expense of heat in the brine which is accordingly chilled. The resulting vapour from the liquid inside the coil is drawn by a pump which then compresses it and delivers it to another pipe coil B in a vessel which is kept cool by constant circulation of water and the vapour is condensed into the liquid form.

The resulting liquid is fed through a regulating valve V to the pipe coil immersed in brine where evaporation takes place again.

This process is repeated and in so doing the brine is cooled to a lower temperature.

The chilled brine, the temperature of which falls bellow 0°C in then circulated round the water in the can which is to be frozen, or sent by pipes through the room or the vessel which is to be kept cool.

The cold brine abstracts heat from the walls and from the substances placed inside the room or the vessel, gets heated and is returned again to the vessel to be chilled further.

Refrigerating machines may be used for (1) cold storage and preservation of food, (2) ice-making and (3) air-conditioning.

SO_2 machine is convenient for dairy installations. CO_2 machines are used on board vessels carrying mutton etc. Ammonia machines are used extensively in land installations.

Modern organic refrigerants include Freon (CCl_2F_8) and several other hydro-carbon derivatives of similar type.

138. The Electrolux Refrigerator: It is a practical application of the cooling due to evaporation and the law of partial pressures.

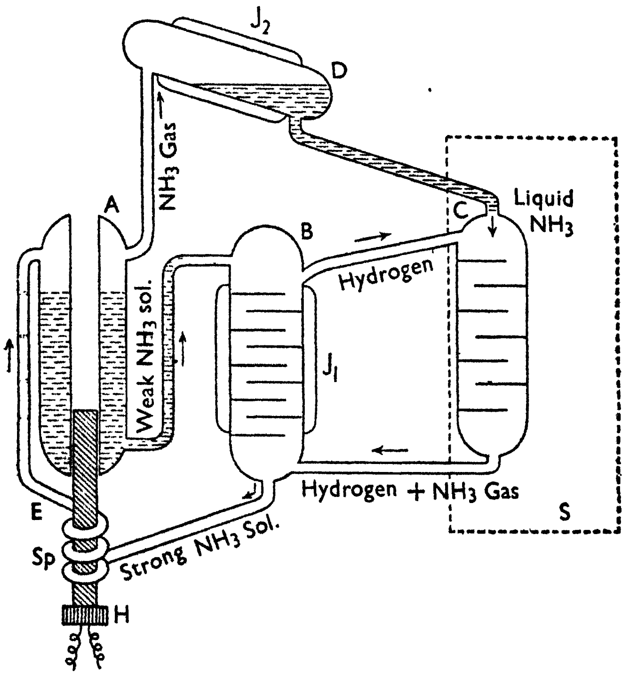


Fig. 55

In it the substances used are liquid ammonia, the refrigerant, hydrogen gas and a solution of ammonia in water. It works practically on the same principle as in other refrigerators but the process is entirely different.

The evaporator in which the liquid ammonia is caused to evaporate is immersed in the brine-bath or the chamber to be cooled. It extracts the latent heat from the surrounding vessel or space and thereby cools it.

The ammonia gas produced in the evaporator encounters a stream of hydrogen and the ammonia gas is then passed through a complicated process to a condenser, the hydrogen in the meantime returning back to the evaporator. The ammonia gas being cooled

in the condenser returns to the evaporator as liquid ammonia to extract a further quantity of heat from the surrounding space or vessel. The process goes on until the vessel or the space is cooled to the desired degree.

The apparatus, a sketch of which is shown in fig. 55, contains a boiler A in which gaseous ammonia is produced from a strong aqueous solution of it by electric heater. D is the condenser surrounded by a cold water jacket in which ammonia gas condenses to liquid state. C is the evaporator placed inside the space to be cooled, in which liquid ammonia evaporates. B is the absorber surrounded by cold water jacket in which ammonia gas is absorbed in a weak aqueous solution of ammonia which therby becomes concentrated. A circulation of hydrogen gas is maintained between B and C. The gas enters into C with liquid ammonia, leaves at its bottom with ammonia gas and returns to C through B unchanged, while the ammonia gas is dissolved. The strong aqueous solution of ammonia goes through the communicating tube into the spiral tube where it is again evaporated by electric heater. At the same time due to larger pressure, weak solution of NH_3 enters into B, the absorber.

139. Air-conditioning : The term *air-conditioning* was originally used in cotton industry for maintaining a uniform degree of dampness in spinning rooms. But now the term *air-conditioning* has come to mean the process by which the air in an enclosure is so conditioned with regard to temperature and humidity that it gives maximum comfort to human beings.

Both *temperature* and *humidity* are equally important in conditioning. Neither reduction in temperature nor a change in humidity of the enclosed air gives us the required comfort.

Temperature should be controlled either by governing the admission of steam by a thermostat or by regulating the supply of cold air to the enclosure as required.

Humidity should also be regulated by spraying water particles from nozzles to the air in the enclosure.

The arrangement for ventilating a room with conditioned air *i. e.*, air which has been cooled by being washed with very cold water is provided here.

The water is cooled by being passed through a pipe immersed in the brine tank of an ice-making machine.

NH_3 gas is compressed by a pump, the heat of compression is removed by passing the compressed gas through a pipe immersed in a cooling tank through which cold water circulates. The NH_3

gas expanding through a valve is liquefied and collected at the bottom of a coil immersed in a brine tank. The liquid NH_3 evaporates and cools the brine. The vapour is drawn off from the upper part of the coil and compressed again and the process is repeated.

Atmospheric air is blown through a wide pipe by a fan. Water cooled in the brine tank is showered on the air through fine holes.

The air in the pipe is thus washed and cooled. On its way to the room to be ventilated the temperature rises and humidity falls to the desired value.

Good thermal insulation is essential and there should be no leakage from the ventilated room through doors and windows.

QUESTIONS

1. Describe the construction and mode of operation some common form of refrigerating machine.
2. Write a full note on refrigeration and air-conditioning. [C. U. 1942.]

CHAPTER XIV

MECHANICAL EQUIVALENT OF HEAT

140. Relation between work and heat: *When mechanical work is completely transformed into heat, or heat is completely converted into mechanical work, for each unit of work that is converted into heat a definite amount of heat is produced, and for each unit of heat converted into work a definite amount of work is obtained.*

The amount of mechanical work equivalent to unit heat is known as **Mechanical equivalent of heat.**

Mathematically, if W units of work are converted into H units of heat, we have

$\frac{W}{H}=J$, or $W=J.H$ (where J is a constant) assuming that all the work is spent in the production of heat, and no part of it is wasted by friction, radiation etc. The constant J is called the mechanical equivalent of heat and the accepted value of J is $4{\cdot}182\times10^{7}$ ergs per calorie.

141. First Law of Thermodynamics : The First Law of Thermodynamics is merely a particular case of the conservation of

energy which states that energy is never lost but can be transformed from one form into another and that the total energy of any material system is always constant. The truth of the law has been proved in various cases such as free motion of a body under gravity and motion of the bob of a simple pendulum.

Now from thermal point of view, when a system absorbs heat, it is, in general, used up partly to increase the **internal energy** and partly to do **external work**.

If the system absorbs a quantity of heat dQ, which produces an increase dU in its internal energy and causes it to do external work of amount dW, then $dQ = dU + dW$

The symbol dU is used for the total internal energy, the sum of the potential and kinetic energies of the molecules, so that

$$dU = dE + dI$$

where dE is the increase in the internal kinetic energy (heat energy) and dI is the gain in internal potential energy.

$$\therefore \quad dQ = dE + dI + dW.$$

This statement is called **Joule's Law.** For a perfect gas $dI = O$.

$$\therefore \quad dQ = dE + dW.$$

142. Joule's Experiment for determination of J: In his experiment Joule determined the quantity of mechanical work equivalent to one thermal unit *i.e.*, a calorie.

The apparatus used by Joule (Fig. 56) consists of a copper calorimeter C fitted internally with radial vanes having spaces to permit the radial paddles attached to a spindle S passing out through the central hole in the water-tight lid of the calorimeter. The spindle is attached by a pin T to a drum D round which two pieces of cord are wrapped in the same direction, which leaving the drum at the same level and at the opposite extremities of a diameter pass round two pulleys P, P and support two weights M, M. The weights are then allowed to descend through a measured height and paddles inside the calorimeter rotate and churn the

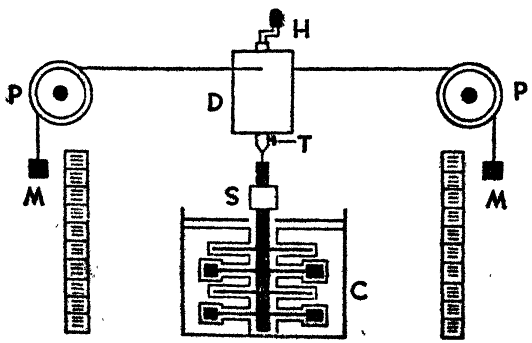


Fig. 56

water inside it. The work done by the falling weights is converted into heat which is evident by the rise of temperature of water in the calorimeter.

When the weights have reached the lowest limit, the drum is disconnected from the spindle and the weights are then raised to their original heights by turning the drum. The drum being again attached to the spindle, weights are allowed to fall through the same height as before and the rise of temperature is noted by a sensitive thermometer inserted into the calorimeter through an opening in the lid. The same operation is repeated several times and the final temperature of the calorimeter and its contents noted.

Let M be the mass of each weight and let each of the weights fall through a height h, n times in succession.

The total potential energy of two weights at start $= 2Mgh$.

Kinetic energy acquired before striking the ground $= 2 \times \frac{1}{2}Mv^2 = Mv^2$,

where v is the steady velocity of each weight when it reaches the ground. The value of v can be found by noting with a stop-clock the time of passage of either weight through a known distance along the scales, and dividing the distance by the time.

Total effective work done in n, falls $W = n(2Mgh - Mv^2)$

Let m be the mass of water taken in calorimeter, w the water-equivalent of calorimeter system, t_1, t_2 initial and final temperatures of calorimeter and contents, then total heat developed is given by

$$H = (w + m.s)(t_2 - t_1), \text{ where } s = \text{sp. ht. of water.}$$

$$J = \frac{W}{H} = \frac{Mn(2gh - v^2)}{(w + m.s)(t_2 - t_1)}$$

whence J can be found out.

In Joule's experiment the water equivalent of the calorimeter and its contents was 6316 grams and the rise of temperature was ·3129°C. The value of J was found to be equal to $4{\cdot}19 \times 10^7$ ergs per calorie or 778 ft. lbs. of work per degree Fah. or 1400 ft. lbs. per degree Cent.

The value of J as obtained by Joule is subject to several corrections and the following corrections were applied to get the exact value.

(1) Heat capacity of the calorimeter.
(2) Kinetic energy of the weight on reaching the bottom.
(3) Frictional resistance of the various bearings.
(4) Energy lost by the vibrations of the rotating paddles.

143. Determination of J by Rowland's Experiment: Rowland modified Joule's experiment with a considerable advantage and carefully determined the value of J.

In his experiment Rowland used a calorimeter which instead of being fixed was suspended from a support h attached to a vertical shaft which in turn is fastened to a torsion wire CD. The vertical shaft passes through a disc of radius r, round the groove of which silk cords are wound in such a manner as to leave it tangentially at the opposite extremities of a diameter and pass over the pulleys P_1 and P_2 (Fig. 57).

At right angles to the shaft a horizontal arm is attached having two movable weights w_1, w_2 for altering the moment of inertia of the suspended parts. The calorimeter C_1 is provided with fixed vanes and between them moves a number of perforated vanes or paddles fitted to a spindle which is driven by an engine.

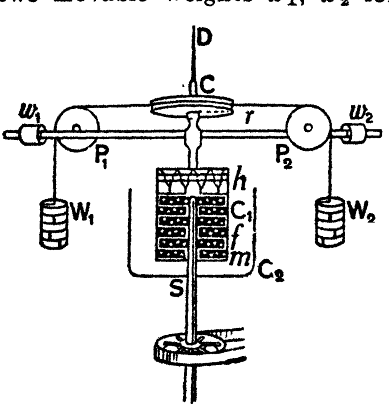


Fig. 57

The calorimeter C_1 is enclosed in another wooden vessel C_2 containing water to reduce the loss of heat due to radiation.

As the spindle S moves, the water in the calorimeter is set in motion by the moving vanes but owing to the viscosity of water, the calorimeter C_1 will tend to rotate in the same direction as the moving vanes, and to prevent this an opposite couple on the torsion wire is exerted through the disc by the weights W_1 and W_2.

Before the calorimeter becomes stationary, the calorimeter would have been rotated through a certain angle due to the rotation of the vanes and the torsion wire twisted. The couple c exerted by the suspending wire is applied as a correction and determined by a separate experiment.

To determine J, we are to know the work done by the rotating vanes against the force tending to oppose their motion.

Let f_1 be the force exerted on a small element of one of the vanes at a distance r_1 from the axis of rotation. Then, in one rotation the distance moved against this resisting force is $2\pi r_1$ and hence, the work done is $2\pi r_1 f_1 n$, where n is the number of

rotations during the experiment. Considering all such elements the total work done $= 2\pi n \Sigma fr$.

The quantity Σfr represents the total couple causing rotation of the vanes. A couple equal and opposite to this tends to rotate the calorimeter. This is again equal to the resisting couple applied by weights W_1, W_2, when the calorimeter is kept stationary.

$$\therefore \quad \Sigma fr = mgd + c,$$

where m = mass attached to each cord, d = diameter of the disc, g = acceleration due to gravity and c = the couple correction due to a twist in the wire which can be found by a separate experiment.

$$\therefore \quad \text{Total work done} = 2\pi n \Sigma fr = 2\pi n(mgd + c).$$

Now, due to the rotation of the spindle and its vanes, water is churned, and thereby becomes heated.

If w be the water equivalent of the calorimeter and its contents, including mass of water taken, t_1 and t_2 its initial and final temperature, then heat generated in water by mechanical agitation $= w(t_2 - t_1)$.

$$\therefore \quad J = \frac{\text{Work done}}{\text{Heat produced}} = \frac{2\pi n(mgd + c)}{w(t_2 - t_1)}$$

Knowing all quantities of the right hand side, the value of J can be found out. The value was found to be $4{\cdot}2 \times 10^7$ ergs/cal.

143a. Points in favour of Rowland's experiment:

1. Joule's mercury thermometer was not compared with an air thermometer and so his readings of temperatures were not accurate. Rowland used a standardised thermometer.

2. In Joule's experiment the rate of rise in temperature was very small *i. e.*, only 0·62°C per hour whereas in Rowland's experiment it was 35°C to 45°C per hour.

3. Joule assumed that the specific heat of water is constant between 0° and 100°C, but it is less than unity between 10°C to 90°C with a minimum value at 40°C. Rowland considered this and got his result.

144. Laboratory Method of finding J: The apparatus devised by Searle consists of a brass cone B (Fig. 58) held firmly in position inside a brass cylinder A by means of non-conducting ebonite block. The cylinder A with the cone B can be rotated by a motor. A second brass cone C fits smoothly into the cone B and is attached to a wooden disc D which has a groove round its circumference. One end of a cord is wound up in the groove and the other end passing over a pulley supports a weight W. When the outer cone is made to revolve rapidly, the inner cone tends to

move due to friction between the two cones, but is held stationary by suspending suitable weight W. The friction between the cones during rotation of the outer cone imparts heat to the water contained in the cone C, the rise of temperature being noted by means of a thermometer inserted in the cone C.

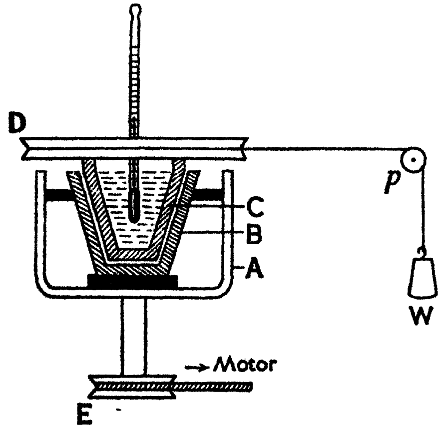


Fig. 58

For equilibrium of the weight W, the frictional couple is balanced by the turning moment exerted by it. If r be the radius of the disc, m the mass of the suspending weight, g the acceleration due to gravity, then frictional couple $= mgr$.

If n be the number of revolutions of the outer cone during the experiment, total work done is given by $W = 2\pi n.mgr$.

If M denote the mass of water taken including water-equivalent of the cone and θ the rise of temperature, then heat developed $H = M\theta$.

$$J = \frac{W}{H} \cdot \cdot \frac{2\pi n.mgr}{M\theta}$$ whence J can be found out.

145. J in different Practical units : $J = 778$ ft. lbs. per lb. degree Fah. $= 778 \times 9/5 = 1400$ ft. lbs. per lb. degree Cent. $= 4{\cdot}2 \times 10^7$ ergs per cal. $=$ **4·2 joules per cal.** (1 joule $= 10^7$ ergs).

146. Calculation of J from the difference between the sp. heats of a gas : We know, $C_p - C_v = \frac{R}{J}$ or $J = \frac{R}{C_p - C_v}$ ...(1)

If in exp. (1) R is the gas constant for the gram-molecular mass, C_p and C_v are really the molecular heats (specific heat × mol. *wt.*) at constant pressure and at constant volume. For air $C_p = {\cdot}2375$, $C_v = {\cdot}1690$, $C_p - C_v = {\cdot}0685$, density of air at N.T.P. $= {\cdot}001293$ gm/c.c.

But $R = \frac{p_0 \cdot v_0}{T_0} = \frac{p_0}{\rho_0 T_0} = \frac{76 \times 13{\cdot}6 \times 981}{{\cdot}001293 \times 273}$ $\therefore J = \frac{76 \times 13{\cdot}6 \times 981}{{\cdot}001293 \times 273 \times {\cdot}0685}$

$= 4{\cdot}16 \times 10^7$ ergs per calorie.

147. Electrical method for J by Callender and Barnes : The method consists in heating up some water by passing an electric current through a wire immersed in it.

The apparatus consists of a thin platinum wire R attached at each end to a thick copper tube (Fig. 59) and mounted inside a narrow glass tube through which a steady stream of water passes, flowing in through A and out through B. The terminals C_1, C_2 and B_1, B_2 are attached to the copper tubes, the former being

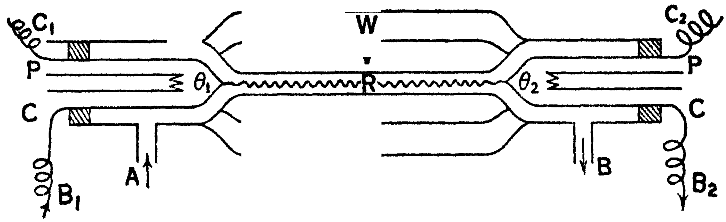


Fig. 59

connected to the source of heating current and the latter to external circuit, for the measurement of P. D. across R and current through it. The potential difference E is obtained by means of a potentiometer with the help of a standard cell. The current I is measured by noting the potential drop across a standard resistance included in the same circuit.

To reduce the external loss of heat the flow tube is surrounded by a vacuum jacket V which is also surrounded by a water jacket W maintained at a constant temperature for ensuring also that the rate of loss of heat due to radiation is constant.

The temperatures of inflowing and outflowing water at A and B respectively are measured by platinum resistance thermometers P, P when the difference of temperature is about 4°C.

The electric current is made to pass through the wire and a flow of water is maintained in the narrow tube. When a steady state is reached, the inflow and outflow temperatures θ_1 and θ_2 are constant.

Let m grams of water flowing through the apparatus in t seconds be collected in a beaker and let $\theta_2 - \theta_1$, be the rise in temperature and h cal. be the heat lost by radiation in t secs.

Then the energy dissipated in t secs. is E It joules, where E is the potential drop across the platinum wire R, and I the current passing. Then we have, $\frac{EIt}{J} = ms(\theta_2 - \theta_1) + h$ (1)

where s is the average sp. heat of water between θ_1 and θ_2.

The heat loss h is eliminated by another experiment by suitably adjusting the electric current so as to secure the same rise of

temperature for different rates of flow at the same time of observation.

For the second experiment, we have $\frac{E_1 I_1 t}{J} = m_1 s(\theta_2 - \theta_1) + h$ ...(2)

Then from (1) and (2) h is eliminated and

$$J = \frac{(EI - E_1 I_1)t}{(m - m_1)s.(\theta_2 - \theta_1)} \text{ joules per cal.} \qquad (3)$$

From this J can be found out.

Advantages.

(1) The thermal capacity of the calorimeter is eliminated.

(2) As the temperatures are steady, the temperatures can be measured accurately.

(3) The radiation correction is eliminated by two experiments.

This is the most accurate method of determining the value of J, since all quantities in R. H. side of the relation (3) can be accurately measured.

QUESTIONS

1. What is meant by mechanical equivalent of heat?

Describe an accurate method of measuring it. [C. U. 1942, '52]

2. State and fully explain the significance of the first law of Thermodynamics. [C. U. 1935, '39, '43, '47, '58]

Explain its importance in the establishment of the Law of conservation of Energy. [C. U. 1935, '39, '47, '58]

3. Describe what you consider the best method for the determination of the mechanical equivalent of heat, giving reasons for your choice. [C. U. 1929, '44]

4. Describe (*a*) an electrical and (*b*) a mechanical method of finding the relation between heat and energy and discuss the accuracy obtainable by the two methods. [C. U. 1935, '46]

Express the value of J in different practical units. [C. U. 1952, '54]

5. Show that J can be obtained if the difference in the molecular heats at constant pressure and constant volume are known. [C. U. 1944]

EXAMPLES

1. A lead bullet, moving with a velocity of 16240 cm. per second, strikes a fixed target and has its temperature raised from 20°C to 120°C by the collision. Calculate the mechanical equivalent of heat, supposing all the energy of motion to be spent in heating the bullet. [Specific heat of lead = ·0314] [C. U. 1914]

Let J be the mechanical equivalent of heat and m the mass of the bullet. Kinetic energy of the bullet $= \frac{1}{2}mv^2 = \frac{1}{2}m(16240)^2$ ergs, and the amount of heat developed in the bullet in being heated from 20°C to 120°C $= m \times {\cdot}0314 \times (120 - 20)$ cal. $= m \times {\cdot}0314 \times 100$ cal.

$\therefore$ **Mechanical equivalent of heat** $= \frac{\frac{1}{2}m \times (16240)^2}{m \times {\cdot}0314 \times 100} = 4{\cdot}2 \times 10^7$ **ergs nearly.**

2. A ball of mass 2000 grammes falls through a height of 2000 cms. and comes to rest at once, all the energy being converted into heat. Find the amount of heat generated. $J = 4{\cdot}2 \times 10^7$ ergs. [C. U. 1916]

Potential energy of the ball $= mgh = 2000 \times 981 \times 2000$ ergs $= 392{\cdot}4 \times 10^7$ ergs.

the amount of heat developed by stopping the ball $= \dfrac{392{\cdot}4 \times 10}{4{\cdot}2 \times 10^7}$ cal. $= 93{\cdot}43$ calories.

3. A cannon ball, the mass of which is 100 kilogrammes, is projected with a velocity of 500 metres per second. Find in C. G. S. units the amount of heat which would be produced if the ball were suddenly stopped. [C. U. 1918]

100 Kilogrammes $= 100 \times 1000$ grammes ; 500 metres $= 500 \times 100$ cms.

Kinetic energy of the ball $= \frac{1}{2}mv^2 = \frac{1}{2} \times 100 \times 1000 \times (500 \times 100)^2$ ergs.

$\therefore$ the amount of heat developed $= \dfrac{\frac{1}{2} \times 100 \times 1000 \times (500 \times 100)^2}{4{\cdot}2 \times 10^7}$ cal.

$= \dfrac{125 \times 10^{12}}{4{\cdot}2 \times 10^7}$ cal. $= 2{\cdot}98 \times 10^6$ calories nearly.

4. A hammer of mass 1000 grammes strikes an anvil with a velocity of 200 cm. per sec., and is brought to rest there. Find the heat produced at each stroke assuming that the whole of energy is converted into heat. Mechanical equivalent of heat $= 4{\cdot}2 \times 10^7$ ergs. [C. U. 1911] (Ans. ·476 cal.)

5. The mechanical equivalent of heat is $4{\cdot}2 \times 10^7$ C. G. S. units. What is the meaning of the statement ? A bullet of mass 20 grammes moving with a velocity of 2×10^3 cm. per second strikes a target. If the whole of the energy is converted into heat, find the amount of heat developed. [C. U. 1913]
(Ans. ·952 cal.)

6. A hammer drops from a known height on a piece of lead placed on an anvil. The lead piece is then immediately transferred into a calorimeter containing a weighed quantity of water. Find an expression for the observed rise of temperature. [C. U. 1932]

Work done by the hammer of mass m in falling through a height h is equal to mgh and since this work is converted into heat in the lead piece, we have $mgh = JH$ ... (1) where J is the Joule's equivalent and H, the amount of heat developed in both the hammer and the lead piece.

If the lead piece be quickly transferred into the water in the calorimeter, the heat of the lead piece passes into the water and raises its temperature.

Now $H = ms(T - t_1) + m_1 s_1 (T - t_1)$ ... (2) where m and s denote the mass and sp. ht. of the hammer, m_1 and s_1 denote the mass and sp. ht. of the lead piece. T, temperature of the hammer and the lead piece when the hammer falls on the lead piece. t_1, the temperature of the water in the calorimeter.

Now since the lead piece is transferred into the water in the calorimeter, we have

$m_1 s_1 (T - t_2) = (M + W)(t_2 - t_1)$ where t_2 = the final temp. of the calorimeter.

M = mass of water and W = water equivalent of the calorimeter.

$\therefore$ the rise of temperature $(t_2 - t_1) = \dfrac{m_1 s_1 (T - t_2)}{M + W}$

From (1) and (2) we have $\dfrac{mgh}{J} = ms(T - t_1) + m_1 s_1 (T - t_1)$

or $\quad m_1 s_1 (T-t_1) = \dfrac{mgh}{J} - ms(T-t_1)$

$$m_1 s_1 = \frac{1}{(T-t_1)}\left\{\frac{mgh}{J} - ms(T-t_1)\right\}$$

$$\therefore \quad (t_2 - t_1) = \frac{\dfrac{(T-t_2)}{T-t_1}\left\{\dfrac{mgh}{J} - ms(T-t_1)\right\}}{M+W}$$

7. A ball of lead strikes a target with a velocity of 400 metres per sec. and stops. The heat produced is equally divided between the ball and the target. Calculate the final temperature of the ball assuming it to be originally at 22°C. ($J = 4{\cdot}2 \times 10^7$ ergs-cal. ; Sp. heat of lead = ·032). [D. U. 1946.]

J. H. = J. M. S.$(t_2 - t_1) = \frac{1}{2}(\frac{1}{2}mv^2) = \frac{1}{4}mv^2$

$$\therefore \quad (t_2 - t_1) = \frac{v^2}{4\ J.S.} = \frac{(400 \times 100)^2}{4 \times {\cdot}032 \times 4{\cdot}2 \times 10^7} = 297^\circ{\cdot}7C$$

$(t_2 - 22^\circ) = 297{\cdot}7 \quad \therefore \quad t_2 = 319^\circ{\cdot}7C \quad$ Here, t_2 = final temp.

8. From what height must a block of ice be dropped so that a tenth part of it would be melted by the heat produced, on the assumption that the whole of the kinetic energy is converted into heat.

(Given $J = 4{\cdot}2 \times 10^7$ ergs-cal, $L = 80$ cal.)

$$mgh = J\frac{m}{10}.\ 80 \qquad = \frac{J \times 8}{980} = \frac{4{\cdot}2 \times 10^7 \times 8}{980} = 342900 \text{ cms. nearly.}$$

9. Find the mechanical energy given out when one horse-power is maintained for an hour. Find the heat equivalent of this energy. State the result in *lb.-deg.-Cent.*, *lb.-deg.-Fah.* and in *Calories.*

1 H.P. = 33000 ft. lb. per minute = 33000 × 60 ft. lb. = 1980000 ft. lb. per hour.

Heat equivalent in different units :—

Heat equivalent in *lb.-deg.-cent.* $= \dfrac{1980000}{1400} = 1414$ *lb.-deg.-cent.*.

$\because$ J = 1400 *ft. lb. per degree cent.*

Heat equivalent in *lb.-deg.-fah.* $= \dfrac{1980000}{778} = 2545$ *lb.-deg.-fah.*,

$\because$ J = 778 *ft.-lb.-deg.-fah.*

Heat equivalent in *Calories* $= \dfrac{1980000 \times 1{\cdot}36 \times 10^7}{4{\cdot}2 \times 10^7} = 641200$ cal.

$\because$ $J = 4{\cdot}2 \times 10^7$ ergs., and 1 ft. lb. $= 1{\cdot}36 \times 10^7$ ergs.

10. In a certain steam plant the engine gives one horse-power for an hour for each $1\frac{1}{2}$ lbs. of coal fed into the boiler furnace. The coal has heating value of 8000 lb. deg. cent. units per pound. What percentage of the heat energy of the coal is converted into mechanical work?

Energy derived from 1 H. P. = 33000 ft. lb. per min.

$$= \frac{33000 \times 60}{J} \text{ per hour} = \frac{33000 \times 60}{1400} \text{ per hour}$$

= 1414 lb. deg. cent. per hour since J = 1400 ft. lb. per deg. cent.

Energy given out by coal per hour $= 8000 \times 3/2 = 12000$ lb. deg. cent.

$\therefore$ Required percentage $= \dfrac{1414 \times 100}{12000} = 11{\cdot}78$

Thus 11·78 per cent of the energy in coal had been converted into mechanical work and the rest wasted in various ways.

11. An engine working at 600 horse-power keeps a train moving at constant velocity on a level ground for 5 minutes. Calculate the amount of heat in calories produced, if all the missing energy is converted into heat.
(1 H. P. = 746 watts ; 1 Joule = 10^7 ergs ; J = $4{\cdot}2 \times 10^7$ ergs per cal.) [C. U. 1939]

We know that W = JH $\quad \therefore \quad H = \dfrac{W}{J} = \dfrac{600 \times 746 \times 5 \times 60 \times 10^7}{4{\cdot}2 \times 10^7}$

$= 319{\cdot}8 \times 10^5$ calories.

12. If an engine working 622·4 horse-power keeps a train at constant speed on the level for 5 minutes, how much heat is produced assuming that all the missing energy is converted into heat?

1 H. P. = 33000 ft. lb. per min. ; J = 788 ft. lb. per degree F.

We know that W = JH. $\therefore \quad H = \dfrac{W}{J} = \dfrac{622{\cdot}4 \times 33000 \times 5}{778} = 132000$ B. T. U.

[1 B.T.U. $= 5/9 \times 453{\cdot}6 = 252$ cal., since $1°F = 5/9°C$ and 1 lb. = 453·6 gms.]

13. A waterfall whose vertical height is 100 metres, discharges 50 litres of water per second, the water falling freely into a pool below the fall. Calculate (a) the quantity of heat produced per second, (b) the rise in temperature of water, assuming all the heat generated remains in water. [C. U. 1952]

We have, gain in Kinetic Energy = Loss in Potential Energy

$= mgh = 5 \times 10^4 \times 981 \times 10^4$ Ergs.

$\therefore$ Work $W = 5 \times 10^4 \times 981 \times 10^4$ Ergs.

(a) Heat produced $H = \dfrac{W}{J} = \dfrac{5 \times 10^4 \times 981 \times 10^4}{4{\cdot}2 \times 10^7} = 11680$ cals.

(b) $m.s.\theta = H$ where θ = rise of temperature

$\therefore \quad \theta = \dfrac{H}{m.s} = \dfrac{11680}{5 \times 10^4} = {\cdot}233°C.$

14. A reversible engine absorbs heat at 160°C and discharges waste products at 100°C and performs work at the rate of 55×10^8 ergs per sec. If one gram of coal can produce $7{\cdot}7 \times 10^2$ ergs by combustion, calculate how much coal must be consumed as fuel per hour, assuming that 60% of the heat of combustion is usefully employed. [D. U. 1948]

Efficiency $= \dfrac{T_1 - T_2}{T_1} = \dfrac{60}{433} = {\cdot}14 \quad \because \quad T_1 = 160 + 273 = 433°A$

55×10^8 ergs per sec. = energy produced

$\dfrac{55 \times 10^8 \times 100}{14}$ per sec. = energy supplied

or $\dfrac{55 \times 10^{10}}{14}$ per sec. = energy supplied.

Let x gms. be the amount of coal consumed per hour. Of this $\cdot 6x$ is used for producing heat of combustion.

$$\frac{\cdot 6x}{60\times 60} \text{ gms. per sec. supplies energy} \quad \therefore \quad \frac{\cdot 6x}{60\times 60}\times 7\cdot 7\times 10^{3} = \frac{55\times 10^{6}}{14}$$

or $$x = \frac{60\times 60\times 55\times 10^{6}}{14\times \cdot 6\times 7\cdot 7\times 10^{3}} = \frac{300}{98}\times 10^{6} = 3\cdot 061\times 10^{6} \text{ gms.}$$

CHAPTER XV

SECOND LAW OF THERMODYNAMICS : CARNOT'S CYCLES : HEAT ENGINES

148. Two Laws of Thermodynamics : The first law of thermodynamics, we have already seen, states that heat applied to a system is equal to the sum of the increase in internal energy of the system and the work done by the system. The law is clearly a statement of the principle of the conservation of heat energy as applied to heat energy. The second law of thermodynamics deals with the condition and possibility of those energy transformations.

149. The Second Law of Thermodynamics : A simple statement of the second law of thermodynamics was made by Rudolf Clausius in the middle of the last century, which runs as :—"*Heat will not of its own accord pass from a cooler to a hotter body.*" Heat therefore behaves with respect to temperature just as masses do with respect to height. Masses cannot of their own accord roll up a hill, but a mass can be made to go uphill by external aid. Similarly by an electric motor heat can be removed from a cold refrigerator to warmer surroundings or outside. The Second law of thermodynamics may be stated generally as :—"*It is impossible for a self-acting machine unaided by external agency to convey heat from a body at a low temperature to one at a higher temperature.*"

150. Reversible and Irreversible processes : Different energy transformations can be classified into two general kinds, reversible and irreversible. A reversible process may be defined as a process which can be performed in either of opposite directions, so that all changes occuring in any part of the direct process are reversed exactly in the corresponding parts of the reverse process. A reversible process is a continuous sequence of equilibrium states.

As for example consider a system consisting of some liquid and its vapour at temperature t. Let the pressure in the system be

P and the vapour pressure of the liquid P′. If P′ be greater than P then evaporation will occur as a natural process. If P′ is less than P the evaporation will not take place. The contrary process namely condensation being then a natural process will occur. If P′ be equal to P, then both condensation of the vapour and evaporation of liquid become reversible processes, since by an infinitesimal increase or decrease of P each process can be made to occur in turn.

151. Carnot's Engine : The Second Law of Thermodynamics governs the conversion of heat into work by considering an ideally simple heat engine, which is free from all imperfections of actual engines. The engine is called the **Carnot's Engine**.

This engine consists of a cylinder C fitted with a non-conducting frictionless piston enclosing a substance called working substance. The walls of the cylinder are perfectly non-conducting and the bottom is a perfect conductor. Three stands are used. (Fig. 60)

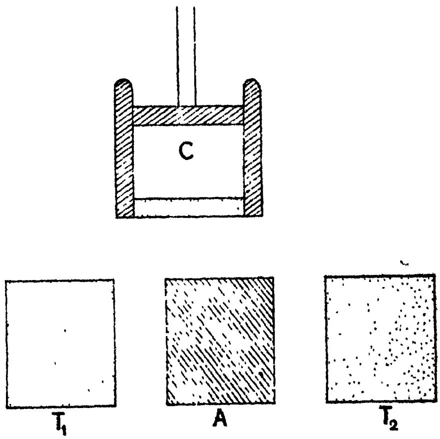


Fig. 60

These consist of (1) a perfectly conducting stand maintained at constant high temperature T_1 called the *source*, (2) a perfectly non-conducting stand A and (3) a perfectly conducting stand at low temperature T_2, called *sink*.

The working substance is taken through a cycle of operations, known as **Carnot's cycle**.

A cycle in which the working substance starting from a given condition of temperature, pressure and volume is made to undergo two successive expansions (one isothermal and another adiabatic), and then two successive compressions (one isothermal and another adiabatic), and then brought back finally to its initial conditions, is called **Carnot's cycle**.

In Carnot's ideal cycle of operation the engine takes the heat at a constant temperature T_1 and discharges it at a constant temperature T_2 and there is no exchange of heat at any other intermediate

temperature. This cycle is a reversible one and the engine is supposed to be such as to enable *isothermal* and *adiabatic* operations to be realised.

151a. Different Operations in Carnot's Cycle : The cycle is bounded by two isothermals DA and CB and two adiabatic AB and DC and the work done by the engine is represented by the area ABCD (Fig. 61).

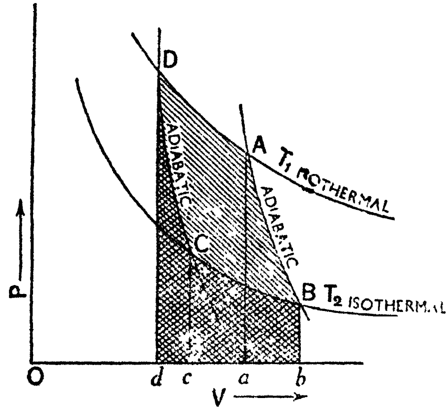


Fig. 61

Starting at the condition represented by the point D the cylinder is placed on the stand at temperature T_1 and the working substance is caused to **expand isothermally** along DA until the volume of the substance has reached the point A. The amount of work done W_1 during this expansion is measured by the area D*Aad*, D*d* and A*a* being drawn normal to the V-axis. The cylinder is removed to the non-conducting stand A. An **adiabatic expansion** along AB is now performed until the working substance has reached the point B at lower temperature T_2. The work done W_2 during this adiabatic expansion is measured by the area AB*ba*, B*b* being drawn normal to the V-axis. The cylinder is then removed to the stand at temperature T_2 and the working substance is made to undergo an **isothermal compression** along BC till it reaches the point C. The work done W_3 during this isothermal compression on the gas is measured by the area B*bc*C, C*c* being drawn normal to the V-axis. The cylinder is then put back on the stand A and the cycle is completed by an **adiabatic compression** along CD where the working substance has reached the point D at high temperature T_1. The work W_4 done during this adiabatic compression on the gas is measured by the area C*cd*D.

The net work done in the cycle $= W_1 + W_2 - W_3 - W_4 =$ area D*Aad* + area A*ab*B − area B*bc*C − area C*cd*D = the area ABCD.

The area enclosed by ABCD is shaded from left to right and other parts *cross-hatched*. The diagram DABC is the **indicator diagram** of the Carnot's engine.

Discussions :—The cycle is completely *reversible*. A reversible cycle is one which consists entirely of reversible changes and the

cycle as a whole can therefore be performed in the opposite direction.

As no work is done at any stage in overcoming friction and no heat is lost to the surroundings, the cycle is completely reversible.

If the Carnot's cycle be reversed the hot body receives a quantity of heat (Q_1) and the cold body loses a quantity of heat (Q_2).

Thus ($Q_1 - Q_2$) represents the heat equivalent of external work done on the working substance.

When working in the reversed direction the engine may be called a **heat pump**.

151b. Work done in different parts of the cycle: The Carnot's cycle is bounded by two isothermals DA and CB and two adiabatics AB and DC. (Fig. 61)

Let us suppose that the state of the gas at start is represented by the point D of the diagram and is taken through the following operations.

(1) Let the temperature of the gas at D be T_1 and let it be allowed to expand isothermally along DA from D to A, the pressure and volume at D and A being p_1, v_1 and p_2, v_2 respectively.

Let Q_1 be the amount of heat absorbed by the gas from the hot body at temperature T_1 in this process.

Then the work done by the gas in this expansion is given by

$$W_1 = Q_1 = \int_{v_1}^{v_2} p\,dv \text{ ; But in isothermal change } pv = k,$$

$$\text{or} \quad p = \frac{k}{v}\text{ ;} \quad \therefore \quad W_1 = k\int_{v_1}^{v_2} \frac{dv}{v} = k\Big(\log v\Big)_{v_1}^{v_2}$$

$$\therefore \quad W_1 = p_1v_1 \log\frac{v_2}{v_1} = p_2v_2 \log\frac{v_2}{v_1} = RT_1 \log\frac{v_2}{v_1}$$

$$[\because p_1v_1 = p_2\,v_2 = RT_1 = k]$$

$$= \text{Area } DdaA$$

(2) The work done in adiabatic expansion from the point A corresponding to pressure p_2, volume v_2 and temperature T_1 to the point B corresponding to pressure p_3, volume v_3 and the lower temperature T_2, is given by

$$W_2 = Q_1 = \int_{v_2}^{v_3} p\,dv \text{ ; But here } pv^\gamma = K = p_2v_2^\gamma = p_3v_3^\gamma$$

$$W_2 = K\int_{v_2}^{v_3} v^{-\gamma}\,.\,dv = K\left(\frac{v^{-\gamma+1}}{1-\gamma}\right)_{v_2}^{v_3}$$

$$= \frac{K v_3^{-\gamma+1} - k v_2^{-\gamma+1}}{1-\gamma} = \frac{p_3 v_3^{\gamma}\,.\,v_3^{-\gamma+1} - p_2 v_2^{\gamma}\,.\,v_2^{-\gamma+1}}{1-\gamma}$$

$$= \frac{p_3 v_3 - p_2 v_2}{1-\gamma} = \frac{p_2 v_2 - p_3 v_3}{\gamma - 1} = \frac{RT_1 - RT_2}{\gamma - 1}$$

$$[\because\ p_2 v_2 = RT_1\ ;\ p_3 v_3 = RT_2\ ]$$

= Area $AabB$.

(3) The work done on the gas in isothermal compression along BC at uniform temperature T_2, the point C corresponding to pressure p_4 and volume v_4, is given as in case (1) above, by

$$W_3 = Q_2 = -\int_B^C p dv = -k\int_{v_3}^{v_4} \frac{dv}{v} = p_3 v_3 \log\frac{v_3}{v_4}$$

$$= p_4 v_4 \log\frac{v_3}{v_4} = RT_2 \log\frac{v_3}{v_4} = \text{Area } BbcC.$$

(4) The work done on the gas in adiabatic compression from C to D is given as in case (2) above, by

$$W_4 = \int_C^D p dv = k\int_{v_4}^{v_1} v^{-\gamma}\, dv = \frac{p_4 v_4 - p_1 v_1}{\gamma - 1} = \frac{R(T_1 - T_2)}{\gamma - 1}$$

= Area $CcdD$. It is seen that $W_2 = W_4$ (numerically).

Therefore the external work done by the gas is given by

$$W = W_1 + W_2 - W_3 - W_4 = W_1 - W_3 = Q_1 - Q_2$$

The net amount of external work is equal also to the area ABCD of the cycle.

Note: Since A and B lie on the same adiabatic we have

$$T_1 v_2^{\gamma-1} = T_2 v_3^{\gamma-1}$$

$$\therefore\ \frac{v_3}{v_2} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} \quad \text{Similarly, } \frac{v_4}{v_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}}$$

$$\therefore\ \frac{v_3}{v_2} = \frac{v_4}{v_1} \quad \text{Or} \quad \frac{v_2}{v_1} = \frac{v_3}{v_4} = \gamma \text{ (isothermal expansion ratio)}$$

We have therefore $Q_1 = RT_1 \log \gamma$, $Q_2 = RT_2 \log \gamma$
and $W = Q_1 - Q_2 = R(T_1 - T_2) \log \gamma$

Hence, $\frac{Q_1}{T_1}=\frac{Q_2}{T_2}=\frac{Q_1-Q_2}{T_1-T_2}=\frac{W}{T_1-T}$ Or $W=Q_1\left(\frac{T_1-T_2}{T_1}\right)$

Efficiency η of the heat engine is given by

$$\eta=\frac{W}{Q_1}=\frac{Q_1-Q_2}{Q_1}=\frac{T_1-T_2}{T_1}=1-\frac{T_2}{T_1}$$

152. Efficiency of a reversible heat engine : The efficiency of a heat engine η is the ratio of the mechanical work done to the heat supplied, both expressed in the thermal units.

If Q_1 and Q_2 be respectively the amounts of heat taken in at temperature T_1 and given out at temperature T_2 by the engine, then Q_1-Q_2 represents the mechanical work done.

Thus $\eta=\frac{Q_1-Q_2}{Q_1}=\frac{T_1-T_2}{T_1}=\frac{W}{JQ_1}$

Here W is the mechanical work done.

The efficiency is greater, the greater is the temperature difference between the hot and cold bodies. If the two bodies are at the same temperature no external work can be obtained.

In Carnot's engine the heat absorbed is taken in at one constant temperature and all the heat rejected to the sink is given out at another constant temperature.

In many actual heat engines, all the heat is not absorbed at the temperature of the hot body, neither all the heat given out at the temperature of the cold body. During the absorption of heat the working substance gets progressively colder since it is doing work faster than it is absorbing heat. The same is true during the rejection of heat to the cold body.

Such an engine is less efficient than Carnot's engine, since it does not utilise the available temperature difference.

Thus it may be said that of all reversible cycles **Carnot's cycle has the greatest efficiency.**

In all reversible engines working on a Carnot's cycle between two given temperatures the efficiency has the same value and is entirely independent of the nature of the working substance.

153. Entropy : The term *entropy* is applied to that thermal property of a substance which remains constant as long as heat is not communicated to or abstracted from it by external bodies.

The increase in entropy is measured by $\frac{Q}{T}$, where Q is the heat absorbed and T is the absolute temperature during the absorption.

The adiabatic lines of any substance are lines of constant entropy.

In any closed reversible cycle

$$\int d\left(\frac{Q}{T}\right)=0 \quad i.e.\ \frac{Q_1}{T_1}-\frac{Q_2}{T_2}=0 \quad \text{or} \quad \frac{Q_1}{T_1}=\frac{Q_2}{T_2}$$

154. Capacity: It is the rate at which the work is done by an engine.

The unit for the measurement of capacity is called a **horse-power** (H. P.).

An engine is said to possess one horse-power if it does 550 *ft-lbs.* of work per second or 33000 *ft-lbs.* of work per minute.

155. Brake horse-power: It is the power which the engine is capable of giving out in the useful form of driving machinery.

Since it is obtained by means of a brake it is called *brake horse-power.*

It is less than the indicated horse-power which is really a measure of the power developed on the piston of the engine.

156. Indicator Diagram: This is a diagram or a curve which represents the work done by a force while its point of application moves in the direction of the force through a certain distance.

In this diagram, the abscissae represents the space in the direction of the line of action of the force and distances measured parallel to the ordinate represent the magnitude (not direction) of the force.

If under the action of the force F the body is displaced through a distance s the work done Fs is represented by the area of the rectangle formed due to the displacement.

If the force is not constant during displacement, the area of the curve contained between any two ordinates, the curve and the abscissae will represent the work done during displacement.

This graphical method of representing the work done by a variable force is used to determine the work done by steam during each stroke of a steam-engine piston.

The curve is mechanically drawn by means of an instrument called an *indicator.* So the curve is called an indicator diagram.

We have seen that the external work done by a gas when it expands from initial condition, p_1, v_1 to final condition p_2, v_2, of pressures and volumes, is expressed as

$$\int_{v_1}^{v_2} pdv \text{ ergs.}$$

whether the expansion takes place under simple conditions or not.

If the changes of pressure and volume of the gas are represented graphically, the curve will be of the types shewn in Figure 62.

Let the pressure at some point such as A on the curve be p and let the volume of the gas increase by dv to the point B. Then the area of the strip ABCD is pdv and represents the external work done by the gas during the expansion dv.

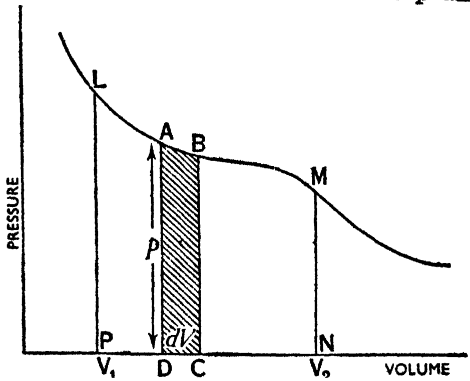


Fig. 62

The whole area LMNP between the curve and the axis of V from V_1 to V_2 represents the external work done by the gas in expanding from V_1 to V_2. Such a diagram is called an **indicator diagram.**

157. Applications of Carnot's Cycle :

(a). Effect of change of pressure on the boiling point of a liquid : Clausius—Clapeyron's Equation :

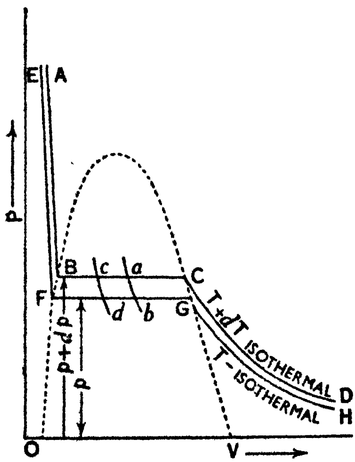


Fig. 63

Let the curves ABCD Fig. 63 and EFGH represent two isothermals of a substance at two temperatures (below its critical temperature) $T+dT$ and T respectively.

At B and F the substance is in the liquid condition while at C and G it is entirely vapour. Along the lines BC and FG the substance exists partly in the state of liquid and partly in the state of vapour.

Hence T and $T+dT$ are the boiling points of the substance under pressure p and $p+dp$ respectively.

Let us start at B and take the unit mass of the liquid round the reversible cycle. Draw two *adiabatics cd* and *ab*.

The heat absorbed along *ca* at $T+dT$ is the latent heat $L+dL$ and the heat given out along *bd* at T is the latent heat L

$$\therefore \frac{L+dL}{T+dT}=\frac{L}{T} \quad \text{or} \quad TdL=LdT$$

The work done in the cycle $cabd = L+dL-L=dL=\dfrac{LdT}{T}$

The external work done is given by the area of the strip *cabd* and is equal to $ca \times$ perpendicular distance between the horizontal portions of the curves $= ca \times dp$ $=(v_2-v_1)\times dp$, where v_2 and v_1 are the volumes of unit mass (*i.e.* specific volumes) of vapour and liquid respectively.

$$\therefore \quad (v_2-v_1)\ dp = dL = \frac{LdT}{T}$$

$$\frac{dp}{dT}=\frac{L.J}{T(v_2-v_1)} \quad \text{(heat units).}$$

This is **Clausius and Clapeyron's Equation.**

(b). Effect of pressure on the boiling point :

The Clapeyron's equation may be written in the form

$$dT=\frac{T\,(v_2-v_1)dp}{L.J}$$

where dT is the depression of the boiling point due to a dimunition of pressure dp. Here v_1, the volume of unit mass of liquid is less than v_2, the volume of unit mass of its vapour.

For water, v_2 = Volume of 1 gm. of steam at 100°C = 1647 c.c.

v_1 = „ „ water at 100°C = 1·04 c.c.

$v_2-v_1=1646$ nearly ; $T=273+100=373$, $dp=1$ mm. of mercury $= \cdot 1\times 13\cdot 6\times 981$ dynes. Again

L = latent heat of steam = 537 Cal.

$\therefore$ the depression of the boiling point of water for a decrease of pressure dp is equal to $dT=\dfrac{373\times 1646\times 13\cdot 6\times \cdot 1\times 981}{537\times 4\cdot 2\times 10^7}=\cdot 0364°C$

(c). Effect of pressure on the melting point :

From Clapeyron's equation, we have $dT=\dfrac{T(v_2-v_1)dp}{L.J}$

In the case of ice which contracts on melting v_1 is greater than v_2 $\quad \therefore \ dT=-\dfrac{T\,(v_1-v_2)dp}{L.J}$

Then the depression of the melting point by an increase of pressure of one atmosphere is calculated as follows.

v_1 = sp. vol. of water at 0°C = 1·000 c.c.
v_2 = „ ice at 0°C = 1·091 c.c.
$dp = 76\times 13\cdot 6\times 981$ dynes
L = 79·6 Cal.

$$\therefore \quad dT=\frac{273\times(1-1\cdot 091)\times 76\times 13\cdot 5\times 981}{79\cdot 6\times 4\cdot 2\times 10^7}=-\cdot 0075°C.$$

158. Heat Engines : Any contrivance which has for its object the transformation of heat energy into mechanical work is called a *heat engine.*

158a. Fundamental facts about a heat engine.

The following are the essential parts of a heat engine.

(1) The furnace *i.e.*, a source maintained at a higher temperature and from which heat is absorbed by the working substance.

(2) A cylinder with a piston in which the working substance expands by doing external work.

(3) A working substance such as vapour, air, kerosene gas, petrol vapour etc. This is necessary to convey the heat into and away from the engine.

(4) A condenser at a lower temperature to which heat is given.

(5) External arrangements of *rods, shafts, cranks and fly wheel*.

In a steam engine the working substance is a vapour (steam). Oil and gas engines use combustible mixture of gases and vapour.

In any heat engine the working substance goes through certain changes of pressure, volume and temperature and returns to the initial condition.

The complete series of operations from the beginning to the end *i.e.*, when the working substance returns to the initial condition constitutes the **cycle** of operations.

Work cannot be obtained unless two parts of the engine *e.g.* the furnace and the condenser, are at two different temperatures. The greater the difference of temperature between the two parts, the greater is the fraction of heat that is converted into work (Carnot's theorem).

The heat is obtained by the combustion of some gases (carbon or hydro-carbon) outside or inside the engine.

159. Different types of Heat Engines: Heat engines can be divided mainly into two classes such as *external combustion* and *internal combustion* engines.

In an **external combustion engine**, the fuel is burnt outside the engine and in the **internal combustion engine** it burns inside.

In a reciprocation *steam engine* which belongs to the first type, heat is used to produce steam at a high temperature and pressure in a boiler. The steam presses alternately above and below a piston which is thus moved to and fro within the cylinder. The steam works by expanding and afterwards passes into a condenser (or the atmosphere) at a lower temperature and pressure. A crank converts this *to and fro* motion of the piston into a circular motion.

When steam is produced at a very high pressure (to increase the efficiency) the expansion is carried in a number of cylinders joined in series (double, triple or quadruple expansion engines).

The steam turbine which is also of same type, is a steam-driven rotary engine in which rotary motion is directly obtained from high

pressure steam. Steam enters the turbine at high pressure and work is obtained by passing steam through various arrangements of nozzles and moving blades. The nozzles are attached to the cylinder or the case and the moving blades to the rotor in successive rings. The steam finally passes into the condenser at a very low pressure.

160. Internal Combustion Engine: Internal Combustion engines may be classified into two types, (1) those in which the fuel is gaseous when it enters the cylinder, (2) those in which it is injected as a liquid.

In an *internal combustion engine* the fuel is burnt within the cylinder of the engine. Internal combustion engines generally work in cycles of four strokes.

The ordinary petrol engine which belongs to type (1) draws its heat supply from the combustion of petrol vapour mixed with certain definite proportion of air. It is designed to work on *four-stroke cycle,* known as **Otto Cycle**. In this engine air is the working substance and petrol vapour is simply there as a source of heat which is supplied at the appropriate stage.

161. Petrol Engine: It consists of (Fig. 64) a cylinder C in which a piston P works. Above the cylinder there is a chamber

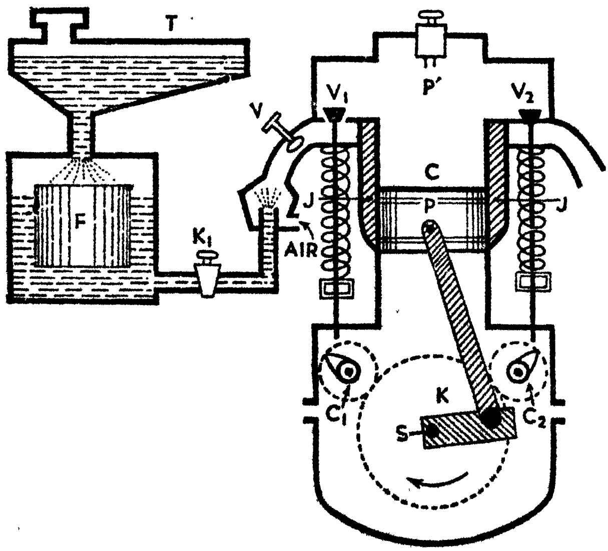


Fig. 64

known as the combustion chamber fitted with a sparking plug P′ and provided with an inlet valve V_1 and exhaust valve V_2. The valves are

placed on their seats by springs and operated respectively by rotating cams C_1 and C_2 fixed on a rotating shaft and driven by the engine.

The cylinder is surrounded by a jacket J through which a continuous circulation of water is maintained for preventing the cylinder from becoming too hot.

Action : The petrol passes from a petrol tank T into a chamber containg a float F and is forced through into the carburator where it is mixed with air and by adjusting the throttle valve V a regulated supply of the mixture forming the fuel is passed into the combustion chamber through the inlet valve V_1 where it is vaporised. In this chamber the mixture is fired or exploded by electric sparks from the spark plug P′, the current being supplied by a device called the magneto. The timing of the spark is regulated.

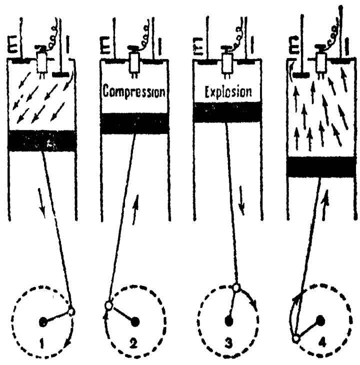


Fig. 65

161a. Cycles of operation : In the **first** or the **suction stroke**, a mixture of air and petrol (hydrocarbon) vapour enters the cylinder through the inlet valve I the exhaust valve E being closed [Fig. 65 (1)].

In the **second** or the **compression stroke**, the mixture is compressed to a small volume and heated by the compression, to about 600°C both the inlet and the outlet valves being closed [Fig. 65 (2)].

At the beginning of the **third** or the **working stroke** the hot compressed **mixture** is fired by an electric spark. A large quantity of gas and vapour is formed at a high temperature and pressure and the piston is pushed out with a great force, both the valves remaining closed [Fig. 65 (3)].

In the **fourth** or the **exhaust stroke**, the outlet or exhaust valve E opens, keeping the inlet valve closed and the piston moves inwards and the waste gases are driven out of the cylinder [Fig. 65 (4)].

161b. Indicator diagram of an Otto Cycle : The entire operation cf a four-stroke cycle of an internal combustion engine is shown in the diagram. (Fig. 66).

First stroke :—An explosive mixture of air and petrol vapour at atmospheric pressure and at a temperature of about 70°C is

drawn by the piston from the curburator along XC through the inlet valve which closes at the point C (Fig. 66).

Second stroke—The piston then moves inwards and compresses the mixture adiabatically along CD to the point D and the temperature rises to about 600°C.

Third stroke—When the mixture had reached the point D, it is ignited by a spark and the fuel is burnt very rapidly, supplying heat very suddenly at constant volume along DA, raising the pressure to the point A and the temperature of the mixture becomes very high.

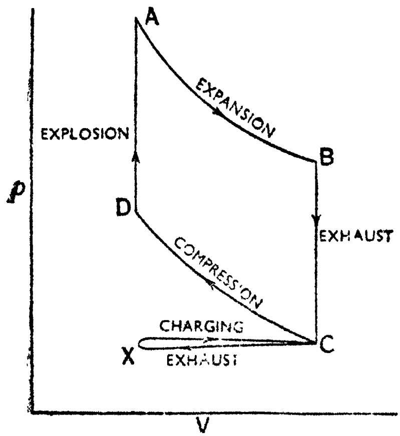


Fig. 66

Adiabatic expansion then takes place along AB till the piston is driven out to the point B at which the exhaust valve opens. The pressure then falls at once to C with the escape of the waste gases to air.

Fourth stroke—The piston then moves inwards along CX expelling the residual waste gases through the exhaust port. Similar cycles are then repeated.

162. Diesel Engine : In the **Diesel** or **Oil Engine** the closed end of the cylinder is strongly heated.

In the *first or the suction stroke*, air is drawn in and then after its compression in second stroke kerosene or some crude oil is injected. The oil vaporises and mixes with the air.

The heat produced by the compression in the second stroke is sufficient to raise the mixture to the ignition point and fire it (no electric spark is required). These engines are called constant volume engines for the combustion of the mixture is so rapid that the piston has no time to move and the volume remains constant during combustion.

In the **third** or **the working stroke,** a jet of oil, generally crude oil is injected in the form of a spray and it burns in hot air. The combustion of the oil spray takes time and the piston moves out under nearly constant pressure.

The fuel supply is then cut off and the heated mixture expands adiabatically, all the valves remaining closed.

In the exhaust stroke the exhaust valve opens with immediate fall of pressure and the burnt gas is forced out.

Note : The **Diesel Engine** is a four stroke constant pressure engine. In the first stroke air is drawn in and in the second stroke air is compressed more highly than in the petrol or oil engine and raised to a very high temperature, say about 600°C.

Note : The Diesel engine has the highest efficiency (about 35%) of all the heat engines and the reciprocating steam engine (without condenser) the lowest (about 5%).

The internal combustion engine is more efficient than a steam engine as the working substance in it can be raised to a much higher temperature.

As it occupies less space it can be used in small power stations.

Uses : It is used in (1) motor cars and lorries, (2) aeroplanes and other airships, (3) in battleships and destroyers.

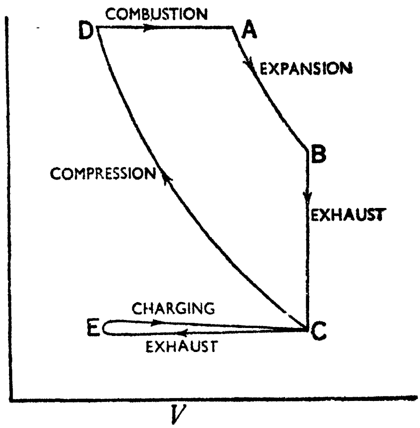


Fig. 67

162a. Indicator Diagram of a Diesel Cycle : The Diesel engine works in four-stroke cycles, known as Diesel cycle.

First stroke—The cylinder is charged with cold air at atmospheric pressure and the state is represented by EC (Fig. 67).

Second stroke—As the piston moves inwards air is compressed adiabatically from C to D and at this point the fuel (heavy oil) is injected.

Third stroke—The piston moves out-wards, the burning fuel is supplying heat to the air at constant pressure from D to A and adiabatic expansion takes place from A to B during the remainder of the working stroke.

Fourth stroke—At B the exhaust valve opens and the pressure drops at once to that of the atmosphere at C. The burnt gases are then expelled as indicated by CE. The operations are then repeated.

163. Steam Engine : The steam engine is a contrivance by means of which heat energy is converted into mechanical work. It consists mainly of two parts, (1) the boiler in which steam is generated and (2) the engine proper in which the alternate expansion and condensation of steam imparts to a piston an alternating rectilinear motion by means of various mechanical arrangements.

The action of a double-acting steam engine may be understood by means of the adjoining diagram.

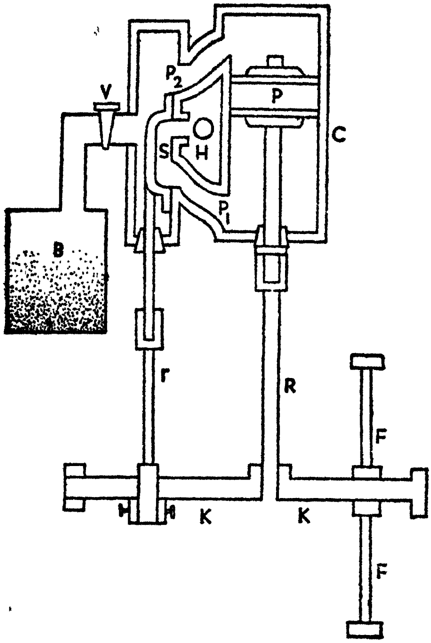


Fig. 68

The steam from the boiler B (Fig. 68) passes through the pipe *j* provided with a valve V into a cast-iron box known as the *valve-chest* fitted on the side of the cylinder C into which the piston P moves from end to end. In the side of the cylinder and inside the valve-chest there are three openings or *ports* P_2, H and P_1 of which P_2 and P_1 communicate with the top and bottom end of the cylinder respectively and allow steam to either enter or come out of it when occasion arises.

The port H permits the discharge of the steam after it has done work on the piston.

The ports P_2 and P_1 are opened and closed at the proper instants by a *slide-valve* S which moves over them by means of an eccentric rod *r* which is attached to the crank-shaft KK.

The piston rod P is connected by a rod R to a crank which is fixed to the revolving crank-shaft KK. A heavy fly wheel FF is usually fixed to the shaft to keep its rotation steady.

Action : The high pressure steam (about 20 atoms. or more) from the boiler passes through the pipe into the valve-chest and enters into the lower part of the cylinder through the port P_1. At that instant the piston P is near the lower port P_1 and the slide-valve S covers the *upper port* P_2 and the *exhaust port* E. [Figs. 68, 69(a)]. The pressure of steam acting on the piston pushes it upward. As the piston moves up the crank-shaft KK connected with the piston rod R rotates causing a movement of the side valve S in the opposite direction by the valve rod.

When the piston during its upward movement reaches the upper port P_2, the slide-valve covers the *lower port* P_1 and the *exhaust*

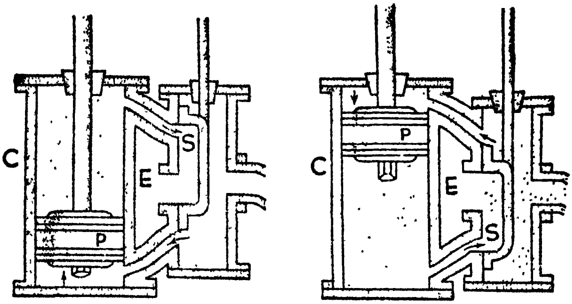


Fig. 69a Fig. 69b

port E [Figs. 68, 69(b)]. The steam now enters through the upper port P_2 and pushes down the piston allowing the exhaust steam to pass through the port E. The cycle of operation is repeated.

Thus by the action of steam the piston is moved forwards and backwards inside the cylinder and this *reciprocating motion* of the piston is converted into *rotational motion* of the crank-shaft through the intervention of the rod and the crank.

It is called a double-acting engine for the steam acts alternately on the two faces of the piston producing the to and fro motion.

164. Efficiency of an engine : It is the ratio of heat transformed into work in a given time to the total heat supplied in the same time.

165. Difference between a Petrol Engine and a Steam Engine : 1. The efficiency of petrol engine is not very high but higher than that of a steam engine.

2. Petrol engine occupies a smaller space than that of a steam engine.

3. Petrol engine can be started very easily but the steam engine requires much time to start.

166. Indicator diagram of an ideal steam engine : Rankine cycle : In this cycle heat is first supplied to the working substance (water and not exclusively steam) at the point A (Fig. 70).

First stage : In this stage, water evaporates to steam at constant temperature T_1 and the steam expands into the cylinder along AB at constant pressure p_1.

Second stage : The steam then expands adiabatically along BC till the temperature T_2 of the sink is reached.

Third stage : The steam then condenses to water at constant temperature T_2 along CD at constant pressure p_2.

Fourth stage : Then there is a vertical jump from D to A along DA and the pressure (saturated vapour pressure at T_2) rises from p_2 to p_1, the saturated vapour pressure at T_1 at constant volume.

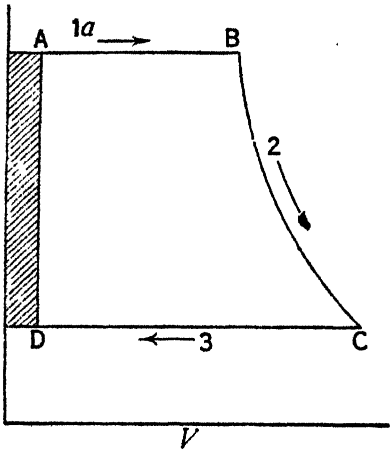


Fig. 70

As no heat is absorbed while water is being raised from T_2 to T_1, it is not identical as the Carnot's cycle.

The area of the figure ABCD represents the work done by the ideal steam engine in each cycle.

In practical engines the actual diagram of work is of smaller area than that of the ideal diagram as the angles at A, B, C and D are not sharp as shewn in Fig. 70. This is due to the fact that the steam cools as it enters the cylinder and the valves do not operate instantaneously.

QUESTIONS

1. Explain what is meant by indicator diagram, break-horse power and reversible cycle and state in general terms the efficiency of a heat engine.

2. Define Carnot's Cycle and shew how the work done in each operation is represented on a pressure volume diagram.

Calculate the work done in each operation of the cycle when the working substance is a perfect gas. [C. U. 1950, '53]

3. Discuss the effect of change of pressure on the boiling point of a liquid shown by Clapeyron. [C. U. 1943]

4. Describe some kind of machine which converts heat into mechanical work and explain, with the aid of diagrams its working and efficiency. What are the practical uses of such machines. [C. U. 1931, '41]

5. Describe some kind of internal combustion engine and explain fully how heat is thereby converted into work. Discuss its advantages over steam engine. [C. U. 1931, '51]

6. Analyse essential parts of a heat engine. [C. U. 1941]

CHAPTER XVI

TRANSMISSION OF HEAT

167. Three modes of heat transference: Heat may be transferred from one place to another place in three ways, namely, **conduction, convection** and **radiation.**

When a metal rod is heated at one end, the other end is found to have become hot sooner or later. The power of transmitting heat in this way is exhibited by all bodies to a more or less extent and the process of such heat transference is called *conduction.* Conduction occurs through the agency of a material medium, but there is no bodily motion of the medium particles in the process.

Thus *conduction* is the mode of transmission of heat in which heat energy travels from particles to particles in the direction of decreasing temperature without any bodily movement of the material particles from their normal positions.

The process of heat transference by convection is very well illustrated by placing a few crystals of magenta gently at the bottom of a beaker containing some water and heating it from below. Heated water at the bottom rises up and colder water at the top goes down. The path of the ascending water particles is rendered visible by the red colour imparted to water. The convection current, as it is called, goes on until the whole of water becomes uniformly heated.

Thus *convection* is the mode of heat transference in which the material particles conveying the heat are carried from one place to another place until the whole mass of the substance becomes uniformly heated.

Transmission of heat either by conduction or convection requires a material medium. But there is yet another mode of transmission of heat in which no material medium is needed. We get heat from the sun on earth, although there is no continuous material medium between the sun and the earth. This mode of transmission of heat is called radiation.

Thus *radiation* is that mode of transmission of heat in which heat energy travels from the source of heat to its recepient without any material medium taking part in it.

Conduction and convection are slow processes owing to the action of the intervening medium, but radiation can travel from place to place with enormously high velocity of light.

168. Variable and Steady state: When heat is applied steadily to one end of a rod, the temperature of the rod begins to rise gradually with the flow of heat, at points situated at different distances from the end which is heated, and also the temperature of any transverse layer is found to rise. This state is known as the *variable state*. After a certain time the temperature of each layer attains a final maximum value and remains stationery at that value so long as the temperature of the heated end remains constant but diminishes gradually in value from the heated end. This state is known as the *steady state*.

In the *variable state* any transverse layer receives heat by conduction from a layer just preceding it, a part of it being absorbed causing a rise of its own temperature, a small portion radiated from its surface and the remainder transferred to the next layer by conduction. So in this stage conduction accompained by absorption is termed *diffusion*.

The rate of diffusion of temperature of a body, say a metal rod depends not only on the conductivity of the material but also on its specific heat since the absorption of heat in the diffusion state depends on the thermal capacity of the substance, per unit volume (sp. heat × density).

169a. Thermal Conductivity: *Thermal conductivity, or coefficient of thermal conductivity of the material of a body is defined as the quantity of heat which would pass in steady state through a slab of the material of unit thickness, unit cross-section in unit-time when the temperatures of the two opposite faces differ by one degree centigrade, the heat flowing normally from one face to the other face.*

Let us consider a slab of the material whose opposite faces are maintained at temperatures θ_1 and θ_2, where θ_1 is greater than θ_2.

Then in the steady state, the total quanity of heat Q flowing through the slab varies directly, (1) as A, the area of the cross-section of the slab, (2) as $\theta_1 - \theta_2$, the temperature difference between the faces (3) as t, the time in second, and varies inversely (4) as d, the thickness of the slab or the distance between the opposite faces.

Then $Q \propto \dfrac{A(\theta_1 - \theta_2)t}{d}$

or $Q = \dfrac{KA(\theta_1 - \theta_2)t}{d}$ ... (1) or $K = \dfrac{Qd}{A(\theta_1 - \theta_2)t}$

where K is the coefficient of thermal conductivity of the material.

It may also be written in the form $Q = -KA.\dfrac{d\theta}{dx}\delta t$.

In this expression the thickness of the slab is dx and the temperature difference between the faces of the slab is $d\theta$.

Temperature gradient is, $-\frac{d\theta}{dx}$. The negative sign indicates that heat, as known by experiments, flow from points of high to points of low temperature or down the temperature gradient. The expression for Q in (1) can be written as, $Q = KAG.t$, where G is the temperature gradient being equal to $(\theta_1 - \theta_2)/d$.

169b. Thermometric Conductivity : Since thermal conductivity K is the quantity of heat which would pass through a slab of material of unit thickness, and of unit cross-section per sec. when the temperatures of the two faces differ by one degree centigrade, this heat would raise the temperature of unit volume of the material through t°C so that the rise of temparature t is given by

$K = \rho.\ s.\ t,$ where s is the sp. heat and ρ the density of the body

$$\therefore\ t = \frac{K}{\rho s} = \frac{\text{Thermal conductivity}}{\text{Thermal capacity per unit volume}} = \textbf{diffusivity}$$

Thus the coefficient of thermal conductivity of a body divided by its thermal capacity per unit volume is called the *diffusivity* or the **thermometric conductivity** of the material.

It may be defined as the rate at which the temperature changes in a substance, in the variable state.

In the *variable state* both thermal capacity and specific heat play important parts but in the *steady state* no more heat is absorbed but the flow of heat depends on the thermal conductivity only.

Thermal conductivity which refers to the rate of flow of heat in steady state, depends on conducting power of a body. Thermometric conductivity which refers to the rate of rise of temperature in variable state, depends on both specific heat and thermal conductivity of a body.

170. Dimensions of K :

(I) We know that $K = \frac{Qd}{A(\theta_1 - \theta_2)t}$ ...(1)

Dimension of Q = H

„ d (thickness) = L

„ A (area) = L^2

„ $(\theta_1 - \theta_2)$ temp. diff. = θ

„ t (time) = T

$\therefore$ Dimensions of K

$$= \frac{HL}{L^2.\theta.T} = HL^{-1}\theta^{-1}T^{-1}$$

This is called **thermal dimensions** of K.

(II) In dynamical system, Q the quantity of heat is measured in energy units, so that dimensions of $Q = ML^2T^{-2}$, the dimensions of other quantities in the right-hand side of (1) remaining same as before,

then **dynamical dimensions** of $K = \dfrac{ML^2T^{-2}.L}{L^2\,.\,\theta\,.\,T} = MLT^{-3}\theta^{-1}$

Note : *Explain* : '*Thermal conductivity of iron* = ·175 C. G. S. units.'

The statement means that when the steady state of temperature gradient has reached, ·175 C. G. S. units of heat (calories) will flow per second through unit area between the faces of a slab of iron 1 cm. apart, the faces differing in temperature by 1°C.

Note : In the steady state, heat is not absorbed by any layer but a part of it is radiated from its surface, the rest being transferred to the next layer by *conduction.*

So to determine the thermal conductivity of a metal the substance is heated till the steady state is attained.

A substance may have a large thermometric conductivity but small *thermal conductivity* and *vice versa.* The thermometric conductivity of Bismuth is greater than that of Iron, though Iron is a better conductor. This happens because Bismuth has a low thermal capacity.

171. Mathematical Investigation of the flow of heat :

Consider a long metal Fig. 71 rod of uniform cross-sectional area A heated at one end. Let us consider two layers A and B at distances x and $x+dx$ from the hot end having temperature

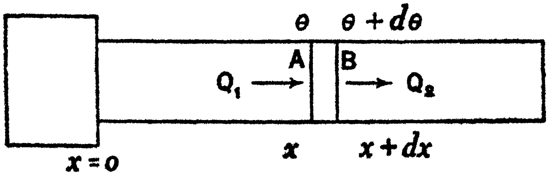


Fig. 71

excesses θ and $\theta + d\theta$ (over that of surroundings) respectively. Let these layers be perpendicular to the length of the rod which is also the direction of the axis of x.

Then the quantity of heat which flows into the surface A per second is equal to $-KA.\dfrac{d\theta}{dx}$.

The negative sign indicates that the temperature falls as we move away from the heated end where $x = 0$.

Again, the quantity of heat which flows out of the surface B is equal to $-KA\dfrac{d}{dx}\left(\theta+\dfrac{d\theta}{dx}.dx\right)$

Therefore the gain of heat by the layer AB

$$=-KA\frac{d\theta}{dx}-\left[-KA.\frac{d}{dx}\left(\theta+\frac{d\theta}{dx}.dx\right)\right]=KA.\frac{d^2\theta}{dx^2}dx$$

In the *variable state*, any transverse layer receives heat by conduction from a layer just preceding it, a part being **absorbed** causing a rise of its own temperature, a small portion **radiated** from its own surface and the remainder transferred to the next layer by **conduction.**

The amount of heat absorbed by the slice of thickness dx raises its temperature and is equal to $\rho A.dx.S.\dfrac{d\theta}{dt}$

where ρ is the density and S, the sp. heat of the material, and $\dfrac{d\theta}{dt}$ is rate of rise of temperature.

The amount of heat lost by radiation from the surface of the slice is equal to

$$Ep\theta.dx.$$

where E is the emissive power of the surface, p its perimeter and θ, the excess of temperature of the surface over that of the surroundings.

Then we have $KA.\dfrac{d^2\theta}{dx^2}.dx=\rho A\ dx.S.\dfrac{d\theta}{dt}+Ep\theta.dx$

$$\text{or}\quad \frac{d\theta}{dt}=\frac{K}{\rho S}.\frac{d^2\theta}{dx^2}-\frac{Ep}{\rho AS.}\theta$$

In the *steady state* when the temperature at every point of the rod is stationary, $\dfrac{d\theta}{dt}=0$

$$\therefore\quad \frac{K}{\rho.S}\frac{d^2\theta}{dx^2}=\frac{Ep\theta}{\rho.A.S};$$

$$\text{or}\quad \frac{d^2\theta}{dx^2}=\frac{Ep.\theta}{KA}=\mu^2\theta,\ \text{where}\ \mu^2=\frac{Ep}{KA}$$

This is a differential equation, the solution of which must involve two arbitrary constants.

Let the required solution be $\theta=ae^{\mu x}+be^{-\mu x}$

where a and b are arbitrary constants to be and determined by the conditions of the experiment.

Suppose that the rod is infinitely long. Then, when $x=0$, $e^{\mu x}=1$ and $e^{-\mu x}=1$, while θ θ_0, θ_0 being the temperature excess at the heated end. So $\theta_0=a+b$

Again if $\theta=0$ when $x=\infty$, the right-hand side of the equation must be infinite unless a is zero. Thus $a=0$.

So $\theta_0=b$ and the solution of equn. is $\theta=\theta_0 e^{-\mu x}$

172. Comparison of thermal Conductivities : Ingen-Hausz's Method :

Theory : Suppose that the rods whose thermal conductivities are to be compared, are extremely long compared with their diameters. Let the rods coated with wax, be heated at adjacent ends at a constant temperature and that the temperatures at their other ends are the same as that of the surroundings. Then the equation for the temperature excess at distance x from the hot end for each is $\theta m=\theta_0 e^{-\mu x}$, where θ_0 is the temperature of the hot bath. θm the temperature of the melting point of wax ; again μ $\sqrt{\frac{Ep}{KA}}$

Here E is the emissive power of the surface, p the perimeter of the rod, K the conductivity, and A the area of the section of the rod. Let l_1, l_2 and l_3 be the lengths of the three rods (a, b, c) from hot end, along which wax melts, then,

For the first rod θm $e^{-\mu_1 l_1}$ (1)

„ second „ $\theta m = \theta_0 e^{-\mu_2 l_2}$ (2)

third „ $\theta m=\theta_0 e^{-\mu_3 l_3}$ (3)

From (1), (2) and (3) $\mu_1 l_1=\mu_2 l_2=\mu_3 l_3$

Since E, p and A are made the same for all the rods.

$$\therefore \sqrt{\frac{Ep}{K_1 A}}.l_1=\sqrt{\frac{Ep}{K_2 A}}.l_2=\sqrt{\frac{Ep}{K_3 A}}.l_3 \text{ or } \frac{l_1}{\sqrt{K_1}}=\frac{l_2}{\sqrt{K_2}}, \frac{l_3}{\sqrt{K_3}}$$

Hence $\frac{K_1}{l_1^2}=\frac{K_2}{l_2^2}..K_3$ $\quad K_1 : K_2 : K_3=l_1^2 : l_2^2 : l_3^2$.

Experiment : Several rods of different metals having equal lengths and diameters are taken and coated uniformly with layers of paraffin wax. One end of each rod such as a, b, c, d, e etc. is inserted

through a hole in the side of a trough T (Fig. 72) and fixed there by means of a cork. The trough is filled with water and boiled by passing electric current through resistance coils immersed in water in the trough.

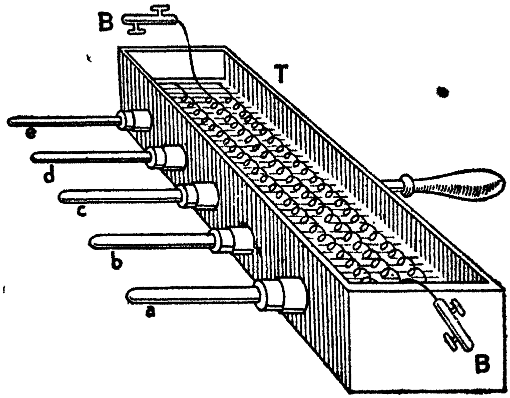


Fig. 72

When water has been boiled for 10 or 15 minutes and a steady state has been reached, the length of each rod from which the wax has melted is measured.

Let K_1, K_2, K_3, etc. be the conductivities of the rods and let l_1, l_2, l_3, etc. be their lengths along which wax has melted.

$$\text{Then } K_1 : K_2 : K_3 = l_1^2 : l_2^2 : l_3^2$$

For any two rods $\frac{K_1}{K_2}=\frac{l_1^2}{l_2^2}$, knowing l_1, l_2, the ratio $\frac{K_1}{K_2}$ giving the comparison, can be found out.

173. Determination of Thermal Conductivity of a metal (good conductor) : The apparatus used for this purpose consists of a cylindrical rod AB (Fig. 73) of copper having copper box soldered near each of its ends. The box C which serves as a steam chamber has two tubes D and E for entrance and exit of steam, respectively. The other box F is fitted with an annular partition which compels the water that enters through a side tube G to circulate round and flow out through another side tube I fitted into the box, in a manner indicated by the arrows. The temperatures of the incoming and escaping water are noted by two thermometers T_4 and T_3. Both the boxes together with the rod are carefully packed round with cotton wool so as to prevent loss of heat as much as possible and enclosed in a wooden box. Two thermometers T_1 and T_2 are inserted into holes bored in the rod at a distance l cm. apart.

As soon as steam passes into the box meant for it, the temperature of the rod gradually rises as is shown by two thermometers T_1 and T_2. A steady current of water is passed through the box F by connecting the side tube G to a

constant pressure tank and carefully adjusting the rate of flow of water so that the rise of temperature of water is from 8 to 10 degrees centigrade.

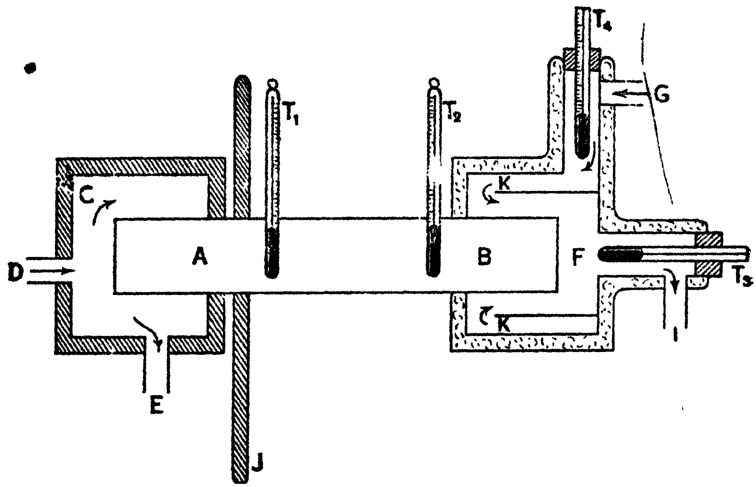


Fig. 73

At the steady state the four thermometers show constant readings. When this is the case, the constant temperatures as indicated by the thermometers T_1, T_2, T_3 and T_4 are noted and a weighed beaker is placed at a noted instant under the tube through which hot water flows out of the box.

A quantity of hot water is collected in the beaker for a known interval of time and its mass is measured. The mean diameter (d) of the rod is then obtained by a slide callipers and the distance between thermometers T_1 and T_2 is carefully determined by divider and scale.

Let area of cross-section of the rod $= A = \pi\left(\frac{d}{2}\right)^2$.

Length of the rod between thermometers T_1 and $T_2 = l$.

Readings of thermometers T_1 and $T_2 = \theta_1$ and θ_2 respectively.

Temperatures of the incoming cold water and outflowing hot water as read by thermometers T_4 and $T_3 = \theta_4$ and θ_3 respectively.

Mass of water collected $=$ M.

Time of collection of water $= t$ (seconds).

Then $Q = \dfrac{KA(\theta_1 - \theta_2)t}{l} = M(\theta_3 - \theta_4)$

$$K = \frac{M(\theta_3 - \theta_4)l}{A(\theta_1 - \theta_2)t} = \frac{M(\theta_3 - \theta_4)l}{\pi\left(\frac{d}{2}\right)^2(\theta_1 - \theta_2)t} = \frac{4M(\theta_3 - \theta_4)l}{\pi d^2(\theta_1 - \theta_2)t}$$

whence the value of K can be calculated.

174. Conduction through Composite blocks : Consider two rectangular slabs B and A (Fig. 74) in contact having the same area of cross-section A. Let their thickness be x_1, x_2 and conductivities K_1 and K_2 respectively. Let the temperatures of the outer faces of B and A be θ_1 and θ_2 respectively θ_1 being greater then θ_2. Suppose the temperature of the interface in steady state be θ.

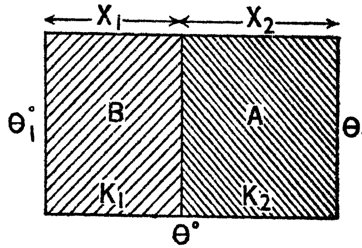


Fig. 74

In the steady state, rate of flow of heat through B and A is the same since there is no absorbtion and hence accumulation of heat anywhere in steady state.

$$\frac{dQ}{dt} = \frac{K_1A(\theta_1 - \theta)}{x_1} = \frac{K_2A(\theta - \theta_2)}{x_2}$$

or $$\frac{dQ}{dt} = \frac{A(\theta_1 - \theta)}{\frac{x_1}{K_1}} = \frac{A(\theta - \theta_2)}{\frac{x_2}{K_2}} = \frac{A(\theta_1 - \theta_2)}{\frac{x_1}{K_1} + \frac{x_2}{K_2}}$$

Rate of flow of heat through the composite

$$\text{blocks} = \frac{A(\theta_1 - \theta_2)}{\frac{x_1}{K_1} + \frac{x_2}{K_2}}.$$

Again $$\frac{K_1A(\theta_1 - \theta)}{x_1} = \frac{K_2A(\theta - \theta_2)}{x_2}$$

or $$\frac{K_1}{x_1}.\theta_1 - \frac{K_1}{x_1}.\theta = \frac{K_2}{x_2}.\theta - \frac{K_2}{x_2}.\theta_2$$

or $$\left(\frac{K_1}{x_1} + \frac{K_2}{x_2}\right)\theta = \frac{K_1}{x_1}.\theta_1 + \frac{K_2}{x_2}.\theta$$

$$\therefore \text{ Interface temp. } \theta = \frac{\frac{K_1}{x_1}.\theta_1 + \frac{K_2}{x_2}.\theta_2}{\frac{K_1}{x_1} + \frac{K_2}{x_2}}.$$

175. Determination of the conductivity of a metal—Forbes' Method : Forbes' method consisted of two experiments known as **statical** and **dynamical** experiments.

In the statical experiment Forbes used a long bar of iron the bent end A of which (Fig. 75) was heated by being placed inside an iron crucible containing molten solder, the other end remaining at the temperature of the atmosphere. A number of thermometers with their bulbs inserted in mercury contained in holes drilled in the bar was used to indicate its temperature at various points along its length. When the temperatures at all points became steady they were read.

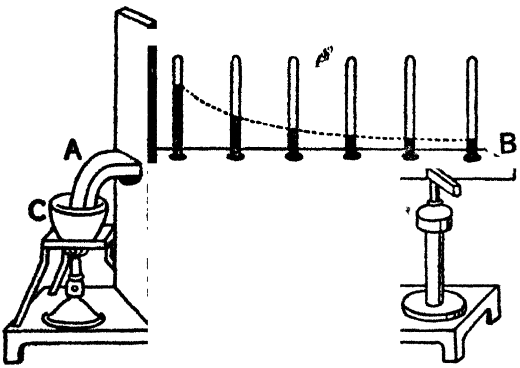


Fig. 75

The temperature distribution is shown by the dotted curve (a) between θ and x in Fig. 76, which follows the exponential law $\theta = \theta_0 e^{-\mu x}$. This experiment is referred as statical experiment as it is concerned with the steady state of flow.

To get the amount of heat that flows across a particular cross-section, Forbes found the amount of heat lost by radiation by the portion of the bar between the said cross-section and the cold end of the bar. These two amounts of heat must be equal in the steady state since no flow of heat occurs from the cold end which is at the room temperature. For this, Forbes performed the Dynamical experiment using a shorter bar of iron which was in all other respects similar to the bar in the statical experiment. The experiment is called dynamical as in this case temperatures were changing. The dynamical bar was heated to a high uniform temperature and then allowed to cool in exactly the same surroundings as the statical bar. A cooling curve (b) between θ and t was then plotted (Fig. 76).

In the *steady state* the heat flowing through the cross-section at $x=x_1$ is equal to the amount of heat lost by radiation by the portion of the bar lying between the above cross-section at $x=x_1$ and the end of the bar.

The heat crossing the section at $x=x_1$ per. sec. is equal to

$$-K.A.\left(\frac{d\theta}{dx}\right)_{x=x_1} \quad \left\{ \begin{array}{l} \text{From } \theta\text{-}x \text{ Curve } (a) \\ \text{[Fig. 76]} \end{array} \right.$$

Now to calculate the amount of heat lost by radiation by the bar in the *Statical experiment* from the point $x=x_1$ to the end of the bar ($x=l$), we are to know the heat lost per second by radiation from the surface of the bar from x to $x+dx$ in the *steady state.* This is again equal to

$$-A\ dx.\rho s.\frac{d\theta}{dt}$$

where A is the area of the cross-section, ρ the density of the substance, s, the specific heat and $-\frac{d\theta}{dt}$ is the rate of cooling of the element.

From *Dynamical experiment the* θ-t curve (b) [Fig. 76] is drawn and

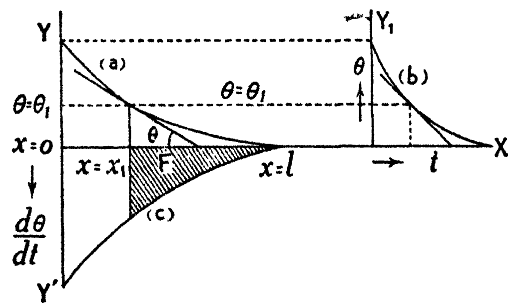


Fig. 76

by drawing tangent at different points on the curve, values of $\frac{d\theta}{dt}$ for different values of θ are computed.

Thus the total heat which will be lost per second by radiation from the surface of the bar between the points where $x=x_1$ and $x=l$ from $\left(\frac{d\theta}{dt} \text{ and } x\right)$ curve (c) in Fig. 76 is equal to

$$-A\rho s\int_{x=x_1}^{x=l}\frac{d\theta}{dt}dx\ ;\ \therefore\ -KA\frac{d\theta}{dx}=-A\rho s\int_{x=x_1}^{x=l}\frac{d\theta}{dt}\,.\,dx$$

$$\text{or}\quad \frac{K}{\rho s}\tan\phi=\int_{x=x_1}^{x=l}\frac{d\theta}{dt}dx=F \qquad \ldots\ (1)$$

where $\tan\phi=d\theta/dx$; and F is denoted by the shaded area, in Fig. 76(c). The area F can be measured by a planimeter and the value of $\tan\phi$ can be obtained from the curve (*a*) in Fig 76. Then knowing also ρ and s, K can be found out.

Note: In Fig. 76 the angle θ between the tangent to $(\theta$-$x)$ curve and the axis OX should be read as ϕ.

Note: It is to be noted that the expression (1) is arrived at on the assumption that the amount of heat radiated from the end surface of the smaller bar used in the Dynamical experiment is neglected.

This method is suitable for a thin bar and gives an absolute value of K.

The method has several disadvantages. The temperature distribution in *statical and dynamical experiments* is not the same and the sp. heat of the material of the bar is not also the same for all temperatures.

176. Radial Flow of heat through the wall of a tube:

When some hot liquid flows through a cylindrical tube, heat flows in all directions along the radius of the section of the cylinder, through the wall of the cylinder, the inner wall being at a temperature higher than that of the outer wall. Such a flow of heat through the wall of a tube through which hot liquid is passing is called *radial flow* of heat.

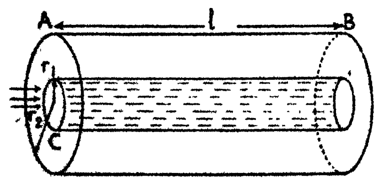


Fig. 77

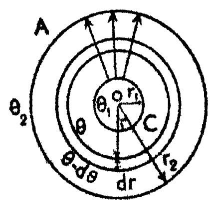


Fig. 78

Let l be the length of the tube (Fig. 77), and r_1 and r_2 its internal and external radii. In steady state, let θ_1 and θ_2 be the temperature of inside and outside wall of the tube θ_1 being greater than θ_2.

Consider a very thin cylindrical element (Section shown in Fig. 78) of radii r and $r+dr$, corresponding temperatures of the two sides of the element being θ and $\theta - d\theta$ respectively. As the element is very thin, heat flows normally to the walls and it may be considered like a parallel-sided slab, so that rate of flow of heat is given by $\frac{dQ}{dt} = -KA\frac{d\theta}{dr}$ ... (1), where K = thermal conductivity of the material of the tube ; A = area of the walls of the element considered. But $A = 2\pi rl$. $\therefore \frac{dQ}{dt} = -K.2\pi rl.\frac{d\theta}{dr} = -2\pi K.l.r\frac{d\theta}{dr}$ ... (2)

In steady state, $\frac{dQ}{dt}$ is the same for all values of r ; then since K and l are constant, $r.d\theta/dr$ is also constant.

$$\therefore \quad r\frac{d\theta}{dr} = c, \text{ where } c = \text{a constant}$$

$$\text{or } d\theta = c.\frac{dr}{r}$$

$$\text{Integrating } \int_{\theta_1}^{\theta_2} d\theta = c\int_{r_1}^{r_2}\frac{dr}{r}$$

$$\text{or } \quad \theta_2 - \theta_1 = c \log\frac{r_2}{r_1}$$

$$\text{or } \quad \theta_1 - \theta_2 = c \log\frac{r_1}{r_2} \quad \text{or } c = \frac{\theta_1 - \theta_2}{\log r_1/r_2}$$

Putting this value of c *i.e.* $rd\theta/dr$ in the relation (2)

$$\frac{dQ}{dt} = -2\pi K.l.\frac{\theta_1 - \theta_2}{\log r_1 - \log r_2}$$

$$= 2\pi K.l.\frac{\theta_1 - \theta_2}{\log r_2 - \log r_1} \quad \ldots \quad \ldots \quad (3)$$

or $K = \frac{dQ}{dt}.\frac{\log r_2 - \log r_1}{2\pi l(\theta_1 - \theta_2)}$; knowing experimentally rate of flow of heat, r_1, r_2, θ_1, θ_2 and l, K can be found out.

Note : Relation (3) may be written as $\frac{Q}{t} = 2\pi Kl\frac{\theta_1 - \theta_2}{\log r_2 - \log r_1}$

$$\text{or } \quad Q = \frac{K.\ 2\pi l.\ (\theta_1 -}{\log r_2 - \log r_1}$$

177. Determination of Thermal Conductivity of a bad Conductor by Lee's Method: The adjoining diagram (Fig. 79) illustrates an experimental arrangement for determination of the conductivity of a bad conductor. In the figure A and A′ are two thin discs of bad conductor of which A is placed between copper discs B and C, and A′ between copper discs B′ and C′, the surface areas of all the discs being the same.

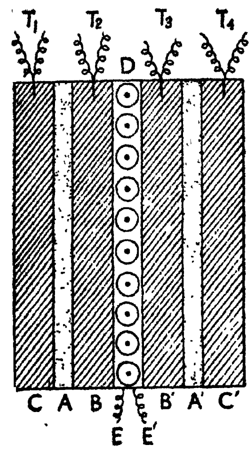


Fig. 79

A flat coil D of insulated platinoid wire is placed between B and B′ such that the opposite sides of the coil are fixed to B and B′. An electric current is passed through the coil by the leads E and E′ and the quantity of heat liberated per second is calculated from a knowledge of current, and resistance of the coil. Let its value be Q. The temperatures of the opposite surfaces of the bad conductors A and A′ in contact with B, C, and B′, C′ respectively are measured by four thermo-couples T_1, T_2, T_3 and T_4 inserted into the copper discs. Let θ_1, θ_2, θ_3 and θ_4 be the temperatures determined by these couples.

Then we have, $$Q=\frac{K.S\,(\theta_2-\theta_1)}{d}=\frac{K.S\,(\theta_3-\theta_4)}{d}$$

where K is the coefficient of thermal conductivity, S the surface area of the bad conductor A or A′, d the thickness of the disc of bad conductor, and $(\theta_2-\theta_1)$ and $(\theta_3-\theta_4)$, the temperature difference between the opposite surface of A and A′. From the above expressions K can be found, knowing the value of Q.

This method is used for determining the conductivities of the bad conductors like rubber, wood, glass etc. It may also be employed for determining the conductivity of a liquid by taking it inside a thin-walled flat-end box.

178. Variation of K of metals with temperature: In case of majority of metals in pure state, K increases with increase of temperature. The electrical conductivity of metals σ also increases with increase of temperature. According to Wiedemann-Franz, the ratio of thermal and electrical conductivities for all metals is same for a given temperature. This is referred to as Wiedemann-Franz's law. The ratio K/σ was further supposed by Lorentz

to be proportional to the absolute temperature T so that $K/\sigma T$ should be same at all temperatures for all metals. Experiments showed that laws of Wiedemann-Franz and of Lorentz hold good at ordinary temperatures, but at low temperatures the value of $K/\sigma T$ shows variations from Lorentz's law.

179. Convection: It is the process by which fluids (liquids and gases) become heated by the actual movement of particles due to the difference of density.

The actual movement of the particles of water due to convection current is visible when a beaker full of water with a few crystals of magenta in it, is heated by a Bunsen flame.

The convection in gases is well illustrated in the formation of wind. The surface of the earth is heated by the rays of the sun and the air in contact with it becomes heated and thus gets lighter. The lighter air ascends and the colder air from surrounding regions flows in to take its place.

180. Radiation: When heat passes from one point to another in a straight line with a great speed without heating the medium through which it passes, it is said to be transmitted by radiation.

If a beam of solar rays or radiation from a heated metal be allowed to fall on the bulb of a thermometer which is coated with lamp-black, the column of mercury in it will be seen to rise at a greater rate than when the bulb is left uncovered and the space surrounding the bulb will remain unaffected by the transference of heat from the hot body to the bulb. The heat thus transmitted is called the **Radiant Heat or** simply **Radiation**.

181. Transmission of Radiant Heat: To explain the propagation of Radiant heat from one place to another, the intervening space is supposed to be filled up with a mysterious medium known as *ether* which is capable of transmitting energy in the form of waves. When the surface of the earth is heated by the rays of the sun, the radiant heat during its passage sets the particles of ether to vibrations which are then propagated in all directions in the form of waves with a very high velocity. These waves carry radiant energy with them and when they fall on a material body the energy is transferred into it and is then converted into **heat.** The waves carrying radiant energy with them are called **Radiations.** The solar radiations mainly consist of three different kinds of vibration. Thus **Radiation** in general may be divided into three distinct parts, such as **Thermal Radiations, Luminous Radiations** and **Actinic Radiations.** All these radiations are nothing but vibrations in ether but the rapidity of vibration increases as

we pass from thermal to actinic radiations. These different radiations having different frequencies produce different effects when they fall on different substances. They excite *sensation of heat* when they fall on material bodies, they cause *vision* when they fall on our eyes and also they produce *chemical effects* when they act on chemical compounds.

Thus we see that those different kinds of radiations are but different varieties of the same physical agent. So then, the invisible heat rays and the visible light rays may be said to be the same in nature being parts of **Radiation** in general.

182. Properties of Thermal Radiations :

1. Thermal radiations can be transmitted through vacuum *i. e.* they are independent of any ponderable medium.
2. They are transmitted in straight lines with the velocity of light.
3. They can be reflected, refracted and polarised and obey the same laws as light.
4. Radiations of particular wave-lengths are absorbed and others transmitted through material substances.

183. Loss of Heat by Radiation : It has been found experimentally that the rate at which heat is radiated by a body depends on (1) the temperature of the body (2) the nature or condition, and extent of its surface and (3) the temperature of the surrounding space into which heat is radiated.

184. Certain laws regarding Radiation :

(1) *Newton's Law of Cooling :*

The heat radiated per second from a body is proportional to the difference of temperature between the body and its surroundings.

This law is applicable for small difference of temperatures, but fails when the temperature of the radiating body is high.

(2) *Dulong and Petit's Law :*

The law stating that the rate R at which energy is radiated from the body at an absolute temperature T is expressed as

$$R = E.1{\cdot}0077^{T}$$

where E is a constant.

This law refers to bodies cooling in vacuo.

(3) *Stefan's Law :*

The law states that the radiating or emissive power E of a body varies as the fourth power of its absolute temperature and is given by, $E = C(T_1^4 - T_2^4)$

where C is a constant and T_1 and T_2 are the absolute temperatures of the body, and its enclosure exhausted of air, respectively.

This law as proposed by Stefan is based on Dulong and Petit's experiments.

(4) *Weber's law :*

The law proposed by Weber is expressed as

$$R = E'.T\,1{\cdot}00043^T$$

where E' is a constant and T the absolute temperatue of the body.

(5) *Wien's Laws :*

The first law states

$$\lambda_{max} T = \text{constant}.$$

where λ_{max} is the wave-length at which E_λ, the emissive power for wave-length lying between λ and $\lambda + d\lambda$ is a maximum at temperature T.

The second law states $E_{max} = \text{constant} \times T^5$.

185. Diathermancy ; Athermancy : When thermal radiations pass through a material substance, certain wave-lengths are absorbed while others transmitted. Substances which transmit radiations of certain wave-lengths are said to be *diathermanous* and substances which absorb radiations of certain wave-lengths are said to be *athermanous* for those radiations.

Air and vacuum are diathermanous while lamp-black, wood, metals, etc., are athermanous.

186. Emissive Power of a Body : It is the ratio of the amount of radiation absorbed in a given time by the body to the total amount incident on the body in the same time by an equal area of a perfectly black surface at the same temperature.

187. Absorptive Power of a Body : It is the ratio of the amount of radiation absorbed in a given time by the body to the total amount incident on the body in the same time.

It has been found by experiment that the surface coated with lamp-black is capable of absorbing practically all the heat energy which falls on it and hence its absorptive power is unity.

Such a body whose absorptive power in unity is called a **perfectly black** body.

According to Fery, a uniformly heated enclosure lamp-blacked on the inside and with a small hole cut in it behaves as a black body as regards emission and the radiation coming out of it will be very nearly black body radiation.

The *hole itself* and not the walls of the enclosure is to be regarded as the surface of the black body.

According to Kirchhoff a **black body** is one which has the property of allowing all incident rays to enter without reflection and not allowing them to leave again.

188. Prevost's Theory of Exchanges: All bodies radiate heat at all temperatures except when they are at the absolute zero of the temperature. In a state of thermal equilibrium, the amount of energy radiated per sec. is equal to the amount of energy absorbed by it per sec. from surrounding bodies. This principle is what is known as Prevost's Theory of Exchanges,

If a hot body be placed inside an enclosure maintained at a constant temperature, the body will gradually cool by radiating heat to the enclosure.

After some time the temperature of the body becomes the same as that of the enclosure and cooling ceases. At this stage it might appear that radiation had ceased. If the temperature of the enclosure be now lowered, that of the body will begin to fall immediately, showing that the radiation has been taking place all the time.

Hence, it may be concluded that every body sends out energy at all times and at all temperatures and its amount depends on the nature of the radiating surface and its own temperature but is not affected by the presence of surrounding bodies.

As all bodies are sending out radiations at all times, each giving out its own radiation and receiving at the same time that from the surrounding objects, a mutual exchange of radiations goes on between the body and the surrounding objects. Hence, this is known as the **Prevost's Theory of Exchanges.**

The body will rise or fall in temperature according as it receives more or less heat from the surrounding objects. When the heat radiated by the body and the surrounding objects will be exactly equal and balance each other, they will be at the same temperature.

It may be shown from this theory, that *the absorbing power of a surface is equal to its emissive power.*

Thus a black surface absorbs more and also radiates more heat than a bright surface.

It may also be shown that a *surface emits the same kind of radiation as it absorbs.*

According to the kinetic theory of matter, the total heat in a body is due to the energy possessed by the vibrating molecules in the body.

When a hot body is suspended in an enclosure, it radiates heat and so its temperature falls. This loss of heat is explained by the kinetic theory of matter.

The movement of the molecules in the body causes a disturbance in the ether in the body and surrounding it, and produces waves which carry away heat energy to other surrounding bodies and so the body cools.

Cooling ceases only when the energy which the body absorbs becomes equal to the energy radiated in the form of waves by the surrounding bodies.

189. Equality of the Emissive and Absorptive Powers of a body : Let a body whose absorptive and emissive powers are each equal to unity, be enclosed in an enclosure of surface area s and whose absorptive power is a and emissive power e.

Let the body lose E units of heat by radiation per second.

Then the walls of the enclosure will absorb Ra units of heat and reflect E $(1-a)$ units of heat per second.

The heat reflected from the enclosure is completely absorbed by the body since its absorptive power is unity.

Thus the heat lost by the body in unit time is $E - E\ (1+a)$ or Ea.

Since its temperature is constant, the enclosure must radiate Ea units which are absorbed by the body. This is in accordance with *Prevost's Theory of Exchanges.*

The amount of heat energy radiated per sq. centimetre of the enclosure is $\frac{Ea}{s}$.

Again if the enclosure be a perfectly black body, then it would absorb E units and radiate E units.

Then the amount of heat energy emitted per sq. centimetre of the black body is $\frac{E}{s}$.

Then the emissive power e is given by, $e = \frac{Ea}{s} \div \frac{E}{s} = a$.

Thus the emissive power of the body is equal to its absorptive power.

190. Transformation into heat of different forms of energy : The energy which a body or a system of bodies possesses may exist in various forms such as mechanical energy, light energy, sound energy, etc., but can always be transformed into kinetic or potential energy or combination of both. Whenever one form of

energy disappears, another appears to take its place and in some cases different forms appear and finally converted into heat.

We will now confine our attention to the transformation into heat of different forms of energy.

When a brass button is sharply rubbed against a table cloth or a piece of wood it becomes heated or when a moving body is suddenly brought to rest as in the case of a stone falling to the ground or a moving train brought to rest by applying brakes to the wheels, the Kinetic energy of the body due to its motion appears as such and reappears in the new form of energy as heat.

Numerous experiments may be performed to prove that mechanical work done by friction, pressure and percussion can be converted into heat.

The barrel of the pump gets heated when the cyclist inflates his tyres and a sheet of flame appears when a bullet strikes a target.

Many chemical actions are accompanied by the production of heat. Carbon or any fuel is a store of potential chemical energy and when carbon combines with Oxygen during the process of combustion, a large amount of heat is developed. Here the chemical energy of carbon is converted into heat energy.

A **loaded cartridge** has also a store of potential energy which can at any moment be converted into kinetic energy of the bullet and finally into heat.

Electric energy of the electric current may also be utilised for the production of heat. When an electric current passes through a conductor it is heated. If the current be strong, and the resistance of the conductor appreciable, a high temperature may be produced. The heat developed in this way may be utilised for cooking and other domestic purposes.

Like electric energy, **magnetic energy** may be converted into heat energy as is evident from the fact that when a piece of iron or steel is magnetised in a strong magnetic field, due to hysteresis it will retain a considerable portion of its magnetisation even after the applied field has been removed and the entire amount of magnetic energy is not recovered, a portion being dissipated into heat.

Again, **light energy** can be converted into heat energy. When light rays are absorbed at a black surface, their energy is converted into energy of molecular motion or into heat energy. This is illustrated by the movement of blackened vanes of the radiometer.

191. The Radiation Pyrometry : In a previous article we have already referred to Gas Pyrometer, Resistance Pyrometer and Thermo-electric Pyrometer and the range of temperature that can

be measured with the help of these instruments. We will now consider how the temperature of a body can be measured by instruments known as **Radiation Pyrometers.** We know that the radiation emitted by a black body depends upon its temperature and nothing else. Hence we can measure the temperature of the body by an examination of the radiation which it emits, by radiation pyrometers. This can be done in two ways ; we can either use Stefan's law for measuring the temperature of the body by total Radiation Pyrometers or use one of the two laws of Wein and determine the temperature by measuring the energy emitted in a particular portion of the spectrum by instruments known as Optical Pyrometers.

The superiority of Radiation Pyrometers over any other method already described is that they can be used to measure any temperature however high that may be or wherever the object may be. The pyrometer itself has not to be raised to that temperature nor need it be placed in contact with the hot body.

The radiation method has, however one disadvantage. The formulæ used for measuring the high temperature apply only to the radiation from a black body. This is by no means necessarily the case and when it is not, the temperature will be too low. The radiation method measures what is called the *black body temperature*. The greater the departure from perfect blackness the greater is the error involved.

192. Fery's Total Radiation Pyrometer : This instrument measures the temperature of a body based on Stefan's law by means of the total radiation which it emits. The radiation from the hot body is focussed by a concave mirror on to a junction of a thermo-couple in front of which is a limited diaphragm. The cold junction of the couple is protected from radiation by a tongue and a box surrounding the couple. The radiation heats up the junction and the electromotive force developed is read on a multivoltmeter connected to the terminals of the couple.

It is important to note that the image of the source must be focussed on the diaphragm in such a way that it fills the hole in the diaphragm completely. This is done by moving the concave mirror and the diaphragm is observed through a hole in the mirror by an eyepice.

The E.M.F. of the couple is given by relation,

$V = a(T^b - T_0{}^b) = aT^b$, neglecting T_0

where T is the temperature of the black body to be measured, T_0 the temperature of the hot junction and b is a constant. Here T_0 is too small and can be neglected in comparison with T.

The constant b varies from 3·8 to 4·2 instead of being exactly 4.

To measure temperatures directly, the millivoltmeter is calibrated by a standard thermo-couple using radiation from a black body.

193. Optical Pyrometer: Of the two types of Optical pyrometers, such as the Disappearing filament type and the Polarising type a brief description of the former is given below.

194. Disappearing filament type: It essentially (Fig. 80) consists of a telescope, the objective of which is placed in front of the source whose temperature is to be measured. The radiation from the source is focussed by the lens on the filament of a standard lamp and viewed by the eye-piece lens through a piece of red filter glass. Besides this, there is a number of diaphragms.

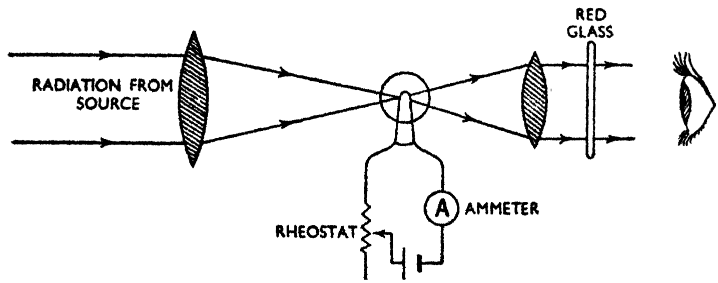


Fig. 80

The current through the filament is adjusted by a rheostat until the filament disappears against the background formed by the image of the source. The instrument is calibrated by sighting it on a black body whose temperature can be measured by a standard thermo-couple, the ammeter which measures the filament current being calibrated directly in degrees.

QUESTIONS

1. Define thermal conductivity and thermometric conductivity of a substance. [C. U. 1929, '34, '36, '40, '42, '45, '46, '47, '52]

2. Find the dimensions of "K"—the thermal conductivity of a substance. "Thermal conductivity of iron = 1·75 C.G.S. units." Explain. [C. U. 1949]

3. Describe a simple experiment for the rough comparison of thermal conductivities of several metal rods. Deduce the formula you would use. [C. U. 1936, '46. '49]

4. Describe Forbes' method for determining the thermal conductivity of a metallic bar. Deduce the formula that you will use for the calculation and comment on the dimension of the conductivity so obtained. [C. U. 1929, '34, '42, '47, '52]

5. Write short notes on :—Thermos flask, Radiant Heat. [C. U. 1941, '45]

6. Describe the common thermos and dewar flask and explain the physical properties on which their utility is based. [C. U. 1932]

7. Describe a method of measuring the thermal conductivity of a bad conductor. [C. U. 1959]

8. State and explain Prevost's theory of exchanges and describe a simple experiment to illustrate it.

Mention and discuss some of the important conclusions drawn from the theory. [C. U. 1937]

EXAMPLES

1. A room has a glass window 2 metres high, 1 metre wide, and 7 mm. thick. The room is kept at a temperature of 15°C and the temperature outside is 0°C. Assuming that the temperature of the two surfaces of the glass are 10°C and 2°C respectively, calculate the quantity of heat which will pass through the glass per hour. The thermal conductivity of the glass is ·0005. [C. U. 1919]

Area of the window, $A=2$ metres $\times 1$ metre $=200$ cm. $\times 100$ cm. $=2\times10^4$ sq. cm. the thickness $d=\cdot7$ cm. Difference of temperature $(t_2-t_1)=(10-2)°C=8°C$ and time $T=1$ hour $=60\times60$ sec. Thermal conductivity $K=\dfrac{Qd}{(t_2-t_1)\,TA}$

$$\therefore\ Q=\frac{KA(t_2-t_1)T}{d}=\frac{\cdot0005\times2\times10^4\times8\times60\times60}{\cdot7}=41\cdot143\times10^4 \text{ cal. nearly.}$$

2. How much heat will be conducted in an hour through each square centimetre of a plate of ice, 2 cms. thick, one side of the ice being at 0°C and the other at -10°C, its conductivity being ·00223 and what volume of water at 0°C will be converted into ice at 0°C by the loss of heat. [C. U. 1924]

$$Q=4\cdot014 \text{ cal. Vol. of ice}=\frac{4\cdot014}{80}=\tfrac{1}{20}\text{ c.c. (approx.)}$$

3. The walls of a room are 20 cm. thick and are of stone, the thermal conductivity of which is 0·009 cal. per cm. per sec. per deg. C. The temperature of the room inside is 20°C, and the temperature outside is 10°C below 0°C. Find the quantity of heat lost by conduction through the wall per hour per sq. metre of the wall. [C. U. 1927]

Area of the wall $=100\times100=10^4$ sq. cm. The thickness of the wall $=20$ cms. Difference of temperature $=20-(-10)=30$°C. Time $=60\times60=3600$ secs.

$$Q=\frac{KA\,(t_2-t_1)T}{d}=\frac{\cdot009\times10^4\times30\times3600}{20}=48\cdot6\times10^4 \text{ cal.}$$

4. Calculate the amount of heat conducted per minute through each square centimetre of an iron plate 5 cms. thick, when the faces of the plate are maintained at 0°C and 100°C, the mean thermal conductivity being 0·16. [C. U. 1936]

From the expression $Q=K.\dfrac{A(t_2-t_1)\,T}{d}=\dfrac{\cdot16\times1\times100\times60}{5}=192$ calories.

5. An ice-box is built of wood 1·75 cm. thick, lined with cork 3·0 cm. thick. If the temperature of the inner surface of the cork is 0°C and that of the outer surface of wood 12°C, what is the temperature of the interface? [C. U. 1946]

The conductivity of wood is 0·00060 and of the cork 0·00012 cal. cm. deg. C units.

Let t = temperature of the interface; t_2 = temperature of the outer surface of of wood; t_1 = temperature of inner surface of cork; at steady state, rate of flow of heat dq/dt per unit area through wood and cork is the same,

$$\therefore\ dq/dt=\frac{t-0}{3}\times\cdot00012=\frac{12-t}{1\cdot75}\times\cdot00060,\ (\because\ t_2=12°C,\ t_1=0°C,),\ \text{whence}$$

$\cdot35t=36-3t$ or $t=10°\cdot75$C.

6. A vessel of surface area 1 sq. metre and of copper sheet of thickness 5 *mm.* is filled with melting ice and is immersed in water at 100°C. Calculate the rate at which ice melts, the conductivity of copper being 0·72 and the latent heat of fusion of ice 80.

[C. U. 1940]

We know that $Q=\frac{K.A.(t_2-t_1)T}{d}=m.\ 80$

where m is the mass of ice melted in one sec.

$$\therefore\quad 80\ m=\frac{\cdot72\times10000\times100\times1}{\cdot5}$$

$$\therefore\quad m=\frac{\cdot72\times10000\times100\times1}{\cdot5\times80}=18000\ \text{gms.}$$

7. A sheet of ice 10 cm. thick covers a lake and the temperature of air is -15°C. How long will it take for the thickness of the ice to be doubled?

[C. U. 1942]

Thermal conductivity of ice=0·004 c. g. s. units; Density of ice=0·92 gm. per c.c.; Latent of fusion of ice=80 cal. per gm., Assuming the steady state,

heat conducted per sq, cm. $Q=K\frac{15}{v}dt=L\times\cdot92dx$ K=coeff. of conductivity, if the thickness x of ice increases by dx in dt secs.

$$\text{or } 15K\int^{t} dt=\cdot92L\int_{10}^{20} xdx \qquad 15Kt=\frac{\cdot92L}{2}(20^2-10^2)$$

$$\therefore\quad t=\frac{\cdot46\times80\times300}{15\times\cdot004}=184000\ \text{secs.}\qquad t=51h-6m-40\ sec.$$

8. The top of a steam chamber is formed of a stone slab 60 cm. long, 5 cm. broad and 1 cm. thick. Ice is piled upon the slab, and it is found that 5 kilograms of ice are melted in half an hour. What is the thermal conductivity of the stone? The latent heat of fusion of ice at 0°C is 80 calories per gram.

[C. U. 1945]

Use the formula $K=\frac{Qd}{A\ (t_2-t_1)T}$ [Ans. ·0074]

9. Water is boiling in a kettle under ordinary pressure and 6 grams are boiling off per minute. If the thickness of the kettle be 2·4 mm., the surface in contact with water 400 sq. cm. and the conductivity of iron be 0·875 in usual c.g.s., units, find the temperature of the outer surface of the bath.

[D. U. 1943]

$$Q=\frac{K.A(t_2-t_1)T}{d}\quad\text{or}\quad t_2-t_1=\frac{Q\ d}{K.A.T.}=\frac{6\times536\times\cdot24}{\cdot875\times400\times60}=\cdot036$$

$t_1=100-\cdot036=99°\cdot964$C. $\quad t_2=100$°C.

10. A wire of resistivity 2×10^{-4} ohms per cm³ and 1 m. m. in diameter carries a current of 10 amps. If it is covered uniformly with a cylindrical layer of insulating material having coefficient of thermal conductivity of 6×10^{-4} calories cm⁻¹ degree⁻¹ sec⁻¹ and a diameter of 1 cm, what is the temperature difference between the inner and outer surfaces of the insulator? (1 cal=4·2 watt-sec.)

[C. U. 1959]

MAGNETISM

CHAPTER I

INTRODUCTORY

1. Natural and Artificial Magnets: The lodestone a *natural magnet* is an iron ore (Fe_3O_4) called magnetite which exhibits two special properties namely: (*a*) it attracts iron filings; (*b*) when suspended so as to be able to rotate freely about a vertical axis, it comes to rest in a direction which is nearly north and south. When pieces of iron and steel are properly treated they acquire same properties as the lodestone, but in a much greater degree and are called *artificial magnets.* Bar-magnet, horse-shoe magnet and compass needle are principal types of artificial magnets.

2. Magnetic poles; Magnetic axis: The two regions near the two ends of a magnet—natural or artificial, where the attractive properties are most strongly exhibited are termed its *poles.* The end setting towards the North is called north-seeking or simply north or N pole and the other end setting towards the South, the south-seeking or simply south or S pole. The line joining the two poles of a magnet is called the *magnetic axis.*

The two opposite poles exist not as a rule at the extremities of a bar magnet, but at two regions near the ends and well inside the magnet. The distance joining the two poles is the *magnetic length* which is about ·85 times the actual length of the magnet.

The two poles north and south which are generally of equal strength, cannot be separated from each other if the magnet be cut into two halves or even further subdivided. In other words a magnet with a single isolated pole (N or S) cannot exist at all.

3. Different ways of magnetisation: A piece of iron can be magnetised by stroking it with magnets in three standard ways called methods of *single touch, separate* or *divided touch,* and *double touch.* A bar or rod of iron can be magnetised by the electro-magnetic method, in which a coil of wire is wrapped round a hollow cylinder, forming what is called a solenoid. The bar or rod to be magnetised is inserted in the cylinder and a strong electric current is passed through the coil. When the current is cut off and the bar or rod is withdrawn it is found to be magnetised. For permanent magnet the bar should be of steel and for electro-magnet a soft iron bar is to be taken. An electro-magnet behaves

like a magnet only when a current is flowing in the coil. A piece of iron can be magnetised also by **inducting influence** of earth which behaves like a magnet) or other strong magnets.

4. **Magnetic Saturation** : The degree of magnetisation of an iron bar depends on the intensity of the magnetising field, on the composition, temper and shape of the bar. When the intensity of the magnetising field exceeds a certain value, degree of magnetisation does not increase further. At this stage the bar is said to have acquired magnetic saturation. It can be explained by molecular theory of magnetism.

5. **Compound Magnets :** As it is difficult to produce magnetic saturation in the interior of a thick iron bar, it is often convenient to magnetise a set of long thin strips of iron, which are then held together at the ends by soft iron bands. Such a combination is a compound magnet.

6. **Preservation of Magnetism** : To preserve magnetisation it is usual to keep bar magnets in pairs [Fig. 1(a)] with their like poles turned in opposite directions, two cross-pieces of soft iron termed **keepers** connecting the unlike poles. Horse-shoe magnets are provided with a single keeper connecting the poles [Fig. 1(b)].

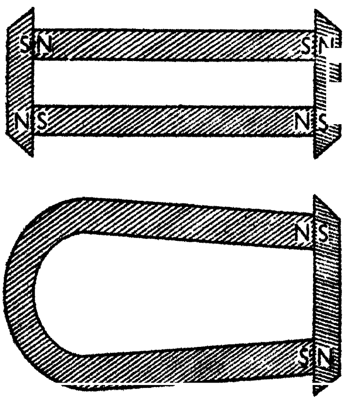


Fig. 1

7. **Destruction of Magnetisation** : An artificial magnet loses much of its magnetism when subjected to rough usage such as hammering, or heated above a certain critical temperature called the **curie point**. For iron this temperatture is 770°C, for nickel 358°C and for cobalt 1120°C.

8. **Portative force** : A general formula for the weight which a horse-shoe magnet can sustain, or in other words, the force which is required to pull away its keeper as given by Hacker, is expressed by $W = am^{\frac{2}{3}}$, where m is the weight of the magnet, W the weight just sufficient to detach the keeper, and a a constant for a certain quality of steel, the magnet being in normal state of saturation.

9. **Non-magnetic bodies** : Bodies which under normal conditions of temperature do not exhibit any magnetic property are

non-magnetic bodies. They can not be magnetised, but magnetic influence can more or less pass through all non-magnetic bodies.

QUESTIONS

1. Define poles and axis of a magnet. What is the relation between magnetic length and geometric length of a magnet? State different ways of making an artificial magnet.

2. Write brief notes on: (*a*) Magnetic saturation; (*b*) Compound magnets; (*c*) Destruction of magnetisation and (*d*) Uses of keepers.

CHAPTER II

INVERSE SQUARE LAW; MAGNETIC FIELD; MAGNETIC POTENTIAL

10. Laws of Action between two poles: The force of attraction (for two unlike poles) or repulsion (for two like poles) between two magnetic poles depends on the strength of the poles, distance between them and the nature of the medium surrounding the poles. *The force (attractive or repulsive) is directly proportional to the product of the strength of the poles and inversely proportional to the square of the distance between them.*

Thus if two poles of strength m_1 and m_2 be at a distance d apart each pole is attracted by the other by a force F given by

$$F \qquad m_1 m_2 \qquad \ldots (1)$$

$$\text{or} \quad F = \frac{m_1 m_2}{\mu d^2} \quad \ldots \quad \ldots (1a)$$

where μ is a constant depending on the nature of the medium in which the poles lie and is termed the **permeability** of the medium.

Let the two poles be placed in air; if now $m_1 = m_2 = 1$, $d = 1$ cm. so that $F = 1$ dyne, then from (1*a*) $\mu = 1$.

Thus, the unit magnetic pole is so chosen or defined that it exerts in air a force of 1 dyne on an equal and similar pole at a distance of 1 cm.

The relation (1*a*) above, therefore reduces to

$$F = \frac{m_1 m_2}{d^2} \quad \ldots \quad \ldots (2)$$

when the medium is air.

Note: From relation (1) above it may be stated. "*The force with which two poles attract or repel each other is inversely proportional*

to the square of the distance apart, the strengths of two poles remaining constant."

This statement is referred to as "Inverse square law".

11. Magnetic Intensity at a point: If a magnetic pole is placed at any point in the space round a pole or a magnet (or a wire traversed by electric current), it experiences a force, and the pole is said to lie in a magnetic field. When the magnetic pole taken is a unit north pole, the force exerted on it is called the magnetic intensity or strength of the magnetic field, or simply *magnetic intensity* at that point. It is usually denoted by H (not to be confused with horizontal component of earth's magnetic Intensity).

Thus magnetic intensity due to a pole m_1 at distance d in air is given by $H = \frac{m_1}{d^2}$; then relation (2) in Art. 10 can be written as

$$F = \left(\frac{m_1}{d^2}\right) \times m_2 = m_2 H.$$

i.e., Force on m_2 pole due to m_1

$=$ magnetic pole strength $(m_2) \times$ magnetic Intensity (due to m_1).

When a bar magnet having pole strength m is suspended freely, the force acting on each pole due to earth's magnetism is mH, where H is the horizontal component of earth's magnetism.

Note: In C. G. S. system unit of magnetic force is 1 dyne but unit of magnetic intensity is 1 dyne per unit pole or 1 oersted. The unit intensity is sometimes called a Gauss.

12. Magnetic Field: The space surrounding a magnet where magnetic attractions and repulsions are observed is called the magnetic field due to the magnet. The force at any point on unit *n* pole in the space is called, as already stated, the strength or the intensity of the magnetic field at that point.

At every point in the field we have a definite force depending on the distance of the point from the poles of the magnet. Since the force is different at different points of the field, the magnitudes and the directions of the resultant force are also different at different points. *The curve, the tangent at any point of which* (Fig 2) *gives the direction of the resultant force at that point is called a* **line of force.** A line of force in a magnetic field is also defined as a path along which a single free north pole moves when placed in the field.

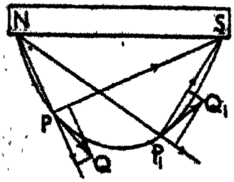


Fig. 2

A line of force may be drawn by means of a small compass needle For any magnet, each line of force arises from a N-pole and terminates on a S-pole. As a compass needle is free to rotate in a horizontal plane, the lines of force drawn correspond tc only the horizontal component of the field. Like field, the lines of force also exist in other planes.

In the case of a bar magnet the lines of force are continuous throughout the material of the magnet (Fig. 3a). When a magnet is bent into the shape of a ring such that two opposite poles are made to touch one another, all lines of force pass round through the ring and no lines of force pass through the air inside it (Fig. 3b).

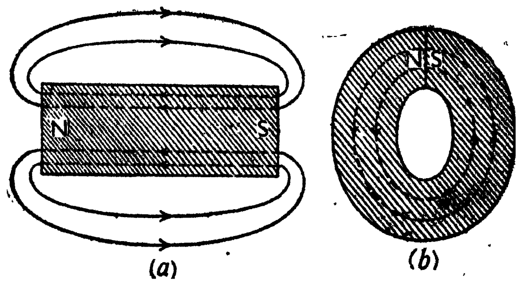


Fig. 3

If lines of force in a magnetic field are parallel, the field is called *uniform.* The lines of force drawn by a compass needle on a sheet of paper fixed on a horizontal board due to earth's field (all magnets and magnetic substances being absent from neighbourhood) are found to be parallel lines extending from geographical south to north. The magnetic field of the earth is therefore uniform in a limited region.

The magnetic effects which occur in a magnetic field, according to Faraday, are due to a strain in the medium consisting of a tension along the lines of force and a pressure at right angles to these directions. The lines of force thus behave as stretched cords.

Note : Two lines of force can never cut one another. For if they cut, then at the point of intersection, two tangents which can be drawn in two different directions will indicate intensity at the said point of intersection, which is absurd (Fig. 4).

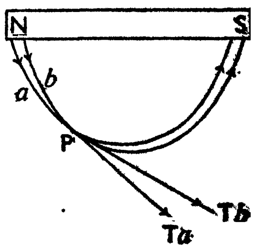


Fig. 4

13. Tube of Force : If a small closed curve is drawn at any region in a magnetic field and a line of force be drawn through every point of it, the lines so drawn will enclose a tubular space termed a *tube of force.* A unit magnetic tube of force is such the magnetic force at any point of the tube multiplied by the cross-section of the tube at that point is equal to unity.

It is supposed that a unit magnetic pole gives out one unit tube of force per square centimetre of the surface of an imaginery sphere of 1 cm. radius, round the pole. To obtain the number of unit tubes of force proceeding from a pole of strength m in air, a sphere of radius r is described round the pole. The intensity of the field at any point P on this sphere is equal to $\frac{m\times 1}{r^2}=\frac{m}{r^2}$. If $\frac{m}{r^2}$ lines of force which radiate uniformly, pass through one sq. cm. round the point P, the total number of lines passing through the surface of the sphere = surface area of the sphere × no. of lines of force per sq. cm. $=4\pi r^2.m/r^2=4\pi m$. The number of lines or tubes of force entering the south pole of same strength m is also $4\pi m$. From this it follows that the number of lines of force proceeding from a pole of strength m is equal to $4\pi m$.

As all these tubes of force come back to the north pole through the material of the magnet there will be evidently $4\pi m$ tubes of force travelling through the magnet, these tubes of force are clearly due to the magnetic property of the magnet.

The intensity of magnetic field at a point from the point of view of tubes of force, may also be expressed by the number of unit tubes of force per unit area placed at right angles to the direction of the lines of force forming the tubes.

14. Flux of Force: If ds be the area in the field and N be the component of intensity perpendicular to ds, then

$$N\times ds=I.$$

Where I is known as the *flux of force*.

In a tube of force, H × S is constant throughout so long as it does not contain magnetisation, where H is the intensity normal to the area S.

The **Gauss** or Oersted is the C. G. S. unit of magnetic field.

The **Maxwell** is the C. G. S. unit of magnetic flux.

15. Mapping of the magnetic field: A bar magnet NS is placed on a sheet of paper (Fig. 5) fixed to a wooden board in such a way that it lies in the magnetic meridian, as determined previously by a small compass needle with its ends pointing north and south. The outline of the magnet is drawn on the paper and is divided by a number of approximately equidistant marks.

The compass needle is now placed with one of its poles S as nearly as possible on one of these marks while the position of the other pole N of the needle is

marked on the paper. The needle is then moved until the first pole S is on the point already marked near the second pole N, and so on until the line traced out reaches the edge of the paper or returns to the magnet. The process is repeated until every mark upon the outline of the magnet is either the beginning or the end of a line of force. Thus the magnetic field is mapped out by a compass needle. A, B and C represent the positions of the compass needle, during the mapping of the field.

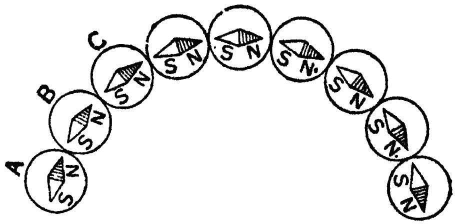


Fig. 5

16. Neutral Points :

In mapping out a magnetic field we really trace out the field due to the resultant action of the magnet and the earth which is itself considered as a big magnet.

Depending on the direction of the axis of a magnet, there are two fixed points in its field where the **magnetic intensity due to the magnet is equal and opposite to the magnetic intensity of the earth's magnetism.** These fixed points are called **neutral points.**

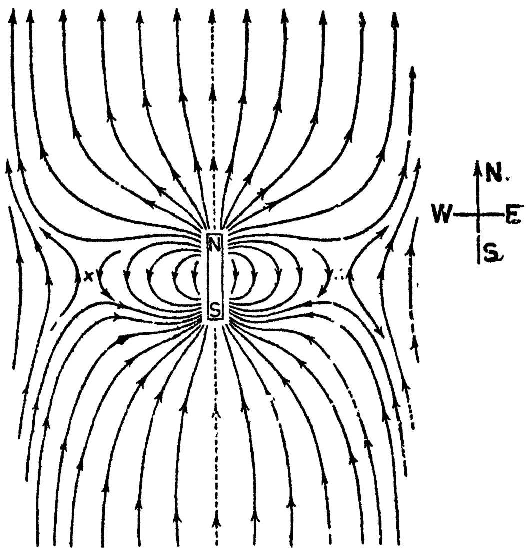


Fig. 6

When this resultant field is mapped out two regions or rather points X, X (Fig. 6) are found in the field in which the forces due to the magnet and the earth exactly neutralise each other and in which a magnetic needle sets itself indifferently in any direction. These regions or points are **Neutral points** in this case. The positions of these neutral points X, X are not fixed, but vary according as the N-pole of the magnet points to the North or South pole of the earth. They are situated on the eastern and western side of the magnet when N-pole points north but when the N-pole points south the neutral points are situated on the axis of the magnet near both

ends of it (Figs. 6, 7). In all cases the two neutral points are equidistant from the centre of the magnet.

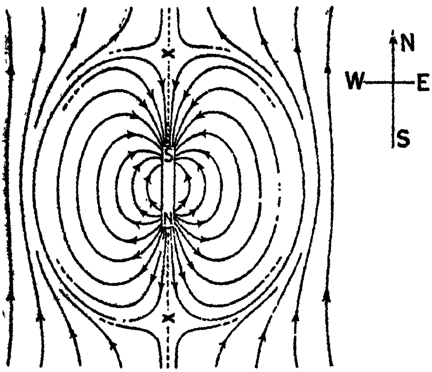


Fig. 7

17. Magnetic Potential : *The magnetic potential at any point is equal to the work done against the magnetic force when a unit north magnet-pole is brought from infinity up to the point.*

The magnetic potential at a point distant r cms. from a north magnetic pole of strength m is equal to $\frac{m}{r}$ (in air).

It is also defined as a quantity whose space rate of variation in any direction is the intensity of the field in that direction.

If V be the potential at any point at a distance x from a magnetic pole, the rate of change of potential as we pass from point to point is equal to $\frac{dV}{dx}$.

Then, by the definition, the strength or intensity of the field at the place considered is given by $F = -\frac{dV}{dx}$, the negative sign indicates that the potential diminishes as the distance x from the pole increases.

The magnetic potential at a point distant r cms. from a north magnetic pole of strength m is determined from the relation

$$F = -\frac{dV}{dx}. \quad \text{Now, work} = \text{Force} \times \text{distance} \quad \text{or} -dV = F.dx$$

$$\text{Hence} \int dV = \int -F.dx \quad \text{or} \quad V = -\int F.dx$$

$$\text{We know that } F = \frac{m}{\mu x^2}. \quad \therefore \quad V = -\int_{\infty}^{r} \frac{m}{\mu x^2} dx = -\frac{m}{\mu}\left[-\frac{1}{x}\right]_{\infty}^{r}$$

$$= -\frac{m}{\mu}\left[-\frac{1}{r} + \frac{1}{\infty}\right] = \frac{m}{\mu r}, \quad \text{since, } \frac{1}{\infty} = 0$$

When $\mu=1$ (in air), $F=\frac{m}{r^2}$ and $V=\frac{m}{r}$.

18. Potential at any point due to a short bar magnet: Let NS be a short magnet of length (Fig 8) $2l$ and pole-strength m and let P be the point at which the potential is to be determined. Join PN and PS and also join P to O, the middle point of the magnet by a straight line which makes an angle θ with the axis of the magnet. Draw perpendiculars, Nn from N on PO, and Ss from S on PO produced. Since for a short magnet, the angles NPO and SPO are very small, we have PN and PS as approximately equal to Pn and Ps respectively.

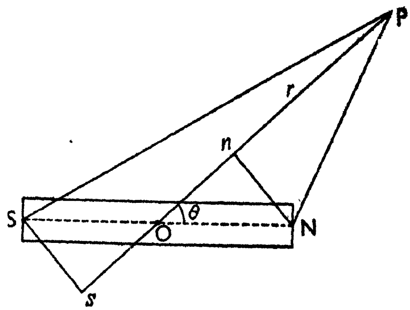


Fig. 8

As triangles ONn and OSs are equal in all respects, On = Os.

The potential at P due to pole m at S $= -\frac{m}{PS}$.

The potential at P due to pole-strength m at N $=\frac{m}{PN}$.

Therefore the resultant potential V at P.

$$\frac{m}{PN}-\frac{m}{PS}=\frac{m}{Pn}-\frac{m}{Ps}=\frac{m}{(PO-On)}-\frac{m}{(PO+Os)}$$

$$=\frac{m}{(PO-On)}-\frac{m}{(PO+On)} \quad \because Os=On.$$

$$=\frac{2m\,On}{PO^2-On^2}=\frac{2ml\cos\theta}{PO^2}, \text{ neglecting } On^2 \text{ since } On \text{ is very small}$$

compared with PO, and putting $l\cos\theta$ for On.

$$V=\frac{M\cos\theta}{PO^2}=\frac{M\cos\theta}{r^2}$$

where M, the magnetic moment of the magnet $=2ml$ (which is the product of the pole strength and the magnetic length of the magnet) and PO $=r$.

Cor. If $\theta=90°$, the potential at a point situated on a straight line at right angles to the axis of the magnet *i.e.* the potential corresponding to broad-side—on position, is zero. This line is called the line of

zero potential. The potential becomes equal to $\frac{M}{r^2}$, when $\theta = 0°$ *i.e.*, when the point P is situated on the axis of the magnet, corresponding to the end-on position.

Cor. *Change of potential* V *with* θ *for a given distance* r.

We have $V = \frac{M\cos\theta}{r^2}$ $\quad \frac{dV}{d\theta} = -\frac{M\sin\theta}{r^2}$ or $dV = -\frac{M\sin\theta}{r^2}d\theta$

or $\int_1^2 dV = -\frac{M}{r^2}\int_{\theta_1}^{\theta_2} \sin\theta d\theta$ *i.e.*, $V_2 - V_1 = \frac{M}{r^2}(\cos\theta_2 - \cos\theta_1)$

18(a). Potential at a point in the End-on position : The end-on position refers to any point on the axis of the magnet. In Figure 9, O is the middle point of a magnet of length $2l$ and moment M and pole strength m, P is a point on the axis at a distance d from O.

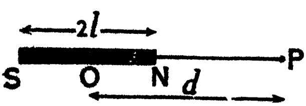


Fig. 9

Potential at P due to $N = \frac{m}{NP} = \frac{m}{d-l}$

Potential at P due to $S = -\frac{m}{SP} = -\frac{m}{(d+l)}$

Resultant potential at $P = \frac{m}{d-l} - \frac{m}{d+l} = \frac{2lm}{d^2-l^2} = \frac{M}{d^2}$,

taking M for $2lm$ and neglecting l^2 for a short magnet.

18(b). Potential at a point in Broad-side-on position : The broad-side-on position refers to any point on the perpendicular bisector of the magnet. In Figure 10, O is the centre of a short magnet NS of length $2l$, pole strength m and moment M. P is a point on the perpendicular bisector of NS, at distance d from O.

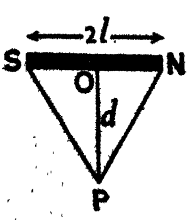


Fig. 10

Potential at P due to N pole $= \frac{m}{NP}$

Potential at P due to S pole $= -\frac{m}{SP}$.

Resultant potential at $P = \frac{m}{NP} - \frac{m}{SP} = 0$,

$\because$ SP = NP

19. Intensity at any point P along any direction OP: Let the point P (Fig. 8) be situated at a distance r from the middle point O of the magnet NS making and angle θ with its axis.

The potential V at P is given by $V = \frac{M\cos\theta}{r^2}$ [Art. 18]

$$\therefore \text{ the intensity, } F = -\frac{dV}{dr} \quad -\frac{d}{dr}\left(\frac{M\cos\theta}{r^2}\right)$$

$$-M\cos\theta\frac{a}{dr}\left(\frac{1}{r^2}\right) : \cdot 2M\cos\theta$$

where M is the magnetic moment of the magent NS.

20. Magnetic moment of a magnet: It is the product of the strength of one of its poles and the distance between them.

If m be the strength of the pole of a magnet of length $2l$, the magnetic moment $M = m \times 2l$.

It is also equal to the couple or rather the moment of the couple required to hold a magnet at right angles to a field of unit intensity.

The moment of a magnet may be resolved like a force. Hence to determine the action of any magnet at an external point, the original magnet may be replaced by a series of magnets, the original moment being the resultant of the moments by which it is replaced.

21. Moment of a magnet resolved like a force: We can prove that potential at a point P due to a short magnet of magnetic moment M is $= \frac{M\cos\theta}{r^2}$ where θ is the angle made by the line joining the point and the centre of the magnet with the axis of the magnet, and r the distance of the point from the centre of the magnet. Again we also know that the potential at any point in the end-on position is $\frac{M}{\cdot}$ where M is moment of the magnet and r the distance of the point from the centre of the magnet. We know also that the potential at any point in the broad-side-on position, due to a short magnet is zero.

Let us now suppose that the moment M of magnet NS be resolved into two components (Fig. 11) one along OP, whose value is $M\cos\theta$, and the other component perpendicular to OP, whose value is $M\sin\theta$.

As P corresponds to end-on position, with respect to the component magnet N_1S_1 of moment $M\cos\theta$, potential at P due to it $=\frac{M\cos\theta}{r^2}$.

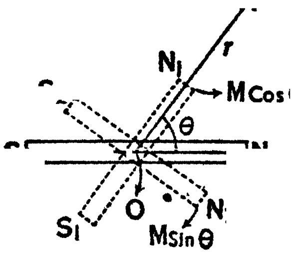


Fig. 11

Again as P corresponds to broad-side-on position, with respect to the component magnet N_2S_2 of moment $M\sin\theta$, the potential at P due to it is zero.

∴ Resultant potential at

$$P=\frac{M\cos\theta}{r^2}+0=\frac{M\cos\theta}{r^2}.$$

The agreement of this with the previous result justifies the conclusion that the moment of a magnet can be resolved like a force.

Note: The above manner of justification is not necessary, since magnetic moment having both magnitude and direction is a vector quantity and such may be resolved or compounded like all othor vector quantities, such as force, velocity etc.

The resultant magnetic moments of two magnets having moments M_1 and M_2 fixed on a common support with angle α between the axis is given by $M=\sqrt{M_1^2+M_2^2+2M_1M_2\cos\alpha}$

If the axes are parallel $M=M_1+M_2$

If the axes are at right angles $M=\sqrt{M_1^2+M_2^2}$

22. Intensity of magnetisation : *It is the magnetic moment of a magnet per unit volume.*

If the magnetic moment of a magnet of cross-section s and of length $2l$ be M, then the intensity of magnetisation *i.e.*, the moment per unit volume is given by $I=\frac{M}{2ls}=\frac{2lm}{2ls}=\frac{m}{s}$

Thus, the *intensity of magnetisation is also equal to the pole-strength per unit area of cross-section.*

It is to be regarded as having direction as well as magnitude, its direction being that of the axis of the magnet.

23. Field strength or Intensity due to a bar magnet :

(a). At any point on the axis of the magnet produced (End-on Position) : Consider a point P (Fig. 12) on the line passing through the poles of a magnet of length $2l$ and pole-strength m

and situated at a distance d from the middle point and in front of the N-pole of the magnet.

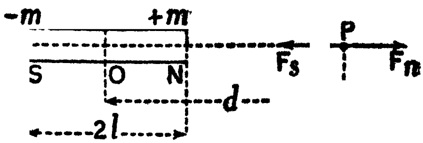


Fig. 12

If a unit N-pole be supposed to be placed at P, then the force on the unit pole due to the *north pole* of the magnet $=\frac{m}{(d-l)^2}$; and force on the unit north pole due to *south pole* of the magnet $= -\frac{m}{(d+l)^2}$. Since the forces are in the same direction but have opposite senses the resultant force on the unit north pole due to the magnet

$$= m\left\{\frac{1}{(d-l)^2}-\frac{1}{(d+l)^2}\right\}=\frac{4mld}{(d^2-l^2)^2}=\frac{2Md}{(d^2-l^2)^2}$$

since $M=2ml.$ = Moment of the magnet.

Therefore the strength of the field *i.e.*, the force on unit pole or simply intensity at P is equal $\frac{2Md}{(d^2-l^2)^2}$ Gauss approximately.

If the point P be at a great distance from the magnet, and magnet be very short, l^2 is neglected in comparison with d^2 and hence the strength of the field at P becomes equal to $\frac{2M}{d^3}$ Gauss.

Thus, intensity at a point due to a very short magnet at the end-on position varies inversely as the cube of the distance of the point from the centre of the magnet.

This is the **cube law** of distance.

(b). At any point on the perpendicular bisector of the magnet (Broad-side-on Position) :

Consider a point P, at which a unit N pole is supposed to be placed on the line bisecting the magnet NS at right angles and at a distances d (Fig. 13) from the middle point of the magnet.

Force F_s due to S pole at P on unit N pole $= -\frac{m}{SP^2} = -\frac{m}{(d^2+l^2)}$

in the direction PB.

Force F_n due to N pole at P on unit N pole $=\frac{m}{NP^2}=\frac{m}{(d^2+l^2)}$

in direction PA.

Since PN = PS, the two forces F_n and F_s are equal in magnitude, the diagonal PK which gives the direction of the resultant F parallel to NS, bisects the angle APB contained by these forces. Now the triangles PKB and SPN are similar so that by triangle of forces,

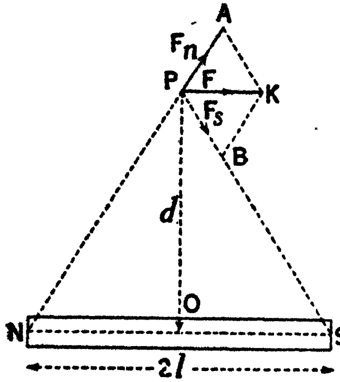


Fig. 13

$$\frac{F}{F_s} = \frac{NS}{SP}, \text{ or } F = \frac{NS}{SP} \cdot F_s = \frac{NS}{SP} \cdot \frac{m}{SP^2}$$

or $F = \dfrac{NS.m}{SP^3}$; but $NS = 2l$,

and $SP^3 = (d^2 + l^2)^{\frac{3}{2}}$

$$\text{Intensity F at P} = \frac{2l.m}{(d^2 + l^2)^{\frac{3}{2}}}$$

$$= \frac{M}{(d^2 + l^2)^{\frac{3}{2}}}, \text{ where } 2lm = M, \text{ the}$$

moment of the magnet.

For a short magnet, l is very small in comparison with d,

$\therefore$ the strength (or intensity) of the field at P $= \dfrac{M}{d^3}$ Gauss.

The **cube law** also holds in this case.

24. Field strength or Intensity due to a magnet at any point : Let NS be magnet of magnetic moment M and let A be the point at which the intensity is required. Let AO make an angle θ with the axis of the magnet NS. (Fig. 14)

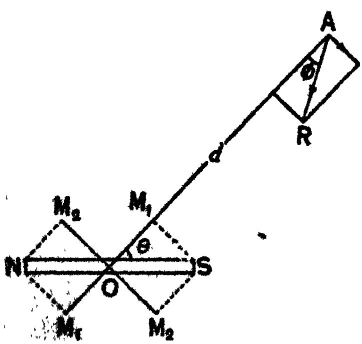


Fig. 14

We know that the effect of the magnet NS at the point A is equivalent to those of two magnets, one M_1OM_1 having its axis along OA and of moment $M\cos\theta$, and the other M_2OM_2 having axis at right angles to AO and of moment $M\sin\theta$.

Let OA be equal to d. Then the intensity at the point A on the axis of the magnet M_1OM_1 of moment $M\cos\theta$ and situated at a distance d from its middle point is by the theory of end-on position,

$=\frac{2M\cos\theta}{d^3}$ along OA, and the intensity at the point A due to the magnet M_2OM_2 of moment $M\sin\theta$ is by the theory of broad-side-on position, $=\frac{M\sin\theta}{d^3}$ at right angles to OA.

Since these component intensities are at right angles to each other, the magnitude of the resultant intensity AR at A is

$$=\sqrt{\frac{4M^2\cos^2\theta}{d^6}+\frac{M^2\sin^2\theta}{d^6}}$$

$$=\frac{M}{d^3}\sqrt{(4\cos^2\theta+\sin^2\theta)}=\frac{M}{d^3}\sqrt{(1+3\cos^2\theta)}$$

Special cases :—1. If $\theta=0$, or π *i.e.*, at a point along the axis the intensity becomes equal to $\frac{2M}{d^3}$ and is greatest,

2. Again if $\theta=\frac{\pi}{2}$ or $\frac{3\pi}{2}$ *i.e.*, at a point on the line at right angles to the axis and through the centre of the magnet, the intensity becomes equal to $\frac{M}{d^3}$ and is least. The maximum value is thus twice the minimum value.

24(a). Alternative Method : Form the definition of potential, the intensity $F = -\frac{dV}{dr}$

But we know that the potential at a point in the field due to a magnet situated at a distance r from the middle point of the magnet and making an angle θ with the axis of the magnet is expressed by $V=\frac{M\cos\theta}{r^2}$, where M is the magnetic moment of the magnet NS.

Therefore $F_1=-\frac{d}{dr}\left(\frac{M\cos\theta}{r^2}\right)=-M\cos\theta\,\frac{d}{dr}\left(\frac{1}{r^2}\right)$

$$=\frac{2M\cos\theta}{r^3}\text{ along OA (Fig. 10)}$$

Again $F_2=-\frac{1}{r}\cdot\frac{d}{d\theta}\left(\frac{M\cos\theta}{r^2}\right)=\frac{M\sin\theta}{r^3}$ **at right angles to OA.**

where F_1 and F_2 are the component forces along OA and at right angles to OA respectively.

Then the magnitude of the resultant intensity at A is equal to

$$\sqrt{F_1{}^2+F_2{}^2}=\sqrt{\left(\frac{2M\cos\theta}{r^3}\right)+\left(\frac{M\sin\theta}{r^3}\right)^2}=\frac{M}{r^3}\sqrt{1+3\cos^2\theta}.$$

Let the resultant force AR at A make an angle ϕ with OA.

$$\tan\phi=\frac{\dfrac{M\sin\theta}{r^3}}{\dfrac{2M\cos\theta}{r^3}}=\tfrac{1}{2}\tan\theta$$

25. Force between two magnets : If a a magnet be placed in a uniform magnetic field as produced by the earth's magnetism, the resultant force upon it vanishes. This is evident by the fact that when a magnet is placed on a piece of cork and made to float on a basin of water, there is no tendency of the magnet to move towards one side of the basin, it simply rotates so as to point in a definite direction, which is nearly north and south.

But if the magnet be placed in the neighbourhood of another magnet, the field is not uniform and consequently there will be a resultant force.

We are to find the resultant force on the magnet at two known positions.

Case I. *When the magnet is placed on the axis of another magnet.*

Let M′ be the magnetic moment of the first magnet N_1S_1 and let a second magnet N_2S_2 (Fig. 15) of moment M and pole strength

Fig. 15

m be placed with its middle point at a distance d from the middle point of the first magnet. Let $2l$ be the length of the second magnet.

Then the force exerted by the first magnet on the S_2 pole of strength m of the second magnet at a distance $(d-l)$ is $\dfrac{2M'm}{(d-l)^3}$, since it corresponds to the end-on position.

Similarly the force exerted on N_2 pole of strength m at a distance $(d+l)$ is $=\frac{2M'm}{(d+l)^3}$

Since these two forces act in opposite direcions, the resultant force R exerted by the first magnet on the second magnet is given by $R=\frac{2M'm}{(d-l)^3}-\frac{2M'm}{(d+l)^3}$

$$=2M'm\left\{\frac{1}{(d-l)^3}-\frac{1}{(d+l)^3}\right\}=2M'm\left\{\frac{2l^3+d^2l}{(d^2-l^2)^3}\right\}$$

$$=2M'm2l\left\{\frac{l^2+3d^2}{(d^2-l^2)^3}\right\}=\frac{2MM'3d^2}{}=\frac{6MM'}{d^4}$$

Here l is taken to be very small in comparison with d and $M=2ml$.

Case II. When the magnet N_2S_2 is placed on a straight line at right angles to the axis of the magnet N_1S_1 (Fig. 16) with its middle point at a distance d from the middle point of the magnet N_1S_1 :

The force exerted by the magnet N_1S_1 on the pole N_2 of strength m of the magnet N_2S_2at distance $(d-l)$ is $\frac{M'm}{(d-l)^3}$ acting in a direction parallel to direction $\overrightarrow{N_1S_1}$, since N_2 corresponds to the broad-side-on position with respect to N_1S_1.

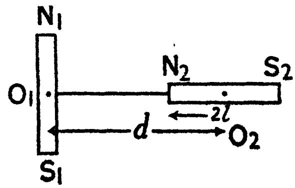


Fig. 16

Similarly the force exerted on the pole S_2 of strength m of the magnet N_2S_2 at a distance $(d+l)$ is $-\frac{M'm}{(d+l)^3}$ acting in a direction parallel to the direction $\overrightarrow{S_1N_1}$.

Therefore the resultant force R exerted by the first magnet on the second magnet is given by $R=\frac{M'm}{(d-l)^3}-\frac{M'm}{(d+l)^3}$

$$=M'm\left\{\frac{1}{(d-l)^3}-\frac{1}{(d+l)^3}\right\}=M'm\left\{\frac{2l^3+6d^2l}{(d^2-l^2)^3}\right\}$$

$$M'm.2l\left\{\frac{l^2+3d^2}{(d^2-l^2)^3}\right\}=\frac{MM'.3d^2}{d^6}=\frac{3MM'}{d^4},$$

Here l being very small in comparison with d, is neglected.

Thus comparing the two cases we see that the force exerted on the second magnet in the first case is double the force exerted in the second case.

26. Couple acting on a magnet in a uniform field : Let N′S′ be the magnetic meridian and let a magnet NS of length $2l$

and pole-strength m be placed (Fig. 17) in uniform field of strength H due to earth. Suppose the magnet is deflected through an angle θ from the magnetic meridian.

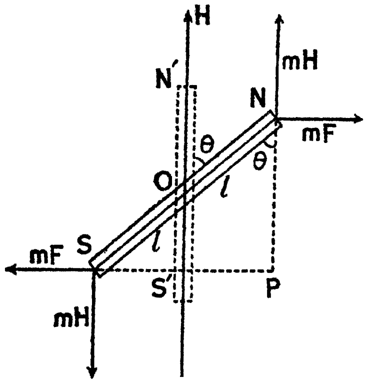


Fig. 17

The two forces acting on the poles of the magnet are each equal to mH and form a couple since they are parallel and point towards the north and south pole of the earth.

The moment of this couple when the angle of deflection is θ is $m\text{H} \times \text{SP} = m\text{H}.2l \sin\theta = \text{MH}\sin\theta$, since $2lm =$ moment M of the magnet.

Now if $\text{H} = 1$ and $\theta = 90°$, the moment of the couple reduces to M, the moment of the magnet.

Thus we see that the moment of a magnet is the same as the moment of the couple required to hold a magnet at right angles to a uniform field of unit intensity.

Let us suppose that another magnetic field of intensity F acts simultaneously on NS in a direction perpendicular to H. This constitutes a couple called deflecting couple, the couple due to mH which tends to restore the magnet to magnetic meridian is the controlling couple. For equilibrium of the magnet at an angle θ with the direction of H, the couple due to $m\text{F}$ = couple due to mH

$\therefore \quad m\text{F} \times \text{NP} = m\text{H} \times \text{SP}$ or $m\text{F}.\ 2l \cos\theta = m\text{H}.\ 2l \sin\theta$

$$\text{or F} = \frac{\text{H}\sin\theta}{\cos\theta} = \text{H} \tan\theta \text{ or } \frac{\text{F}}{\text{H}} = \tan\theta$$

Thus the ratio of the deflecting fi ld to the controlling field is the tangent of angle deflection of the magnet. This is Tangent law.

Note :—In a *uniform* field the magnet has no motion of translation but a couple acts upon it tending it to turn round and set itself with its axis parallel to the field. In a *non-uniform field*, the two poles of the magnet placed in it experience *unequal forces* and so they not only tend to rotate it but also tend to move it bodily. So the magnet in a non-uniform field sets itself with its axis parallel to the field.

27. Work done in deflecting a magnet :

The work done in turning (Fig. 18) the magnet so that the angle made with H by its axis changes from θ to $\theta + d\theta$ is $cd\theta$, where c is the moment of the couple when the angle of deflection is θ.

The total work done in moving it from the equilibrium position to angle θ

$$\int_0^\theta c\,d\theta = \int_0^\theta \text{MH} \sin\theta d\theta$$

$\because$ $c = \text{MH}\sin\theta$

$= \text{MH}\ (1 - \cos\theta)$

So when $\theta = 90^\circ$, work done $=$ MH

when $\theta = 180^\circ$, work done $=$ 2MH

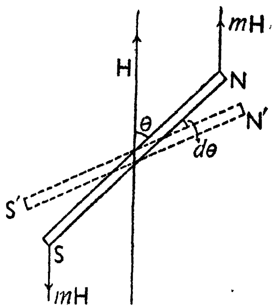


Fig. 18

28. Couple between two magnets in two special positions :

(1) Tangent A position corresponding to end on-position : In this position the magnet is situated so that its axis is perpendicular to the magnetic meridian and in line with the centre of the second magnet which is kept in a suspended position.

Let the strength of the field due to the first magnet, in which the second magnet (Fig. 19) is suspended be F. Since F acts parallel to the axis of the first magnet *i.e.*, perpendicular to the meridian, the two forces each of value mF acting on the ends of the suspended magnet in opposite directions are parallel and therefore form a couple.

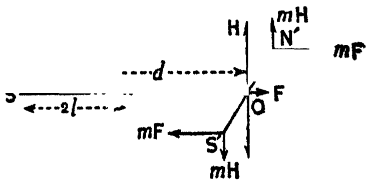


Fig. 19

This couple acting on the suspended magnet tries to place it at right angles to the magnetic meridian, but the couple due to earth's magnetic field tends to rotate it into the magnetic meridian. The resultant effect of these two couples is to place the magnet in an equilibrium position at an angle θ with the meridian when the moment of the two couples are equal and opposite.

The moment of the couple of forces mF, mF, $= m\text{F} \times 2l' \cos\theta$.

The moment of the couple of forces mH, $m\text{H} = m\text{H} \times 2l'\sin\theta$ where H is the intensity of the earth's magnetic field ,$2l'$ the length of the suspended magnet.

In equilibrium position, $m\text{F} \times 2l' \cos\theta = m\text{H} \times 2l \sin\theta$

or $\quad \text{F} = \text{H} \tan\theta$

We know that the strength of the field F due to a magnet of length $2l$ and pole-strength m in the End-on position is equal

to $\frac{2Md}{(d^2-l^2)^2}$. Therefore $\frac{2Md}{(d^2-l^2)^2} = H\tan\theta$

or $\frac{M}{H} = \frac{(d^2-l^2)^2}{2d}\tan\theta = \frac{d^3}{2}\tan\theta$, for a short magnet.

(2) Tangent B position corresponding to Broad-side-on Position : In this position (Fig. 20) the suspended magnet is so placed that its axis is on the line bisecting the axis of the other right angles.

We know that in the broad-side-on position the strength of the field due to a magnet of length $2l$ and pole strength m at points near the suspended magnet is equal to $\frac{M}{(d^2+l^2)^{\frac{3}{2}}}$

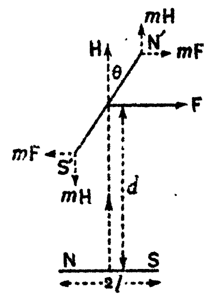


Fig. 20

Therefore, when the suspended magnet is in equilibrium at angle θ with the meridian we have as before $F = H\tan\theta$

That is $\frac{M}{(d^2+l^2)^{\frac{3}{2}}} = H\tan\theta$

or $\frac{M}{H} = (d^2+l^2)^{\frac{3}{2}}\tan\theta = d^3\tan\theta$, for a short magnet.

(3) Sine Position : In this position the deflecting magnet is so placed that its axis is perpendicular to that of the suspended magnet. This is done by rotating the whole instrument with the deflecting magnet in the Tan A position.

Here $\frac{2Md}{(d^2-l^2)^2} = H\sin\theta$

or $\frac{M}{H} = \frac{(d^2-l^2)^2}{2d}.\sin\theta = \frac{d^3}{2}\sin\theta$, for a short magnet.

29. Gauss's method of proving the Inverse Square Law : Gauss at first assumed the truth of the Inverse Square Law and deduced mathematically the values of the couples exerted by a magnet on a needle in two positions known as the Tangent A and the Tangent B position, and then he verified the law indirectly from the results obtained in the magnetometer experiment.

The magnetometer is so arranged that the magnet is placed on one of its arms at a distance of d *cms* from the magnetometer needle in the Tangent A position.

In this position $\frac{M}{H} = \frac{(d^2 - l^2)^2}{2d} \tan\theta_1$

$$= \frac{u}{2} \tan\theta_1, \text{ for a short magnet} \qquad (1)$$

The magnetometer is then rotated so that the arms are in the magnetic meridian and the magnet is placed at a distance of *d cms.* from the needle with its broad side towards the needle.

In this Tangent B position we have,

$$\frac{M}{H} = (d^2 + l^2)^{\frac{3}{2}} \tan\theta_2 = d^3 \tan\theta_2, \text{ for a short magnet} \quad \ldots \quad (2)$$

Thus from (1) and (2) we have $\frac{\tan\theta_1}{\tan\theta_2} = 2.$

The formulæ (1) and (2) are derived on the assumption of the truth of the Inverse Square Law. Hence if it can be proved that the ratio of (1) and (2) is 2 : 1, the law is verified.

If the law varies inversely as any other power of the distance, say n, the ratio $\frac{\tan\theta_1}{\tan\theta_2}$ would be n instead of 2.

But from experiments in the two positions $\frac{\tan\theta_1}{\tan\theta_2}$ has been found to be equal to 2. Hence the inverse Square Law is verified.

The experiment of Gauss proved the validity of the law with a fair degree of accuracy.

The significance of the Inverse square law lies in the fact that effects calculated on the assumption of its truth are in accordance with experimental results.

30. Magnetometer : It consists (Fig. 21)of a small magnetic needle pivoted or suspended at the centre of a circular scale which is placed

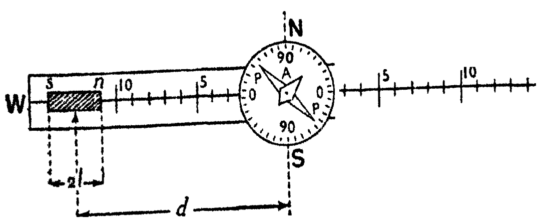


Fig. 21

over the middle part of wooden board. The needle and the circular scale are generally enclosed in a circular brass box with a glass cover.

A metre scale is fixed on each of the arms of the board prolonged on the opposite sides of the circular scale so that the centre of the suspended needle is just on the zero marks on the metre scales. At right angles to the length of the needle there is a light pointer to indicate the deflections of the needle.

30(a). Adjustments for correct deflections :

In all magnetometer experiments the following adjustments are generally made.

1. Both ends of the pointer should point to $0°-0°$ of the circular scale when there is no deflecting magnet near the instrument. This is done by turning round and levelling the instrument.

2. The point of suspension *i.e.*, the needle point on which the needle is supported may not be at the centre of the circular scale and therefore the deflections indicated by both ends of the pointer should be observed.

3. The deflecting magnet may not be symmetrically magnetised. This error is eliminated by turning the magnet over so that its N and S poles change places and deflections for both ends of the pointer are observed.

4. The point of suspension *i.e.*, the middle point of the suspended needle may not be at the zero of the long straight scale. To correct this error the magnet is now removed to an equal distance according to the scale on the other side of the needle and the deflections are again observed.

The mean of these 8 deflections gives the true deflection of the magnetometer needle due to any deflecting magnet placed outside the instrument.

31. Comparison of magnetic moments :

(1) By Deflection method :

A magnet of moment M_1 is placed at a distance of d_1 on one of the arms of a magnetometer so that its middle point coincides with some fixed mark on the same metre scale, the magnetometer being so arranged that the arms point in a direction as right angles to the magnetic meridian *i,e.*, in the Tangent A position. The needle is deflected through an angle θ_1. Another magnet of magnetic moment M_2 is placed at a different distance d_2 and deflection of the needle in this case is θ_2.

We know that

For the first magnet $\quad \dfrac{M_1}{H}=\dfrac{d_1^3}{2}\tan\theta_1 \quad ... \quad (1)$

For the second magnet $\quad \dfrac{M_2}{H}=\dfrac{d_2^3}{2}\tan\theta_2 \quad ... \quad (2)$

Dividing (1) by (2) $\quad \dfrac{M_1}{M_2}=\dfrac{d_1^3\tan\theta_1}{d_2^3\tan\theta_2}$

If more exact results are required, the general expression for $\dfrac{M}{H}$ should be used. The deflections θ_1 and θ_2 are corrected and the mean values for θ_1 and θ_2 are obtained according to methods described before.

(2) Null method :

The second magnet of moment M_2 is placed on the other side of the needle at such a distance d_2 that the deflection produced by the first magnet at a distance d_1 is completely annulled and the pointer points to $0° - 0°$ of the circular scale.

In this case

For the first magnet $\frac{M_1}{H} = \frac{d_1^3}{2} \tan \theta_1$ (1)

For the second magnet $\frac{M_2}{H} = \frac{d_2^3}{2} \tan \theta_1$ (1)

From (1) and (2) $\frac{M_1}{M_2} = \frac{d_1^3}{d_2^3}$.

The experiment can be repeated with several values of d_1 and the corresponing values of d_2.

(3) Neutral Point Method :

A short bar magnet is placed in the magnetic meridian with its N pole pointing towards the north on a white sheet of paper and the lines of forces in the field are traced by means of a small compass needle. Two neutral points are observed on both the sides of the magnetic axis where the horizontal component due to earth's magnetic field is balanced exactly by the magnetic force of the magnet.

Thus if H be the horizontal component of the earth's magnetic field and F, the field due to the magnet at the neutral points, then F=H.

But since $F = \frac{M_1}{d_1^3}$, where d_1 is the distance of each neutral point which corresponds to broad-side-on position, from the middle point of the magnet of moment M_1 we have $\frac{M_1}{d_1^3} = H$ or $M_1 = Hd_1^3$

For another magnet of moment M_2, we have similarly, $M_2 = Hd_2^3$.

$$\therefore \frac{M_1}{M_2} = \frac{d_1^3}{d_2^3}.$$

Note : When the magnet is placed in the magnetic meridian with its N-pole pointing towards the south, two neutral points are observed on the axis of the magnet produced and near the ends of it.

At these two neutral points, we have F=H

But since in this position (Tangent A position) $F = \frac{2M}{d^3}$, where d is the distance of the neutral points from the middle point of the magnet, we have $\frac{2M}{d^3} = H$ or $M = \frac{Hd^3}{2}$.

Again from the knowledge of H and d, the magnetic moment M can be easily calculated.

Note: For a very accurate work a reflecting magnetometer is generally used. In this form a concave mirror is attached to the needle which consists of a few pieces of magnetised watch spring and the whole system is suspended by a torsionless thread over the centre of a graduated circular scale enclosed in a glass cylindrical box. A beam of light from a lamp stikes the mirror and the angle through which the reflected beam moves over a scale placed below the lamp is twice the angle of deflection θ of the needle which is given by $\theta = \frac{d}{2D}$ where d is the distance through which the spot of light moves over the scale and D, the distance of the scale from the mirror.

(4) Oscillation Method :

Two magnets of different magnetic moments are made to oscillate alternately and the time of oscillation T is determined in each case.

If M_1 and M_2, I_1 and I_2 and T_1 and T_2 are respectively the magnetic moments, the moments of inertia and the times of a single oscillation for the two magnets, then from the formula

$T = 2\pi\sqrt{\frac{I}{MH}}$ where H is the intensity of the earth's field,

we get the relation $\frac{M_1}{M_2} = \frac{I_1}{I_2} \times \frac{T_2^2}{T_1^2}$

and with its help the magnetic moments of the magnets are easily compared.

32. Factors on which the periodic time of a vibration magnetometer depends: The periodic time (T) of vibration of a magnetic needle is given by $T = 2\pi\sqrt{\frac{I}{MH}}$, where I is the moment of inertia of the needle, M, the magnetic moment of the needle and H, the horizontal component of earth's magnetic field.

Hence, periodic time of vibration of a magnetic needle depends on (i) moment of inertia of the needle, (ii) moment of the needle, (iii) horizontal intensity of earth, being directly proportional to the square root of the former and inversely proportional to the square root of each of the latter.

32(a). Precautions in the vibration magnetometer experiment. The following precautions are to be taken in using the instrument.

1. The needle should be enclosed in a wooden box with glass windows to protect it from the disturbing effects of air current on the oscillations of the needle.

2. The needle should be suspended horizontally from a torsion head fitted at the top of a vertical cylindrical glass tube fixed to the upper side of the wooden box by a *fine unspun torsionless silk thread*. Any torsion in the thread will cause irregular oscillations and vitiate the result.

3. The instrument should be properly levelled by the levelling screws at the base of the instrument.

4. The needle should be placed in the magnetic meridian.

5. To start the oscillations in the needle, an auxiliary magnet is to be taken near the box and then taken away.

[**Note**: The value of I the moment of inertia of any magnet is obtained from the expression $I = M\left(\frac{a^2+b^2}{12}\right)$, where M is the mass of the magnet (rectangular) and a and b are the adjacent edges of the faces through which the axis of rotation passes.

33. Proof of the relation $T = 2\pi\sqrt{\frac{I}{MH}}$

The Formula for the simple pendulum is

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{ml^2}{mgl}} = 2\pi\sqrt{\frac{I}{mlg}} \qquad (1)$$

where I or ml^2 is the moment of inertia of the magnet.

The product mg in the expression for the simple pendulum is replaced by an attraction of one pole $+m$H and a repulsion $-m$H of the other *i.e.*, by a total force $2m$H.

So the expression (1) becomes

$$T = 2\pi\sqrt{\frac{I}{2mHl}} = 2\pi\sqrt{\frac{I}{MH}} \text{ since } M = 2ml.$$

or $\frac{1}{n} = 2\pi\sqrt{\frac{I}{MH}}$ where n is the frequency, *i.e.*, the number of oscillations per second.

$\therefore \quad n^2 \propto H$

Thus the intensity at any point in the magnetic field is proportional to n^2.

33(a). Direct proof :

When a magnet oscillating freely in a field of uniform intensity H is displaced through a small angle θ (Fig. 22) from its equilibrium position, a couple acting on it tending to restore it to its original position is of moment $MH\sin\theta$.

But the couple acting on a rigid body when it rotates is of moment $I.\frac{d^2\theta}{dt^2}$, where I is the moment of inertia of the body and $\frac{d^2\theta}{dt^2}$ is the angular acceleration.

Then the equation of motion of the magnet is given by

$$I.\frac{d^2\theta}{dt^2} = -MH\sin\theta$$

The negative sign indicates that the couple MH $\sin\theta$ acts in a direction tending to decrease the angle of rotation.

When θ is very small, $\sin\theta = \theta$.

Therefore $I.\frac{d^2\theta}{dt^2} = -MH\theta$

$$\frac{d^2\theta}{dt^2} = -\frac{MH}{I}\theta$$

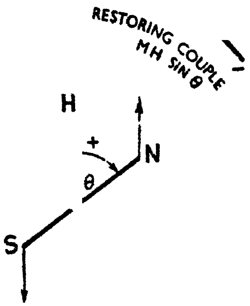


Fig. 22

$$\therefore \quad \frac{d^2\theta}{dt^2} = -\omega^2\theta, \text{ where } \omega^2 = \frac{MH}{I}.$$

That is, angular acceleration is proportional to the angular displacement.

Hence, the motion of the magnet is simple Harmonic and the period of osciliation, T of the magnet is given by

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{MH}}$$

Note : In the **deflection and null methods**, the result will be affected if a magnet be brought from outside, as the intensity of magnetic field as well as the relative position of the needle with respect to the line on which the experimental magnets are to be placed, will change. But if the external magnet be far away from the magnetometer, the field may remain unaltered and the result may not be affected if the relative positions of the needle and the line be maintained the same by rotating the magnetometer.

In **neutral-point method**, the result will be affected by the presence of a magnet in the neighbourhood as the field and consequently the positions of the neutral points will change and therefore the formula used in this method will not be applicable.

In the **Oscillation method**, the result will remain unaffected as the field over which the magnets oscillate although altered due to an external magnet, remains the same in both the cases and affects the time of oscillation equally.

34. Determination of Earth's horizontal Intensity (H) : To determine H, two experiments one with deflection magnetometer, and another with oscillation magnetometer are to be performed, for the same magnet as before.

Let M be the moment of a magnet of length $2l$ and let a deflection θ of a small suspended magnet placed in the tan A position (say) be noted. If the distance of the suspended magnet from the centre of the first magnet be d, then for equilibrium of the suspended magnet.

$$\frac{2Md}{(d^2-l^2)^2}=H\tan\theta\ldots(1),$$ where H = earth's horizontal intensity.

The period of oscillation of the same magnet of moment M is found by an oscillation magnetometer. The period T is given by

$$T=2\pi\sqrt{\frac{I}{MH}}\ldots(2),$$ where I = moment of inertia of the magnet.

The relation (1) and (2) can be written respectively as

$$\frac{M}{H}=\frac{(d^2-l^2)^2}{2d},\ \tan\theta \qquad \ldots(1a)$$

$$\text{and}\quad MH=4\pi^2.\frac{I}{T^2} \qquad \ldots(2a)$$

Eliminating M from ($1a$) and ($2a$) the value of H can be found out. The value of I can be obtained from a knowledge of its mass, length and breadth. The value of M also can determined by eliminating H from ($1a$) and ($2a$).

35. Magnetic Shell: If A thin sheet or lamina of a magnetic substance be magnetised at right-angles to its surface so that north pole is developed over one face and south pole over the other, it is called a magnetic shell. A magnetic shell may be either plane or curved and of unequal thickness (Fig. 23).

If σ be the surface density of magnetism *i.e.*, pole strength per unit area of the shell and t its thickness then strength of the shell $\phi=\sigma\times t$.

Thus, strength of the shell is defined as the product of the pole strength per unit area and the thickness of the shell.

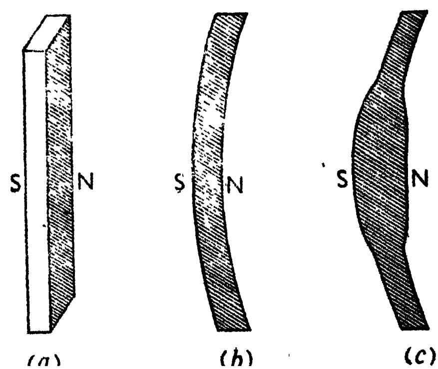


Fig. 23

Again if A be the area of each face of the shell, the moment of the shell $=\phi.A=\sigma\times t\times A$.

If the strength of the shell $(\sigma \times t)$ be same at all points on the surface, the shell is called uniform. For a magnetic shell to be uniform, neither t nor σ need be constant individually, at all points of the shell; what is necessary is that the product $\sigma \times t$ must be constant.

35(a). Potential at a point due to Magnetic Shell. Let ABCD be a small portion of a uniform magnetic shell of area A and thickness t, the face AB having north and the face CD south polarity. P is a point at distances d_1 and d_2 from the faces A and C respectively (Fig. 24).

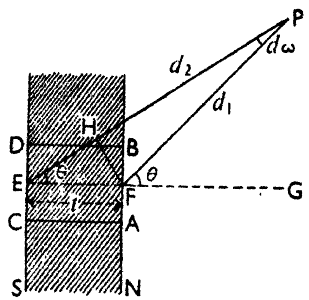


Fig. 24

Draw EFG perpendicular to the faces and through the centre of the faces BA and DC. Join PE and PF. Let θ be the angle PFG. Then since the thickness of the shell is very small angle PEF $= \theta$.

If $d\omega$ be solid angle subtended by the face AB of the shell, then

$$d\omega = \frac{\text{Area of symmetrical base FH}}{d_1^{\,2}}$$

$$= \frac{A \cos\theta}{d_1^{\,2}}, \qquad \text{or } d_1^{\,2} = \frac{A \cos\theta}{d\omega} \qquad ..(1)$$

If m be the quantity of magnetism on each face, resultant potential at P is given by

$$V = \frac{m}{d_1} - \frac{m}{d_2} = m.\frac{d_2 - d_1}{d_1 d_2} \qquad ...(2)$$

But $d_2 - d_1 = t\cos\theta$ (approximately), and $d_1 d_2 = d_1^{\,2}$ (approximately)

Hence, resultant potential $V = \dfrac{m \times t \cos\theta}{d_1^{\,2}}$

$$\text{or} \quad V = \frac{m \times t \cos\theta}{A \cos\theta} \times d\omega = \frac{m}{A} \times t \times d\omega$$

But $m/A = \sigma$, the surface density of magnetism.

$\therefore$ $V = \sigma \times t \times d\omega = \phi . d\omega$, since $\sigma \times t = \phi$ the strength of the shell.

Then the resultant potential at P due to the whole shell

$= \int \phi d\omega = \phi\omega$, where ω is the solid angle subtended at P by the boundary of the whole shell.

If the point P faces the south pole of the shell, the potential at P will be $-\phi\omega$.

Thus the potential at any point due to a Magnetic shell is equal to the product of the strength of the shell and the solid angle subtended at the point by the boundary of the shell.

Note : It is to be noted that for uniform magnetic shell of given strength ϕ, the potential at a point is proportional to the solid angle and hence the area of the face of the shell.

The magnetic potential V due to any distribution of poles is changed from V to $\dfrac{V}{\mu}$ when the surrounding medium has a permeability μ.

Therefore the magnetic potential becomes $\dfrac{1}{\mu}\phi\omega$.

Cor. I. If the point P be placed very close to the N seeking face of the shell, the potential at $P = 2\pi\phi$ since the solid angle subtended at P by the shell is 2π.

Cor. II. If P be situated near the opposite face the potential $= -2\pi\phi$,

So if a north-seeking pole be moved from the north pole to the south pole of the shell, the work done (change in potential) $2\pi\phi - (-2\pi\phi) = 4\pi\phi$.

35(b). Alternative method :—

Let a very small area CD equal to ds of the shell AB form an elementary magnet (Fig.25) whose end faces are CD and EF and whose length is t the thickness of the shell, the faces CD and EF having N and S poles respectively. If σ be the pole strength per unit area, the moment of the elementary magnet $= \sigma . ds \times t = \sigma t \times ds = \phi ds$, where ϕ is the strength of the shell.

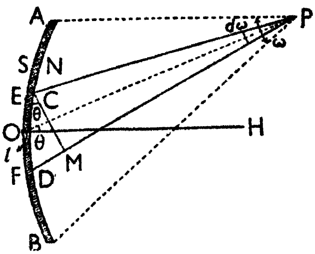


Fig. 25

If r be the distance (OP) of any point P from the element and θ the angle between OP, and OH the normal to CD or ds, the potential at the point can be found as follows. Join PC, PD, PO and draw CM perpendicular to PD.

Potential at P due to the elementary shell $CD = \dfrac{\phi\ ds \cos\theta}{r^2}$

[compare the expression $\frac{M\cos\theta}{r^2}$ the potential at a point due to a short magnet]. As OP may be taken normal also to CD, the angle DCM is equal to θ.

$\angle DCM = \theta \quad \therefore \; ds \cos\theta = CM$, the base of the cone CNP. Hence the potential at P due to whole shell

$$= \int \frac{\phi \, ds \cos\theta}{r^2} = \int \frac{\phi . CM}{r^2}$$

$= \int \phi . \, d\omega$ where $d\omega \left(= \frac{CM}{r^2} \right)$ is the solid angle subtended at P by area ds.

$= \phi \int d\omega$ $\because$ for uniform shell ϕ is constant.

$= \phi\omega$ where ω = solid angle subtended by the whole shell AB at P.

36. Equipotential lines and Surfaces :

In a magnetic field, a line or a surface passing through points having the same potential is called an equipotential line or surface. As the difference of potential between any two points on this line or surface is zero no work is done in moving a pole along an equipotential line or over an equipotential surface.

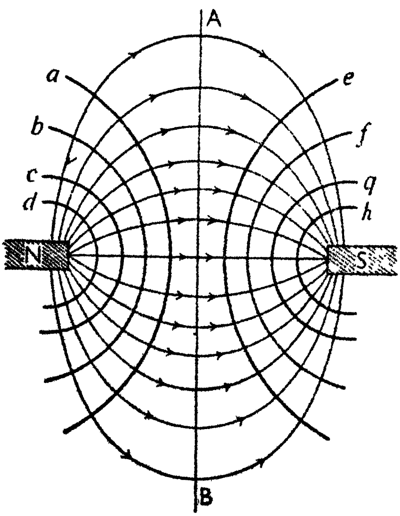


Fig. 26

The lines of force in any magnetic field always cut the equipotential surfaces or lines at right angles.

The equipotential lines may be mapped in a horizontal plane, by means of a compass needle having a small aluminium pointer at right angles to the length of the needle. The direction of the pointer gives the direction of the equipotential line, the direction of the compass showing the direction of the lines of force. In Fig. 26 the fine lines show the lines of force and the bold lines *a*, *b*, *e*, *f* etc. show the equipotential lines.

QUESTIONS

1. Define Intensity of a magnetic field.
How would you determine the strength of a given magnetic field?
[C. U. 1940]

2. Define magnetic potential and find its value at any point due to a small magnet. [C. U. 1937, '43, '47, '51, '53, '55]

Prove that the perpendicular bisector of the axis of a short magnet is a line of zero potential.

3. Obtain an expression for the magnetic potential at a point due to a short magnet.

Determine the magnitude and direction of the field of a short bar magnet at a distance 'r' from the centre on a line making an angle "θ" with the axis of the magnet. [C. U. 1959]

4. Show that the moment of a small magnet can be resolved like a force.
[C. U. 1943, '47, '53]

5. Discuss the significance of the "inverse square law" as applied to electricity and magnetism. Devise an experiment to prove the validity of the law for a magnetic field. [C. U. 1944]

6. Deduce an expression for the couple acting on a magnet freely suspended in a uniform field. [C. U. 1945, '50]

7. What are meant by magnetic moment and intensity of magnetisation?
[C. U. 1956]

Prove that the intensity of magnetic field due to a small bar magnet "end on" is twice that due to the same magnet "broad side on" at the same distance.
[C. U. 1946]

8. Describe experiments to compare the magnetic moments of two magnets and two magnetic needles. [C. U. 1947]

9. Describe a magnetometer and explain its magnetic experiments.
[C. U. 1942]

10. Obtain an expression for the period of an oscillating magnet.

11. Define the factors on which the periodic time of a vibration magnetometer depends.
What precautions would you take in using the instrument and why?
[C. U. 1956]

12. Find the work done in deflecting a small magnet through 130° when suspended in the field supposed to be uniform.

13. Explain what is meant by magnetic potential at any point due to a magnetic shell. [C. U. 1952]

14. Find the expression for the magnetic potential at any point due to a magnetic shell the surrounding medium having a permeability μ.
[C. U. 1952, '56]

EXAMPLES

1. A short magnet 30 cms. to the west of a compass needle deflects it through 45°, while another magnet in the same position deflects it through 30°. Compare their magnetic moments.

Let M_1 and M_2 be the moments of the first and the second magnet respectively.

Since in both cases, the magnets and the needle are in the tangent position of Gauss, we have, neglecting the length of the magnets,

$$\frac{M_1}{H}=\frac{d^3}{2}\tan\theta_1 \quad =\frac{(30)^3}{2}\tan 45^\circ$$

$$\frac{M_2}{H}=\frac{d^3}{2}\tan\theta_2=\frac{(30)^3}{2}\tan 30^\circ \qquad \frac{M_1}{M_2}=\frac{\tan 45^\circ}{\tan 30^\circ}=\frac{1}{\frac{1}{\sqrt{3}}}=\sqrt{3}$$

2. The neutral point of a short magnet is 24 cm. from the centre of the magnet, which lies with its axis north and south and the N-pole pointing to the north. If the value of H be 0·21 C. G. S. unit, what is the moment of the magnet? [C. U. 1934]

At the neutral point, we have $\frac{M}{(d^2+l^2)^{\frac{3}{2}}}=H$: Here l is small,

$$\frac{M}{d^3}=H \quad \text{or} \quad M=Hd^3=\cdot 21\times(24)^3=2903\cdot 04 \text{ C. G. S. unit.}$$

3. A uniformly magnetised bar magnet 10 cm. long and of moment 200 C. G. S. units is placed horizontal with its axis in the magnetic meridian and the north pole pointing towards north. A small compass needle, placed at a distance of 10 cm. east of the centre of the bar, is observed to be in neutral equilibrium. Find the horizontal intensity of the earth's field. [C. U. 1939]

We know that the horizontal intensity H is given by

$$H=\frac{M}{(d^2+l^2)^{\frac{3}{2}}}$$

where M is the magnetic moment, $2l$, the length of the magnet and d, distance of the neutral point from the middle point of the magnet. Here $M=200$,

$$d=10 \text{ cm. } l=5 \text{ cm, } \therefore H=\frac{200}{(100+25)^{\frac{3}{2}}}=\cdot 143 \text{ Gauss.}$$

4. "The strength of horizontal component of the earth's field in Calcutta is ·37 Gauss." Explain fully the meaning of the statement. If a small magnet makes 12 oscillations per minute in the field, what additional field will be necessary for it to make 20 oscillations per minute at the same point? [C. U. 1942]

From Art. 33 we have $n^2 \propto H$

Therefore $n_1^2 \propto H_1$; $n_2^2 \propto H_2+H_1$ $\quad \frac{n_2^2}{n_1^2}=\frac{H_2+H_1}{H_1}$;

$$\frac{20^2}{12^2}=\frac{H_2+\cdot 37}{\cdot 37}=\frac{25}{9}; \quad \therefore \frac{H_2}{\cdot 37}=\frac{16}{9} \qquad H_2=\cdot 66 \text{ Gauss nearly.}$$

5. Find the value of the potential at a point situated on a line passing through the middle point of a magnet of moment 30, at an angle of 60° with its axis, the point being 5 cms. away from the mid-point of the magnet. [C. U. 1943, '51, '55]

From Art 18. we have $V=\frac{M\cos\theta}{}=\frac{30\cos 60}{5^2}=\frac{30\times 0\cdot 5}{25}$

$$=\frac{3}{5} \text{ ergs.}$$

6. A magnet 10 cm. long, with poles of unit strength, is freely suspended in a horizontal uniform field of intesity 0·18 unit. Find the moment of the couple tending to restore the magnet to its original position of rest when it is deflected in a horizontal plane through 30° from that position. [C. U. 1945, '50]

Moment of the couple on a magnet in a uniform field
$= m.2l.\ H \sin\theta. = 1 \times 10 \times {\cdot}18 \sin 30° = 10 \times {\cdot}18 \times \frac{1}{2} = {\cdot}9$ C. G. S. unit.

7. Two short bar magnets of moments 108 and 192 units are placed along two lines drawn on the table at right angles to each other. Find the intesity of the field at the point of intersection of the lines, the centres of the magnets being respectively 30 and 40 cm. from this point. [C. U. 1946]

From Art. 23(a). the intensity of the field F in the tangent A position $2M$

For the magnet of moment 108 the intensity $F_1 = \frac{2 \times 108}{(30)^3} = \frac{2 \times 108}{27000} = \frac{4}{500} = {\cdot}008$

For the magnet of moment 192 units

the intesity $F_2 = \frac{2 \times 192}{(40)^3} = \frac{2 \times 192}{64000} = \frac{3}{500} = {\cdot}006$

$\therefore$ the resultant intensity $R = \sqrt{F_1^2 + F_2^2} = \sqrt{({\cdot}008)^2 + ({\cdot}006)^2} = {\cdot}01$ dynes.

The resultant intensity will be inclined at an angle θ such that $\tan\theta = \frac{1}{125} / \frac{3}{500} = \frac{4}{3}$.

CHAPTER III

MAGNETIC PROPERTIES OF SUBSTANCES

37. Introductory : We know that when a long cylindrical rod of soft iron is placed parallel to the direction of lines of force in a magnetic field, it becomes magnetised by induction and the number of lines of force per sq. cm. of cross section of the field in the space occupied by the iron is increased being due to the number of lines added by the molecular magnets inside the soft iron rod.

Thus within any magnetisable medium we have to consider two sets of tubes of force :—

(1) The tubes which are due to the magnetising field and which would exist if the magnetic medium be replaced by a non-magnetic medium, are called **Lines of force.**

(2) The tubes which exist inside the magnetic medium and which are due to the magnetism of the molecules of the medium are called **Lines of magnetisation.**

Now if S be the area of cross section of the soft iron rod placed in a magnetic field of intensity H and if m be the strength of the pole induced in the iron, the number of lines which pass through the iron is SH, due to the inducing field, and $4\pi m$ due to induced magnetism (Art. 13) and therefore the total number of lines or tubes of magnetic flux which crosses the soft iron rod is equal $SH + 4\pi m$. Thus the number of lines of force which crosses unit area of the cross section of the rod is called magnetic flux and is equal to $H + 4\pi \frac{m}{S}$ or $H + 4\pi I$, where I is the intensity of magnetisation. In the case of soft iron the magnetisation is always parallel to and in the same direction as the magnetic force.

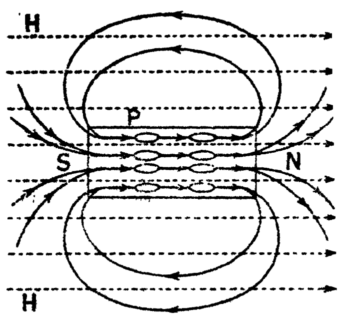


Fig. 27

The resultant **(Lines of Induction)** of the field produced by the magnet and the original field is obtained from the knowledge that the field at points above and below SN, is weakend and at points on the right and left of SN, strengthened.

The quantity $H + 4\pi I$ is generally represented by the letter B and is called **Induction.** It is defined as the number of tubes of force which cross unit area at right angles to the tubes. The unit of magnetic Induction is **Maxwell.**

Thus **Lines of Induction** may be regarded as the whole group of lines as being partly lines of force and partly lines of magnetisation, which would traverse a gap if cut across the iron placed in a magnetic field.

A line of induction is a curve drawn such that its tangent at any point is parallel to the magnetic induction at the point. In non-magnetisable substances the lines of magnetic induction coincide with the lines of magnetic force.

In air $B = H$

In the case of a magnet placed in a region where there are no magnetic forces, the lines of induction form a series of closed curves all passing through the magnet and then spreading out in the air, the lines running from the north to the south pole in air and from the south to the north pole inside the magnet.

38. (a) Magnetic Force and (b) Magnetic Induction :

(a). Magnetic force : The magnetic force at any point in air or any other non-magnetisable substance is defined as the force

exerted on a unit (north) pole placed at that point. But in order to determine the magnetic force inside a magnetisable substance such as a piece of soft iron, a cavity is to be made inside the iron rod and the force acting on a unit pole placed inside this cavity gives a measure of the magnetic force. But since the magnitude of this force depends on the shape of the cavity it is necessary to specify shape of the cavity to give a definite meaning to the term **magnetic force** inside the iron.

If, inside the iron a long narrow cylindrical cavity RS be made with its axis parallel to the direction in which the iron is magnetised, there will be no free poles developed on the sides but only on the ends. If the length of the cylinder is very great in comparison with its diameter, and if a unit pole be placed at the middle point of the cavity the force due to the free poles on the ends will be negligible and the only force which the unit pole experiences is due to the inducing field and is called the magnetic force at any point inside the magnetisable substance. It is denoted by H.

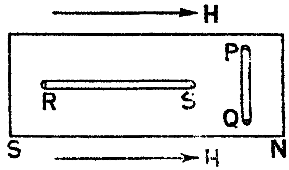


Fig. 28

The magnetising force H' in the iron is really the resultant of the force H due to the field acting from left to right and the force h due to the end poles of the iron (acting from right to left) *i.e.* $H' = H - h$.

(b). Magnetic Induction: If instead of a long cylindrical cavity, a cavity in the shape of a crevasse PQ in which the length is very small in comparison with its diameter is made in the iron, the force exerted on a unit pole placed inside this cavity is the resultant of the magnetic force at the point at which the pole is placed and the force exerted at the same point by the magnetism induced on the ends of the crevasse. We know that the total number of lines of force crossing a unit area at right angles to the crevasse *i.e.* induction is equal to $H + 4\pi I$, where I is the intensity of magnetisation under consideration. Thus the resultant force on a unit pole *i.e.* the strength of the field in the crevasse is known as magnetic induction and is expressed by $H + 4\pi I$. It is denoted by B. Thus $B = H + 4\pi I$.

If the cavity be spherical, the force due to magnetisation is $\frac{4}{3}\pi I$.

In the case of a bar magnet the magnstism is merely residual magnetism produced by magnetic forces which have been removed.

If the magnetic force be restricted to that due to the poles, the induction $B = 4\pi I - H$.

39. Certain terms : In the relation $B = H + 4\pi I$, B is total induction, H the magnetising force, and I the intensity of magnetisation.

(a). Total induction is the total number of lines of force passing through unit area of the medium.

(b). Magnetising force is the magnetic force to which magnetic substance is subjected.

(c). Intensity of magnetisation is the magnetic moment per unit volume or pole strength per unit area.

40. Demagnetisation : If a cylindrical rod of soft iron be placed in a magnetic field of strength H, with its axis parallel to the lines of force, owing to induction poles will be induced at the ends of the iron and these poles produce a force within the material of the iron which will tend to diminish the strength of the field.

The resultant of the two fields, one due to the magnetising field of strength H when the iron is not present, and the other due to the poles induced at the ends is expressed in the following equation $H' = H - NI$

whrere H' is the field actually existing in the interior of the body and NI is the demagnetising field proportional to the strength of the pole *i.e.* to the intensity of magnetisation I. Here N is a constant depending on the form of the magnetised body.

In the case of sphere of permeability μ, $N = \frac{4}{3}\pi$.

In the case of a magnetic sheet perpendicular to the field $N = 4\pi$.

If the rod be very long NI is very small and in this case $H' = H$.

If the external field be removed as in the case of a bar magnet, the force which arises from the free poles in the magnet itself, only is directed from the north to the south Pole both within and in the space outside the magnet. The force due to the induced poles will tend to demagnetise the rod. The longer the bar, the smaller will be the demagnetising force and so for making permanent magnets long thin rods rather than short and broad bars are generally used.

So permanent magnets which have free poles have a tendency to demagnetise themselves and to remove the effect of demagnetisation the magnets are usually provided with soft iron keepers, the keepers producing poles equal and opposite to those of the free poles of the magnet and being very nearly coincident in position with them, produce a field equal and opposite to the demagnetising field.

41. Permeability and Susceptibility :

(a). Permeability : The equation for magnetic force between magnetic poles as expressed by $F = \frac{m_1 m_2}{d^2}$, where m_1 and m_2 are the

strengths of the two poles and d, the distance between them is strictly true only when they are situated in vacuum and very nearly true in air or in any other non-magnetic material.

If the poles are situated in a magnetic material a new quantity μ is introduced into the expression for force, and we have

$$F = \frac{m_1 m_2}{\mu d^2}$$

This quantity μ is termed permeability and is also expressed as the ratio of $\frac{B}{H}$

Hence permeability may be defined as the ratio of total induction to the magnetising force.

Note : If $m_1=1$ and $m_2=m$, the field intensity $H=\frac{m}{\mu d^2}$.

Magnetic Induction $B=\mu H=\frac{\mu m}{\mu d^2}=\frac{m}{d^2}$.

Thus the field intensity H depends on the strength of the pole, on the distance and on the medium.

But induction B depends on m and d but not on the medium.

The greater the permeability of a substance, the greater would be the concentration of lines of force in it when placed in a magnetic field and it is for this reason substances having greater permeability are generally used to protect suspended needle galvanometers from external magnetic disturbances.

In Prof. Boy's radio-micrometer this property of magnetic substance is utilised.

(b). Susceptibility : The magnetic susceptibility (k) of a specimen is a measure of the ease in magnetising the specimen and is expressed by the relation $k=\frac{I}{H}$.

Hence susceptibility may be defined as the ratio of the intensity of magnetisation to the magnetising force.

It is not constant for a specimen but varies with the magnitude of H.

If magnetism is entirely induced in a rod of soft iron placed with its axis parallel to the direction of the field, and if H denote the magnetising force, I, the intensity of magnetisation and B, the induction, the ratio of I to H is termed ***susceptibility*** **(k)**, and the ratio of B to H is termed ***permeability*** **(μ)**.

Thus we have $k = \frac{I}{H}$ and $\mu = \frac{B}{H}$.

Since $B = H + 4\pi I$ we have $\frac{B}{H} = 1 + 4\pi \frac{I}{H}$

or $\mu = 1 + 4\pi k$; $k = \frac{\mu - 1}{4\pi}$

42. Retentivity and Coercivity :

(a). Retentivity : It is the property of a magnetic substance by which it retains a large portion of magnetism when subjected to a magnetising force and not disturbed by any external agency.

(b). Coercivity : It is the property of a magnetic substance by which it retains magnetism inspite of any subsequent treatment such as rough handling, heat or any demagnetising influence.

It is usually measured by the amount of negative magnetising force required to deprive the magnetic substance of the whole of its original magnetisation.

For a good magnet it must have a large retentivity and large coercivity. Hence, these two properties are utilised in making good permanent magnets for galvanometers, ship's compass magnetos etc.

43. Cycles of magnetisation. Hysteresis : If a curve is plotted with I or B as ordinate and H, the magnetising force as abscissae, it will be noticed (Fig. 29) that as the magnetising force H increases, the intensity of magnetisation I increases from O to A, and at points near A where the curve is practically parallel to the axis of H, the substance is said to be approximately *saturated*. On decreasing H, the value of I decreases slowly but remains greater than when H was increasing and the path AB is followed. Now if H be reduced to zero value, the curve cuts the axis of I at B. OB then represents the amount of **residual magnetism** in the magnetised substance. Then, on applying a negative force, I is gradually reduced and becomes zero when H = OC. This negative magnetising force required to reduce I to zero value is called the **Coercive force**.

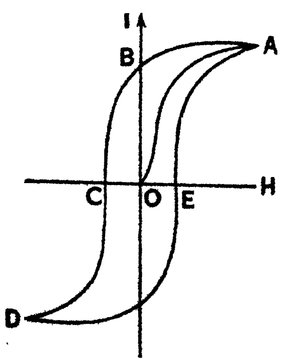


Fig. 29

Now if the negative magnetising force be gradually increasing up to a value equal numerically to the maximum positive value

applied before and then gradually diminished to zero value and reversed up to the maximum positive value, the curve will follow the path CD and DEA and enclose a certain area forming a closed curve known as **hysteresis curve**.

It is observed that the magnetic condition set up in the magnetised substance tends to persist *i. e.* the value of I when H is diminishing is always greater than when H is increasing. Owing to this lag of I behind H in the cycle of changes, the phenomenon is known as **hysteresis**.

The **term hysteresis means the lag** of the induced magnetism behind the magnetising force and this is due to the fact that μ, the permeability of the substance does not remain constant as H, the magnetising force varies, which is evident from the nature of the curve. For if μ were constant the reletion $\mu=\frac{B}{H}$ or $\mu H=B$ would be represented graphically by a straight line and there would not have been any lag or hysteresis.

But since $B=H+4\pi I$ or $\frac{B}{H}=1+4\pi\frac{I}{H}$ or $\mu=1+4\pi k$, we have μ depending on H, and I the intensity of magnetisation and therefore the phenomenon of hysteresis is caused by the variation of μ due to the change of the magnetising force.

If instead of plotting I and H, we plot corresponding values of B and H, a similar hysteresis curve will be obtained and the area of the loop in $B-H$ curve will be 4π times the area of the loop in $I-H$ curve.

The hysteresis curve thus obtained is characteristic of the material giving an idea of its retentivity and coercivity and its area gives a measure of the dissipated energy in unit volume of the material during a complete cycle of changes.

44. Magnetic Condition of substances : The permeability μ is expressed as B/H

If a curve is plotted with μ as ordinates and H as abscissa it will be observed that in weak magnetic fields the permeability rises rapidly to a maximum and then falls. But in strong magnetic fields, μ for all magnetic substances falls except in the case of manganese steel for which μ is constant.

The magnetic condition or properties of a material is easily determined from a complete diagram of the cycle *i. e.* of the hysteresis curve. If the curve be not completely drawn, a knowledge of the intensity of *magnetisation* near *saturation* together with

the values of the *residual magnetism* and *coercive force* is sufficient to give a very good idea of the magnetic properties of the material.

The curve obtained with annealed *soft iron wire* is different in shape from the curve obtained with the same wire after being hardened by stretching. The shapes of these curves are not also the same as that of the curve for a pianoforte steel wire. If we carefully study the shapes of all these curves we will notice that the harder the material, the less is the residual magnetism and greater the coercive force.

The effect of any mechanical disturbance such as tapping, is to make the ascending and descending branches for soft iron very nearly coincide *i.e.* the residual magnetism and coercive force are practically zero. In the case of steel the effect is in the same direction but is not so very marked.

45. Soft Iron and Steel : In the case of **soft-iron**, the hyteresis curve encloses a much **smaller area** than in the case of **steel**, consequently the loss of energy as represented by the area of the curve is greater in the case of steel than in the case of soft iron and this appears as heat in the substance. For this reason annealed soft-iron is used in the armature core of a transformer as it has high permeability and low hysteresis loop.

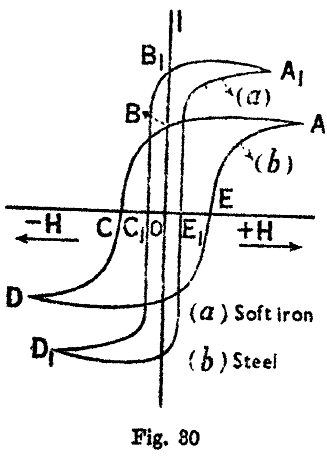


Fig. 30

In Fig. 30 (a) denotes loop for soft iron and (b) that for steel. It is evident that coercive force for steel given by OC is much greater than that for soft iron given by OC_1. Residual magnetism denoted by OB_1 for soft iron is again much larger than that for steel as given by OB.

The act of taking a magnetic body through a cycle of magnetisation involves some expenditure of energy, for the energy required to magnetise the body is not recoverable on removing the magnetic field since the amount of magnetism does not fall to nothing.

The complete demagnetisation of a specimen can be effected by taking it through a series of magnetic cycles which gradually decrease in magnitude to zero. It has been found that the hysteresis loop becomes smaller and smaller until no magnetism remains in the specimen.

If a conductor which is *ferro-magnetic* be placed in a rapidly alternating magnetic field, the loss of energy in the form of heat will be due to two distinct causes, one due to the generation of eddy current and the other due to hysteresis in the magnetic substance.

If a conductor which is non-magnetic is placed in the neighbourhood of a changing current or of a rapidly alternating magnetic field, eddy current or Foucault current is generated in it and the energy of the current is dissipated in the form of heat.

The suitability of a magnetic material for use as the core of a transformer, as a permanent magnet or as the diaphragm of a telephone ear-piece, depends on low hysteresis and eddy current losses and also on a high value of permeability at a low value of H, the magnetising field.

Cobalt Steel and *Ticonal* are suitable for making permanent magnets and *Stalloy* is suitable for diaphragms of telephone ear-pieces.

46(a). Loss of energy due to hysteresis : It has been stated already, that the act of taking a magnetic body through a cycle of magnetisation involves some expenditure of energy and this loss of energy is represented by the area enclossed by the hysteresis curve.

To prove this, let us consider a small magnet AB whose intensity is I, cross-section α and length l and that it is placed in a field of strength H.

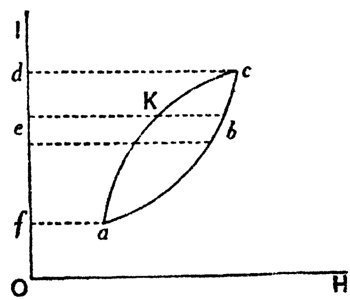


Fig. 31

Let V_A and V_B be the magnetic potentials at A and B of the magnet and let m be the magnetic charge at the end of the magnet.

Now if a small magnetic charge dm be transferred from A to B, the work done $= dm\,(V_A - V_B) = dm.H.l$

Then the work done per c. c. of the substance $= \dfrac{dm H.l}{V} = \dfrac{dm.\ H.\ l}{\alpha.\ l} = dI.\ H.$ since $dI = \dfrac{dm}{\alpha}$

In Fig. 31 the area of the strip be represents the work done $H.dI$ for the small change of intensity dI. For, if the intensity of the field H changes from H to $H + dH$ during the change of intensity dI, the work done $= dI.\ (\ H + dH\ )$

$$= dI.H$$

since $dI.dH$ is negligibly small.

So the total work done by the magnet in passing from c to a along cKa is represented by the area $cdefaKc$ and similarly the work done on the magnet in passing from a to c along abc is represented by the area $abcdefa$.

Thus the excess of the work done on the magnet of unit volume over that done by the magnet when the body is taken through a complete cycle is represented by the area of the loop $abcKa$. This

work appears as heat in the body and represents an irrecoverable loss of energy.

46(b). From B—H Curve : For a B − H curve, the area of the loop is 4π times the corresponding area of the loop on the I − H curve and consequently the loss of energy is 4π times in I − H curve.

$$\text{For} \quad B = H + 4\pi I$$

$$\therefore \quad H.dB = HdH + 4\pi H.dI$$

$$\int H.dB = \int HdH + 4\pi \int HdI$$

$$\text{or} \quad \int H.dB = 4\pi \int H.dI$$

Since $\int HdH = 0$ as the initial and final values of H are equal.

47. Molecular Theory : The magnetic behaviour of a magnetic substance when carried round a cycle of magnetisation is explained satisfactorily by Ewing in his theory of induced magnetism. According to him the molecules of the magnetic substance are permanent magnets arranged in a haphazard manner forming more or less complicated groups so as to produce no external magnetic effects, and the observed magnetic effects are explained by means of the magnetic forces exerted between the neighbouring molecules.

To explain the above effects let us consider four molecular magnets arranged in a group so as to place the north and south poles of the consecutive magnets very near to one another and to produce no external effect. Fig. 32(*a*)

When the group is subjected to a weak magnetic field H the molecules rotate slightly towards the direction of the field and the first portion of OA of the hysteresis curve is obtained, Fig. 32 (*b*). If H is increased gradually the group breaks up and assumes the arrangement in which the couple acting on the magnets becomes greater than the restoring couple due to the force between the molecular magnets and the magnets turn towards the field and remain stable. In this case a small increase in H produces a large increase in the intensity of magnetisation I and the steep portion of OA in fig. 29 is explained.

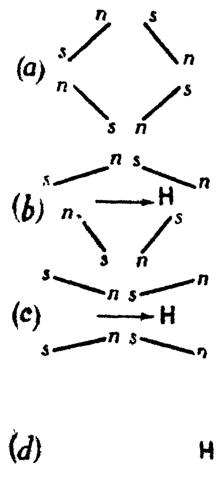


Fig. 32

Any further increase of H rotates the magnets slightly more in line with the direction of the field H and the curve becomes parallel to the axis OH and at this stage the substance is said to be saturated with magnetism. Fig 32(*d*). Fig. 32(*c*).

If after this stage H is gradually reduced to zero the molecules will be slightly disturbed and a certain amount of magnetism known as residual magnetism represented by OB of the curve (Fig. 29) will be left.

Again when H is reversed *i.e.*, a negative magnetic field represented by OC is applied, the molecules return to the group arrangement and the substance is deprived of its magnetism and no external effect is produced.

48. Measurement of Magnetic Permeability :

(a). Magnetometer Method. To determine the permeability of any material it must be long and narrow so that the effect of demagnetisation is negligible and must be placed in a magnetic field of known strength produced by means of a solenoidal coil through which an electric current is passed.

The specimen of the material, say iron, is inserted in the coil AB which is placed with its axis East and West of the magnetometer needle. When a current is passed through the coil the iron wire is magnetised and the deflection in the magnetometer needle due to the induced magnetism in the iron is observed after compensating the direct action of the solenoid by an auxiliary coil CD placed on the opposite side of the needle and through which the current which passed through the solenoid is also passed.

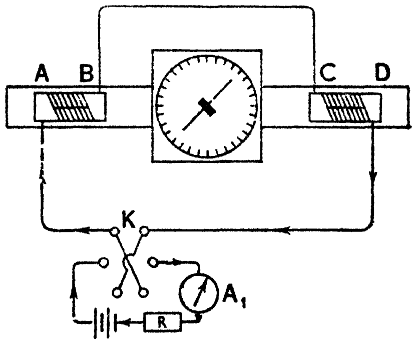


Fig. 33

The magnetising field F is easily determined from the knowledge of the number of turns per centimetre of the solenoidal coil and the strength of the current passing through it and is given by the expression.

$$F = 0{\cdot}4\pi ni. \quad ...(1)$$

where n is the number of turns per cm. and i, the current in amperes.

We know that the induction within the iron is given by the expression

$$B = F + 4\pi I.$$

and permeability μ is given by $\mu = 1 + 4\pi k$. (2) Thus if we know k we can determine μ from (2).

To determine the value of k which is equal to $\frac{I}{F}$ we must know the value of I for a particular value of F.

Let 2L be the length of the wire of cross-section S and let I be the intensity of magnetisation. The magnetic moment of the iron is equal to 2LSI.

If M′ be the moment of the magnetometer needle, the moment of the couple exerted by the iron on the needle when it is deflected through an angle θ is $= \frac{4M'SLID}{(D^2 - L^2)^2} \cos\theta$

where D is the distance of the needle from the middle point of the wire.

In the equilibrium position, this couple is balanced by the couple due to earth's magnetic force H.

$$\text{Therefore} \quad \frac{4M'LSID}{(D^2 - L^2)^2} \cos\theta = M'H \sin\theta$$

$$\text{or} \quad I = \frac{H(D^2 - L^2)^2}{4LSD} \tan\theta \quad \dots \quad (3)$$

Thus I is determined and since F is calculated from the expression $F = 4\pi ni$ the value of $\frac{I}{F}$ or k is also determined.

Therefore from the expression $\mu = 1 + 4\pi k$, the permeability μ is obtained.

To determine the cycle of magnetisation, the magnetisting field F is varied by altering the strength of the current i with the help of the adjustible resistance R and its value for each current as obtained from the ammeter A is determined from (1).

The intensity of magnetisation I corresponding to different values of θ is determined from (3).

Then proceeding as in Art. 43 the hysteresis curve is plotted.

(b). Second Method : The substance whose permeability is to be determined is taken in the form of a wire AB and placed vertically inside the magnetising solenoid so that the upper end A is on a level with the needle of the magnetometer M.

If I be the intensity of magnetisation and α, the area of the cross section of the wire, then the intensity of the field due to the pole A at the magnetometer needle M is $\frac{I\alpha}{d^2}$

where d is the distance between the pole A and the needle M. The intensity of the field at M due to the pole B is

$$-\frac{I\alpha}{d^2 + l^2}$$

where l is the length of the magnet. The horizontal component of the field due to the pole B is $-\frac{I\alpha d}{(d^2 + l^2)^{\frac{3}{2}}}$

Thus the resultant horizontal field at M due to AB is

$$I.\propto\left\{\frac{1}{d^2}-\frac{d}{(d^2+l^2)^{\frac{3}{2}}}\right\}$$

and this is equal to H $\tan\theta$ where H is the controlling field which

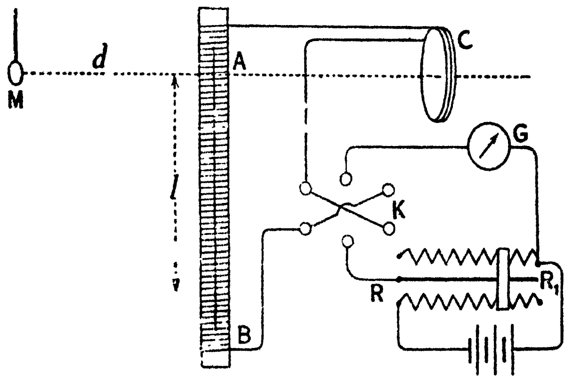


Fig. 34

is generally the earth's horizontal intensity, θ, the angle of deflection. Then

$$I\propto\left\{\frac{1}{d^2}-\frac{d}{(d^2+l^2)^{\frac{3}{2}}}\right\}=H\ \tan\theta.\quad\ldots\quad(1)$$

Thus if θ is observed and H be known I can be calculated. To determine H, the Earth's horizontal intensity, the compensating coil C is disconnected from the solenoid and a current is passed through the coil. A deflection θ_1 is observed when a current i_1 is passed in the coil C and if the distance between the coil and the magnetometer needle be x, the field at the needle M is

$$\frac{2\pi n i_1 a^2}{(a^2+x^2)^{\frac{3}{2}}},\text{ where } a \text{ is the radius of the coil.}$$

$$\therefore\quad\frac{2\pi n i_1 a^2}{(a^2+x^2)^{\frac{3}{2}}}=H\ \tan\theta_1\text{ from which H is determined.}$$

To trace the curve of magnetisation, values of current i with corresponding values of deflections θ are obtained by gradually altering the resistance of the rheostat with the slider.

The current is increased step by step to a maximum, then diminished to zero, then reversed and increased to the positive maximum value.

Again from the expression $B = F + 4\pi I$, the induction B is determined if I, and F the magnetising force are known.

The intensity of magnetisation I is obtained from the expression (1) the magnetising force F from the expression $F = \cdot 4\pi ni$, where n is the number of turns per centimetre length of the coil of magnetising solenoid and i, the strength of the current expressed in amperes passing through it.

Thus from the formula $\mu \quad \frac{B}{F}$

The value of susceptibility k is also determined from that expressions $k \cdot \frac{I}{F}$, and $\mu = 1 + 4\pi k$.

The value of permeability μ is obtained from the knowledge of B and F.

The experimental arrangement is shewn in figure 34.

In this experiment several disturbances are to be allowed for.

(1) The magnetic field produced at the magnetometer needle by the magnetising solenoid before AB is introduced into it is eliminated by adjusting the position of the coil C whose axis is in line with A and M and which is connected in series with the solenoid until on passing the current the magnetometer needle remains undisturbed.

(2) The specimen of wire AB is always magnestised by the vertical component of the earth's field.

This effect is neutralised by adjusting the current through a second coil wound round the first coil (not shewn in the figure).

[**Note 1:** It has been noticed that the permeability of iron depends on the strength of the magnetising force. When the magnetising force is high the permeability diminishes with the rise of temperature and when the magnetising force is small the permeability increases with the rise of temperature.

Note 2: It has also been found out that the rate of increase of permeability is very great as the temperature approaches the *critical temperature* (a temperature at which the substance ceases to exhibit any magnetic properties) which for iron is about 785°C but after passing this temperature the rate of fall of permeability is very rapid and few degrees above, it becomes zero and the substance ceases to be magnetic.]

49. Measurement of Susceptibility: We know that when a magnetic substance is in a uniform magnetic field it does not move as a whole in any direction but tends at first to rotate. But if the field is not uniform a paramagnetic substance moves from the weakest to the strongest part of the field whereas a diamagnetic substance moves from the strongest to the weakest part of the field.

Let us consider a specimen of length δx and cross-sectional area δA and of susceptibility k placed in a non-uniform field whose mean value in the space occupied by it is H.

Then the pole strength of the specimen is equal to I. $\delta A = kH.\delta A$ where I is the intensity of magnetisation.

Difference in the field at the two poles $= \delta x.\dfrac{dH}{dx}$

Resultant force on the specimen $= kH.\delta A \times \delta x.\dfrac{dH}{dx}$

$$= kH.\frac{dH}{dx}.\delta A.\delta x = kH.\frac{dH}{dx} \text{ per unit volume.}$$

Thus to measure susceptibility k, a specimen of the substance is placed in a non-uniform field H produced between the poles of an electromagnet, the poles being inclined towards one another.

In Curie's experiment the specimen is suspended in the field from one arm of a rod which is in turn suspended from the torsion fibre. The specimen of volume v experiences a force of $vkH.\dfrac{dH}{dx}$ which is measured by the torsion balance.

H and $\dfrac{dH}{dx}$ are measured by an exploring coil and hence the value of k can be determined.

Note: We know that permeability μ is related to susceptibility by the expression

$$\mu = 1 + 4\pi k$$

So if μ is determined by experiments described in Art. 48. k is easily found out.

50. Paramagnetic, Ferro-magnetic and Diamagnetic Bodies: Bodies such as Platinum, Aluminium, copper, oxygen, air, solution of iron and copper salts whose permeabilities are greater than unity *i.e.* greater than that of air are known as **paramagnetic bodies**. These bodies when freely suspended in a magnetic field set themselves in the direction of the lines of force and tend to move from the weaker to the stronger part of the field.

As far as the inside of the paramagnetic body is concerned the flux-density is increased *i.e.*, the induction B is greater than the magnetising field H.

Since $\mu = 1 + 4\pi k$ and since for paramagnetic substances μ is greater than 1, k for them is positive. This is interpreted physically by the fact that the induced magnetism is in such a

direction as to *assist* the flux producing it. So the lines of force become more closely packed within the body than in the space outside [Fig. 35(*a*)].

Materials such as iron, nickel, cobalt and alloys of these elements are known as **ferromagnetic substances** and in distinction to paramagnetic substances, these metals have permeabilities which are so very great in comparison with others that they form a class by themselves. Ferromagnetic substances are crystalline in nature Curie showed that with a given magnetising force, the magnetisation for paramagnetics varies inversely as the absolute temperature over a wide range and above the Curie point (critical temperature) ferromagnetics pass into the paramagnetic state.

Liquids and gases have no definite structure and so they can never be ferromagnetic.

Bodies such as Antimony, Bismuth Zinc, Silver, Quartz, Mercury, water, Alcohol etc. and monatomic gases are **diamagnetic substances.** These bodies when freely suspended in a magnetic field set themselves at right angles to the direction of the field and move from stronger to the weaker part of the field, though the effect is generally too feeble. The polarity induced in the diamagnetic body is the reverse of that created in the para magnetic body.

The flux density inside the body is decreased *i.e.* $B<H$ for the induced magnetism is in such a direction as to *oppose* the flux producing it.

Since $B<H$ *i.e.* permeability μ is less than unity and so the susceptibility is negative.

Hence the lines of force inside the body are fewer than in the surrounding space [Fig. 35(*b*)].

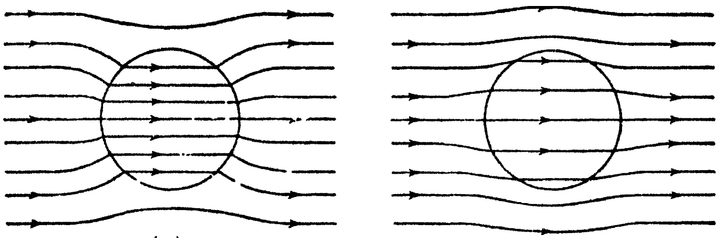
Diamagnetism is independent of temperature while paramagnetism is influenced by temperature.

The behaviour of *paramagnetic* substance may be explained by Weber-Ewing Theory in which molecules are considered as permanent magnets with currents circulating round them and that of *diamagnetic substances* may be explained by Weber-Maxwell Theory in which molecules are considered as electrical conductors which generate induced currents when placed in a magnetic field.

We know that when a conducting circuit is moved in a magnetic field or when a magnetic field changes through the circuit an induced current is generated in the circuit in such a direction as to produce a field opposite to the direction of the original field. The magnetic induction is thus reduced by the presence of the conducting molecules and the presence of such conducting molecules would give to the material the diamagnetic property. If the electrical resistance of the conductor be zero, the current once started will be maintained and the molecules will acquire polarity opposite to that due to the original field. Hence the presence of such conducting molecules in the material would give rise to the diamagnetic property.

The para and diamagnetic properties of substances can also be explained by considering the atoms with their **rotating electrons.**

When both a paramagnetic and a diamagnetic body are magnetised, the polarity created in the diamagnetic body is the reverse of that created in the paramagnetic body.



(a) (b)

Fig. 35

Since in a diamagnetic body the permeability is less than that in air, the induction B through the body is less than the value of the field which would exist if the body were removed. So when a diamagnetic body is introduced into a uniform magnetic field, the lines of force within this space occupied by the body are fewer and in the space outside more closely packed than they would be in the absence of the body. Fig. 35(*b*).

The case is reversed if a paramagnetic body be introduced into a uniform field. In the space occupied by the body lines of force will be more closely packed than in the space outside. Fig. 35(*a*).

QUESTIONS

1. Define the terms magnetic induction B, intensity of magnetisation I and magnetic force H and show how to obtain the relation $B = H + 4\pi I$. [C. U. 1935. '42, '45, '49]

Shew that $\mu = 1 + 4\pi k$. [C. U. 1945, '51]

2. Explain the terms Magnetic force and magnetic induction, permeability and susceptibility, coercivity and retentivity. [C. U. 1941, '48, '51]

3. What is meant by the magnetic permeability of a substance and how can it be measured?

Describe generally how the permeability of a piece of soft iron varies with the intensity of the magnetising force. [C. U. 1939]

Show how the permeability of a given magnetic material is related to its susceptibility. [C. U. 1951]

4. What is meant by magnetic susceptibility? Describe some methods for its determination. [C. U. 1940, '51]

5. Explain the meaning of the terms—Hysteresis; Retentivity and coercive force in the case of ferro-magnetics. [C. U. 1941, '58]

Shew that the area of the hysteresis loop is proportional to the work done in carrying a magnetic substance through a cycle of magnetisation. [C. U. 1941]

6. What is meant by 'Hysteresis'' when used in connection with the magnetisation of iron ?

Describe generally the from of the B – H curve for soft iron. What is the significance of the hysteresis loop ? [C. U. 1943, '47]

7. Explain the use of the terms dia-magnetism, para-magnetism and ferro-magnetism in case of magnetic substances. Give examples of each and shew how they are distinguished. [C. U. 1935, '38, '42]

8. In what respects do the magnetic properties of iron and steel differ ?

What is the general character of the magnetic permeability of iron in strong fields ? [C. U. 1942]

CHAPTER III

TERRESTRIAL MAGNETISM

51. Earth as a Magnet : The behaviour of a magnetic needle when suspended freely and pointing approximately towards the north and south pole of the earth suggests that the earth itself is a large magnet having poles situated near the geographical poles of the earth or a short magnet placed at the centre of the earth along its magnetic axis. Lines of force start from the magnetic north pole of the earth situated near the south geographical pole and end in the south magnetic pole near the north pole of the earth.

As the distance between the magnetic poles of the earth is very great the magnetic field may be said to be uniform at places where the lines of force are parallel but for places situated near the poles the field is not uniform.

The earth's field is directive and the three magnetic elements such as the horizontal component of earth's field, dip and declination have different values at different places on the surface of the earth.

For some places on the surface of the earth the horizontal intensity has the same value and for similar other places the declination has also the same value. The dip angle is maximum near the magnetic poles of the earth.

It may also be noted that the value of these elements for any place does not always remain constant but changes periodically.

52. Magnetic Elements : To specify completely the earth's magnetic field at any point the following quantities should be determined.

(1) The **Horizontal Component** of the earth's magnetic force.
(2) The **Dip.**
(3) The **Declination.**

53. Definition of Certain Terms :

(a) Magnetic Meridian : It is a vertical plane DBC in which the magnetic axis of a freely suspended magnet comes to rest. It is also the vertical plane which passes through the magnetic poles of the earth and the zenith of the place.

(b) Dip : The angle (δ) formed between the horizon and the magnetic axis of the needle when it is freely suspended about its centre of gravity is the dip of the place.

It is the angle which the direction of earth's resultant magnetic force makes with its horizontal component.

(c) Declination : It is the angle (θ) between the magnetic and geographical meridian planes DBC and ABC of the place.

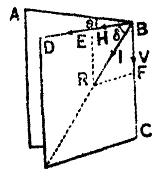


Fig. 3

Let the resultant magnetic intensity I act on either pole of the magnetic needle in the direction of the dipping needle *i.e.*, in the direction of the lines of forces of the earth's field. Let δ be the dip *i.e.*, the angle of inclination of the lines of force to the horizon.

Let I be resolved into two components, H and V of which H is horizontal and V is vertical.

Then $H = I \cos \delta$, $V = I \sin \delta$

$$\text{Therefore } \frac{V}{H} = \tan \delta \text{ and } V^2 + H^2 = I^2.$$

When a magnetic needle is mounted on a horizontal axle passing through the centre of gravity of the needle and resting on horizontal knife-edges placed near the centre of a vertical circle graduated in degrees and coincident with the magnetic meridian plane, the direction of the axis of the needle represents the resultant direction of the earth's magnetic field. The vertical and horizontal components of the earth's resultant field are respectively expressed by V and H. This vertical component V is constant for all vertical planes but the horizontal intensity changes according to the cosine of the angle of inclination of any other plane in which the needle rests with the magnetic meridian plane.

So if θ be the angle of inclination between the planes, the horizontal component acting on the needle is $H \cos \theta$, the vertical component remaining unchanged.

54. To determine the total intensity of earth's magnetic field : The total intensity I of earth's magnetic field is given by

$$I = \frac{H}{\cos\delta} \text{ or } \frac{V}{\sin\delta}$$

where H and V are respectively the horizontal and vertical components of earth's magnetic field and δ, the angle of dip.

If H is determined by oscillation and deflection experiments (Art. 55) and δ determined by dip circle the total intensity I is found out.

The vertical and the horizontal components may also be determined by an earth inductor and a ballistic galvanometer. (*Consult Current Electricity*).

55. Determination of the Horizontal Component of the Earth's Magnetic Field :

Horizontal Intensity : It is the resolved part of the earth's resultant magnetic intensity at a place in the horizontal direction along the magnetic meridian.

The method generally used to perform this experiment involves two separate experiments which, of course, must be done at the same place. The first experiment is known as the **oscillation experiment** and the second experiment is known as the **deflection experiment.**

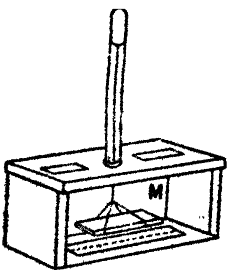


Fig. 37

(*a*) In the **oscillation experiment** a magnet is freely suspended by means of a torsionless fibre inside a rectangular box with glass sides so that the vibration can be counted without being disturbed by any external air current.

The magnet is allowed to swing through a small angle by suddenly bringing a magnet near the suspended one and taking it away. When the oscillations are steady and uniform the time taken to execute fifty oscillations are noted and from which the time of a single oscillation is calculated. Thus from the formula

$$T = 2\pi\sqrt{\frac{I}{MH}}$$

where H = the horizontal component of the earth's magnetic field, M = Magnetic moment of the magnet and I = Moment of Inertia of the magnet.

$$\text{or } MH = \frac{4\pi^2 I}{T^2}$$

the value of MH is calculated from the knowledge of I which is given by the expression $K = m\left(\frac{a^2+b^2}{12}\right)$

where m is the mass of the magnet (rectangular) and a and b are the adjacent edges of the faces through which the axis of rotation passes.

(*b*) **Deflection Experiment** : In this experiment the same magnet is placed in the Tan A position with its axis east and west of the magnetic meridian so that its middle point is at a distance d from the centre of the magnetometer needle.

The needle comes to rest making an angle θ with the magnetic meridian under the action of two equal and opposite couples, one due to the field F created by the deflecting magnet, and the other due to the earth's magnetic field H.

We have, in the equilibrium position of the needle

$$mF \times 2l' \cos\theta = mH \times 2l' \sin\theta \quad \ldots (1)$$

where m is the pole-strength and $2l'$, the length of the needle.

From (1) $F = H \tan\theta$.

But since in the Tan A position, $F = \frac{2Md}{(d^2-l^2)^2}$

we have, $\frac{2Md}{(d^2-l^2)^2} = H\tan\theta$, or $\frac{M}{H} = \frac{(d^2-l^2)^2}{2d}\tan\theta$.

Thus from the knowledge of d, θ, and $2l$ which is approximately equal to $\frac{5}{6}$th of the distance between the ends of the magnet, the value of $\frac{M}{H}$ is determined.

Then, from MH and $\frac{M}{H}$ the value of H is determined.

Thus $\quad MH = \frac{4\pi^2 I}{T^2} \ldots (2)$, $\frac{M}{H} = \frac{(d^2-l^2)^2}{2d}\tan\theta$, $\ldots (3)$

Dividing (2) by (3) $\quad H^2 = \frac{4\pi^2 I . 2d}{T^2 . (d^2-l^2)^2 \tan\theta}$

or $\quad H = \frac{2\pi}{T(d^2-l^2)}\sqrt{\frac{2Id}{\tan\theta}}$

Note : The length of a bar magnet may be determined by two sets of observations with the same magnet in the Tangent A position. If d_1 and d_2 are the distances and θ_1 and θ_2 are the corresponding deflections, we have,

$$\frac{(d_1^2 - l^2)^2}{2d_1} \tan \theta_1 = \frac{(d_2^2 - l^2)^2}{2d_2} \tan \theta_2. \quad [\because \text{ each is} = M/H]$$

$$\text{or } l^2 = \frac{d_1^3 \tan \theta_1 - d_2^3 \tan \theta_2}{2(d_1 \tan \theta_1 - d_2 \tan \theta_2)}$$

56. Another Method of determining H : A magnet is freely suspended by means of a torsionless thread and made to execute a certain number of oscillations and the times for these oscillations are noted.

The value of MH is determined from the expression

$$MH = \frac{4\pi^2 I}{T^2} \quad ...(1)$$

in the way described in the **Oscillation experiment**.

To determine the value of $\frac{M}{H}$ the same magnet is placed a white sheet of paper with its axis and the edge of the paper in the magnetic meridian and the field round the magnet is traced when either the North or the South pole of the magnet points towards the North pole of the earth.

In the case when the North pole of the magnet points northwards the two **neutral points** in the field are situated on the eastern and the western sides of the magnet and at each of the neutral points the expression $\frac{M}{d^3} = H$ ...(2) is satisfied, where M is the magnetic moment of the magnet, and H and d are the horizontal component of the earth's magnetic field and the distance of the neutral points from the middle of the magnet. Thus from the two equations (1) and (2) the value of H, the horizontal component of earth's magnetic field is easily determined.

Note : To determine the vertical component of earth's magnetic field consult Article on Earth Inductor (Current Electricity).

57. Measurement of Magnetic Dip : The magnetic dip of any place is measured by an instrument known as the **dip circle**. It consists of a vertical graduated circle SS mounted on the frame which can rotate in a circular direction its rotation being indicated by a pointer E moving on a horizontal circular scale PP provided with three levelling screws. At the centre of the vertical circle is a magnetic needle AB supported by a cylindrical axle resting on agate knife-edges (Fig. 38).

At first the instrument is properly levelled and then the vertical circle is rotated until the needle is vertical *i.e.*, points to 90° of

the vertical scale. The plane of the circle is then at right angles to the magnetic meridian and by rotating the vertical circle through 90° on the horizontal scale, the plane of the vertical circle *i.e.* the plane of rotation of the needle is made to coincide with the magnetic meridian plane. The angle which the axis of the needle now makes with the horizontal line through $0^\circ - 0^\circ$ of the vertical scale is the value of the dip of the place.

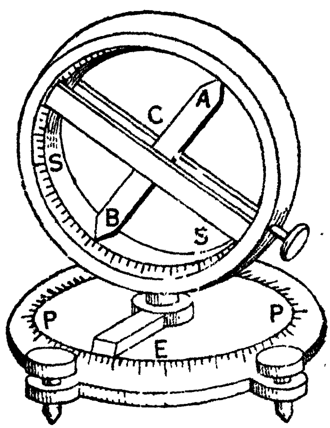


Fig. 38

Note : It is to be noted that at the position where the plane of rotation of the needle is at right angles to the magnetic meridian, the horizontal component of the earth's field H becomes ineffective, the vertical component being effective sets the needle in the vertical direction.

The magnetic dip δ is determined by observing the angles δ_1 and δ_2 which the axis of the needle makes with the horizontal line through $0^\circ - 0^\circ$ of the vertical circle for any two of its positions between which the angle is 90°, by the expression $\cot^2\delta_1 + \cot^2\delta_2 = \cot^2\delta$.

Let θ be the angle between the plane of rotation of the needle *i.e.*, of the vertical circle and the magnetic meridian and let δ be the angle made by the axis of the needle with the horizontal in this position.

The component of the resultant magnetic force acting on the needle in this position are $H\cos\theta$ and V in the horizontal and vertical directions respectively.

Then, $$\mathrm{Cot}\ \delta_1 = \frac{H\cos\theta}{V} \qquad (1)$$

The vertical circle is rotated through 90° and the angle between the axis of the needle and the horizontal line is found to be δ_2. In this position the component forces acting on the needle are $H\cos(\theta + 90)$ and V and we have therefore,

$$\mathrm{Cot}\ \delta_2 = \frac{H\cos(\theta + 90)}{V} = -\frac{H\sin\theta}{V} \qquad (2)$$

Squaring (1) and (2) and adding, we have,

$$\mathrm{Cot}^2\delta_1 + \mathrm{Cot}^2\delta_2 = \frac{H^2}{V^2} = \mathrm{Cot}^2\delta$$

for when the angle between the vertical circle and the magnetic meridian is zero, the angle between the axis of the needle and the horizontal line is δ which is the true dip of the place and in this position the component forces are H and V and therefore,

$$\text{Cot } \delta = \frac{H}{V} .$$

Sources of Error :

(1) The axis of rotation of the needle may not be at the centre of the vertical circle. The error due to this is eliminated by taking the mean of the readings at both ends of the needle for it will be found that the reading for one pole will be too high and that for the other pole too low to an equal extent. [Fig. 39(*a*)].

(2) The zero line of the circle may not be truly horizontal. The error is corrected by taking readings for both ends of the needle and then by rotating the instrument about a vertical axis through 180°, two more readings for the ends are taken. The mean of these readings will give the correct reading. [Fig. 39 (*b*)].

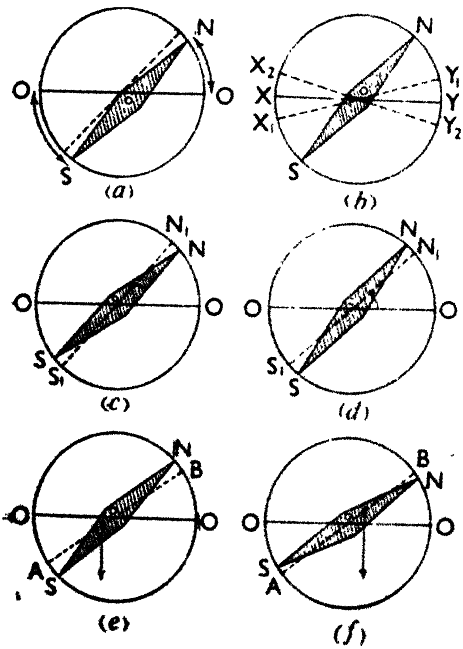


Fig. 39

(3) The magnetic axis of the needle may not coincide with its geometric axis. To avoid this error the needle is reversed in its bearings and the previous readings (1) and (2) are repeated. In one case, the readings are too low and in the reversed position, the readings are too high. The average of the former readings gives the correct reading. [Fig. 39 (*c*), (*d*)].

(4) The centre of gravity of the needle may not coincide with its axis of rotation. The error is corrected by remagnetising the needle in the opposite direction so that the end which dipped previously now points upward and by repeating the observations (1), (2) and (3). [Fig. 39 (*e*), (*f*)].

In the first case the dip is too low and in the second case when the poles are reversed the dip is increased.

The mean of all these sixteen readings is taken as the true value of the dip.

58. Measurement of Declination :

Since declination is the angle between the geographical and the magnetic meridian of the place, the experiment involves the determination of both the magnetic meridian and the geographical meridian. For accurate determination of declination a Kew magnetometer is employed.

The Kew magnetometer consists (Fig. 40) of a hollow cylindrical magnet C having a fine transparent scale S at one end and a lens L at the other, the focal length of the lens being equal to the length of the hollow magnet. The magnet is thus a collimator. Hence when a telescope, focussed for infinity is placed coaxially with the magnet, an image of the fine scale will be seen in the focal plane of the telescope. The cylindrical magnet is suspended inside a box with glass sides fixed to the centre of a divided circle in a horizontal plane. Round the circle there is a moveable arm carrying a telescope whose position on the circle can be read off by means of a vernier attached to it. The telescope is turned until the image of the middle division of the fine scale coincides with the vertical cross wire and the reading on the divided circle gives the direction of the geometric axis of the hollow magnet. The magnet is then turned over and the reading for the geometric axis again determined. The mean of these readings gives the reading corresponding to the magnetic meridian on the divided circle.

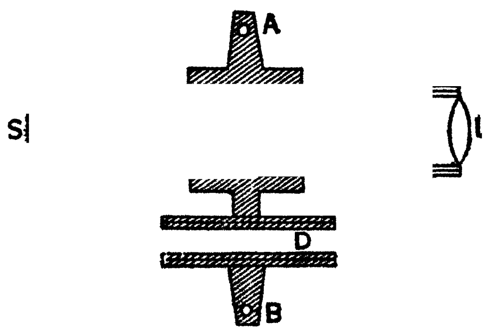


Fig. 40

To determine the reading on the circle corresponding to the geographical meridian, the telescope is turned to observe the time of transit of a star or the sun over the vertical cross-wire of the telescope and the time reading for the transit is noted. Then calculating the angular distance, East or West of the meridian, of the sun *i.e.* the azimuth, from the time of transit, the latitude and the north polar distance of the sun, the

telescope is rotated through the angle calculated and its axis then coincides with the geographical meridian. Thus the *declination* is equal to the difference between the last scale reading and that corresponding to the magnetic meridian.

59. Certain Terms :

(1) **Isogonic Lines** are lines drawn though places at which the magnetic declination has the same value. Line of no declination is called *Agonic Line.*

(2) **Isoclinic Lines** are lines drawn through places at which the magnetic dip has the same value. The line of no dip is called the **Aclinic Line** or the **Magnetic equator**.

(3) **Isodynamic Lines** are lines drawn through places at which the horizontal magnetic force is the same.

(4) **Secular Variation**. The gradual and continuous changes observed in the magnetic elements are called secular variations.

Shorter cyclic changes are called annual or diurnal variation according to the periodic time of the cycle.

59(a). Annual and Diurnal Variation : The variation of declination in a periodic time of one year is known as *annual variation* and occurs simultaneously in opposite directions in the northern and southern hemispheres.

The variation of declination having a period of 24 hours is known as *diurnal variation.*

60. Magnetic Storms : The sudden and non-periodic changes of the magnetic elements are known as *magnetic storms* which are usually accompanied by the display of *aurora borialis.* The phenomenon may be due to the projection of streams of electrons which pass through space in the neighbourhood of the earth.

61. Magnetic Maps : The magnetic elements of a point on the earth are the *angle of dip, the angle of declination and the value of earth's horizontal intensity.*

The values of these elements have been determined at different places on the earth and many places have been found to have equal values of these elements.

Magnetic Maps have been drawn by joining places in which a magnetic element has equal values.

In the maps the following lines are drawn :

(1) Isogonic lines or lines of equal declination.
(2) Agonic line of zero declination.
(3) Isoclinic lines or lines of equal dip.
(4) Aclinic line or line of no dip.

(5) Isodynamic lines or lines of equal horizontal intensity.

Three maps are required, one for the representation of each element or these may be represented on one map.

62. Measurement of magnetic elements by continuous recording: The instruments used for measurement of variations of magnetic elements are called magnetographs.

62(a). Declination magnetograph. The declination magnetograph is an appartus (Fig. 41) in which a beam of light, reflected from a mirror attached to a suspended magnet falls on a sensitized photographic paper wound upon a drum rotating at a constant speed. The curve traced upon the paper indicates the variations in the declination. Thus a continuous record of variation of declination is obtained.

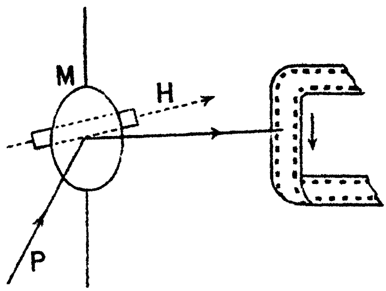


Fig 41

62(b). In the instrument devised by Watson the magnet which consists of nine small permanent magnets is cemented in an aluminium centre piece (Fig. 42) is suspended by a phosphor bronze wire from a fixed point, the magnetic system being enclosed inside a rectangular copper box B to cause oscillations of the magnet to be rapidly damped, by the induced current generated in the copper of the box. Rigidly attached to the magnet is a semicircular mirror M_1, and immediately below this there is another similar semicircular mirror M_2 fixed to the base. A beam of light rays reflected by these mirrors is ultimately reduced to two points by a convex lens. These luminous point images fall on a photographic paper wound on a drum. If the drum be slowly rotated by a clockwork so that the photographic paper moves in the vertical direction, the point image produced by the light reflected from the lower fixed mirror traces a straight line, while the other luminous point produced by the other mirror traces a curved line. The distance

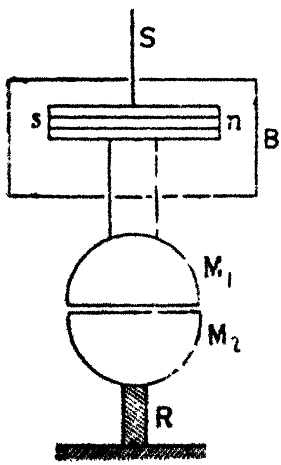


Fig. 42

between two corresponding points of these lines which is proportional to the angle through which the upper mirror rotates gives a measure of the change in the position of the magnet and therefore the change in the magnetic meridian. The geographical meridian being fixed the change of declination is known from the change of the magnetic meridian.

63. Horizontal Variometer : The magnetograph used for recording changes in H is called horizontal variometer. In this instrument a magnet is suspended by bifiliar suspension. The torsion head is turned until the magnet is very nearly perpendicular to the magnetic meridian. Any change in the value of H will produce a corresponding change in the direction of the magnet. The arrangement of mirrors and light-focussing being done as before, changes in H are recorded continuously on a photographic paper wound on a rotating drum in a similar way, as in the case of declination.

64. Vertical Variometer : The magnetograph used for recording changes in V, the vertical component of earth's intensity, is called vertical variometer. In this instrument the magnet is balanced in a horizontal position over a knife edge, so that the centre of gravity of the magnet is slightly towards one side of the magnet and consequently the couple due to vertical component balances the bending moment due to the weight of magnet. Any variation in V makes a proportional change in the inclination of the magnet to the vertical. A beam of light is reflected by a mirror attached to one end of the magnet and is then concentrated to a point by a convex lens. A photographic paper wound on a uniformly rotating drum receives the point image. The photographic paper moves in the horizontal direction and the variation of V is continuously recorded on the paper.

65. The ship's Compass. Kelvin's Pattern : It consists of a thin aluminium rim on which is pasted a scale divided into 32 main divisons and from these divisions 32 silk threads run out radially and connect an inverted sapphire cup resting on a vertical iridium point. In the middle of the rim is placed a system of magnets of various lengths, six in number and fastend together like the steps of a ladder by two silk threads which are attached to the threads connected with the central support. This system has a high magnetic moment and a very small weight and consequently any oscillations in the magnet due to the movement of the ship are damped out easily. This compass is supported inside a copper bowl on *gimbals* consisting of two concentric rings connected in such a manner that the bowl is carried by the horizontal axes

at right angels to one another. This keeps the bowl always horizontal.

In good compasses the compass card with the small magnets is immersed in metheylated spirit to buoy up the card and take part of its weight off the needle point.

The liquid also serves to reduce any oscillations of the compass that would otherwise be troublsome.

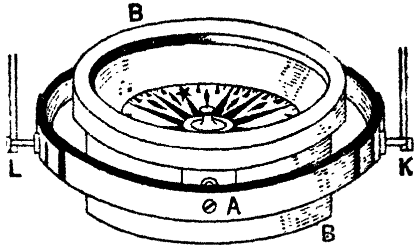


Fig. 43

The compass box contains a vertical mark on the side next the ship's bow and this mark serves as an index for reading off on the card the direction to which the ship's bow is turned.

The magnetic bearing of the ship being obtained from the compaes, its geographical bearing is determined by adding or subtracting the declination of the place from the magnetic bearing.

Note :—The adjoining figure 43 illustrates the old type of ship's compass. In it there is a circular card divided into 32 main divisions and underneath it is firmly fixed a magnetic needle resting on a pivot fixed vertically to the base of a copper bowl B on *gimbals* which consist of an outer ring AA suspended between two diametrically opposite points K and L. This gimbal arrangement keeps the bowl always horizontal.

66. Errors of ship's compass and their corrections :

(a) Errors due to permanent magnetism : In ships built chiefly of iron and steel there are many sources of magnetic disturbances of which two are mentioned for convenience.

(1) The *permanent magnetism* acquired at the time of building of the ship.

(2) The temporary magnetism induced by the earth's magnetic field.

To consider the error due to permanent magnetism we are to take into account the horizontal as well as the vertical component of the field produced by it.

The effect due to the horizontal component on the needle may be compared to a long steel magnet placed beneath the needle and in a direction from bow to stern.

If the ship points towards N and S no error is introduced and if it points E and W the errror is maximum. Since the error remains the same for a change of 180°, it is called a *semicircular* error. This error is corrected by placing two magnets below the

compass needle horizontally and with their two poles pointing in a direction opposite to those of the permanent magnetism of the ship's magnet.

The error due to the vertical component of the ship's magnetism may be compared to a vertical magnet placed near the compass needle and the error in this case is also *semicircular*. It is corrected by suspending a magnet vertically below the needle.

(b). Errors due to temporary magnetism : The error due to the horizontal component of the magnetism induced by earth's magnetic field may be compared to that of a soft iron bar placed below the needle.

If the ship points N and S or E and W no error is produced and it is maximum when the ship points N. E., N. W., S. E. and S. W.

This error is called the *quadrantal error* for the error changes in sign from quadrant to quadrant.

This error is corrected by placing two large soft iron balls on each side of the compass needle.

The error due to the vertical component of the induced magnetism is rectified by placing a soft iron bar vertically with its upper end just above the level of the compass needle. This bar is known as the *Flinder's bar* as the method was suggested by Captain Flinder.

The adjustment of the compass needle is done by the process of swinging the ship in which it is moored to a buoy and moved successively into the various directions of the compass and in each position the true magnetic bearing and the observed compass bearing are observed. The difference is the error of the compass and the correction is applied to obtain the true magnetic bearing.

67. Theories of terrestrial magnetism : Various theories have been put forward from the time of Gilbert regarding the origin of terrestrial magnetism. But none of them could be used to explain all phenomena regarding magnetic property of earth. It was suggested that the magnetic field of the earth may be due to (i) a permanent magnet within the earth, (ii) a magnetic field of which the origin is at a great distance from the earth, (iii) electric currents circulating within the earth, or (iv) electric currents produced due to the ionisation of the layers of air surrounding the earth's surface.

QUESTION

1. Define magnetic elements at any place on the earth's surface with a short account of the methods for their measurement. [C. U. 1939, '41 '45, '50, '52, '56]

Explain how the daily variation of the intensity of earth's field may be continuously recorded. [C. U. 1941, '50]

2. Explain what observations are necessary for the determination of the total intensity of earth's magnetic field at a given place, with the full description of any method for the determination of the vertical component. [C. U. 1938, '47]

3. Explain what you mean by a magnetic map. How is it prepared ?

Describe an accurate method for finding the declination of a place. How would you arrange for observation of its daily variation ? [C. U. 1944, '52]

4. Describe the Dip circle and explain how you would determine by means of it. (a) the magnetic meridian. (b) the dip. [C. U. 1954, '58]

What are the various errors which may arise in the determination of dip by means of a dip circle ? And show how these are eliminated.

5. What is meant by the horizontal intensity of the earth's magnetic field ? Explain what observations are necessary for the determination of total intensity of earth's magnetic field at any given place. [C. U. 1947]

6. If θ_1 and θ_2 are the readings of a dip circle at any place when it is set in two vertical planes mutually at right angles, show that the true angle of dip θ at the place is given by $\cot^2\theta = \cot^2\theta_1 + \cot^2\theta_2$.

7. A sensitive balance has a steel pointer which is a magnet. How will this affect the accuracy of the balance ? How will you determine the magnetic moment of the pointer without removing it from the beam of the balance ?

8. If θ' be the apparent dip when the plane of the dip circle makes an angle ϕ with the magnetic meridian, prove that the true dip θ is given by the relation, $\tan\theta = \tan\theta' . \cos\phi$.

9. Define with the help of a diagram the elements that characterise earth's magnetism at any point on its surface.

Explain how the horizontal component of the earth's magnetic field and the dip are measured in the laboratory. [C. U. 1958]

EXAMPLES

1. Find the resultant force of the earth's magnetism at a place at which the dip is 30° and the horizontal force is 1·8.

Let I be the resultant intensity at the place, then

$$\frac{H}{I} = \cos\theta, \text{ where } \theta \text{ is the dip}$$

$$\text{or } I = \frac{H}{\cos\theta} = \frac{\cdot 18}{\cos 30^\circ} = \frac{\cdot 18}{\frac{\sqrt{3}}{2}} = \frac{\cdot 36}{\sqrt{3}} \text{ dynes.} \quad \therefore \quad I = \cdot 208 \text{ dynes nearly.}$$

2. What is the total intensity at a place where the horizontal component is 0·21 *c. g. s.* unit and the dip is 60° ? *Ans.* ·42 *c. g. s.* unit. [C. U. 1929]

3. At a place where the angle of dip is 45° and the total intensity ·4 *c. g. s.* unit, a magnet vibrating horizontally makes 10 oscillations per minute. Calculate the number of oscillations it would make at a place where the dip is 60° and the total intensity ·5 *c. g. s.* unit.

$$H_1 = I_1 \cos\theta = \cdot 4 \cos 45 = \cdot 4 \times \frac{1}{\sqrt{2}} \text{ and } H_2 = I_2 \cos\theta = \cdot 5 \cos 60^\circ = \cdot 5 \times \tfrac{1}{2}$$

Again $H_1 \propto n_1^2$

$$H_2 \propto n_2^2 \text{ or } \frac{H_1}{H_2} = \frac{n_1^2}{n_2^2} = \frac{\cdot 4 \times \sqrt{2}}{\cdot 5} \text{ or } n_2^2 = \frac{\cdot 5 \times 100}{\cdot 4 \times \sqrt{2}} \text{ or } n_2 = 9\cdot 4 \text{ per min.}$$

(3) If the frequency of a magnet oscillating horizontally be 60 per minute at a place, where dip is 60°, and 40 per minute at another place where value of dip is 40°, compare the total intensities of earth's magnetism at the two places under consideration.

Let R_1 ond R_2 be total intensities at the two places where frequencies of oscillations are respectively 60 per minute and 40 per minute.

Then $R_1 \cos\delta_1 = H_1$ where δ_1, H_1 = dip and horizontal intensity corresponding to R_1; and $R_2 \cos\delta_2 = H_2$, where δ_2, H_2 dip and horizontal intensity corresponding to R_2.

We have $n^2 = KH$ $\quad \therefore \left(\frac{60}{60}\right)^2 = K.\, R_1 \cos 60$

$$\text{or} \quad 1 = K\, R_1 \cos 60. \qquad \cdots \qquad (1)$$

$$\text{Again} \quad \left(\frac{40}{60}\right)^2 = K\, R_2.\, \cos 40$$

$$\text{or} \quad \frac{4}{9} = K\, R_2 \cos 40 \qquad \cdots \qquad (2)$$

$$\text{Dividing} \quad \frac{9}{4} = \frac{R_1 \cos 60}{R_2 \cos 40} \quad \text{or} \quad \frac{R_1}{R_2} = \frac{9 \times \cos 40}{4 \times \cos 60}$$

$$\therefore \frac{R_1}{R_2} = \frac{9 \times \cdot 7660}{4 \times \cdot 5000} = \frac{9 \times 766}{4 \times 500} = 3\cdot 44$$

$$\therefore R_1 : R_2 = 3\cdot 44 : 1$$

4. The apparent dip indicated by a dip circle in any position, is 60°. If the dip circle be rotated through 90°, the apparent dip changes to 45°. Find the true dip at the place.

Let δ be the angle the vertical coil make with magnetic meridian. If θ_1 be apparent dip, V, and H vertical and horizontal components of earth, we have

$$\tan\theta_1 = \frac{V}{H \cos\delta} \quad \ldots \quad (1) \text{ for the first position}$$

For the second position, if θ_2 be apparent dip

$$\tan\theta_2 = \frac{V}{H \cos(90° + \delta)} = \frac{V}{H \operatorname{Sin}\delta} \qquad (2)$$

Then from (1) & (2) $\quad \cot^2\theta_1 + \cot^2\theta^2 = \frac{H^2}{\;} = \cot^2\theta$

where θ is the true dip at the place.

In the problem $\theta_1 = 60°$; $\theta_2 = 45°$

$$\therefore \cot^2\theta = \cot^2 50 + \cot^2 45$$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 + (1)^2$$

$$= \frac{1}{3} + 1 =$$

or $\cot\theta = \frac{\;}{\sqrt{3}}$ $\quad$ **or** $\tan\theta = \frac{\sqrt{3}}{\;} = \cdot 8650$ **(nearly)**

Whence $\quad \theta = 40°52'$

STATICAL ELECTRICITY

CHAPTER I

FUNDAMENTAL FACTS

1. Introductory : In about 650 B.C. Thales of Miletus discovered that amber when rubbed with fur acquires the property of attracting light bodies. It was later shown by W. Gilbert, a physician to Queen Elizabeth, that many other substances behave in the same way. As for examples, a glass rod rubbed with silk, an ebonite rod rubbed with flannel exhibited this effect markedly. The property thus acquired by the bodies was called electric charge and the bodies were said to be **electrified** or **charged.** The charge acquired by ebonite was found to be opposite to that produced on glass, the former being arbitrarily called *negative* and latter, *positive.*

2. Electrification by rubbing : In the process of electrification by rubbing both the rubber (say flannel) and the rubbed (the ebonite) are electrified, one positively and the other negatively. A list of substances have been arranged such that if any two of them are rubbed together, the one prior in the list acquires a positive charge and the latter in the list a negative charge. The list forming what is called Electrostatic series is as follows :—Fur ; Flannel ; Glass ; Mica ; Silk ; Wood ; Amber ; Resin ; Metals Sulphur ; Ebonite.

A simple device consisting of a ball of pith wrapped with tin-foil and suspended by a silk thread (called Pith ball electroscope) can show that electric charge produced on glass rubbed by silk is opposite to that produced on ebonite rubbed by silk or flannel. The former is conventionally called positive and the latter negative. In the list of things mentioned in the previous paragraph, metals are electrical **conductors** and others are called **non-conductors** or insulators. Both conductor and non-conductor can be charged by rubbing ; but when a metal rod is rubbed at one part the charge spreads over the whole rod whereas, when an ebonite rod is rubbed at one part the charge remains confined to that part only. Human body, and earth especially when moist, are good conductors of electricty. When a charged body is connected to earth, the charge whether positive or negative goes to earth without affecting its electrically neutral condition as it is a conductor of vast size.

3. Modern theory of electrification by rubbing: The electron theory of the structure of atom readily explains the process of electrification by rubbing (friction). Atoms of all elements according to this theory consist of one or more negatively charged particles called **electrons** moving in shells round the positively charged particle or particles called **protons** which form the nucleus. The nucleus of elements except hydrogen contain another particle called neutron which has no charge but has mass equal to that of the positively charged particle. The number of electrons appears to range between 1 and 92 in different elements. As an atom is ordinarily neutral electrically there should be in it an equal number of electrons and protons. When one or more electrons are removed temporarily from an atom of a body by friction or otherwise, the remaning portion is said to be positively charged. (Fig. 1) Similarly when atom of an element captures electrons temporarily it becomes negatively charged.

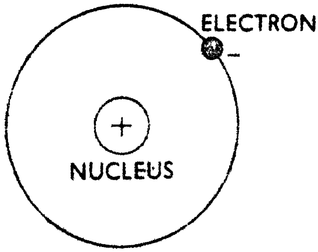


Fig. 1

4. Conductors and Insulators: In certain bodies the electrons are losely bound to their atoms, so that the electrons are free to move under some exciting cause. These bodies are called good conductors. Bodies in which the electrons are rather firmly bound to the nucleus and very few of them are free to move, are non-conductors or insulators.

When ebonite is rubbed by silk electrons move from silk to ebonite. An excess of electrons on ebonite makes it negatively charged; a deficit of electrons in silk makes it positively charged.

5. Electrification by Conduction and Induction: When an electrified body is placed in contact with a conducting body the latter takes a part of the charge of the electrified body and is said to be charged by **Conduction.** A charged body A is brought to a conducting body B on an insulating support; B is touched momentarily and the charged body A is moved away from B, when the latter is found to have a charge of opposite sign. This is electrification by **induction.**

6. Distribution of electric charge on a Conductor: The charge of an electrified body resides entirely on its outer surface. This can be verified by a charged hollow conductor of which the inner wall shows no sign of electrification and the entire charge is found to be on the outer surface.

The surface density of electric charge at a point on a charged conductor is the amount of charge per unit area containing that point. In case of a charged sphere as the distribution of charge is uniform surface density is same at all points. If q be the charge, r the radius of the sphere the surface density of the sphere, or surface density (δ) at any point on the outer surface is $q/4\pi r^2$. The surface density of charge depends on shape of the conductor, being inversely proportional to its radius of curvature. In a pear shaped charged conductor surface density is maximum at the sharp end (A) and minimum at the flat sides B and C. (Fig. 2)

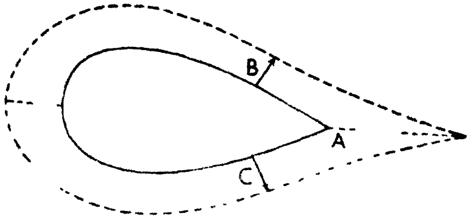


Fig. 2

7. Laws of Action between two electric charges :

(a) Like charges repel and unlike charges attract.

(b) The force of attraction or repulsion exerted between two electric charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

From (b) it follows that for two given charges the force exerted between them varies inversely as the square of the distance between them. This is **Inverse Square Law.**

Let q_1 and q_2 be two charges r distance apart, then each charge is acted upon by a force F given by,

$$F \propto \frac{q_1 q_2}{r^2} \quad \text{or} \quad F = \frac{q_1 q_2}{Kr^2}, \quad \ldots (1)$$ where K is a constant.

The constant K depends on the nature of the medium in which the charges are situated, and is called the **dielectric constant or Specific Inductive Capacity** (S. I. C.) of the medium. The unit of charge in c. g. s. system is so defined for air as medium, that K is unity.

In relation (1) let in air, $q_1 = q_2 = 1$, $r = 1$cm when F = 1 dyne, then K = 1

8. Unit of Charge: The unit of electrical charge is that amount of charge which situated one centimetre apart from a similar and equal charge in air, will repel it with a force of one dyne.

Then for air, relation (1) becomes $F = \frac{q_1 q_2}{r^2}$ **dynes ... (2)**

The above unit is called c. g. s. electrostatic unit of charge. (E. S. U). There is another unit called electro-magnetic unit. (E. M. U). The practical unit of charge is a **Coulomb.**

One coulomb $= 10^{-1}$ E. M. unit of charge

One E. M. unit of charge $= 3 \times 10^{10}$ E. S. unit of charge

Therefore one coulomb $= 3 \times 10^{9}$ E. S. unit of charge

Note : Let the force between two charges q_1, q_2 at a given distance r apart in air, be F_a and in a medium of dielectric constant K, F_m. Then $F_a = q_1q_2/r^2$ and $F_m = q_1q_2/Kr^2$. Whence $K = F_a/F_m$.

Hence dielectric constant or S. I. C of a medium is the ratio of the force between two charges a given distance apart in air, to the force between them at same distance apart in the medium under consideration.

8(a). Electric Intensity : The electric intensity at a point due to a charge is defined as the force experienced by a unit positive charge situated at that point.

Thus if P be a point at a distance r from a charge q placed in a medium of S.I.C., equal to K, then intensity I at P is given by $I = \frac{q \times 1}{Kr^2} = \frac{q}{Kr^2}$. Intensity in air $= \frac{q}{r^2}$, since $K = 1$ for air. The unit of intensity is dynes per unit charge. Electric intensity is a vector quantity having both direction and magnitude. Hence the resultant intensity at any point due to a number of charges is found by parallelogram law of vectors.

Note : If I be intensity at a point due to a charge q, then another charge q' placed at the same point is acted upon by a force $q'I$.

$\therefore$ Force = Intensity × Quantity of charge, or **Intensity = Force/Quantity** of charge.

The force on a unit negative charge at the point considered above will be of same magnitude but acting in direction opposite to that of the intensity *i. e.* force on unit positive charge.

8(b). Dimension of Force and Intensity :

Dimension of Force (F) is MLT^{-2}

Dimension of Intensity : Intensity $= \frac{\text{Force}}{\text{charge}} = \frac{F}{Q}$; $F = \frac{Q^2}{Kr^2}$,

$Q = r\sqrt{FK}$. Now intensity $= \frac{F}{r\sqrt{FK}} = F^{\frac{1}{2}}.r^{-1}K^{-\frac{1}{2}}$

Hence dimension of intensity may be written

$(MLT^{-2})^{\frac{1}{2}}L^{-1}.K^{-\frac{1}{2}} = M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}K^{-\frac{1}{2}}$

9. Inverse Square Law. Its verification :

(a). Coulomb's Torsion Balance : The law was first verified by Coulomb by his torsion balance in which he balanced the forces of repulsion between two charges on two gilt pith-balls against the force produced by the torsion in a fine wire.

The torsion balance consists of a cylindrical glass vessel round the side of which a scale '*s*' is fixed or etched.

A narrow glass tube with a brass *torsion-head* D (Fig. 3) at its top is fixed to the middle part of the upper side of the cylindrical vessel. A fine silver wire is suspended inside the glass tube and carries a horizontal insulated rod *ab* with a small gilt pith-ball at one of its ends. The upper end of the wire is fixed to the torsion-head whose rotation can be observed by a circular scale etched round the rim of the torsion-head.

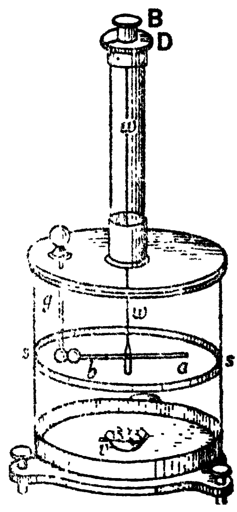


Fig. 3

An insulated rod *g* with a gilt pith-ball at its lower end is placed vertically inside the cylindrical vessel in such a way that the gilt pith-ball approximately coincides in position with the pith-ball *b* at the end of the horizontal rod.

Before the introduction of the vertical rod inside the vessel, the pith-ball in the rod is charged, introduced into the vessel vertically and then placed in contact with the pith-ball at the end of horizontally suspended rod. The charge is shared between the two pith-balls and mutual repulsion takes place.

The rod *ab* rotates and the suspension wire becomes twisted, the rod taking a position of equilibrium when the couple due to repulsive force balances the torsional couple. Let θ be the deflection produced and hence the torsion in the wire is also θ.

The torsion head is rotated through α_1° to reduce the deflection to θ_1, then the torsion in the wire is $(\alpha_1+\theta_1)$. If the inverse square law is true then Force $\times$ (distance)^2 = a constant.

If θ be small, θ may be taken as a measure of force, and also θ may be taken as a measure of linear distance between charged balls. Therefore by Inverse Square law we should have,

$$\theta \times \theta^2 = (\alpha_1+\theta_1)\times\theta_1{}^2 = (\alpha_2+\theta_2)\times\theta_2{}^2 = \text{constant.}$$

By altering θ and α for a number of times in actual experiment, the above fact may be approximately proved.

The above formula is a bit modified if instead of the angular distance the true distance (d) between the balls is considered.

In this case $d=2l\sin\frac{\theta}{2}$ where l is the length of the arm carrying the suspended ball.

Again the component of the repulsive force F tending to twist the wire is $F\cos\frac{\theta}{2}$

Therefore the moment of this force given by $Fl\cos\frac{\theta}{2}$ balances the torsion in the wire.

$$\therefore \quad F_1 l\cos\frac{\theta_1}{2}=K(\alpha_1+\theta_1) \text{ and } F_2 l\cos\frac{\theta_2}{2}=K(\alpha_2+\theta_2)$$

where K is the moment of the couple for unit twist in the wire, F_1 and F_2 are the repulsive forces and $(\alpha_1+\theta_1)$ and $(\alpha_2+\theta_2)$, the corresponding torsions in the wire.

By Inverse Square Law $F\times d^2=C$ a constant.

$$\text{or} \quad \frac{K(\alpha_1+\theta_1)}{l.\cos\frac{\theta_1}{2}}\times 4l^2\sin^2\frac{\theta_1}{2}=C \qquad \frac{K(\alpha_2+\theta_2)}{l.\cos\frac{\theta_2}{2}}\times 4l^2\sin^2\frac{\theta_2}{2}$$

If θ is small $Kl(\alpha_1+\theta_1)\theta_1^{\,2}=Kl(\alpha_2+\theta_2)\theta_2^{\,2}=C$

Defects in the method :

The charges are not situated at points but are distributed over spheres.

This would not matter if the charges were uniformly distributed. This is not possible since the distribution of the charges is altered by the presence of the charged spheres.

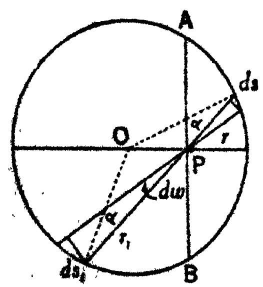


Fig. 4

(b). Cavendish's Experiment : (Indirect Method)

It can be proved that if the electric forces between charged bodies obey the Inverse Square Law, the electric intensity within a charged conductor would be zero.

Let P be any point within a positively charged spherical conductor and let a cone having vertex at P with a small solid angle $d\omega$ be drawn through P so as to cut sphere in two small areas ds and ds_1. (Fig 4).

The area of the right section of the cone at ds is $r^2 d\omega$ and makes an angle α with ds.

$$ds = \frac{r^2 d\omega}{\cos\alpha}.$$

$$\text{Similarly, } ds_1 = \frac{r_1{}^2 d\omega}{\cos\alpha}$$

Let the sphere be uniformly charged and let σ be the surface density ; then charge on $ds = \frac{\sigma r^2 d\omega}{\cos\alpha}$; the charge on $ds_1 = \frac{\sigma r_1{}^2 d\omega}{\cos\alpha}$

If the field due to the charges varies inversely as the nth power of the distance,

the field at P due to the charge on ds is $\sigma r^2 d\omega$

at P due to the charge on ds_1 is $\frac{\sigma r_1{}^2 d\omega}{r_1{}^n \cos\alpha}$

The resultant field at P due to charges on ds and ds_1 is given by $\frac{\sigma d\omega}{\cos\alpha}\left(\frac{1}{r^{n-2}} - \frac{1}{r_1{}^{n-2}}\right)$

The expression is only equal to zero if $n=2$. The same theory nly can be applied to other cones into which we can imagine the phere to be divided.

It can be shown that no other law except the *inverse square law* ould give the same result.

The two fields are oppositely directed and they are equal if $n=2$.

The whole sphere may be divided by cones into pairs of surfaces n the same manner and consequently the electrical intensity at P ue to the whole charge on the sphere is zero.

Experimental arrangement :

In his experimental verification of the Inverse Square Law avendish and at a later date Maxwell used a form of apparatus hewn in the figure.

The sphere A is supported concentrically inside the bigger sphere B so that the two are independently insulated except when he electric connection is made between them by the hinged wire at B.

Let the sphere B be positively charged and then A is connected to B by the wire. (Fig. 5) The connection is broken and A is left insulated.

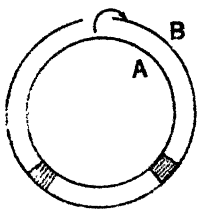


Fig. 5

From the reasoning given before if n the inner sphere A will remain uncharged.

On testing the sphere A by using a pith-ball electroscope no appreciable charge was detected. From his experiment Cavendish concluded that n must be equal to 2 or it must be within one per cent of the value 2.

Maxwell repeated the experiment and concluded that n can not differ from 2 by more than 1 in 21600.

QUESTION

1. Describe the experiments which have established the inverse square law of force between charged bodies. [C. U. 1941]

2. Define electrostatic unit of charge. What is the relation between the c. g. s. electrostatic and practical unit of charge ?

3. Define electric intensity at a point due to a charge. What is meant by dielectric constant of a medium ?

EXAMPLES

1. Three charges $+10, -10$ and $+10$ are placed at the three corners of a square of side 5 cms. Find the intensity at the fourth corner.

Resultant intensity at A due to $+10$ at B and $+10$ at D (Fig. 6)

$$=\sqrt{\left(\frac{10}{25}\right)^2+\left(\frac{10}{25}\right)^2}=\sqrt{\frac{2\times100}{625}}=\frac{2}{5}\sqrt{2};$$

this acts in a direction opposite to AC.

Intensity at A due to -10 at $C=\frac{10}{50}=\frac{1}{5}$; this acts in a direction AC.

$\therefore$ Resultant intensity at A due to all three charges

$$=\frac{2}{5}\sqrt{2}-\frac{1}{5}=\frac{2\times1{\cdot}41}{5}-\frac{1}{5}=\frac{2{\cdot}82-1}{5}=\frac{1{\cdot}82}{5}={\cdot}364 \text{ units}$$

which acts in a direction opposite to AC.

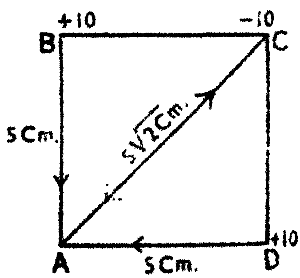


Fig. 6

2. Charge 1, 2 and -3 units are placed at the corners of an equilateral triangle, taken in order. If the length of each side of triangle is 10 cms., find the force, in magnitude and direction at the middle point of the side joining the charges 1 and 2 units. [C. U. 1941]

Let the charges 1, 2 and -3 units be placed at the angular points A, and B and C of the triangle ABC. (Fig. 7)

Let D be the middle point of AB.
The intensity at D due to the charge +1 at A

$$F_b = \frac{1\times 1}{5^2} \text{ dynes along DA}$$

" " " due to the charge at B

$$F_a = \frac{2\times 1}{5^2} \text{ dynes along DB}$$

Thus at D the resultant of these two forces is

$$\frac{1}{5^2} \text{ dynes acting along DA.}$$

Then the intensity at D due to the charge at C

$$F_c = \frac{-3\times 1}{CD^2} \text{ dynes along DC}$$

$$= -\frac{3}{5^2\times 3} = -\frac{1}{25} \text{ dynes along D}$$

Since $DC = AC \sin 60 = 10\frac{\sqrt{3}}{2} = 5\sqrt{3}$

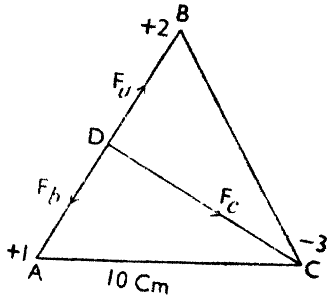


Fig. 7

Thus at D the forces, each of value $\frac{1}{25}$ dynes act at right angles and the resultant is of value $\frac{\sqrt{2}}{25}$ dynes acting at D in a direction making an angle of 45° with AB.

CHAPTER II

ELECTRIC FIELD; GAUSS'S THEOREM AND ITS APPLICATIONS

10. Electric Field: The space surrounding a charged body where electrical attraction or repulsion are observed is called the electric field of force. If the electric intensity at every point in the field is the same, the field is said to be **uniform**, otherwise it is **variable**.

10(a). Electric Intensity: The electric intensity or field strength at a point in an electrostatic field is the force which a unit positive charge experiences when placed at the given point. It is a vector quantity.

It is also defined as the number of Maxwell unit tubes of force over unit area enclosing the point.

11. Lines of force: If a unit positive charge be brought close to a charged body it will experience a force, which at every point

in the field surrounding the charged body will have a definite magnitude and direction. The direction is indicated by a line of force. A line of force may be defined as a curve indicating the path in which a unit positive charge would move and is such that the tangent at any point of this curve gives the direction of the electric intensity at that point.

Note : The existence of the lines of force can be demonstrated by placing a strongly charged body under a horizontal glass plate on which some crystals of gypsum or dust particles are sprinkled. These particles acquire electric charges and set themselves along definite curves.

11(a). Properties of lines of force :—

(1) They are in a state of tension *i.e.*, they tend to shrink in length.

(2) They repel one another *i.e.*, they exert pressure perpendicularly to the direction of their length.

(3) They start from a positively charged body and end on a negatively charged body.

(4) They never intersect one another.

(5) They leave or terminate on a conductor normally in steady state of charge of the conductor. No charge moves across a charged conductor and therefore there is no component force acting parallel to the surface of the charged conductor.

Magnetic lines of force do not necessarily leave or end on a magnet normally. Again magnetic lines of force are closed, whereas electric lines of force are not closed.

(6) The lines of force do not pass through the interior of a conductor.

Magnetic lines of force are continuous and exist inside a magnetic material. Electric lines are continuous in an isotropic dielectric but end on opposite charges.

Various electrical phenomena especially those of attraction and repulsion between charged bodies can be explained by the idea of the existence of lines of force between them. If the bodies are oppositely charged, the lines of force will start out from the positive charge and pass into the negative charge and the tension in the lines will tend to pull the oppositely charged bodies together.

Again, if the bodies are similarly charged *i.e.*, if both of them are positively charged lines of force starting out from the charged bodies will repel one another and the lateral push of the lines to each side will tend to push the two similarly charged bodies apart.

Two lines of force as stated above, can never intersect one another For, if they do, then at the point of intersection there would be two

values for the force acting in different directions given by two different tangents, which is impossible. That is, the lines of force can never intersect one another.

12. Lines of force in Uniform Field : A field in which the force on a small electrified body (a small charge) is everywhere the same both in magnitude and in direction is called the uniform field. Hence in a uniform field, the lines of force are everywhere parallel and the tubes of force have everywhere the same cross section.

13. Mapping of lines of force due to charged bodies :

(*a*) **A single charged body :** Since the surface of a charged body is an equipotential surface (see Chapter on Potential), no work is performed when a charge is moved about upon it and since the direction of lines of force cuts the equipotential surface at right angles, it follows that the **lines of force due to a spherical charged body will proceed radially from its surface** and in the case of bodies of different shapes lines of force will travel outwards in directions normal to the surfaces at different parts of the body.

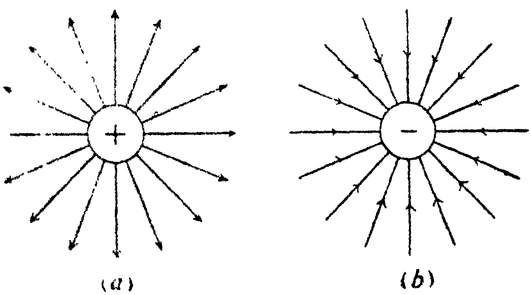


Fig. 8

Fig. 8(a) and 8(b) show lines of force due to positively and negatively charged spheres respectively. The arrow marks show the direction of lines of force along which a free unit positive charge will move.

If the body be charged with 5 or 6 units of positive electricity we may draw 5 or 6 straight lines from the body outwards, each unit of charge representing a line but if the body be charged with 5 or 6 units of negative electricity, 5 or 6 lines are to be drawn towards the body.

(*b*) **Two spherical bodies oppositely charged :** If we consider two bodies oppositely charged and placed at some

distance apart lines of force will start out from the positively charged body and end on the negatively charged body and the space between the bodies will be crowded with a large number of lines of force. (Fig. 9)

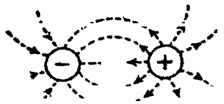


Fig. 9

If the charges are not of the same magnitude *i.e.*, if the positive charge be greater than the negative one, some of the lines of force originating from the positive charge will fall on the negative charge and the rest will travel off to an inifinite distance. The separation between the lines which pass between the charged bodies and those which originate from the positive charge and travel off to an infinite distance is marked by a line of force passing through a point known as the point of equilibrium.

(*c*) **Two spherical bodies charged with the same kind of electricity** : In the case of two bodies charged with the same kind of electricity, say positive electricity, lines of force starting out from the charged bodies will repel one another and travel off to an infinite distance. It is to be noted that a point or a region denoted by sign × in figure 10 is found between the charged bodies at which the electric intensity vanishes and consequently there is no line of force. If the charges are equal in magnitude the point which is called neutral point will be midway between two charges.

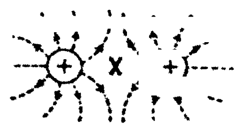


Fig. 10

14. Explanation of electrical induction in terms of lines of force :

When a positively charged body is held in front of another insulated conductor, some of the lines of force from the charged body passes into the nearer surface of the conductor and the rest proceeding directly to the walls of the room without interruption. The side of the conductor opposite to the charged body and at which the lines of force terminate becomes oppositely charged and the side remote from it and from which the lines run out and pass into the walls gets the same kind of charge.

If the conductor be now touched by the finger the lines of force which run out from the conductor to the wall vanish and those which previously passed into the wall from the charged body now fall into the nearer side of the conductor and thereby increase the amount of charge induced there.

Again if the charged body be now removed to a greater distance, the lines of force reaching the conductor now distribute uniformly over the conductor and

If the charge q be **outside**, the cone from q of solid angle $d\omega$ cuts the surface at e, f, g and h, *i.e.* an even number of times. Hence, total normal induction across the surfaces e, f, g and h

$$= -qd\omega + qd\omega - qd\omega + qd\omega = 0,$$

Note : Total normal induction is taken +ve or −ve according as it is directed outwards or inwards.

20. Electric intensity inside a tube of force : If F and F′ be normal intensities for a pair of small areas, S, S′ respectively and K the dielectric constant, then KF′S′ − KFS = O or F′S′ = FS. It follows from this that the product intensity × cross-section is constant along a tube of force which contains no charge, inside. If two small areas or sections of the tube of force, S, S′ are equal, *i.e.*, if the lines of force are parallel, then F = F′, *i.e.*, the field is uniform.

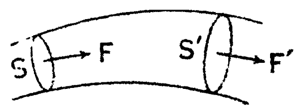


Fig. 16

20(a). Field inside a charged hollow sphere : Consider a point P inside the charged hollow sphere A of radius r, (Fig. 17) at a distance a from the centre O. Draw a sphere of radius a around O. Since there is no charge inside or on this spherical surface, the total normal induction over it is zero so that intensity inside the charged sphere A is also zero.

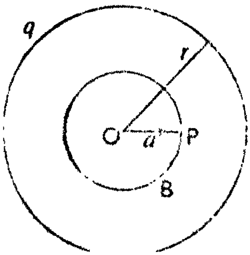


Fig. 17

21. Uniformly charged sphere : Let a small sphere A be charged uniformly with q units of charge (Fig. 18). Let F be the intensity at an external point P due to this charge, P being at a distance r from the centre O. With the centre O and radius OP equal to r describe a sphere through From symmetry of the sphere, the intensity at such point on this sphere is F and is directed normally to the surface of the sphere.

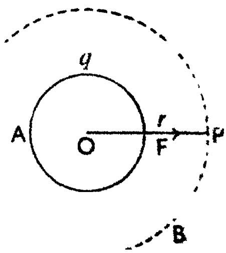


Fig. 18

Total normal induction over the surface of this sphere = KF × area = KF.$4\pi r^2$, where K = dielectric constant.

∴ By Gauss's Theorem KF.$4\pi r^2 = 4\pi q$ or $F = \dfrac{q}{Kr^2}$.

This will also be the intensity at P if a point charge of same amount as q be placed at O. Thus the electric intensity at a point outside a uniformly charged sphere is the same as if the

charge on the sphere were concentrated at the centre. In othe words, *a uniformly charged sphere acts to an external point a if the whole of its charge were concentrated at the centre.*

22. Coulomb's Theorem : The theorem states that th **electric intensity near to the surface of a charged body i equal to 4π times the surface density of the charge.**

Consider any small area S (Fig. 19) on the surface of a charged conductor having a surface density of charge equal to σ. Draw lines o force through every point on th boundary of the small area s as to form a tube extending a short distance outside and insid the conductor and having tw sections ds and ds' drawn perpendicular to the lines o force, the former near to th outer surface and the latter nea to the inner surface.

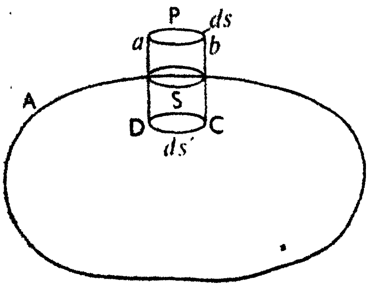


Fig. 19

The cylinder formed of th lines of force contains inside it a charge equal to σ.S. As no line of force cuts the curved walls of the cylindrical tube, and also the section ds' being inside the conductor, the total normal induction through the curved wall and the section ds' is zero and the only induction is through the section ds outside the surface If F be the intensity at any point in ds, then the induction through ds is KF$\times ds$, since F cuts the section normally.

But by Gauss's theorem $KF \times ds = 4\pi\sigma S = 4\pi\sigma ds$ Since $S = ds$ (approx.). Therefore, $F = \frac{4\pi\sigma}{K}$. In air $F = 4\pi\sigma$, as K is equal to unity.

23 (a). If the charged conductor be a sphere, the intensity at any point at distance r from the centre of the sphere is equal to $\frac{Q}{Kr^2}$, where Q is the charge on the sphere.

If the point be very near the surface, r is very nearly equal to the radius of the sphere. Therefore Q, the charge on the sphere $= 4\pi r^2\sigma$, where σ is the surface density of the charge. Then the intensity at a point near to the surface of the charged conductor $= \frac{Q}{Kr^2} = \frac{4\pi r^2\sigma}{Kr^2} = \frac{4\pi\sigma}{K}$.

23 (b). Infinite cylinder uniformly electrified : Consider an infinitely long cylinder uniformly electrified and let Q be the charge *per unit length* of it (Fig. 20). Through the point P at whic

the intensity is to be determined, describe a circular cylinder, co-axial with the charged one, and draw two planes at right angles to the axis of the cylinder and at unit distance apart. Since this cylinder is infinitely long and symmetrical about its axis, the electric intensity is parallel to the plane ends and perpendicular to the curved surface of the cylinder. Therefore, the total normal induction through the ends of the cylinder will be zero and the only induction will be through the curved surface.

Let F be the electrical intensity at each point of the curved wall of cylindrical surface and perpendicular to it and let r be the distance of the point P or any other point on the curved surface of the external cylinder from the axis of the charged cylinder.

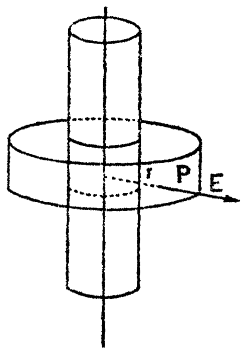


Fig. 20

Then if K be the dielectric constant of the medium the induction over the curved surface of the external cylinder of unit length $= KF \times 2\pi r \times 1$ ($2\pi r \times 1 =$ area of the curved surface). Since Q, *i.e.* the charge per unit length on the electrified cylinder is enclosed within the external cylinder of unit length, through the surface of which the induction is considered, the induction according to Gauss's theorem is $4\pi Q$.

Therefore, $KF \times 2\pi r = 4\pi Q$ or $F = 2Q/Kr$.

If σ be the charge per unit area *i.e.*, the surface density, then $2\pi r \times \sigma = Q$.

Therefore $F = \dfrac{2Q}{Kr} = \dfrac{2 \times 2\pi r \times \sigma}{Kr} = \dfrac{4\pi\sigma}{K}$

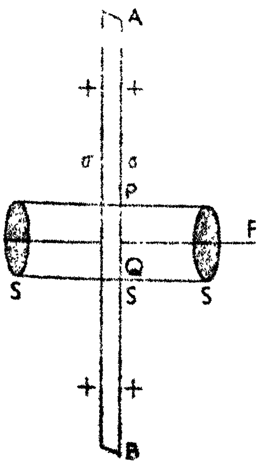


Fig. 21

23(c). An infinite plane sheet uniformly electrified : Consider a point in front of a uniformly electrified infinite plane and through the point draw a straight line FF (Fig. 21) normal to the plane and produce it to an equal distance behind it. With this straight line as axis describe a circular cylinder bounded by planes S,S parallel to the electrified plane. Here the electric intensity F acts along the axis from left to right in front of the plane and from right to left behind it. The induction through the sides of the cylinder is zero and the only induction is

through the plane ends on either side of the sheet. If F be the intensity at any point in the field and S, the area of either of the plane ends of the cylinder, the induction through the ends is 2KFS since the induction through the ends is along the outward drawn normal for both. Again since the normal intensity vanishes over the curved surface, the total normal induction over the closed cylindrical surface is equal to 2K.F.S.

This cylinder contains a charge σS, where σ is the surface density of the charge.

Therefore by Gauss's theorem, the total normal induction is equal to $4\pi\sigma$S.

$$\text{Hence, } 2KFS = 4\pi\sigma S \quad \text{or} \quad F = \frac{2\pi\sigma}{K}$$

That is, the electric intensity due to a uniformly electrified plane is half that just outside a charged cylindrical surface having the same surface density of charge.

[**Note** : The expression for the intensity in various cases considered above is simplified if the medium surrounding the charged body is air for which K = 1.]

24. Electrostatic pressure. Mechanical force exerted on each unit area of a charged conductor.

(a). Electrostatic Pressure : When a conductor is charged with electricity a pressure or a mechanical force per unit area acts on the conductor in the direction normal to the surface and causes it to expand. This pressure or the force per unit area being of the same nature as the hydrostatic pressure is called the electrostatic pressure. The fact that a soap-bubble expands when charged shews the existence of this electrostatic pressure.

(b). Mechanical force per unit area : ABC is a charged conductor (Fig. 22) and P and Q are two points infinitely close to the outer and inner surfaces respectively. We may consider the forces at these points to be made up of two parts, namely the force due to the portion AC and that due to the portion ABC of the charged conductor. Since the points P and Q are situated very near to each other, the forces at these two points due to the part ABC will be very nearly the same and let its value be F_1. Again since the two points are very close to one another and situated on the opposite sides of the part AC, the forces due to the part AC at these two points are equal but opposite in sign. If the force at P be F_2, that at Q is $-F_2$. Thus by Coulomb's theorem the force at P due to the parts AC and ABC, *i.e.* due to the charged conductor is

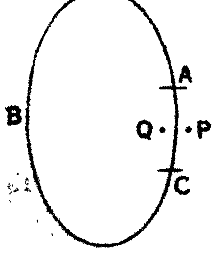


Fig. 22

$F = F_1 + F_2 = \frac{4\pi\sigma}{K}$ where K is the dielectric constant of the medium.

The force at Q is equal to $F_1 - F_2 = 0$, since the point Q is inside the charged conductor.

That is, $F_1 = F_2$. Therefore, since $F_1 + F_2 = \frac{4\pi\sigma}{K}$, $2F_1 = \frac{4\pi\sigma}{K}$

That is $\quad = F_1 = F_2 \cdot\cdot \frac{2\pi\sigma}{K}$

Now if the portion AC be disconnected from the charged conductor and situated in the field F_1 due to the part ABC the mechanical force acting on the charged portion AC is equal to $\sigma\, dS.F_1$ where dS is the area of the portion AC, and σ the surface density of the charge.

Therefore, the mechanical force $= \sigma dS.\frac{2\pi\sigma}{K}$ $\frac{2\pi\sigma^2}{K}dS$ $\cdot$ F $\cdot dS$

$\frac{KF^2}{8\pi}dS \left(\text{Since } F \cdot \frac{4\pi\sigma}{K}\right)$

Therefore, the mechanical force per unit area of the surface of the charged body, *i.e.* the electrostatic pressure is $\frac{2\pi\sigma^2}{K}$ or $\frac{KF^2}{8\pi}$

If the charged conductor be surrounded by air of S. I. C., $K = 1$, the force per unit area is $= 2\pi\sigma^2 = \frac{F^2}{8\pi}$.

25. Energy of an Electrostatic Field : We know that the mechanical force per unit area of the surface of a charged body *i.e.*, the electrostatic pressure is equal to $\frac{KF^2}{8\pi}$ when the surrounding medium is other than air.

If, now, the surface be displaced through a distance dl in the direction of its normal the work done per unit area of the surface $= \frac{KF^2}{8\pi} dl$.

But the volume swept out by the unit area is equal to dl since the area of the cross-section is unity. The work is stored up as energy in this volume of the field and therefore, the energy associated with unit volume of the dielectric (air) is equal to $\frac{F^2}{8\pi}$ ergs.

In the case of a dielectric of Specific Inductive capacity K, the energy per cubic centimetre of the dielectric is $\frac{KF^2}{8\pi}$ ergs.

26. *The electrical intensity at any point in the field is proportional to the number of Faraday tubes passing through a unit area drawn at right angles to the direction of the electric intensity.*

Consider a small area s drawn at right angles to the electric intensity and let a tubular space be formed round the area s and bounded by tubes of force drawn through every point on its boundary and produced up to the positively electrified body from which they start. Let s' be the area of the surface of the charged body cut off by the tube.

Then applying Gauss's theorem to the tubular space formed we have $Fs - F's' = 0$

where F and F′ are the electric intensities at the areas s and s' respectively. $\therefore$ $Fs = F's'$

But by the Coulomb's theorem F′, the electric intensity near the charged body $= 4\pi\sigma$

where is σ the surface density of electrification. $\therefore$ $Fs = 4\pi\sigma s'$.

But $\sigma s'$ is equal to N, the number of Faraday tubes which start from s'. $\therefore$ $F = 4\pi N$ where s is unity.

Thus the electric intensity in air is proportional to the number of Faraday tubes passing through unit area drawn at right angles to the electric intensity.

QUESTIONS

1. Define lines of force and state their properties. What do you mean by tubes of force? Explain the significance of Maxwell and Faraday tubes of force.

2. State and explain Gauss's theorem regarding induction over a closed surface in an electric field. [C. U. 1938, '45, '51 '56

3. What is meant by the total normal induction over a surface? Find the intensity at a point outside an insulated charged conducting sphere at a distance r from centre. [C. U. 1951

4. Determine the magnitude of the electric intensity at a distance r, from the axis of a uniformly charged infinite cylinder. [C. U. 1938, '45

5. Prove that the electrostatic field near the surface of a hollow conductor is equal to 4π times the surface density of the charge.

Hence, show that the mechanical force per unit area of a charged surface surrounded by air is $2\pi\sigma^2$. [C. U. 1939, '46, '59

5. Apply Gauss's theorem to calculate the intensity of the electric field close to the surface of a charged conductor.

Hence, find the force acting per unit area on the surface of a charged conductor. [C. U. 1950

CHAPTER III

ELECTROSTATIC POTENTIAL

27. Meaning of Potential: When two charged bodies are placed in contact or are connected to one another by a conducting wire, electricity may or may not flow from one to the other. The transference of electricity will depend on the electric condition of the two bodies. If the electric condition of the bodies be the same, there will be no flow of electricity but if it is different, electricity will flow from one body to the other and the body from which electricity flows is said to be at a higher potential than the body to which it flows.

So the potential of any charged body may be defined as that relatively electric condition which determines the direction of the transfer of electricity. Thus we see that the term potential is a relative one and the absolute potential of a charged conductor can not be readily measured. A place situated at an infinite distance from any charged body may be considered to be at zero potential since electric forces vanish at that place. The potential of the earth is always taken as the zero of potential for, since its dimension is very large, no charge can produce a marked change in potential.

28. Analogy between Potential in Electricity, Level of water in Hydrostatics and Temperature in Heat: The idea of potential can be made clear by comparing it with *level* in Hydrostatics and *temperature* in Heat. As the difference of level of water in two cisterns connected together by a short pipe or the difference of temperature between two hot bodies determines the direction of flow of water or heat so the difference of potential between two charged bodies determines the direction of flow of electricity.

Again if water be poured into a vessel it spreads out until the level is everywhere the same or if a certain quantity of heat be imparted to a body, it passes into every portion of it and makes the temperature uniform all over the body. In electricity we have noticed that when a charge is given to a conductor it spreads out until the potential becomes everywhere the same.

Thus we see that in comparing electricity, water and heat we find a resemblance between the three quantities, potential, level and temperature. But the analogy between electricity, water and heat can not be pushed further. For we know that two similar bottles of hot water or two bodies similarly heated do not repel one onother.

Again if electricity be gradually introduced into a conductor its potential rises and when it becomes very high electricity leaks out without producing any change in the conductor but when heat is imparted to a body its temperature rises and when it becomes very high no heat is given out but the body passes into the liquid condition.

The analogy between temperature and potential becomes satisfactory if the following points are considered.

A body may have a high temperature with a small quantity of heat or a low temperature with a large quantity of heat. Similarly, a body may have a high potential with a small amount of charge and *vice versa*.

Both temperature and potential are *scalar* quantities.

Although temperature and potential are analogous to some extent, the difference between the two is also marked.

The temperature to which a body is raised by heat depends on the quantity of heat, the mass of the body and also on the nature of the body but the potential of a charged body does not depend on the mass or the nature of the body but on the quantity of charge imparted to the body and also on its size.

An insulated conductor when placed in an electric field acquires some potential although the resultant charge in it may be zero but a body can not have any temperature when it has got no heat.

29. Definition and Measurement of Potential at a point: We know that difference of level between two stations is determined from the amount of work done in lifting a certain mass from one station to another against the force of gravity.

Hence, potential at a point near to a charge is defined and measured by the amount of work done in bringing a unit positive charge from infinity (region of zero potential) to that point.

If the charge be negative the potential is negative and is measured by the work done to bring the unit positive charge from the point to infinite distance away.

30. Potential at a point due to a positive charge: Let us consider a certain number of points A, B, Z in a straight line with a charge Q, the distance of A, B, Z from Q being r, x and r_1 respectively, so that $r_1 > x > r$.

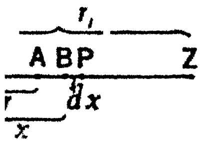


Fig. 23

Then at x, intensity $F = \frac{Q}{x^2}$ (in air)...(1)

If P be a point very close to B at a distance dx apart, then F may be taken for intensity along a very small length dx (Fig. 23). If dV be the difference of potential between B and P then

$$-dV = \text{work done in conveying unit} + \text{charge from B to P}$$
$$= \text{Intensity} \times \text{distance} = F \times dx \ldots \quad (2)$$

$$F = -\frac{dV}{dx} \quad .. \quad (2a)$$

$$-dV = F.dx \text{ or } \int dV = \int -Fdx \quad \ldots(3)$$

Putting the value of F from (1) $dV = -\frac{Q}{x^2}dx$.

Then the work done in carrying unit positive charge from r_1 to r in opposition to the force is the difference of Potential at r from that at r_1.

$$\text{Hence,}\quad V_r - V_{r_1} = -Q\int_{r_1}^{r}\frac{dx}{x^2} = -Q\left[-\frac{1}{x}\right]_{r_1}^{r} = \frac{Q}{r} - \frac{Q}{r_1}$$

where V_r and V_{r_1} are the potentials at distance r & r_1 respectively. If $r_1 = \infty$ (infinity), the potential at a distance $r = \frac{Q}{r}$. This becomes $\frac{Q}{Kr}$ in a medium of S. I. C equal to K.

If instead, the unit charge were carried from point at a distance r to the point at a distance r_1, the work would be done by the field on the charge and in this case $V_r - V_{r_1}$ would be negative.

31. The difference of potential between two points is equal to the work in ergs, done by the electrical force when one unit of positive electricity is conveyed from the point of higher potential to that of lower potential.

From relation (2a) in Art. 30, we find that, the potential at a point is such that its negative space rate of variation at the point gives the intensity at that point.

31(a). Relation between Intensity and Potential : Let us consider two points at a distance dx apart in front of a charged body and let F be the intensity along the very small length dx. If dv be the difference between the two points then,

$-dv$ = work done in conveying unit + charge from one point to another

= Intensity × distance = $F \times dx$

$$\text{or}\quad F = -\frac{dv}{dx}.$$

Thus we find that the potential at a point is such that its negative space rate of variation gives the intensity at the point.

Note : The use of negative sign indicates that the potential diminishes as the distance of the point from the charge increases as the force between the like charges is a repulsion.

Note : The quantity $\frac{dv}{dx}$ is called the potential gradient in the direction of the normal to the equipotential surface passing through the point at which the intensity of the field F acts.

The electrostatic unit (e. s. u) of potential difference is defined as the P. D. between two points such that 1 erg of work is done when 1 e. s. u. of charge is taken from one point to the other of higher potential.

The *practical unit* of potential difference is the *Volt* which is defined as the *p. d.* between two points when 1 joule of work is done in taking 1 coulomb between them

1 **coulomb** $= 3 \times 10^9$ e. s. u. of charge
1 **joule** $= 10^7$ ergs

$$1 \textbf{ volt} = \frac{1 \text{ joule}}{1 \text{ coulomb}} \quad \frac{10^7}{3 \times 10^9} \text{ e. s. u.} = \frac{1}{300} \text{ e. s. u.}$$

32. Electron-Volts : Instead of erg or joule, the electron-volt is generally used in atomic physics. Electron-volt is expressed as the work done in moving an electron (of charge $= -4.8 \times 10^{-10}$ e. s. u.) between two points differing in potential by one volt (1/300 e. s. u.) and is equal to $4.8 \times 10^{-10} \times 1/300 = 1.6 \times 10^{-12}$ ergs.

Thus 1 electron-volt is defined as the kinetic energy (1.6×10^{-12} ergs) acquired by an electron in falling through a p. d. of one volt.

33. Intensity inside a hollow conductor : If there be no charge inside the hollow conductor, no normal induction is possible over any closed surface drawn inside it and so the intensity at any point inside the hollow conductor is zero.

Now, the electric intensity can only be zero at different points in a field, if the potential is constant.

$$\text{Since } F = -\frac{dV}{dx}; \text{ then } F = 0, \text{ if } \frac{dV}{dx} = 0.$$

This means that potential at all points inside a charged hollow sphere has the same value, which is again equal to the potential at the surface of the conductor.

Note : Let us imagine an equipotential surface, passing through a point inside a charged conductor, of potential V_1 which is different from that of the charged conductor itself and let the potential of the conductor be V. Let the potentials of the two equipotential surfaces (the surface of the charged conductor and that imagined inside the conductor) be different. Some work must then be done if a unit charge be moved from the surface of the charged conductor to that imagined inside it. The expression for the work done is $V - V_1 = F \times d$ where F is the electric intensity inside the conductor and *d*, the distance between the two surfaces. But since F inside the charged conductor is zero, therefore $V - V_1$ is zero, *i. e.* $V = V_1$. Similarly it can be shown that all points inside the conductor are at the same potential and equal to that of the conductor itself. Thus the potential inside a charged conductor is constant.

34. Equipotential surface: It is a surface on which the potential is the same at all points.

The surface of a charged conductor is an equipotential surface. For, if there be a difference of potential between two different points on the conductor, electricity will begin to flow from one point to the other until the potential is the same at the two points.

Again *two equipotential surfaces can not intersect one another.*

If, otherwise, the potential at the point of intersection of the two surfaces will have two different values, which is impossible. So two equipotential surfaces can never intersect.

35. Lines of force cut the equipotential surface at right angles: Suppose that the lines of force do not cut the equipotential surface at right angles. Let a line of force cut the surface at an angle θ. Then the force F which acts along the direction of the line of force can be resolved into two directions, one acting in a direction normal to the surface at the point of intersection and the other along the surface. The component parallel to the surface is equal to $F\cos\theta$. Then the work done if an unit charge be moved through a distance d on the surface and in the direction of the component force is equal to $F\cos\theta \times d$. But since every point of the equipotential surface is at the same potential, no work can be done by moving the charged body from one point to another on the surface.

That is, $F\cos\theta \times d = 0$. Since neither F nor d is zero, θ must be equal to 90° so that $\cos 90^\circ = 0$.

That is, the lines of force cut the equipotential surface at right angles.

So the *lines of forces radiate normally from a charged conductor* since its surface is equipotential.

36. Mapping of Equipotential surfaces due to spherical charged bodies: Since we know that the equipotential surface cuts the lines of force at right angles, equipotential surfaces due to a charged body spherical in shape and from which the lines of force start out radially will be spherical surfaces concentric with the charged body. This is evident from the fact that the value of potential at any point at a distance r from the body charged with Q units of electricity is expressedby $\frac{Q}{r}$.

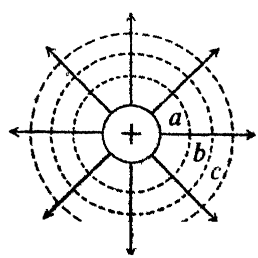


Fig. 24

Consequently, we will several points situated at a distance r round

the body and having the same value of potential. So if a surface be drawn through all these points having the same potential, the surface will be concentric with the spherical charged body and will be called the equipotential surface. For different values of r different equipotential surfaces may be drawn. In figure 24 each of a, b and c is an equipotential surface. For bodies of different shapes different equipotential surfaces may be drawn from the knowledge that equipotential surfaces always cut the lines of force at right angles.

37. Explanation of electrical induction in terms of Potential : If an insulated conductor be placed near a positively charged body, it is said to be situated in a field of positive potential and the potential of its part opposite to the charged body is higher than that of its remotest part. Due to this difference of potential between the two parts of the same body electricity will flow from the part which is at a higher potential to the part at a lower potential until the potential of the whole conductor is uniform.

If the conductor be now touched with a finger, its potential will be zero and no line of force will pass from the conductor to the walls of the room and so there will be no charge at the side of the conductor remote from the charged body. The zero potential of the side nearer to the charged body is the resultant of its positive potential due to the charged body and of its negative potential due to its own charge.

Now if the charged body be removed, the negative charge of the conductor instead of being confined to one side now spreads over the whole conductor and gives to the body a negative potential.

38. Explanation of spark discharge between two oppositely charged bodies in terms of lines of force : When two oppositely charged conductors are placed before one another, the space between them becomes over-crowded with tubes of force which then exert a continual stress tending to produce a strain in the medium. When the stress becomes greater than what the medium can support due to the over-crowding of the increasing number of lines of force, the medium breaks down, the opposite charges neutralising one another and a **spark discharge** passes.

When the charged bodies are moved nearer, the tubes of force due to their tension will shorten and tend to crowd in the space between the charged bodies. But when the charged bodies are actually made to touch each other and meet at the point of contact where the opposite charges destroy each other, the conductors are then said to be discharged. The discharging of a conductor by touching it with a finger is explained in the same way.

QUESTIONS

1. Define the term potential as used in electrostatics. [C. U. 1944, '48]
2. Show how from a knowledge of intensity at a point you can find the potential at that point.
3. Show that the potential at any point distant r cm. from a point charge Q is $\frac{Q}{r}$. [C. U. 1948]

Explain what is meant by electric intensity and potential at a point in an electrostatic field.

Deduce a relation between these quantities. Calculate the intensity of the electric field close to the surface of a charged conductor. [C. U. 1957]

EXAMPLES

1. A sphere of radius 3 cms. and charged to potential of 9 C. G. S. units is placed with its centre A at a distance of 118 cms. from the centre B of another sphere of radius 4 cms. and charged to a potential of 8 C.G.S. units. Find the potential at O in AB such that AO = 54 cms. [C. U. 1918]

Since AB = 118 cms. and OA = 54 cms., OB = 64 cms.

∴ Capacity of the first sphere = 3 units and that of the second sphere = 4 units. Charge on the first sphere = capacity × potential = 3 × 9 = 27 C. G. S. units and that on the second sphere = 4 × 8 = 32 C. G. S. units.

Again, potential due to a charged sphere at a point outside it $= \frac{Q}{r}$, where Q is the charge on the sphere and r the distance of the point from the centre of the sphere.

∴ The potential at O due to the first sphere $= \frac{27}{54} = \frac{1}{2}$ C. G. S. units and potential at O due to the second sphere $= \frac{32}{64} = \frac{1}{2}$ C. G. S. units.

∴ the potential at O due to the two spheres $= \frac{1}{2} + \frac{1}{2} = 1$ C. G. S. unit.

2. A sphere of radius 5 cm. is charged positively to a potential of 10 C. G. S. units. Calculate the intensity due to it at a point whose distance from the centre is 10 cms. [C. U. 1920]

Capacity of a sphere is numerically equal to its radius.

∴ Capacity of the given sphere = 5 C. G. S. units.

Charge on the sphere = capacity × potential = 5 × 10 = 50 C. G. S. units. Now a charged sphere acts with respect to an external point as if the whole charge were concentrated at its centre.

∴ Intensity at a distance of 10 cm. from the centre of the sphere = Force on unit positive charge placed at the point $= \frac{50 \times 1}{(10)}$ = 5 dynes. per unit charge.

3. A sphere of radius 5 cm., charged to a potential of 20 C. G. S. units, is placed with its centre A at a distance of 100 cm. from the centre B of another sphere of radius 4 cm., charged to a potential of 25 C. G. S. units. Find the intensity at the middle point of AB.

Let O be the middle point of AB. Then OA = OB = 50 cm.

Capacity of the first sphere = 5 C.G.S. units.

∴ The charge of the first sphere = capacity × potential = 5 × 20 = 100 C.G.S. units.

Capacity of the second sphere = 4 C.G.S. units.

∴ The charge of the second sphere = capacity × potential = 4 × 25
= 100 C.G.S. units.

Intensity at O due to the first sphere $= \frac{100}{(50)^2}$ dynes along OA.

Intensity at O due to the second sphere $= \frac{100}{(50)^2}$ dynes along BO.

$\therefore$ Intensity at O due to both the spheres $= \frac{100}{(50)^2} - \frac{100}{(50)^2} = 0$.

4. A metal sphere of radius 10 cms. is charged with 10 units of electricity. What is the resultant potential? How will the potential be affected if the sphere be brought close to the earth without actually touching it?

[C. U. 1929. *Ans.* V = 1.

[The potential will be diminished if the charged sphere be brought near the earth.]

5. A spherical conductor of radius 5 cms. and charged to a potential of 48 E. S. units is placed at a great distance from another of radius 8 cm. and charged to potential of -30 E. S. units. Calculate the potential at a point midway between the centres of the spheres. C. U. 1932. [*Ans.* V = 0]

Let Q_1 and C_1 and Q_2 and C_2 be respectively the charge and capacity of the spherical conductors of radii 5 cms. and 8 cms.

Then the potential V_1 due to the first conductor is $\frac{Q_1}{d}$, where $2d$ is the distance between the centres of the two conductor and similarly the potential V_2 due to the second conductor at a point midway between the conductors is $\frac{Q_2}{d}$.

But $Q_1 = C_1V_1$ and $Q_2 = C_2V_2$

$$V_1 = \frac{Q_1}{C_1} \quad \text{and} \quad V_2 = \frac{Q_2}{C_2}$$

Again $Q_1 = 5 \times 48$ units $= 240$ units and $Q_2 = -8 \times 30 = -240$ units

$$\therefore \quad V_1 - V_2 = \frac{240}{d} - \frac{240}{d} = 0$$

i.e. the resultant potential $= 0$.

6. Assuming that the earth is a negatively charged sphere of radius $6{\cdot}37 \times 10^8$ cm. placed in space, find the density of charge per square metre of its surface on a day when the fall of potential in the air around it is 300 volts per metre.

[C. U. 1916, '36

The field of the earth is supposed to be uniform and since the rate of fall of potential is 300 volts per metre, the rate of fall of potential per cm. $= \frac{300}{100} = 3$ volts.

But we have $F = -\frac{dV}{dr} = 3$ volts $= \frac{3}{300}$ E.S.U. Since 1 volt $= \frac{1}{300}$ E.S.U.

Again we know that the intensity $F = 4\pi\sigma$

$$\therefore \quad \tfrac{1}{100} = 4\pi(-\sigma).$$

$$\therefore \quad \sigma = -\frac{1}{4\pi \times 100} \text{ per sq. cm.} = -\frac{10^4}{4\pi \times 100} \text{ per sq. metre} = -\frac{25}{\pi} \text{ E.S.U.}$$

$$= -7{\cdot}956 \text{ E.S.U.}$$

7. Calculate the potential, in electrostatic units, to which a spherical conductor of unit radius has to be raised, in order that the mechanical pressure may be equal to the normal atmospheric pressure *viz.* 10^6 dynes per sq. cm.
[C. U. 1939]

The mechanical pressure $=2\pi\sigma^2$; We have $\sigma=\dfrac{Q}{A}=\dfrac{CV}{4\pi r^2}=\dfrac{V}{4\pi}$

Since capacity $=1$, for radius $r=1$

$\therefore$ The mechanical pressure $=2\pi\sigma^2=\dfrac{2\pi V^2}{16\pi^2}=\dfrac{V^2}{8\pi}$ or $\dfrac{V^2}{8\pi}=10^6$

or $V=\sqrt{8\pi.10^6}=2\times10^3\sqrt{2\pi}$ E.S.U.

CHAPTER IV

CAPACITY, CONDENSER, SPECIFIC INDUCTIVE CAPACITY

39 Capacity . The capacity of any conductor is measured by the quantity of electricity with which it must be charged in order to have its potential raised from zero to unity.

If Q be the quantity of charge on a conductor and if its potential be V then

$$\text{Capacity (C)}=\frac{\text{Quantity}}{\text{Potential}}=\frac{Q}{V}$$

[**Note** : The term capacity is also expressed by the term **capacitance.**]

The capacitance C is measured in e. s. u. when Q and V are in e. s. u. But when Q is in Coulombs and V in Volts, the capacitance is expressed in Farads (practical unit).

39(a). Relation between 1 Farad and 1 E. S. U. of capacitance :

$$1\ \textbf{Farad (F)}=\frac{1\ \text{Coulomb}}{1\ \text{volt}}=\frac{3\times10^9\ \text{e. s. u. of charge}}{1/300\ \text{e. s. u. of potential}}$$
$$=9\times10^{11}\ \text{e. s. u. of capacitance.}$$

$$1\ \textbf{micro-farad}\ (\mu F)=\frac{1}{10^6}F.=10^{-6}F$$

$$1\ \textbf{micro-micro farad}\ (\mu\mu F)=\frac{1}{10^{12}}F.=10^{-12}F$$

40. Capacity of a spherical conductor : Consider a sphere of radius r charged with Q units of electricity and situated at a considerable distance from all other conductors.

The potential V of the charged sphere is, according to the definition of potential, equal to Q/r.

But the capacity C of the sphere $= Q/V = Q \div Q/r = r$, since $V = Q/r$.

That is, the capacity of the sphere in numerically equal to its radius.

• **41. Condenser** : It is an arrangement for condensing a large quantity of electricity on a comparatively small surface. It is essentially an arrangement in which the capacity of a charged conductor is increased by the presence of a conductor preferably earth-connected. In its simplest form it consists of two parallel metal plates close together, with an insulating medium between them. The medium, *e.g.* air or mica is known as the *dielectric* of the condenser. The degree of condensation on a surface increases with the increase of its capacity and since the capacity of a conductor varies inversely as its potential (*i. e.*, $C = Q/V$) it will increase if the potential of the conductor be diminished anyhow.

An earth-connected plate held in front of a charged plate lowers its potential and thereby raises its capacity and the charged plate can receive a greater amount of charge.

42. Simple Parallel Plate Air Condenser : Two metal plates A and B separated by air form a simple air condenser. Let the plates A and B (Fig. 25) be at potentials V and zero respectively and let the plates, each of area A, be d cms apart. Generally the tubes of force running between the plates are straight except near their ends but if the plates are very close to one another we may suppose that the tubes are all straight and of uniform cross-section and distributed uniformly between the plates. The intensity F is, therefore, the same at all points in the field between the plates.

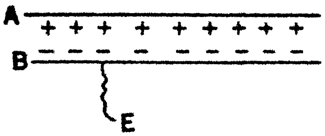


Fig. 25

Now if a unit charge be conveyed from one plate to the other the work done $= F \times d$

That is, $V - O = F \times d$ or $V = F \times d$ ... (1)

But $F = 4\pi\sigma = \dfrac{4\pi Q}{A}$ since $\sigma = \dfrac{Q}{A}$

where σ is the surface density of charge Q on the plate of area A

Hence, $V = \dfrac{4\pi Q}{A} \times d$; Therefore C (capacity) $= \dfrac{Q}{A} = \dfrac{A}{4\pi d}$

Note : In the above case when one of the plates is earth-connected the potential gradually decreases from the charged plate and finally becomes zero at the

earth-connected plate. But when the earth-connected plate is insulated and removed to greater distance the negative charge induced on it gives it a negative potential and therefore the potential at some intermediate point between the plates has the zero value.

Again from the expression $C = \frac{A}{4\pi d}$, we see that the capacity of a condenser is inversely proportional to the distance between the plates of the condenser. So by putting different values to d we get different values of C and if C be plotted against d the curve obtained will be a rectangular hyperbola.

If medium have a dielectric constant K the relation (1) becomes

$$V = \frac{F}{K} \times d$$

whence a capacity is given by $C = \frac{KA}{4\pi d}$

43. Practical Condensers :

(1) In the *mica condenser*, mica plates are used as dielectric and placed between two sets of metal plates.

(2) In the *paper condenser*, two strips of paraffined waxed paper and two strips of tin foil are lightly pressed in a cylindrical form.

(3) In the *variable air condenser*, there are two sets of metal plates, one set consisting of fixed plates and the other set movable between the fixed plates and insulated from each other. The principle of the variable condenser is simply that the capacity varies with the common area between the plates with the distance between them remaining constant.

(4) In the *electrolytic condensers* the plates are the aluminium case and the paste of borates (ammonium) with which contact is made by the inner rod. The dielectric is an extremely thin layer of aluminium oxide formed on the aluminium case by electric deposition. It is a compact form of condenser of relatively large capacity.

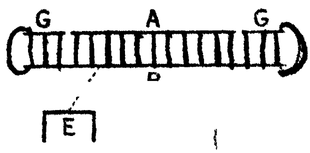


Fig. 26

44. Guard-ring Condenser : Of all the standard condensers Kelvin's Guard-ring Condenser is the most satisfactory type. In a simple plate condenser the field near the edges of the plates is not uniform and the lines of force there are not straight but curved.

Lord Kelvin got over this difficulty by making the insulated plate A circular and surrounding it by a flat circular ring G, known

as a guard-ring in the same plane as the plate A (Fig. 26). If the gap between the plate and the ring is not very wide the value of A, the area of the plate in the expression for the capacity is the mean area of the plate and of the opening in the ring.

45. Capacity of a Spherical Condenser : A spherical condenser consists of two concentric spheres, the inner and outer ones A and B being of radii a and b respectively (Fig. 27). Let the inner sphere be insulated and charged with Q units and the outer one earth-connected and separated from the inner sphere by air. Due to induction the inner surface of the outer sphere is charged with $-Q$ units since it surrounds the inner sphere charged with Q units completely.

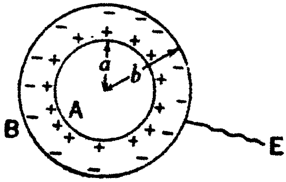


Fig. 27

We know that the potential at any point inside a charged conductor is constant and of value equal to that of the conductor. Therefore the potential of the inner sphere due to charge $-Q$ on the outer sphere is the same as that of the outer sphere and, therefore, of value equal to $-\frac{Q}{b}$.

Again the potential of the inner sphere due to its own charge is equal to $\frac{Q}{a}$.

Therefore the resultant potential V of the inner sphere $=\frac{Q}{a}-\frac{Q}{b}$

That is, $V=Q\left(\frac{1}{a}-\frac{1}{b}\right)=Q\left(\frac{b-a}{ab}\right)$

Therefore, Capacity $(C)=\frac{Q}{V}=\frac{ab}{b-a}$.

Case 1. If a and b differ by a very small amount, then $ab=a^2$ (approximately) anb $b-a=x$

$\therefore\ C=\frac{a^2}{x}=\frac{4\pi a^2}{4\pi x}=\frac{\text{Surface area of the inner sphere}}{4\pi\times\text{thickness of the dielectric}}$

For unit area of the surface the capacity is equal to $\frac{1}{4\pi\times\text{distance between the conductors}}$. The case of two spheres whose distance apart is very small compared with their radii is approximately the same as the capacity of two parallel plates per unit area of surface and is equal to $\frac{1}{4\pi d}$ where d is the distance between the plates.

Case II. If b is very large compared with a, then $C=\frac{ab}{b}=a$.

In this case the capacity of the condenser is the same as that of the inner sphere.

Note : It is to be noticed also that the effect of surrounding a sphere by an earthed concentric sphere is to increase its capacity from a to $\frac{ab}{b-a}$ as in the case considered above.

Case III. Let us now consider the case **when the inner sphere is earthed.** Let the total charge on the outer sphere be Q and the induced charge on the inner sphere be $-Q_1$. Then the charge on the inner surface of the outer sphere is $+Q_1$, thus the charge on the outer surface of outside sphere is Q_2 such that $Q=Q_1+Q_2$

In this case we are to deal with two condensers.

(1) The inner sphere and the inner surface of the outer sphere with air as dielectric.

(2) The outer surface of the outside sphere and infinity (or earth).

The capacity of the condenser (1) is $\frac{ab}{b-a}$ and that of the condenser (2) is b, the radius of the outside sphere.

Therefore the capacity of the concentric spheres when the inner sphere is earthed is given by

$$C=\frac{ab}{b-a}+a$$

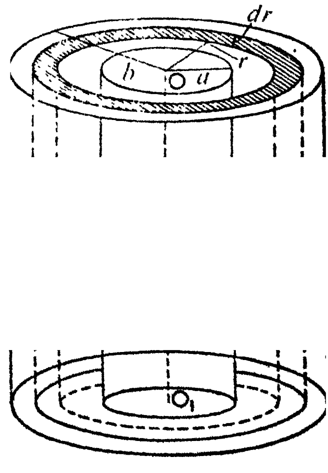


Fig. 28

46. Capacity per unit length of two co-axial cylinders placed in air *i.e.* of a cylindrical condenser :

When a metal cylinder is placed co-axially inside a hollow metal cylinder of larger radius, and the space between the cylinders is filled with a dielectric, a cylindrical condenser is formed.

Consider two co-axial cylinders of radii a and b respectively (Fig. 28). Let the inner cylinder of radius a be charged to Q units per unit length and the outer one be earth connected. If K be the dielectric constant of the medium between the two cylinders, the intensity at a point P at a distance r from the common axis is given by,

$$F = \frac{2Q}{Kr},$$

If this intensity be taken uniform along a very small distance dr then P.D, dV across dr is given by

$$dV = -F.dr = -\frac{2Q}{Kr}.dr \text{ where } dr = r_2 - r_1$$

V_a and V_b be the potentials of the inner and outer cylinders of radii a and b respectively [Sectional Fig. 28(a)], then the difference of potential between the surfaces of the two cylinders is given by,

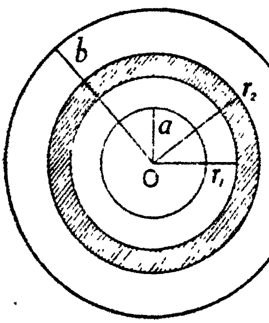


Fig. 28(a)

$$V_a - V_b = -\int^{a} \frac{2Q}{Kr}.dr$$

$$= \frac{2Q}{K}\left[\log_e r\right]_a^b = \frac{2Q}{K}\log_e \frac{b}{a}$$

As the outer cylinder is earthed, $V_b = 0$

the capacity $C = \frac{Q}{V_a} = \frac{K}{2 \log}$ per unit length.

For a length l of each cylinder, $C = \frac{Kl}{2 \log_e}$

A submarine cable is a practical example of a cylindrical condenser, in which the inner conductor consists of copper cable and the surrounding sea-water forms the outer earth-connected cylinder, the insulating sheet forming the dielectric.

46(a). Capacity of Co-axial Cylinders with Compound Dielectric : If now a co-axial cylindrical shell of a substance of dielectric constant K and thickness t be inserted between the

cylinders, then the pot. difference between the cylinders of radii a and r_1

$$= V_a - Vr_1 = 2Q \log \frac{r_1}{a}$$

$$\ldots \quad \ldots \quad r_1 \text{ and } r_2 = Vr_1 - Vr_2 = \frac{2Q}{K} \log \frac{r_2}{r_1}$$

$$\ldots \quad \ldots \quad r_2 \text{ and } b = Vr_2 - V_b = 2Q \log \frac{b}{r_2}$$

Therefore the pot. difference between the cylinders of radii a and b

$$V_a - V_b = (V_a - Vr_1) + (Vr_1 - Vr_2) + (Vr_2 - V_b)$$

$$= 2Q \left\{ \log \frac{r_1}{a} + \frac{1}{K} \log \frac{r_2}{r_1} + \log \frac{b}{r_2} \right\}$$

$\therefore$ the *capacity per unit length* when *the outer cylinder* is *earthed*, *i.e.* when $V_b = O$, is given by

$$C = \frac{Q}{V_a} = \frac{1}{2\left\{ \log \frac{r_1}{a} + \frac{1}{K} \log \frac{r_2}{r_1} + \log \frac{b}{r_2} \right\}}$$

where V_a is the pot. difference between the inner and outer cylinders.

47. Comparison of Capacities--Kelvin's Method : In this method four condensers of capacities C_1, C_2, C_3 and C_4 are arranged as shewn in figure 29 and of which one is a standard variable.

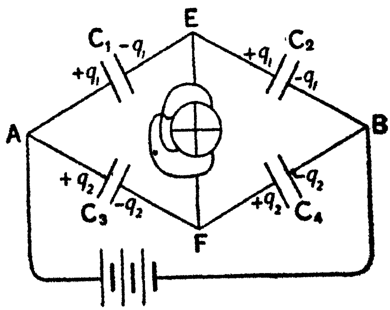


Fig. 29

The two points A and B are connected to the terminals of a battery and the points E and F to the opposite quadrants of an electrometer.

When the connections are made as in the figure, the electrometer needle is disturbed and a deflection is observed. The variable condenser is then adjusted until there is no deflection of the needle. The points E and F are then at the same potential.

The potential difference between A and $E = \frac{q_1}{C_1}$

... ... A and $F = \frac{q_2}{C_3}$

... ... E and $B = \frac{q_1}{C_2}$

... ... F and $B = \frac{q_2}{C_4}$

Therefore $\frac{q_1}{C_1} = \frac{q_2}{C_3}$ $\frac{q_1}{C_2} = \frac{q_2}{C_4}$; $\frac{C_1}{C_2} = \frac{C_3}{C_4}$.

48. Specific Inductive Capacity : The ratio between the capacity of a condenser when any dielectric other than air is used and its capacity when air is used is termed the Specific Inductive Capacity of the dielectric or the dielectric constant. Dielectric is the medium between the two conductors of a condenser.

For crystalline substances it is considered as an absolute constant but for many substances the value S. I. C. varies with their different physical conditions and with different specimens of the substances.

49. Effect of dielectric on capacity : The effect of dielectric other than air separating the plates of the condenser, as already mentioned, is to increase its capacity.

The field F between the plates when a dielectric of S. I. C., K is substituted for air $= \frac{4\pi\sigma}{K}$

$$\therefore \quad V = \frac{4\pi Q}{KA} . d \quad \because \quad V = F \times d \quad \text{and } \sigma = \frac{Q}{A}$$

$$\therefore \quad C = \frac{Q}{V} = \frac{KA}{4\pi d}.$$

50. Faraday's Experiment : Faraday in his expriments used a special form of spherical condenser to demonstrate the function of a medium other than air between the conductors. The outer sphere B is made of brass and exhausted and the inner sphere A is also made of brass and suspended concentrically within the outer by means of an insulating rod (Fig. 30). Faraday used two exactly similar condensers the dielectric in both of them being air. He charged the inner sphere of one of the condensers and observed its potential by touching its knob by a proof plane and obtaining a measure of

the charge removed, by a torsion balance. The charged inner sphere A was then connected with the inner sphere A of the second condenser so that the two shared the charge and by measurement, the charge on each of them was equally divided as might have been expected from the equality in the dimensions of the condensers and, therefore, of their capacities.

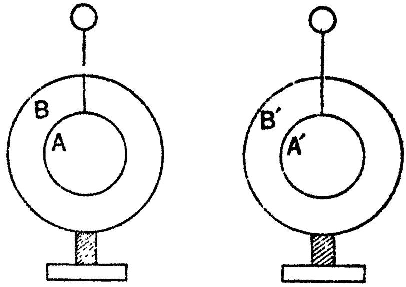


Fig. 30

The condensers being discharged, the space between the inner and outer sphere of one of the condensers was filled with sulphur or shellac and the same experiment repeated. It was then found that the charge in the condenser having sulphur as dielectric was much greater than that in the condenser having air as dielectric. That is, the capacity of the condenser had been increased by the substitution of sulphur for air and the mutual potential considerably reduced. If C_1 be the capacity of the air condenser and V_1 its potential when charged, the charge in the condenser is equal to C_1V_1. If C_2 be the capacity of the condenser with sulphur as dielectric and V_2 the mutual potential of the condensers when they are connected, then $C_1V_1 = C_1V_2 + C_2V_2$

Therefore $\dfrac{C_2}{C_1} = \dfrac{V_1 - V_2}{V_2}$

The ratio $\dfrac{C_2}{C_1}$ is the Specific Inductive Capacity K of the dielectric (Sulphur) and is determined from the knowledge of the potentials V_1 and V_2.

51. Parallel Plate Condenser with Compound Dielectric :

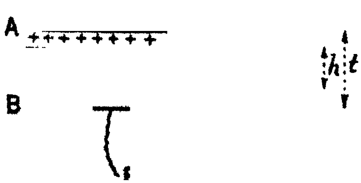


Fig. 31

Let t be the distance between the plates A and B of a parallel plate condenser differing in potential by $V_1 - V_2$ (Fig. 31). If a slab of dielectric of S. I. C., K and thickness h be inserted between the plates, the work done in carrying a unit charge across the air gap between the plates is $F \times (t-h)$ where F is the electric intensity in air.

Again since the electric intensity in the dielectric of S. I. C., K is $\frac{F}{K}$, the work done through the dielectric of thickness h is

$$\left(\frac{F}{K}\times h\right)$$

Therefore the total work done in carrying unit + ve charge from the plate A of potential V_1 to the plate B whose potential V_2 is

$$V_1 - V_2 = F(t-h)+\frac{F}{K}h \qquad V_1 - V_2 = F\left[(t-h)+\frac{h}{K}\right]$$

As $V_2 = 0$ when B is earthed and taking $V_1 = V$ then,

$$V = F\left[(t-h)+\frac{h}{K}\right]$$

By Coulomb's theorem $F = 4\pi\sigma = \frac{4\pi Q}{A}$, where σ is the surface density of charge, Q the charge on the plate of area A.

Therefore $V = \frac{4\pi Q}{A}\left[(t-h)+\frac{h}{K}\right]$, *i.e.* $C = \frac{Q}{V} = \dfrac{A}{4\pi\left[(t-h)+\frac{h}{K}\right]}$

If $h=0$, *i. e.* if the dielectric be air only, the capacity of the condenser is equal to $\frac{A}{4\pi t}$.

52. Determination of S. I. C., (K) of a dielectric : The insulated plate of a simple air condenser is connected to a gold-leaf electroscope and the divergence of the leaves is noted when a suitable charge is given to the condenser. A slab of dielectric of thickness h is then introduced into the gap between the plates of the condenser and the distance of the air is adjusted until the divergence is the same as before. Let t be the distance of the air gap before the slab is introduced and t_1, the adjusted distance after the introduction of the slab.

If C_1 and C_2 are respectively the capacites of condensers with air alone as dielectric and with compound dielectric, then,

$$C_1 = \frac{A}{4\pi t} \text{ and } C_2 = \frac{A}{4\pi\left[(t_1 - h)+\frac{h}{K}\right]}$$

Since the divergence is the same in both the cases we have $C_1 = C_2$. That is, $\dfrac{A}{4\pi t} = \dfrac{A}{4\pi\left\{(t_1 - h) + \dfrac{h}{K}\right\}}$

Therefore $t = (t_1 - h) + \dfrac{h}{K}$ or $K = \dfrac{h}{t - (t_1 - h)}$

53. Relation between Intensity of Field and Specific Inductive Capacity : The intensity F_K of the Field in a dielectric of S. I. C., K. is $\dfrac{1}{K}$ of intensity F_A of the field in which air is the dielectric, the charges and the dimensions of the condensers being the same.

Thus $F_K = \dfrac{F_A}{K} = \dfrac{4\pi\sigma}{K}$, where $F_A = 4\pi\sigma$.

We have, the *mechanical force* per unit area of the plate of the condenser separated by a dielectric of S. I. C., K and of thickness t

$$\frac{2\pi\sigma^2}{K} = \frac{\sigma F_K}{2} = \frac{4\pi\sigma^2}{2K} \text{ dynes per sq. cm.}$$

$$= \frac{K F_K^2}{8\pi}, \text{ since } F_K = \frac{4\pi\sigma}{K}$$

$$= \frac{KV^2}{8\pi t^2}, \text{ since } V = F_K \times t$$

Thus the mechanical force per unit area

$$\frac{KV^2}{8\pi t^2} \text{ dynes per sq. cm.}$$

54. Energy of a Charged Conductor : Suppose a conductor carrying a charge Q be at potential V and let its capacity be C. Let us imagine that the total charge Q is given to the conductor gradually by bringing infinitely small charge dq from infinity to the conductor. Let at any stage of the process of imparting charge, the charge on the conductor be q and its potential v. Evidently $q = Cv$. When the next instalment of charge dq is brought to the conductor, the work done is equal to $v.dq$, since the potential v denotes the work done in bringing unit positive charge to the conductor from infinity.

Hence, the total work done and hence the potential energy

$$\int_0^Q v\,dq = \int_0^Q \frac{q}{C}\cdot dq \qquad \left[\because v = \frac{q}{C}\right]$$

$$\frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 \qquad \left[\because Q = CV\right]$$

$$= \frac{1}{2}QV.$$

54(a). Energy of a Charged Condenser : Let C be the capacity of a condenser (say parallel plate type), and Q be the charge on its positive plate at potential V, the other plate being earth-connected and, therefore, at zero potential. Then proceeding as in article 54, we can show that energy of the charged condensor $= \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$.

Note : In deducing above relations replace the term "Conductor" in Art. 54 by the term "positive plate of the condenser."

55. Loss of Energy on sharing of charges between two Conductors or Condensers : Let us suppose that a conductor (or the positively charged plate of a condenser) of capacity C_1 at potential V_1 (the other plate being earth-connected and, hence, at zero potential, in case of condenser) be connected to another conductor (or positively charged plate of another condenser) of capacity C_2 at potential V_2, (the other plate being at zero potential), so that charges are shared between them.

Then assuming $V_1 > V_2$ and denoting the common potential after connection by V, we have,

$$Q = C_1V_1 + C_2V_2 = C_1V + C_2V = V(C_1 + C_2) \quad \ldots \quad (1)$$

since total charge Q remains same before and after connection.

From (1) above, $V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$

Now total original energy of the two conductors (or condensers) $= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$

Their final total energy after connection $= \frac{1}{2}(C_1 + C_2)V^2$

$$= \frac{1}{2}(C_1 + C_2)\frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)^2} = \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2}$$

Difference of energy = Original energy − Final energy.

$$=\tfrac{1}{2}(C_1V_1{}^2+C_2V_2{}^2)-\tfrac{1}{2}\frac{(C_1V_1+C_2V_2)^2}{C_1+C_2}$$

$$=\tfrac{1}{2}\frac{(C_1+C_2)(C_1V_1{}^2+C_2V_2{}^2)-(C_1V_1+C_2V_2)^2}{C_1+C_2}$$

$$=\tfrac{1}{2}\frac{C_1{}^2V_1{}^2+C_1C_2V_2{}^2+C_1C_2V_1{}^2+C_2{}^2V_2{}^2-C_1{}^2V_1{}^2-C_2{}^2V^2-2C_1C_2V_1V_2}{C_1+C_2}$$

$$=\tfrac{1}{2}\frac{C_1C_2V_2{}^2+C_1C_2V_1{}^2-2C_1C_2V_1V_2}{C_1+C_2}=\tfrac{1}{2}\;\frac{C_1C_2(V_1-V_2)^2}{C_1+C_2}\quad\ldots(2)$$

In this relation C_1 and C_2 are positive quantities and $(V_1-V_2)^2$ being a perfect square, is also positive, so that right hand side of relation (2) is also positive. Thus final energy is less than original energy and, hence, there is a loss of energy. Loss of energy is zero when V_1 and V_2 are equal, but in that case no charge flows from one conductor (or, condenser) to another after connection. The energy lost in above case reappears as heat in the connecting wire or spark between two conductors (or condensers).

56. Energy in the Medium : Let us now consider an element of a tube of induction in the medium of specific inductive capacity K.

Let the length dl of the tube be so small that the intensity F and the area of the cross-sections S, be considered to be constant and that they are at right angles to F, the intensity.

If $+\sigma$ be the surface density of charge on one of the sections S, and $-\sigma$ on the other, the charges on the sections are $+\sigma.S$ and $-\sigma.\ S$ respectively.

But energy $E=\tfrac{1}{2}QV=\tfrac{1}{2}\sigma S.F.\ dl.$ Since $V=F.\ dl.$

$$=\tfrac{1}{2}.\frac{K.F}{4\pi}.S.\ F.\ dl\left(\text{Since } F=\frac{4\pi\sigma}{K}\right)=\tfrac{1}{2}.\frac{KF^2}{4\pi}S.\ dl$$

Since $S.dl$ is the volume of the tube and so the energy per unit volume of the medium is $\dfrac{K.\ F^2}{8\pi}$ *.ergs.*

57. Dielectric in a Condenser : The dielectric is an insulating medium situated between the plates of a condenser. It has been found by the experiment of the dissected Leyden jar that the charge of the condenser resides in the dielectric but not in the metal plates and that the dielectric becomes strained due to the absorption of the charge during the process of charging.

Hence, the total work done and hence the potential energy

$$\int vdq = \int \frac{q}{C} \cdot dq \qquad \left[\because v = \frac{q}{C}\right]$$

$$= \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 \qquad \left[\because Q = CV\right]$$

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Difference of energy = Original energy − Final energy.

$$= \tfrac{1}{2}(C_1V_1^2 + C_2V_2^2) - \tfrac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{C_1 + C_2}$$

$$= \tfrac{1}{2}\frac{(C_1 + C_2)(C_1V_1^2 + C_2V_2^2) - (C_1V_1 + C_2V_2)^2}{C_1 + C_2}$$

$$= \tfrac{1}{2}\frac{C_1^2V_1^2 + C_1C_2V_2^2 + C_1C_2V_1^2 + C_2^2V_2^2 - C_1^2V_1^2 - C_2^2V^2 - 2C_1C_2V_1V_2}{C_1 + C_2}$$

$$= \tfrac{1}{2}\frac{C_1C_2V_2^2 + C_1C_2V_1^2 - 2C_1C_2V_1V_2}{C_1 + C_2} = \tfrac{1}{2}\,\frac{C_1C_2(V_1 - V_2)^2}{C_1 + C_2} \quad \ldots(2)$$

In this relation C_1 and C_2 are positive quantities and $(V_1 - V_2)^2$ being a perfect square, is also positive, so that right hand side of relation (2) is also positive. Thus final energy is less than original energy and, hence, there is a loss of energy. Loss of energy is zero when V_1 and V_2 are equal, but in that case no charge flows from one conductor (or, condenser) to another after connection. The energy lost in above case reappears as heat in the connecting wire or spark between two conductors (or condensers).

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$$= \tfrac{1}{2}.\frac{K.F}{4\pi}.S.\ F.\ dl \left(\text{Since } F = \frac{4\pi\sigma}{K}\right) = \tfrac{1}{2}.\ \frac{KF^2}{4\pi}S.\ dl$$

Since $S.dl$ is the volume of the tube and so the energy per unit volume of the medium is $\dfrac{K.\ F^2}{8\pi}$ *ergs*.

57. Dielectric in a Condenser : The dielectric is an insulating medium situated between the plates of a condenser. It has been found by the experiment of the dissected Leyden jar that the charge of the condenser resides in the dielectric but not in the metal plates and that the dielectric becomes strained due to the absorption of the charge during the process of charging.

According to Faraday the dielectric or rather the insulating medium is supposed to consist of a number of insulating particles of matter charged inductively by the electric intensity acting between the plates of the condenser and successive layers in the medium become alternately positively and negatively charged and no free charge is detected in the dielectric, since the opposite charges of the contiguous particles on layers neutralise one another. The layers of the dielectric which exhibit free charges are in contact with the plates of the condenser where the tubes of force originate and terminate. Thus we see that the dielectric in the condenser is the real seat of the electric phenomena and that the conducting plates merely serve as origins or terminations of the tubes of force.

58. Condensing Electroscope : It is used to detect the presence of electricity from a weak but continuous source. It is an ordinary gold-leaf eletroscope provided with a metal plate, instead of a knob at the top of the metal rod which carries the gold-leaves. An additional metal plate fitted with an insulating handle is placed on the plate of the electroscope and the surfaces of contact of the two plates are coated with shellac vernish which acts as a dielectric between them.

Action: The plate of the electroscope is momentarily connected with the positive pole of a voltaic cell, the negative pole being connected to the earth and the upper plate is touched by the hand. By induction the charge on the lower plate attracts opposite but sensibly equal charge in the upper plate and repels the charge of the same kind through the body to the earth. The hand is then removed and the upper plate lifted by the insulating handle. The condensed electricity on the lower plate being free spreads over the rod and the gold-leaves which then diverge widely.

The separation of the plates diminishes the capacity and strengthens the potential of both, one becoming more strongly positive and the other more strongly negative since the separation does not produce any change in the quantity of charge and so the increased potential causes the increased divergence in the leaves.

59. Spark Discharge : When a condenser is charged and the conducting plates are brought very close to one another, the dielectric becomes overcrowded with tubes of force which then exart a continual stress tending to produce a strain in the medium. The medium assumes a special state and when the stress becomes greater by the overcrowding of the increasing number of lines of force than what the medium can support, the medium breaks down, the opposite charges neutralise one another and a **Spark discharge passes.**

60. Residual Charge : If a condenser is discharged after the charge has been retained for some time it is found that after some time, subsequent discharges of diminishing magnitude can be obtained.

The phenomenon may be explained by the strained condition of the dielectric after charging. When a condenser is discharged, the dielectric takes time to recover from this strained condition as a deformed elastic body slowly recovers from its condition and consequently a number of feeble discharges can be obtained one after another until the medium returns to its original unstrained condition. The residual discharge can be obtained with condensers having paraffined paper as dielectric.

QUESTIONS

1. Show that for a spherical conductor, the capacity is proportional to its radius [C. U. 1936]

Find an expression for the energy of a charged conductor.

2. What is condenser and why is it so called? [C. U. 1938]

Explain the action of a condenser and define (a) the capacity of a condenser, (b) the specific inductive capacity of a dielectric. [C. U. 1947]

3. Shew that the energy W of charged condenser is given by $=\frac{1}{2}CV^2$ where C and V have their usual significance. [C. U. 1943, 47, '53, '55]

4. Show that the energy of a condenser $W=\frac{1}{2}CV^2$ where C = capacity, V = potential difference. Hence, show that the force acting between the two plates of a plate condenser is given by $F=\frac{KV^2}{8\pi d^2}A$ where A = area, d = distance between the plates and K = specific inductive capacity of the medium between the plates. Do you know of any practical application of this force of attraction? [C. U. 1937]

5. How would you determine the specific inductive capacity for solid substance being given a slab of the material in question? [C. U. 1937, '39, '40, '42, '46]

6. Find the expression for the capacity per unit length of two co-axial cylinders placed in air. Investigate the effect of inserting between the cylinders a co-axial cylindrical shell of a dielectric substance of thickness less than that of the layer of air. [C. U. 1952, '54]

7. Deduce an expression for the capacity of a parallel plate condenser. Show how this capacity will be affected if a slab of a dielectric material is interposed between the plates. [C. U. 1950, '35 '58]

8. A condenser is made to share its charge with another uncharged condenser of twice its capacity. Find the sum of the energy of the two condensers. Account for any loss of electrical energy which occurs in the process. [C. U. 1942, '53]

9. State what happens when two charged conductors or condensers at two different potentials are connected together. Give mathematical proof.

EXAMPLES

1. Compare the energy of discharge of two otherwise similarly parallel plate condensers, one of 15 sq. decimeters, the other of 60 sq. decimeters. [C. U. 1912]

Energy of a condenser $=\frac{1}{2}CV^2$

Let V be the potential and C_1, C_2, the capacities of the two condensers.

Then the energy of discharge of the first condenser $= \frac{1}{2}C_1V^2$ and that of the second condenser $= \frac{1}{2}C_2V^2$.

If A_1 and A_2 be the areas of the plates in the first and the second condenser respectively and d, the distance between the plates in each, then

$$C_1 = \frac{KA_1}{4\pi d} \text{ and } C_2 = \frac{KA_2}{4\pi d}.$$

$\therefore$ The energy of discharge of the first condenser : the energy of discharge of the second condenser $= \frac{1}{2} C_1 V^2 : \frac{1}{2} C_2 V^2$,

$$= C_1 : C_2 = \frac{KA_1}{4\pi d} : \frac{KA_2}{4\pi d} = A_1$$

= 15 sq. decimetres : 60 sq. decimetres = 1 : 4.

2. An insulated metal sphere of radius 5 cm. is charged with 5 units of electricity. It is made to touch another insulated metal sphere of 10 cm. radius and then removed. Compare their charges. [C. U. 1913]

Capacity of a sphere is numerically equal to its radius.

$\therefore$ Capacity of the first sphere = 5 units and that of the second = 10 units.

Let V be the common potential after touching.

Then the charge on the first sphere = capacity × potential = 5V units and that on the second = 10V units.

$\therefore$ Charge on the first sphere : charge on the second sphere = 5V : 10V = 1 : 2.

3. Two brass plates of area one square metre are placed parallel at a distance of 10 cm. from each other. Calculate the capacity. Find also the change in the capacity when a slab of glass of thickness 5 cm. is placed parallel between the two plates. Specific inductive capacity of glass = 8. [C. U. 1919]

1 sq. metre = 1 metre × metre = 100 cm. × 100 cm. = 10^4 sq. cm.

The capacity of the parallel plate condenser with air as dielectric $= \frac{A}{4\pi d}$

$$\frac{10^4}{4 \times \frac{22}{7} \times 10} = 79{\cdot}6 \text{ E. S. Units.} = \frac{79{\cdot}6}{9 \times 10^5} \text{ microfarads.}$$

Again the effective air thickness corresponding to the slab of glass = $\frac{5}{8}$cm.

$\therefore$ The total effective air thickness of air between the plates in the second case $= (5 + \frac{5}{8})$ cm. $= \frac{45}{8}$ cm.

$\therefore$ The capacity of the condenser $= \dfrac{10^4}{4 \times \frac{22}{7} \times \frac{45}{8}} = 141{\cdot}5$ E. S. Units

$\therefore$ The change in capacity $= (141{\cdot}5 - 79{\cdot}6) = 61{\cdot}9$ E. S. Units

$$= \frac{61{\cdot}9}{9 \times 10^5} \text{ microfarads.}$$

4. The inducing plate of a charged parallel plate condenser is connected to an electroscope, the distance between the plates being 5 cms. The divergence of the leaves of the electroscope is halved by introducing an ebonite slab of thickness 4 cm. between the plates. Calculate the S. I. C. of ebonite assuming the divergence in the electroscope to be proportional to its potential [C. U. 1923]

Let V be the potential of the inducing plate in the first case and Q, the charge on it. Then the potential of the same plate in the second case $= \frac{V}{2}$. Let K = S. I. C. of ebonite.

apacity of the condenser in the first case $= \frac{A}{4\pi d} = \frac{A}{20\pi}$ E. S. units. The effective air thickness corresponding to the slab of ebonite $= \frac{4}{K}$ cm.

$\therefore$ The total effective air thickness $= 1 + \frac{4}{K}$ cm.

Hence the capacity of condenser in the second case

$$= \frac{A}{4\pi\left(1+\frac{4}{K}\right)} = \frac{KA}{4\pi(K+4)} \text{ E. S. units.}$$

Now, Q = capacity × poetntial $= \frac{A}{20\pi} \times V$ from the first case

$= \frac{KA}{4\pi(K+4)}$ from the second case.

$\therefore \frac{KAV}{8\pi(K+4)} = \frac{AV}{20\pi}$ or $5K = 2K + 8$ or $3K = 8$ $K = \frac{8}{3} = 2{\cdot}66$.

5. Calcuate the capacity of a condenser, consisting of two parallel circular plates 10 cm. diameter and ·6 cm. apart, the space between the plates being occupied by a plate of shellac (S. I. C = 3·3)

Area of each plate $= \pi r^2 = \pi \times 25$ sq. cm

$\therefore$ Capacity of the parallel plate condenser

$$= \frac{KA}{4\pi d} = \frac{3{\cdot}3 \times \pi \times 25}{4 \times \pi \times {\cdot}6} = \frac{3{\cdot}3 \times 25}{4 \times {\cdot}6} = 34{\cdot}375 \text{ E. S. U.} = \frac{34{\cdot}375}{9 \times 10^5} \text{ microfarads}$$

6. You are provided with thin zinc sheet and also ebonite sheet 2 cm. thick. Describe how you would proceed to coustruct a condenser of capacity 100 cm. S. I. C of ebonite = 3·14 [C. U. 1922, '58]

Capacity $C = \frac{KA}{4\pi d}$

or $100 = \frac{3{\cdot}14 \times A}{4\pi \times 2}$ $A = 800$ sq. cm

That is, the effective area of the plate should be 800 sq. cm.

7. Calculate the capacity of a condenser formed by two silver rupees placed in air 1 cm. apart (Diameter of a rupee in 3 cm). [*Ans.* C = ·6 cm. E. S. uuit.]

8. The thickness of the air layer between the two coatings of a spherical air condenser is 2 cm. The condenser has the same capacity as that of a sphere of 120 cm. diameter. Find the radii of the surfaces of the air condenser. [C. U. 1933]

Capacity of spherical condenser is expressed by $C = \frac{ab}{b-a}$

and the capacity C′ of a sphere is equal to its radius 60 cm.

and since $b = a + 2$

we have $C = \frac{a(a+2)}{b-a} = \frac{a^2+2a}{2}$

and since $C = C' = 60$ cm. $a^2 + 2a = 120$

$a = 10$ cm. and $b = 12$.

9. A capacity of 1 *m. f.* is charged to 10,000 volts and suddenly discharged through a fine copper wire. If all the energy went to heating the wire, how many calories would be adiabatically liberated ? [C. U. 1943]

Work or Energy $= \frac{1}{2}CV^2$ ergs.

Here C and V are expressed in electro-static units.

1 micro-farad $= 9 \times 10^5$. E. S. U.

1 volt $= \frac{1}{300}$. E. S. U.

10,000 volts $= \frac{10000}{300} = \frac{100}{3}$ E. S. U. $=$ V

$\therefore$ Energy $= \frac{1}{2}CV^2 = \frac{1}{2} \times 1 \times 9 \times 10^5 \times (\frac{100}{3})^2$ ergs. $= \cdot 5 \times 10^9$ ergs.

But we know that W = JH, where W is the work or the energy converted into heat H in calories and J, the Joule's equivalent. ($J = 4{\cdot}2 \times 10^7$ ergs)

Therefore $\cdot 5 \times 10^9 = 4{\cdot}2 \times 10^7 . H$ $\therefore$ $H = 11{\cdot}9$ Calories.

10. Two spheres of radii 5 and 10 cms. respectively have equal charges of 50 units each. They are then joined by a thin wire so as to be able to share the charges between them. Calculate the total energy of the conductors before and after sharing. What becomes of the difference of energy ? [C. U. 1944]

For the first sphere $C_1 = 5$, $Q_1 = 50$, $V_1 = \frac{Q_1}{C_1} = \frac{50}{5} = 10$ e. s. u.

„ second „ $C_2 = 10$, $Q_2 = 50$, $V_2 = \frac{Q_2}{C_2} = \frac{50}{10} = 5$ e. s. u.

$\therefore$ Energy before connection $= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 = \frac{1}{2}(5 \times 100 + 10 \times 25)$
$= 375$ ergs

Total capacity of the spheres after connection $= 5 + 10 = 15$
„ charge „ „ „ $= 50 + 50 = 100$

$\therefore$ the common poetential „ „ $= \frac{100}{15}$

$\therefore$ Energy after connection $= \frac{1}{2}CV^2 = \frac{1}{2} \times 15 \times \left(\frac{100}{15}\right)^2 = 333{\cdot}3$ ergs.

Loss of energy $= 375 - 333{\cdot}3 = 41{\cdot}7$ ergs

11. The plates of a parallel plate condenser are 2 cms. apart. A slab of dielectric of S. I. C = 5 and thickness 1 cm. is placed between the plates with its faces parallel to them, and the distance between the plates is altered so as to keep the capacity of the condenser unchanged. What is the new distance between the plates ? [*Ans.* 2·8 cm.] [C. U. 1946]

12. Two spheres of diameters 6 and 10 cms. respectively placed at a distance from each other are charged with 8 and 12 units of positive electricity respectively. They are connected by a fine wire. Does any spark pass ? If so how much energy is dissipated ? [C. U. 1947]

Ans. Energy before connection $= 25{\cdot}066$ ergs.
„ after „ $= 25$ ergs.
Loss of energy „ $= 25{\cdot}066 - 25 = \cdot 066$ ergs.

13. A parallel plate condenser of one microfarad capacity is to be constructed, using paper sheets of 0·05 mm. thickness as the dielectric. Find how many sheets of circular metal foil of diameter 20·0 cm. will be needed for the purpose. Dielectric constant of paper = 4·0 [C. U. 1950]

(2) Thomson's Quadrant Electrometer : This instrument mainly consits (Fig. 33) of four cylindrical quadrants A. A and B, B made from a flat cylindrical box made of brass and insulated from one another on being mounted on amber pillars. A dumb-bell shaped needle cut of a thin aluminium sheet is suspended by a quartz fibre made conducting by dipping it into a strong solution of C_aCl_2, inside the quadrants, of which the opposite pairs are metalically connected. When the two pairs of quadrants are at the same potential the needle hangs symmetrically between them. If the needle is charged to a very high potential (positive or negative) and a difference of potential be maintained between the quadrants, the needle will swing round and move inside the pair of quadrants whose potential differs most widely from its own

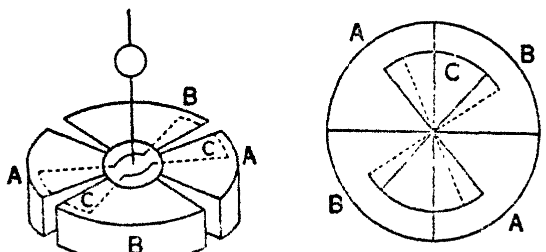


Fig. 33

until the couple due to the field is balanced by the couple due to the torison in the fibre, The angle through which the needle is deflected is observed by reflecting light from a mirror attached to the suspending fibre and gives a means of determining the potential difference between the quadrants

The quardrants A and a part of the needle C lying within them form a parallel plate condenser whose area diminishes as the needle rotates causing a loss of electric energy. The effective area of the condenser formed by a part of the needle and the quadrants B is proportionally increased and there is a gain of electric energy. The resultant is a gain of electric energy in the whole system, for the conductors such as the quardrants A, B and the meedle C are maintained at constant potentials V_1, V_2, and V respectively and the battery with which the quadrants A and B are connected is called upon to furnish energy equal to the sum of the mechanical work done during displacement and the increase in the electric energy.

Let θ be the angle of deflection, then the area of the diminution of the A – C condenser for one quadrant is $\pi r^2 \frac{\theta}{2\pi} = \frac{r^2\theta}{2}$ and for the pair of quadrants is $\frac{2r^2\theta}{2} = r^2\theta$ where r is the radius of the

needle. The capacity of the condeneser when both the faces of the needle are used is equal to $\frac{2r^2\theta}{4\pi d} = \frac{r^2\theta}{2\pi d}$ where d is the thickness of the dielectric between the plates of the condenser.

The energy in a charged A − C condenser $= \frac{1}{2}C(V - V_1)^2$. Then the loss of energy due to the diminution of the area of the A − C condenser $= \frac{1}{2}\frac{r^2\theta}{2\pi d}(V - V_1)^2$ and similarly the gain of energy due the the increase of the area of the B − C condenser is $\frac{1}{2}\frac{r^2\theta}{2\pi d}(V - V_2)^2$.

Supposing V to be greater than V_1 and V_1 greater than V_2 the resultant gain in energy $= \frac{r^2\theta}{4\pi d}\{(V - V_2)^2 - (V - V_1)^2\}$

$$= \frac{r^2\theta}{4\pi d}\{2V - (V_1 + V_1)(V_1 - V_2)\} = \frac{r^2\theta}{2\pi d}(V_1 - V_2)\left(V - \frac{V_1 + V_2}{2}\right)$$

The gain in electric energy produced by the displacement of the needle through the angle θ when the potentials V, V_1 and V_2 remain constant is equal to the work done by the electrical forces against the torsional forces of the suspension.

Let the couple exerted by the supension for one radian twist be C, then, for a twist θ the couple is $C\theta$ and work done for an additional twist $d\theta$ is $C\theta.d\theta$. Then the total work done for producing a deflection θ is $\int_0^\theta C\theta.d\theta = \frac{1}{2}C\theta^2$

Then equating this to the resultant gain in energy, we have

$$\frac{1}{2}C\theta^2 = \frac{r^2\theta}{2\pi d}(V_1 - V_2)\left(V - \frac{V_1 + V_2}{2}\right)$$

$$\theta = \frac{r^2}{\pi C d}(V_1 - V_2)\left(V - \frac{V_1 + V_2}{2}\right)$$

$$\therefore\ \theta \propto (V_1 - V_2)\left(V - \frac{V_1 + V_2}{2}\right)$$

If V is very large compared with V_1 or V_2 then $\frac{V_1 + V_2}{2}$ is negelected in comparison with V. That is, $\theta \propto (V_1 - V_2)V$.

Thus the difference of potential between the quadrants is proportional to the angle of deflection.

62. Application of the above formula :

(*a*) *Comprison of small difference of potential.*

The above formula is used to be determine the small difference of potential, say $(V_1 - V_2)$. This is done by previously connecting the oppôsite pair of quadrants to two points having a known difference potential $(V_3 - V_4)$, say the terminals of a standard cell.

If θ and θ_1 be respectively the deflections corresponding to the difference of potential $(V_1 - V_2)$ and $(V_3 - V_4)$

then $\quad \theta \propto (V_1 - V_2)V, \quad \theta_1 \propto (V_3 - V_4)V$

$$\text{Therefore } (V_1 - V_2) = (V_3 - V_4)\frac{\theta}{\theta_1}$$

For **measuring large differences of potential**, the needle is connected to one pair of quadrants, say A and therefore $V = V_1$.

Then substituting in the expression

$$\theta \propto (V_1 - V_2)\left(V - \frac{V_1 + V_2}{2}\right), \text{ we have } \theta \propto (V_1 - V_2)^2.$$

Since θ is proportional to $(V_1 - V_2)^2$ it is immaterial whether $(V_1 - V_2)$ is positive or negative. This method is used for *rapidly alternating potentials.* When V, V_1 and V_2 are all different the arrangement is said to be *heterostatic* and when $V = V_1$ or V_2 the arrangement is said to be *idiostatic.*

(*b*) *Comparison of Capacities,*

One condenser of capacity C_1 is connected to the electrometer and charged to potential V_1 the resultant deflection θ_1 is noted. The charge is then shared with a second condenser of capacity C_2, the potential falling to V_2. The deflection θ_2 is again observed.

Since the total charge remains constant,
$(C_1 + C)V_1 = (C_1 + C_2 + C)V_2$ where C is the capacity of the electrometer.

$$\text{Hence } \frac{C_1 + C}{C_2} = \frac{V_2}{V_1 - V_2} = \frac{\theta_1}{\theta_1 - \theta_2}$$

C being small in comparison with C_1 and C_2, the capacities are easily compared,

Note : The shape of the needle is immaterial provided the change in the area within each pair of quadrants is proportional to the deflection. This is possible when the outer edge of the needle lies well within the quadrants.

63. Electrostatic Voltmeter: The principle of electrostatic voltmeter is same as an idiostatically used quadrant electrometer. It is made of a single pair of electrically connected quadrants mounted in a vertical position. A plate of aluminium used to serve as the needle of the electrometer is pivoted on a horizontal axis in such a way that it can freely rotate in a vertical plane between the pair of quadrants. A pointer attached to the upper part of the needle moves over a graduated scale, and to its lower part is fixed a carrier on which suitable weights can be placed.

When the needle and the pair of quadrants are maintained at two different potentials, the needle will be turned by the electrostatic force. This turning force is balanced by the couple produced by the weights placed on the carrier. Then from the calibration of the instrument previously made with known potential differences, the potential difference between the pair of quadrants and the aluminium needle can be measured.

By placing different weights on the carrier, the instrument can be used to find different ranges of voltages up to thousands of voltages. As it is used idiostatically, the electrostatic voltmeter can be used to measure alternating potentials. Again since no current flows through the instrument, while in action, no power is wasted.

63(a). Multicellular Voltmeter: It is a modification of the electrostatic type described above. It consists of a number of paddles attached horizontally at equal distances on a vertical spindle of aluminium wire to the top of which a pointer is attached and which moves over a scale calibrated in volts. The paddles are fixed to the spindle in such a way that they move in the spaces between a number of fixed quadrants.

It is used for measuring small potential differences and can also be used for alternating current circuit.

64. Advantages of Electro-static Voltmeter:

(1) They take no current and waste no energy.

(2) They are used either with direct or alternating pressure.

(3) In a laboratory when battery cells are under test, *i.e.*, when constancy of cells is essential, an electro-static voltmeter is desirable, since as it takes no current, it neither alters the potential difference nor causes polarisation in the battery.

(4) There are no errors due to variation in temperature and magnetic field as its action is independent of the passage of current.

65. Instruments for measuring Electric Potential and Charge : An *electrometer* is an instrument for measuring electrical potential in absolute units or for comparing the potential difference between any two bodies. But the ordinary gold-leaf electroscope is generally used to indicate a rise or fall of potential without measuring it and also to detect and understand quantitatively the presence, the nature, and the quantity of charge in a body. In C. T. R. Wilson's modern tilted gold-leaf electroscope potential difference can be measured quantitatively with the help of a previously calibrated scale and also the ionisation current can be measured with it.

Of the different forms of electrometers, quadrant electrometer can be used not only to measure the difference of potential but also to measure the quantity of charge and current.

Both the electrometer and the electroscope are used for the same purpose, but they differ in construction and sensitiveness. Electrometer is more sensitive regarding potential measurement but for current measurement electroscope is more sensitive.

A *galvanometer* is an instrument generally used for measuring current passing through a conductor and differs in construction from either an electroscope or an electrometer. It can be used to measure potential difference by connecting a high resistance in series and with the help of the scale previously calibrated. Electroscopes and electrometers measure potential difference in either open or closed circuits but galvanometers when converted into a voltmeter measure potential difference in closed circuits only. The galvanometer is also used to measure quantity of electricity as is done by a ballistic galvanometer.

Tilted *Electroscope.* (C. T. R. Wilson). It consists mainly of a leaf of gold supported by a wire which is put into connection with a conductor by a wire carried by a spring. The conductor is insulated from the box surrounding the leaf by an ebonite plug.

The movement of the leaf is observed by a microscope having a finely divided scale through a circular window in the box.

QUESTIONS

1. Describe and explain the action of any type of electrometer. [C. U. 1943, '47]

2. Describe some form of absolute electrometer and give the theory of its action. [C. U. 1944, '49]

EXAMPLES

1. An insulated plate 10 cms. in diameter is charged with electricity and supported horizontally at a distance of 1 mm. below a similar plate suspended from a balance and connected to earth.

If the attraction is balanced by the weight of one decigram, find the charge on the plate. (g=980 C.G.S.) [C. U. 1949]

We know that the mechanical force per unit area of a charged conductor of surface density is given by $f=2\pi\sigma^2$

$\therefore$ Total force on area $\pi\times 5^2=2\pi\sigma^2\times\pi\times 25$

Since the force is balanced by the weight of $\frac{1}{10}$ gm.

we have $\frac{1}{10}\times 980=2\pi^2\sigma^2\times 25=50\times\pi^2\sigma^2$ $\quad\therefore\quad \sigma=\sqrt{\frac{98}{50\times\pi^2}}$

$\therefore$ Charge $Q=\sigma\pi.5^2=\sqrt{\frac{98}{50\times\pi^2}}\times\pi\times 25=35$ e. s. u.

2. A potential difference of 1000 volts is established between two parallel plates 1 *mm.* apart in vacuo. Calculate the force of attraction per sq. cm.

[*Ans.* 44·2 dynes per sq. cm.]

We know that $f=\frac{V^2 s}{8\pi d^2}=\frac{10^6\times 1}{300\times 8\times 3{\cdot}142\times{\cdot}01}=44{\cdot}2$ dynes per sq. cm.

since 1 volt $=\frac{1}{300}$ E. S. unit

3. A condenser is made of two 8×12 cm metal plates separated by a thin layer of air. What is the field intensity between the plates near their centres where they have opposite charges of 163 e. s. u. each ? What would it be if they were separated by crown glass of S. I. C. 6 ?

4. Calculate the force of attraction between the lower and upper discs of an electrometer when a potential of 1000 volts between them, given that they are ·5 cm. apart and of area 10 sq. cm. [Ans. 17·69 dynes]

CURRENT ELECTRICITY

CHAPTER I

PRIMARY CELLS AND EFFECTS OF CURRENT

1. Preliminary Considerations : In order to obtain a continual flow of electricity from one point to another point some conducting path is to be provided between them, and suitable means must be adopted to maintain the two points at different potentials. One method of producing the difference of potential necessary for a continual flow of electricity is by chemical action, and the device first constructed on this principle is a **Simple Voltaic Cell.** A Simple Voltaic Cell consists of a copper (Fig. 1) and a zinc plate immersed in dilute sulphuric acid, the copper plate being at a higher potential than the zinc plate. The difference in potential between the zinc and the copper plate when they are simply immersed in the acid and not connected externally, *i.e*, when the cell is on open circuit, is called Electro-motive force (E. M. F.) of the cell.

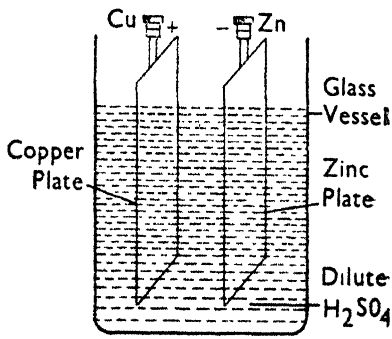


Fig. 1

A simple Voltaic Cell cannot give steady uniform current owing to two defects namely (1) Local action and (2) Polarisation. Local action is due to the presence of impurities in commercial zinc used in Simple Voltaic Cell, and Polarisation is due to accumulation of hydrogen bubbles on the copper plate when the cell is in action. The local action is avoided by using amalgamated zinc plate (*i.e.* zinc plate coated with mercury) and polarisation is removed by using suitable oxidising agents called depolariser.

The accumulation of hydrogen has two effects. Firstly, it reduces the effective area of the copper plate by covering a portion of the exposed surface of the plate. Since these bubbles of Hydrogen are bad conductors, they offer a great resistance to the passage of the electric current and hence current is reduced. Secondly, hydrogen produces an electro-motive force tending to send a current through the cell in a direction opposite to that in the cell.

2. Various Forms of Primary Cells :

(1) **Daniell Cell** : It (Fig. 2) consists of an outer cylindrica vessel of copper serving as positive plate. The vessel contains concentrated copper sulphate solution which acts as depolariser. In the

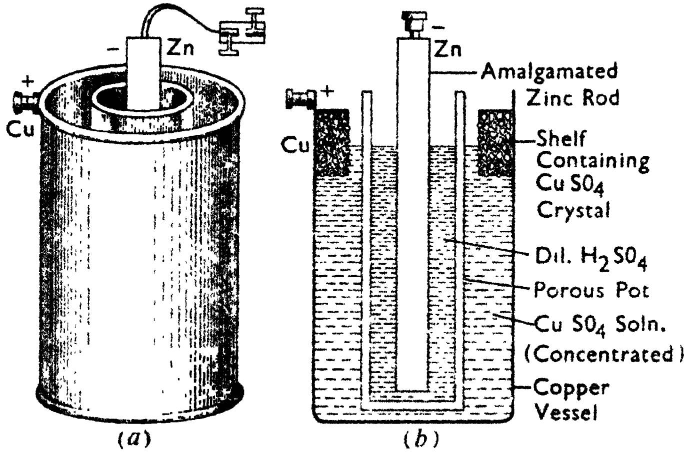


Fig. 2

copper vessel stands a porous pot containing dilute H_2SO_4 and an amalgamated zinc plate or rod. Near the top of the copper vessel a perforated shelf is attached filled with copper sulphate crystals. The crystals which are partially covered by the solution, by gradually dissolving keep the strength of the latter unchanged. The E. M. F. of Daniell is about **1·1 volts** and the resistance rather high. But both being fairly constant, the cell is useful when small constant current is required. Hydrogen evolved by the action of H_2SO_4 on zinc passes out through the porous pot and is acted on by $CuSO_4$. The chemical reactions in the cell are:

$$Zn^{++} + H_2SO_4 = ZnSO_4 + 2H^{++}$$
$$2H^{++} + CuSO_4 = H_2SO_4 + Cu^{++}$$

∴ Thus copper and not hydrogen is deposited on the high potential copper plate and hence polarisation is prevented.

(2) **In Grove's Cell** **the positive plate is a sheet of platinum and the negative plate is amalgamated zinc plate in the form of**

U or a cylinder. The exciting liquid and the depolariser are respectively dilute H_2SO_4 and strong HNO_3. Hydrogen liberated by the action of the exciting liquid on zinc is oxidised by strong nitric acid, the depolariser and thus polarisation is prevented. The chemical actions are given by :

$$Zn + H_2SO_4 = ZnSO_4 + H_2$$
$$H_2 + 2HNO_3 = 2H_2O + 2NO_2$$

Thus hydrogen being oxidised to water cannot collect on the platinum plate ; the nitrogen peroxide dissolves in HNO_3 and cannot deposit on platinum. Polarisation is thus absent. The E. M. F. of this cell is about **1·9 volts**, and the resistance fairly low and constant. Hence it is useful when strong and fairly steady current is required. The drawbacks are however that platinum makes the cell expensive and fumes of nitrogen peroxide are very disagreeable.

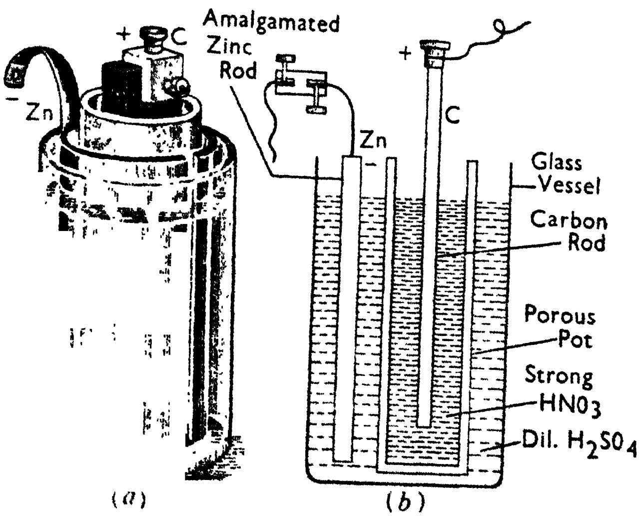


Fig. 3

(3) **Bunsen's Cell :** It (Fig. 3) is a modification of the Grove's cell. In place of platinum cheap gas coke is used as positive plate. The liquids used, the chemical reactions and e. m. f. etc. are the same.

(4) **Leclanche cell :** In this cell (Fig. 4) the positive plate is a carbon rod surrounded by a mixture of MnO_2 and charcoal powder

contained in a porous pot. The porous pot stands inside a glass vessel containing a solution of ammonium chloride in which is

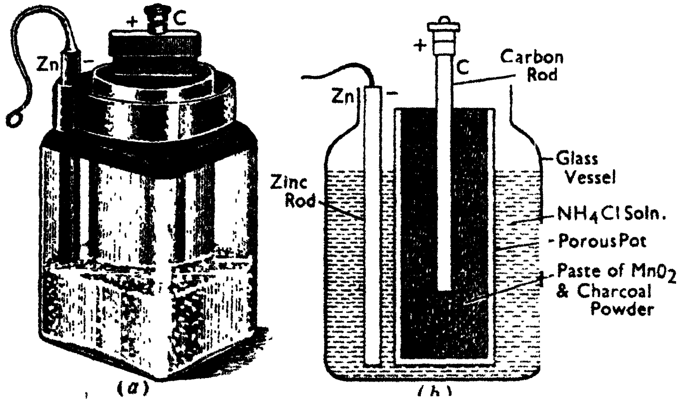


Fig. 4

immersed an amalgamated zinc rod, the negative plate. The hydrogen liberated by the action of the exciting liquid NH_4Cl solution on zinc is oxidised by MnO_2, the depolariser and thus polarisation is prevented. The chemical reactions are :

$$Zn + 2NH_4Cl = ZnCl_2 + 2NH_3 + 2H \quad \ldots \quad (1)$$
$$2H + 2MnO_2 = Mn_2O_3 + H_2O. \quad \ldots \quad (2)$$

The hydrogen is liberated at a greater rate than the MnO_2 can use it up, so that polarisation again sets in and current strength falls. If allowed to rest for a short time, MnO_2 does its work according to equation (2) and the cell returns to its normal strength. Leclanche cell is therefore employed for intermittent work such as in electric bells and telephone calls. The **E. M. F.** is about **1·5 volts** and the resistance is frequently high.

3. Latimer Clark standard cell : It (Fig. 5) consists of a wide test tube about 2 cm. in diameter and about 5 cm. deep. Some mercury is placed at the bottom of the tube and this forms high potential element or positive plate of the cell. Above the mercury is a paste of mercurous sulphate and zinc sulphate solution. Above the paste is a layer of crystals of zinc sulphate and saturated solution of zinc sulphate. The low potential element or negative plate is an amalgamated zinc rod which is dipped up to the paste in the tube. Contact with mercury is made with the help of a

platinum wire protected inside a glass tube. The open end is sealed with glue coated with sodium silicate. The **E. M. F.** of this cell is taken as **1·435 volts** at 15°C and it falls in value with rise of temperature.' The effect of rise of temperature is given by the relation:

$E_t = 1·434\{1 - ·00077(t - 15)\}$ volts, where t is centigrade temp.

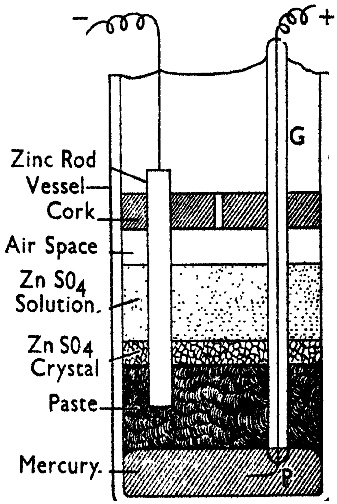


Fig. 5

4. E. M. F. and resistance of a cell: The E. M. F. of a cell depends on the temperature and on the materials used in making it. It does not depend on the size of the plates and the distance between them for a given type of cell. The internal resistance however depends on these two factors. The bigger the area of the plates the less the resistance, while the less the distance apart the less the resistance. The internal resistance depends also on the concentration of the liquid or liquids used. Evidently a cell with larger plates has the same E. M. F. of a smaller one of the same type, but the larger one has the lesser resistance.

4(a). Requisites of a good Voltaic Cell: It should be free from local action and polarisation. The E.M.F. should be high and constant but the internal resistance small. A steady current for a considerable time should be supplied by it. No chemical action should take place when the cell is not giving a current. The cell should be economical and convenient in use.

5. Effects of Current: When positive and negative poles of a primary cell are connected externally by a wire through a key, an electric current flows along the wire from the higher to the lower potential plate. The existence of the electric current along the wire is known by various effects produced by it. The wire traversed by the current becomes heated—this is **heating effect.** If the wire in question traversed by current be placed near a compass needle, it is deflected—this is **magnetic effect.** If the current be passed through liquids, they are decomposed—this is **chemical effect.** These various effects will be dealt in details in due course.

QUESTIONS

1. Explain local action and polarisation and show how these are avoided in a Leclanche cell.

2. Explain the statement "E. M. F. of a given kind of cell is constant". How does a Bunsen cell of large size differ from a similar cell of smaller size ?

3. Make a comparative study of the common primary cells. Give a brief account of any type of standard primary cell.

CHAPTER II

MAGNETIC FIELD DUE TO CURRENT

6. Action of Current on magnet: In 1819 Oersted, a scientist of Copenhagen first observed that a wire carrying an electric current produces a magnetic field in its neighbourhood, so that if a magnetic needle (Fig. 6) be placed very near to the wire, the magnetic needle becomes deflected. The movement of the magnetic needle indicates the presence of magnetic field round the current.

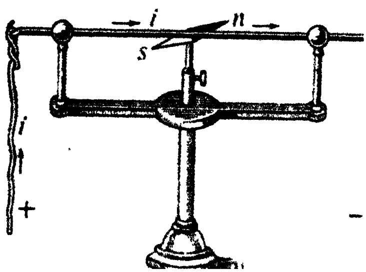


Fig. 6

The direction of the field round the wire is given by either (*a*) Ampere's rule or (*b*) Maxwell's Corkscrew rule.

(a) Ampere's Rule: Suppose a man to be swiming along the wire in the direction of the current with his face towards the needle (placed above or below the wire carrying the current) then the direction of his left hand gives the direction of the field, *i.e.*, the direction of motion of the *n*-pole of the needle.

(b) Maxwell's Corkscrew Rule: Suppose (Fig. 7*a*) a corkscrew is driven along the wire in the direction of the current, the direction of rotation of the thumb indicates the direction of the field.

7. Lines of force due to Linear and Circular Current: A straight conductor, say a wire (Fig. 7) carrying a current is surrounded by circular lines of force with their centres lying on axis of the wire, their planes being perpendicular to it.

For a circular wire (Fig. 8) carrying a current, the magnetic lines of force are perpendicular to the plane of the coil and for a

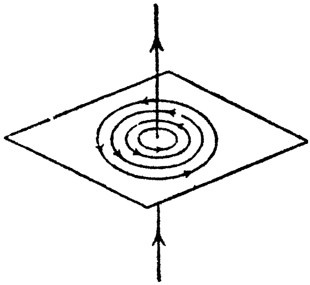


Fig. 7

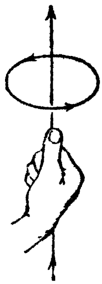


Fig. 7(a)

small region near the centre of the circle, the field is uniform and the lines of force are parallel. In circular current, the shape of the lines of force is quite independent of the strength of the current, but the strength of the magnetic field produced, depends on the current strength.

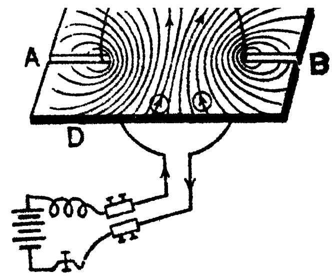


Fig. 8

The magnitude of the magnetic field at any point, he may be obtained by considering each element of the conductor conveying the current according to a law known as Laplace's law.

8. Laplace's Law : The law states that "the strength of the magnetic field (F) at a point P due to the current in a very small element of a conductor is directly proportional to (i) the length (ds) of the element, (ii) the strength (i) of the current, (iii) the sine of the angle (α) between the direction of the element and the line joining the point to the midpoint of the element, and (iv) inversely proportional to the square of the distance (r) of the point from the element.

Let AB (Fig. 9) be a wire carrying current of strength i. Consider an elementary length ds at any point O of this wire. Then the strength of the field at any point P, due to the current in the element ds may be expressed as

$F \propto \frac{i\,ds \sin\alpha}{r^2}$, where $r = PO$, α is the angle between the element

ds and the line PO, and *ds*.sinα is the component of *ds* norma to OP.

The direction of the magnetic field at P due to *ds* is perpendicula to the plane passing through the element and th line OP (known as radius vector). The resultan field strength at P due to the whole conducto AB may be obtained by integration or by suitabl combining the effects of fields of similar element into which AB may be divided.

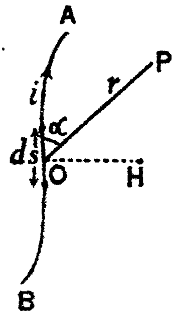


Fig. 9

9. Unit of Current : Now, from Art. 8 above

$$F \propto ids \sin\alpha$$

$$F = K.ids \sin\alpha$$ ...(1), where K is a constant depending on the way in which *i* is measured. It is to be noted that the expression (1) is independent of the nature of the medium in which the field is produced The strength of the magnetic field due to a magnetic pol however depends on the nature of the medium.

If in relation (1), $ds = 1$, $r = 1$, $\alpha = 90^\circ$, and the current flowing under these condition be called unit current *i.e.* if $i = 1$, then constant $K = 1$. These give us a definition of the current of unit strengt in c. g. s. electromagnetic unit (*e.m.u.*).

The C.G.S. **electromagnetic unit of current** is that curren which flowing in a wire of length one cm., bent into an arc o a circle of 1 cm. radius, will produce a magnetic field of intensity one Oersted (1 dyne per unit *n* pole) at the centre of the circle The angle α is 90° for each element of the conductor, as the point where intensity is considered is the centre of the circular arc.

The above is **absolute unit** of strength of current. The practical unit called an **Ampere** is one-tenth of 1 *e.m.u.*

Therefore, 1 ampere $= \frac{1}{10}$ *e.m.u.* of current $= 10^{-1}$ *e.m.u.*

10. Intensity of Field at a point due to Straight Current by Laplace's Law : Let a straight wire AB (Fig. 10) carry a current of *i e.m.* units and the point P at distance *a* from the wire, at which intensity is required. Take a small element CD of length *d*

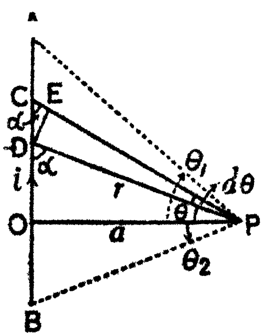


Fig. 10

and draw normal PO of length a on AB. Join PA, PC, PD and PB. Let PD $= r$; $\angle$DPO $= \theta$; $\angle$DPC $= d\theta$; $\angle$ODP $= \alpha$, and $\angle$OPA $= \theta_1$, and $\angle$OPB $= \theta_2$.

Draw an arc of circle DE with centre P and radius r. Since the angle CPD or $d\theta$ is very small, DE may be taken as normal to both PD and PC.

Magnetic field at P due to element CD of length ds is given by

$$\delta F = \frac{ids \cdot \sin\alpha}{r^2} \quad \ldots \quad \ldots \qquad (1)$$

Now $\angle$OCP $= \angle$ODP $= \alpha$, since each is compliment of $\angle$CDE. Then we have, $\theta + \alpha = 90^\circ$ $\therefore$ $\sin\alpha = \sin(90 - \theta) = \cos\theta$......(2)

Again DE $= rd\theta =$ CD$\sin\alpha = ds.\cos\theta$......from (2)

$$\text{Or} \quad ds.\frac{a}{r} = r.d\theta \qquad \cos\theta = a/r.$$

$$r^2.d\theta \qquad \frac{ids \cdot \sin\alpha}{r^2} = \frac{i.r^2 d\theta}{a.r^2} \cos\theta = \frac{i}{a}\cos\theta d\theta$$

Thus $\delta F = \frac{i}{a}\cos\theta d\theta$. Intergrating this between the limits $-\theta_1$ and $+\theta_2$, the intensity at P due to the whole wire AB is given by.

$$F = \int_{-\theta_1}^{+\theta_2} \frac{i}{a}\cos\theta.d\theta. = \frac{i}{a}\Big[\sin\theta\Big]_{-\theta_1}^{+\theta_2} = \frac{i}{a}(\sin\theta_2 + \sin\theta_1)$$

In the case of an infinitely long conductor $\theta_1 = \theta_2 = 90$; Therefore $F = \frac{i}{a}(\sin 90 + \sin 90) = \frac{2i}{a}$

11. Field due to Circular Current at a point on the Axis: Consider an element ds of a conductor bent into the form of a circle of radius a. Let A be a point on the axis of the circle and situated at a distance x from O, the centre of the circle. (Fig. 11).

If a current of strength i e.m.u. be made to flow round the circular conductor, the magnetic intensity due to the element ds at A and at a distance r from the element ds is equal to $\frac{ids. \sin\alpha}{r^2}$, where α is the angle between the element ds and the direction of r and acts in a direction Af at right angles to the plane containing the element ds and r. Here $\alpha = 90^\circ$, since the line joining P and the mid-point of ds is perpendicular to element ds.

This force Af is resolved into two compoments $Af.\cos\theta$ and $Af.\sin\theta$ at right angles to one another, where θ is the angle between r and x, one component being along Av and the other along Ah.

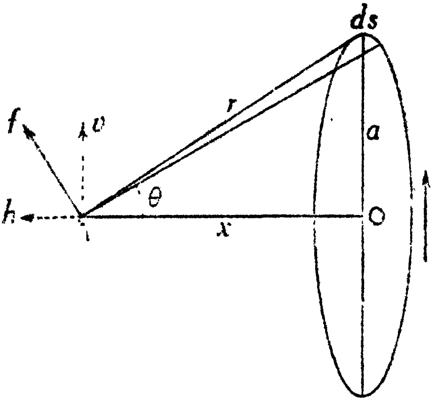


Fig. 11

An element diametrically opposite to ds will produce a force at A which will have components, one equal and opposite to $Af\cos\theta$ and the other equal and in the same direction as $Af.\sin\theta$. So the components along the axis will be added, the components normal to the axis being neutralised by each other.

Now if the whole circle be divided into pairs of such elements, the resultant intensity parallel to the direction OA is

$$\frac{2\pi ai}{r^2}\sin\theta$$

$$\frac{2\pi ai}{r^2}\cdot\frac{a}{r}=\frac{2\pi a^2 i}{(a^2+x^2)^{\frac{3}{2}}}\qquad \sin\theta=\frac{a}{r},\ \text{and } r^2=(a^2+x^2)$$

Thus intensity at A is given by $F=\dfrac{2\pi a^2 i}{(a^2+x^2)^{\frac{3}{2}}}$ Oersted.

At a point $x=0$, *i.e.* at the centre of the coil

$$\frac{2\pi a^2 i}{a^3}=\frac{2\pi i}{a}$$

Note: It is evident from the above expression that as a increases, the intensity decreases.

If there be n number of turns of the coil, then intensity at $A=\dfrac{2\pi na^2 i}{(a^2+x^2)^{\frac{3}{2}}}$, and intensity at the centre of coil $=\dfrac{2\pi ni}{a}$.

The direction of the intensity at A is along the axis and directed away from the coil if the current be anti-clockwise as seen by an observer at A.

Cor. Intensity at the centre of a circular coil carrying a current can be found directly as follows :

The radius r is perpendicular to any elementary length ds of the circular (Fig. 12) coil, in this case. Therefore the angle between r and any element is 90°. Then by Laplace's Law, intensity at the centre O due to the whole wire is given by

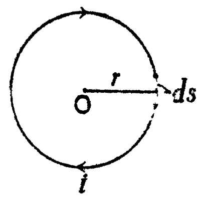


Fig. 12

$$F=\Sigma\frac{ids\sin 90}{r^2};\qquad \Sigma ds=\frac{i}{r^2}.2\pi r=\frac{2\pi i}{r}$$

for n turns of the coil, $F=\frac{2\pi ni}{r}$. The direction of the intensity is normal to the plane of the coil, and when the current is clockwise the field is directed towards the plane of the paper.

12. Intensity at any point on the axis of a solenoid : When a current is flowing in a cylindrical sheet in such a way that its direction is everywhere perpendicular to the axis, it is said to be *solenoidal*.

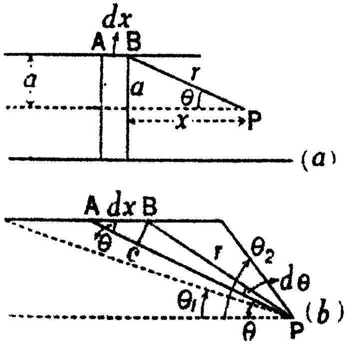


Fig. 13

The condition is fulfilled if the current traverses a wire coiled round a cylinder when the thickness of the wire is small compared with the radius of the cylinder.

The solenoid shown in Fig. 13(a) is represented as a continuous tube in which P is a point on the axis. Figure 13(b) is a section of the solenoid along the axis, only upper half being shown.

If i be the current per unit length of the solenoid, then the current in a thin section of length dx is idx, [Fig. 13 (a)].

Since dx may be regarded as a small element of a circular coil of radius a and of current element $i.dx$.

The intensity of field due to this at $P=\frac{2\pi a^2 idx}{(x^2+a^2)^{\frac{3}{2}}}$

where x is the distance of the plane of the circle from P.

But, we have from the Fig 13(*b*), $BC = rd\theta = dx.\ \sin\theta$

$$\therefore \quad dx = \frac{rd\theta}{\sin\theta}$$

$\therefore$ The intensity due to this section

$$= \frac{2\pi a^2 i.\ rd\theta}{\sin\theta} = \frac{2\pi a^2 i d\theta}{r^2 \sin\theta} = 2\pi i \sin\theta\ d\theta.$$

$$\left[\text{since } \frac{a}{r} = \sin\theta\ ;\ r^2 = a^2 + x^2\right]$$

Then the intensity for the whole solenoid

$$= 2\pi i \int_{\theta_1}^{\theta_2} \sin\theta\ d\theta \quad = 2\pi i \Big[\cos\theta\Big]_{\theta_2}^{\theta_1}$$

where θ_1 and θ_2 are the values of θ at the ends of the solenoid.

If the solenoid consists of n turns of wire per centimetre length, the current in each turn being i, then

$$F = 2\pi ni \Big[\cos\theta\Big]_{\theta_2}^{\theta_1} = 2\pi ni\Big[(\cos\theta_1 - \cos\theta_2\Big]$$

If the point P be within the solenoid $\cos\theta_2$ is $-ve$ and the intensity $= 2\pi ni\ (\cos\theta_1 + \cos\theta_2)$.

If the **solenoid is very long** and if the point be inside it and at a considerable distance from the ends, $\cos\theta_1 = 1$,

$$\cos\theta_2 = -1$$

$\therefore$ The intensity $= 4\pi ni$.

At a point at the extreme end of a very long solenoid $\cos\theta_1 = 0$, and θ_2 is practically π and $\cos\theta_2 = -1$, so that $F = 2\pi ni$.

In the case of a ring solenoid the number of turns of wire are closer on the inside than on the outside of the ring so the intensity is greater towards the inside.

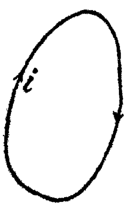


Fig. 14

13. Ampere's Theorem : The theorem states that a closed conducter, say a wire carrying a current **is equivalent to a simple magnetic shell**, the bounding edge of which conicides with the wire, the moment of the shell per unit area, that is, the strength of the shell being proportional to the strength of the current.

When the current is thus replaced by the magnetic shell the magnetic potential at any point due to the shell may be obtained and

then differentiating the potential with respect to distance, the intensity may be determined.

14. Equivalence of Current and Magnetic Shell in any medium: It has been found that a closed circuit conveying a current produces a magnetic field identical with that due to a shell whose boundary coincides with that of the current circuit, the strength of the current being equal to the strength of the shell,

By Laplace's Law, the force exerted by the current i in the circuit on a unit pole situated at a point on the axis of the circuit at a distance d from its centre *i.e.*, from all parts of the circuit (since it is considered small) is equal to $\frac{2\pi a^2 i}{d^3}$, the circuit being in the from of a circle of radius a.

If the closed circular circuit be replaced by a shell of strength ϕ and of magnetic moment M, the force on a unit pole at the same point on the axis of the shell is equal to $\frac{2M}{d^3}$

Thus the shell becomes equivalent to the circuit conveying the current i when $M = \pi a^2 i$.

That is, the magnetic moment of a circular current is equal to its area multiplied by the strength of the current in it (in c. g. s. unit).

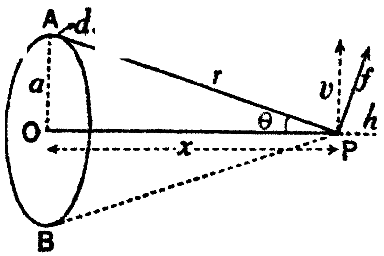
But since the magnetic moment of the shell of area equal to the area of the circit $= \pi a^2 \phi$.

Hence, we have $\quad \pi a^2 \phi = \pi a^2 i \quad i.e.\ \phi = i$

Thus the magnetic field at any point due to a current in a closed circuit is identical with that of a magnetic shell of the same contour when the strength of the current is numerically equal to the strength of the shell and the north pole of the shell coincides with the face of the circuit in which the current flows in the anti-clockwise direction.

The condition of equivalence will be a bit modified if the whole of the space is filled with a medium of permeability μ differing from unity. The field due to the current at any point in the space is unchanged but that due to the shell is reduced to $\frac{1}{\mu}$ of the value when the permeability of the medium is unity. So, in this case, the shell will be equivalent to a circuit conveying a current having the same boundary as the shell if $\phi = \mu i$.

15. Intensity at any point on the axis of a circular coil carrying current by Ampere's Theorem: Let P be a point (Fig. 15) at a distance x from the circular current of radius a. Let i be the strength of the current and ω the total solid angle subtended by the coil at P. Let θ be the semi-vertical plane angle subtended at P.



Fis. 15

If the circular current be replaced by a magnetic shell, the potential V at P due to this shell is given by, V = strength of the shell × solid angle

$$V = i\,\omega, \text{ since strength of the shell} = i$$

or $$V = i.2\pi(1-\cos\theta), \quad \because \quad \omega = 2\pi(1-\cos\theta)$$

$$= 2\pi i\left(1-\frac{x}{r}\right) = 2\pi i\left[1-\frac{x}{(a^2+x^2)^{\frac{1}{2}}}\right]$$

$$r^2 = a^2 + x^2$$

Hence intensity $$F = \frac{dV}{dx} = 2\pi i\frac{d}{dx}\left[x.(x^2+a^2)^{-\frac{1}{2}}\right]$$

$$= 2\pi i\left[\frac{1}{(x^2+a^2)^{\frac{1}{2}}} - \tfrac{1}{2}x.\ 2x.\frac{1}{(x^2+a^2)^{\frac{3}{2}}}\right]$$

$$= 2\pi i\left[\frac{1}{(x^2+a^2)^{\frac{1}{2}}} - \frac{x^2}{(x^2+a^2)^{\frac{3}{2}}}\right]$$

$$= 2\pi i\frac{a^2}{(a^2+x^2)^{\frac{3}{2}}} \text{ Oersted.}$$

$$= \frac{2\pi n i a^2}{(a^2+x^2)^{\frac{3}{2}}} \text{ Oersted for } n \text{ turns of the wire.}$$

At centre of the coil $x = 0$, then $F = \dfrac{2\pi n i}{a}$ Oersted.

Note: To determine the solid angle ω subtended by the circular coil at the point P let us describe a sphere of unit radius round P as centre. Then the area on the sphere's surface enclosed within the solid angle is the measure of the solid angle ω.

16. Mathematical deduction for solid angle : Let DCN in Fig. 16 represent a sphere of unit radius and CPD the solid angle subtended by the circular coil CD to the left of P.

If the small element ds of the surface at C revolves round E as centre, the area of the surface of revolution is equal to $2\pi \text{CE}.ds = 2\pi \sin\theta d\theta$.
since $a\sin\theta = \text{DE}$ and $ad\theta = ds$, where the radius of the sphere a is unity.

$\therefore$ The total area on the sphere's surface enclosed within the solid angle

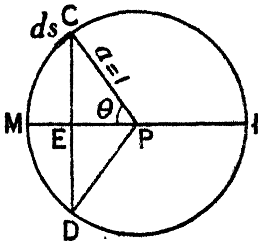


Fig. 16

$$= 2\pi\int_0^\theta \sin\theta d\theta = -2\pi\left(\cos\theta\right)_0^\theta = 2\pi(1-\cos\theta)$$

17. Intensity at a point on the axis of, and inside a solenoid by Ampere's Theorem : The resultant magnetic field inside the solenoid may be compared to that due to a row of magnetic shells with faces of opposite polarities in contact, each turn of the wire being replaced by its equivalent magnetic shell.

In this case the strength ϕ of the shell is equal to the current strength i.

Let n be the number of turns of wire per cm. length of the solenoid, then the thickness of the shell is $\frac{1}{n}$ and $\phi = \frac{\sigma}{n}$, where σ is the pole-stregth per $(\text{cm.})^2$,

$$\text{Thus } \phi = \frac{\sigma}{n} = i \text{ or } \quad \sigma = ni$$

If a be the radius of each turn of the coil, *i.e.*, of the equivalent magnetic shell, the pole-strength at each end of the solenoid $= \pm\pi a^2\sigma = \pm\pi a^2 ni$.

But we know that the number of lines originating from a pole of strength m is equal to $4\pi m$.

Therefore, the number of lines originating from ends of the solenoid $= 4\pi \times \pi a^2 ni = 4\pi^2 a^2 ni$

$\therefore$ **Intensity of the field within the solenoid**

$$= \frac{4\pi^2 a^2 ni}{\pi a^2} = 4\pi ni \text{ C.G.S. units.}$$

18. Work done in carrying a magnetic pole round a circuit : We know that the difference of potential between two points is the

work done in carrying a unit N pole from one point to the other independent of the path along which the pole is carried.

Let us consider two points P and Q (Fig. 17) very close to but on opposite sides of a magnetic shell AB of strength ϕ. Then the magnetic potential at P is $2\pi\phi$ and at Q is $-2\pi\phi$, where 2π is the solid angle subtended by the face of the shell.

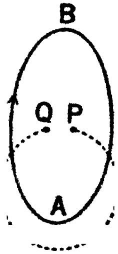


Fig. 17

So if a unit N pole be carried from the N pole to the S pole of the shell, the work done (change of potential)

$$= V_p - V_q = 2\pi\phi - (2\pi\phi) = 4\pi\phi.$$

If, however, the shell is replaced by its equivalent current $(i = \phi)$ flowing round the boundary of the shell, the work for the external path from P to Q as in the case of a shell is $4\pi\phi$ or $4\pi i$.

As P and Q are adjacent, there being no magnetic material to traverse, $4\pi i$ is the *work done* in carrying a unit pole round a closed path linked once with a current i.

This is Ampere's theorem.

19. Magnetic field due to a straight current: We know that the magnetic field due to a linear current through a long straight wire is everywhere at right angles to the wire.

Let the intensity at a distance R be H (Fig. 18). Then on carrying a unit N pole once round the wire in a circle of radius R in a plane at right angles to the current I, the field is everywhere in the same direction as the path, and by symmetry, it is constant.

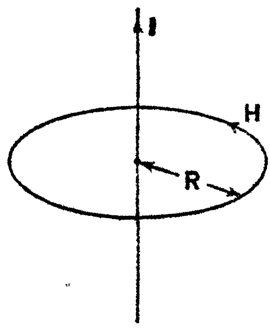


Fig. 18

Work done in taking unit pole round the circular path =

force × distance = $H \times 2\pi R$ ergs.

Work done in linking current $I = 4\pi I$ ergs.

Therefore, $H \times 2\pi R = 4\pi I$ or $H = \dfrac{2I}{R}$ gauss.

If the current is I amp., $H = \dfrac{2I}{10R} \quad \dfrac{I}{5R}$ gauss.

QUESTIONS

1. Describe the action between a current and a magnet and explain how it has been used in the construction of a very delicate instrument for measurement of the strength of electric current. [C. U. 1940]

2. Obtain an expression for the magnetic intensity at a point on the axis of a circular turn of wire carrying an electric current. Deduce therefrom an expression for the reduction factor of a Helmholtz double-coil galvanometer. What is its advantage over the single coil type? [C. U. 1950, '55]

3. Find the expression for the magnetic potential at any point due to a closed circuit carrying a current. [C. U. 1952]

4. Find an expression for the strength of the magnetic field near the middle of a long uniform solenoid.

5. What is meant by C.G.S. electromagnetic unit of current?

6. What do you understand by the eqivalence of current and magnetic shell in any medium?

CHAPTER III

GALVANOMETERS

20. Introductory: The principle of the action between a current and a magnet has been utilised in the construction of galvanometers of different types.

In one type, the current or rather the coil carrying the current is fixed and the magnet movable and in the other type, the magnet is fixed and the coil carrying the current movable. A **galvanometer** is an instrument which is used for the measurement or detection of very small currents. When comparatively large currents are to be measured, instruments known as ammeters are used for direct reading of the current in amperes on a scale fixed inside the instrument.

In galvanometers in which simply the passage of a current is detected, the chief requisite is sensitiveness, *i.e.*, a very small current shall produce measurable deflection in the needle, while in other galvanometers in which the strength of the current is measured, the sensitiveness is not a very important requirement to calculate the current strength.

The term **senstiveness** or **sensibility** of a galvanometer is the deflection produced by one micro-ampere, *i.e.*, 10^{-6} amp. current on a scale placed at a distance of 1 metre from the mirror and is generally expressed by $\frac{\theta}{i}$, where θ is the deflection produced by a current of strength i.

The **Figure of merit** of a galvanometer is the current in micro-amperes required to produce a deflection 1 *mm.* on a scale placed at a distance of 1 metre from the mirror of the galvanometer and is expressed by $\frac{i}{N}$ where N is the number of scale divisions in millimetres by a current of strength i in mircro-amperes.

21. Classification of Galvanometers :

Galvanometers are generally classified as follows :—

I. According to the nature of the system :

(a). Suspended Needle Galvanometers (Fixed Coil Type) : Two different types of Galvanometers are commonly used. In one type a single magnetic needle is suspended by a silk thread or pivoted on a fine point in the magnetic field due to an electric current, as in the **Tangent Galvanometer.**

In another type a compound magnetic system consisting of two light needles almost equally magnetised and rigidly connected with their magnetic axes in opposite directions is suspended by silk thread so that one of the needles is mounted inside the coil conveying the current and the other outside it.

(b). Suspended Coil Galvanometers (Moving Coil Type) : In this type, a coil wound on a frame of a conducting material and conveying a current is suspended by phosphor-bronze strip between the poles of a very powerful magnet. The coil is practically *dead-beat* and the galvanometer is known as Dead-beat Galvanometer.

If the frame over which the coil is wound be of a non-conducting material, the galvanometer becomes suited for ballistic experiments and is known as Ballistic Galvanometer. **D'Arsonval type** of suspended coil galvanometer is commonly used.

II. According to the nature of the controlling force :

The controlling force is generally of two kinds :—

(1) Force exerted by a magnet suitably placed or that due to earth's magnetic field.

(2) Force due to torsion in the suspending fibre.

For galvanometers in which the controlling force is that due to the field created by a magnet or the earth's magnet we have two types—(1) Tangent Galvanometer, (2) Astatic Galvanometer.

For galvanometers in which torsion is the controlling force, we have the following types :—

(1) Astatic Galvanometer, (2) Dead-beat and Ballistic Galvanometers.

III. According to their specific uses :

(a) For the detection of currents, the following types are generally used. (1) Galvanoscope, (2) Astatic Galvanometer, (3) Dead-beat Galvanometer.
(b) For measurement of current—(1) Tangent Galvanometer Ammeter.

We know that for ordinary tangent galvanometer, $F' = H. \tan\theta$ where H is the earth's horizontal intensity.

or $\frac{32\pi ni}{5r\sqrt{5}} = H \tan\theta$ $\qquad i = \frac{5\sqrt{5}}{32} \cdot \frac{rH}{\pi n} \tan\theta$ in e. m. u.

$$\frac{50\sqrt{5}}{32} \cdot \frac{rH}{\pi n} \tan\theta \text{ in amperes.}$$

The reduction factor K of the double-coil galvanometer

$$= \frac{5\sqrt{5}}{32} \cdot \frac{rH}{\pi n} \text{ in e. m. units.}$$

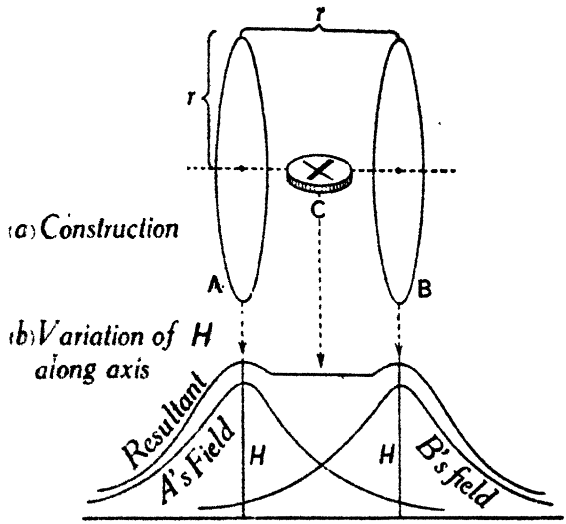


Fig. 22

To prove that $x = r/2$: If F be the intensity due to a single coil t C, then,

we have at the point C, $\frac{dF}{dx} = k$ a constant,

So that $\frac{d^2F}{dx^2} = O$

Now we have $F = \frac{2\pi nir^2}{(x^2 + r^2)^{\frac{3}{2}}} = A(x^2 + r^2)^{-\frac{3}{2}}$

where $A = 2\pi nir^2$ = a constant, for a given current.

Then differentiating F with respect to x

$$\frac{dF}{dx} = A(-\tfrac{3}{2})(x^2+r^2)^{-\frac{5}{2}}.\ 2x,$$

$$= -3Ax(x^2+r^2)^{-\frac{5}{2}}$$

Differentiating again,

$$\frac{d^2F}{dx^2} = -3A\left\{(x^2+r^2)^{-\frac{5}{2}} - 5x^2(r^2+x^2)^{-\frac{7}{2}}\right\} \quad \dots \quad (1)$$

But $\frac{d^2F}{dx^2} = 0$, $\therefore$ from (1) $(x^2+r^2)^{-\frac{5}{2}} = 5x^2(x^2+r^2)^{-\frac{7}{2}}$

or $\quad x^2 + r^2 = 5x^2$

or $\quad r^2 = 4x^2 \quad \therefore \quad x =$

(D) The Sine Galvanometer : The galvanometer is similar to the tangent galvanometer with the difference that the vertical coil is capable of rotation about a vertical axis and its position is determined by means of a graduated horizontal circle at the base of the instrument with the help of a vernier scale fitted to the frame of the coil.

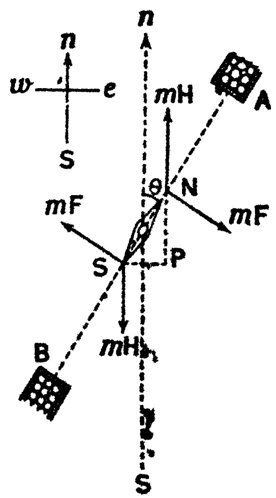


Fig. 23

At first when no current passes through the coil, both the coil AB and the needle NS lie in the magnetic meridian plane NS, (Fig. 23), but when a current passes, the needle is deflected. Then the coil is rotated until the needle is again in the plane of the coil.

In this position the deflecting field is perpendicular to the coil and therefore to the needle as well.

We have then, $mF.l = mHl.\sin\theta$

or $\quad \frac{2\pi ni}{r} = H\sin\theta$

or $\quad i = \frac{rH}{2\pi n}\sin\theta = \frac{H}{G}\sin\theta = K\sin\theta$

As the maximum value of $\sin\theta$ is 1, it is not possible to measure currents greater than K. Again when a tangent galvanometer produces a deflection of 45° for a given current, a sine galvanometer with same value of G will produce a deflection of 90°. Hence, for weak currents

ne galvanometer is more suitable than the tangent galvanometer. or heavy currents tangent galvanometers should be used.

23. Sensitive Moving Magnet Galvanometer :

In both tangent and sine type, the galvanometer will be sensitive K is small, for in that case θ will be fairly large although the rrent is very feeble and small. Now $K = rH/2\pi n$. We may decrease but this is limited by mechanical considerations of the size the magnet and the coil. Again by decreasing r, the field at ne suspended magnet will not be quite uniform and the tangent law ill not then apply. To decrease K we may also increase n. But this gain can not be increased indefinitely, for increase of n will mean crease in the resistance of the coil and hence no advantage is rived from that. The couple due to controlling field H can owever be reduced to a very large extent by using, what is called n **astatic pair** of needles. The astatic pair consists of two agnets SN and N′S′ of very nearly equal strengths m and m' espectively fixed to a rigid support so that their axes are parallel ith opposite poles near each other.

24. Astatic Galvanometer :

In this galvanometer an astatic air is used instead of a single agnetic needle. The coil of the alvanometer surrounds only ne needle of strengh m. The ther needle N′S′ of strength ′ is made to move over a raduated circle (Fig. 24).

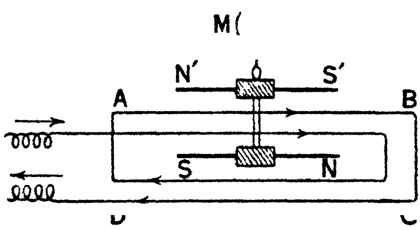


Fig. 24

Here the couple due to he current depends on mF ut couple due to earth's field epends on $(m \sim m')$ H. If a urrent is passed through the oil, the astatic pair will rotate through an angle, say θ.

In equilibrium position of the pair, the couple due to the urrent is balanced by the couple due to earth's field.

Therefore, from figure 25 we have,

$mF.2l \cos\theta = (m \sim m')H2l.\sin\theta$ where $2l$ = length of the needle ound which current travels.

or $mF = (m \sim m')H \tan\theta$

or $m.\frac{2\pi ni}{-} = (m \sim m')H \tan\theta \qquad F = \frac{2\pi ni}{-}$, for circular coil

where r, n are radius and number of turns of the coil, the coil used being circular.

$$\therefore \quad i = \frac{r(m \sim m')\mathrm{H}}{2\pi n m} \tan\theta.$$

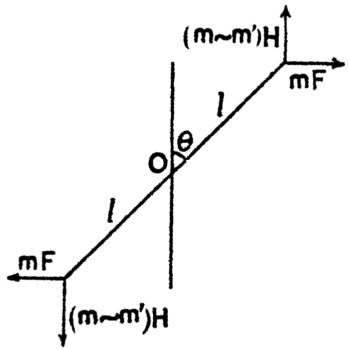


Fig. 25

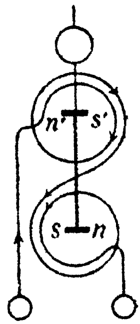


Fig. 26

Thus the reduction factor K can be considerably reduced by making m and m' almost equal. When the coil is wound round both the needles of the pair (Fig. 26) in such a manner that current flows round them in opposite directions the effect becomes twice that of the former.

25. Action of Magnet on current : We know from Laplace's law that the magnetic field at a point P due to a small element ds at O traversed by current i is given by $F = \frac{ids \sin\alpha}{r^2}$, where r is the distance of P from O and α is the angle between the element ds and r. If now a magnetic pole of strength m be held at P, the force, exerted on it due to the current in ds is $= mF = \frac{m\, ids \sin\alpha}{r^2}$. By Newton's third law the force on the elementary current along ds due to pole m at P is also equal to $\frac{mi\, ds \sin\alpha}{r^2}$. Of this quantity m/r^2 is the intensity, say H, of the field at O due to the pole m placed at P. Thus the force exerted on the elementary current along ds is $\mathrm{H}ids \sin\alpha$ and its direction is along r.

Hence we see that if a conducting straight wire of length l traversed by a current i be placed in a magnetic field of intensity H, then the force experienced by it is equal to $Hil\sin\alpha$, where α is the angle between the wire and the direction of the magnetic field. If direction of H be normal to the wire, $\alpha = 90$, so that force becomes $Hil\sin 90$ or Hil. If the field H be along the current, the force on the current becomes zero, since in this case $\alpha = 0$ and $\sin\alpha = 0$.

25(a). Fleming's Left-hand Rule : The direction of the force may be ascertained by Fleming's Left-hand rule. The rule is as follows : "Let the observer stretch the thumb, the fore-finger and the middle finger of his left hand, so that each is perpendicular to the other two. Then if the fore-finger be directed along the lines of force, the middle finger along the current, then the thumb indicates the direction of motion, *i.e.*, the direction in which the force acts.

26. Suspended Coil Galvanometer (D'Arsonval type) : The above principle has been made use of in the construction of a suspended or moving coil galvanometer. A moving coil galvanometer essentially consists (Fig. 27) of a rectangular coil G (circular coil may also be used,) of insulated wire suspended between the two massive soft-iron pole pieces attached to the poles of a permament horse-shoe magnet. The coil is usually suspended by a phosphor-bronze strip which also serves as the lead for the current to the coil, and is finally joined to the terminal at the base of the instrument. The other end of the coil is connected to one end of a spiral wire the other free end of which is connected to another terminal at the base. A soft iron cylinder A is placed midway between the poles of the magnet and within the frame of the coil and fixed to an upright of the galvanometer but quite detatched from the coil. Either a horizontal pointer is attached to the coil, or a small circular mirror M is attached to the suspension wire to note the deflection by suitable **light and scale arrangement.**

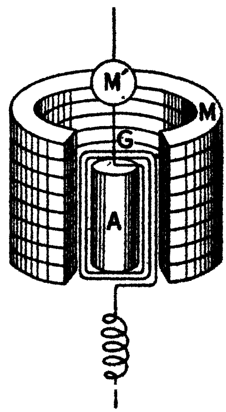


Fig. 27

To use the instrument, the horizontal base board is levelled by levelling screws with which the board is provided. The plane of the coil is made parallel to the direction of the magnetic field of the horse-shoe magnet. When a current is passed through the

coil, the force on each of the vertical wires AB and DC (Fig. 28) by Ampere's theorem $= Hil$, where H is the strength of the magnetic field, i the current strength, and l the length of the coil. The currents in the two vertical wires are oppositely directed, and hence the forces on them also being oppositely directed constitute a couple. The two horizontal sides AD, BC being along the direction of the magnetic field experience no force. The moment of the couple acting on the vertical sides

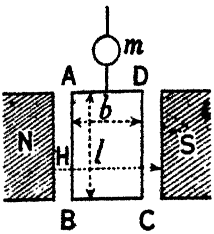


Fig. 28

$= Hil \times b$, where b is the breadth of the coil

$= HiA$, where A is the face area of the coil $(= l \times b)$.

For n turns of the coil, total moment $= nAHi$. As the coil is free to rotate, it is deflected till its rotation is balanced by the torsional couple of the suspension. The moment of the couple when the coil is deflected through θ is

$= nHil \times b\cos\theta$, since in the deflected position the arm of the couple becomes $b\cos\theta$.

Or, moment of the couple in the deflected position $= nHiA\cos\theta$

Again, the torsional couple $= C\theta$, where C is the coefficient of torsion or torsional couple for unit twist. Hence, for equilibrium of the coil,

$$nAHi\cos\theta = C\theta \quad \text{or} \quad i = \frac{C}{nAH\cos\theta}.\theta.$$

When θ is small, which is usually the case, $\cos\theta$ becomes unity

$$\therefore\ i = \frac{C}{nAH}\theta = K\theta, \quad \text{where } K = \frac{C}{nAH} = \text{a}$$

constant for a given galvanometer.

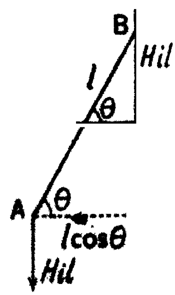
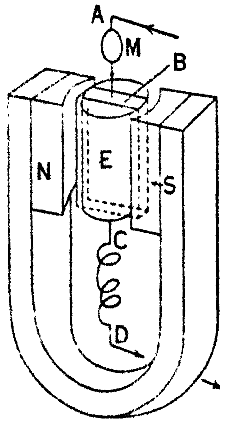


Fig. 29

The current is, therefore, proportional to the angle of deflection. In order to measure deflection θ, a ray of light from a fixed source is made to fall on the mirror attached to the suspension wire. The reflected ray is then received on a scale. When the coil rotates, the spot of light is displaced and the deflection θ is measured from the [illegible]ent.

In some forms (Fig. 30) of the galvanometer the pole-pieces are curved and a soft iron cylinder R is situated inside the curved

pole-pieces N, S in such a way that the sides of the rectangular coil can move freely between the cylinder and the pole-pieces. The curved pole-pieces and the soft iron cylinder serve the purpose of making the field stronger and at the same time radial, so that the coil is always in the magnetic field of the same strength.



30

The radial field (Fig. 31) has the advantage that the couple due to the current does not depend on the position of the coil as it is in the uniform field.

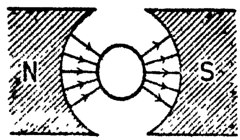


Fig. 31

If the field be *radial*, in which case the vertical sides of the coil experience forces of value Hil at right angles to the plane of the coil, the deflecting couple equal to HiA, and is independent of the position of the coil. Hence the assumption that deflection θ is small becomes unnecessary.

26(a). Important points :

(1) The deflections are scarcely affected by the external magnetic field.

(2) The instrument may face any direction since the zero position of the coil is independent of the direction of magnetic field in which it is suspended, and earth's field does not come into consideration.

(3) The deflections are damped very easily as the coil is generally wound on a *conducting* frame of copper sheet.

(4) The damping effect can be minimised by winding the coil on a frame of any *non-conducting* sheet and therefore the galvanometer may be used for **ballistic** experiments.

26(b). Sensitiveness :

The sensitiveness of the galvanometer is increased if C is diminished or n, A and H increased. But C cannot be decreased without limit as the suspending fibre must be strong to carry the coil. The suspending fibre is generally made of phosphor-bronze for it has a high tensile strength and is not readily oxidisable.

If the number of turns in the coil be increased, the sensitiveness will be increased to the same extent but the large number of turns decreases the sensitiveness due to the high resistance of the coil. The loss in sensitiveness due to a large number of turns is compensated by the use of a very strong magnetic field.

The D' Arsonval galvanometer has several advantages over the suspended needle galvanometer. The following are the advantages.

(1) The suspended coil galvanometer is not susceptible to any disturbance by varying external magnetic field.

(2) The suspension of the coil is much stronger.

Note : Its sensitiveness cannot be varied at will and the damping effect is much greater. The latter is not always a disadvantage, In ballistic experiments it is a great disadvantage no doubt ; but in determining the balance point in Wheatstone's bridge experiment it is really a great advantage.

27. Distinction between Suspended Needle Galvonometer and Suspended Coil Galvanometer :

(1) A suspended needle galvanometer may be used to measure transient current as well as steady currents just as a suspended coil galvanometer. The first type is always ballistic, while the second type can be made ballistic by special device.

(2) H (magnetic field) is fixed in the second type, but it can be varied in the first type.

(3) In the suspended needle galvanometer, the coil should be placed in the magnetic meridian, whereas in the suspended coil galvanometer the coil need not be placed in the magnetic meridian.

(4) In the suspended needle galvanometer the deflecting force is due to the current passing through it and the restoring force is due to the earth's magnetic field, whereas in the suspended coil galvanometer the deflecting force is due to the magnet and the restoring force is due to the torsion in the suspending fibre.

(5) In the suspended needle galvanometer the oscillations of the needle are not damped easily whereas in the suspended coil galvanometer the oscillations of the coil can be damped easily by a special device.

(6) The sensibility of the suspended needle galvanometer is low whereas the sensibility of the suspended coil galvanometer is high.

28. Figure of merit of a galvanometer :

The figure of merit of a galvanometer is defined as the current in micro-ampere for producing a deflection of one millimeter on a scale placed perpendicular to the beam at a distance of one metre from the mirror of the galvanometer.

To find it experimentally, the galvanometer is shunted by a low resistance and the shunted galvanometer is placed in a circuit containing a storage battery, a key and a high resistance. The scale is placed at a distance of 1 metre from the mirror of the galvanometer.

When a current is passed through the galvanometer by closing the circuit with the key, we have the current i_g flowing through the golvanometer given by

$$i\frac{S}{S+G}, \quad i\frac{S}{S+G} \cdot \frac{E}{R+\frac{SG}{S+G}} = \frac{ES}{R(S+G)+SG}$$

where S is the resistance of the shunt, R the high resistance, and G, the resistance of the galvanometer and i is the main current.

Then the figure of merit $F\left(=\frac{i_g}{d}\right)$ is obtained from a knowledge of the deflection d in millimeters of the scale.

29. Ammeter : An ammeter is a galvanometer with a fixed scale calibrated to read currents directly in amperes or fractions of an ampere by means of a pointer (Fig. 32) moving over a graduated scale. It must have a low resistance so that the value of the current in the circuit in which it is inserted, is not altered and that the heat produced is negligible.

For measuring large currents, the instrument is provided with a shunt or sets of shunts which increases enormously the range, in some cases, to several thousand amperes.

Various forms of ammeter based on different principles have been constructed of which the Moving Coil Ammeter is the commonest form.

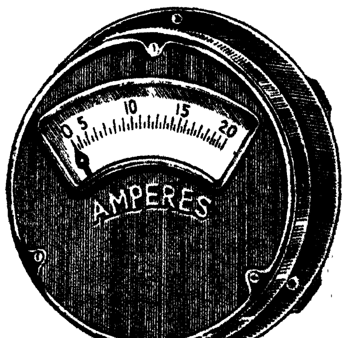


Fig. 32

29(a). Moving Coil Ammeter : It is a suspended coil galvanometer except in its mode of suspension. In this instrument, the coil to which a pointer P is fixed (Fig. 33) is pivoted between two needle points and controlled by hair-springs which serve to conduct the current to and away from, the coil.

The curved pole-pieces N, S between which the coil is placed and the soft iron cylinder are arranged as in the Suspended Coil Galvanometer and help to make the field radial and so the scale is uniform and the deflections give directly the current in amperes.

29(b). To increase the range of an Ammeter: We know that an ammeter is a low-resistance galvanometer with a shunt *Ch* **arranged in parallel** with it between the points A and B.

Let the scale of the ammeter read from 1 to 10 amperes. Now when the pointer points to any division say one, the current in the main circuit is one ampere but when the range of the scale is increased 10 times or rather n times by adjusting the resistance of the shunt and the pointer points to the division 1, the value of the main current becomes 10 amps. or n amps.

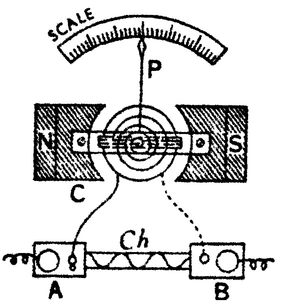


Fig. 33

Thus we see that although the pointer points to the same division, the current measured has different values.

It is also clear that in both the cases the current passing through the galvanometer is the same.

To obtain the resistance of the shunt to increase the range n times, the shunt resistance is altered in such a way as to keep the galvanometer current the same in both the cases.

Let i_g be the current in the galvanometer and i, the main current. Then $i_g = \frac{S}{S+G} \cdot i = \frac{S}{G} \cdot i$

where S, the shunt resistance is small in comparison with the Galvanometer resistance G.

Now when the range is increased n times *i.e.* when the main current is ni, the resistance of the shunt should be made $\frac{1}{n}$th its former value so that i_g remains the same in both the cases.

Let S′ be the value of the shunt when the main current is ni. In order that the galvanometer current i_g remains the same as when the shunt was S

$$i_g = \frac{S'}{S'+G} \cdot ni = \frac{S'}{G} ni = \frac{S}{G} i,$$

S' being small in comparison with G. $\therefore \quad S' = \frac{1}{n} S.$

29(c). Example : A milliammeter of 25 milliampere range is to be made to read up to 10 amps., the resistance of the milliammeter being 5 ohms.

A shunt S is to be used in parallel with terminals, say A and B of the ammeter. Maximum current that can be passed through the ammeter = 25 milli-amperes = ·025 amp.

∴ Current through the shunt = 10 − ·025 = 9·975 amps.

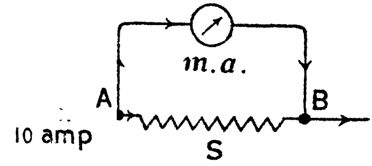


Fig. 34

Then, $V_A - V_B = 9{\cdot}975 \times S$ (across the shunt)

$V_A - V_B = {\cdot}025 \times 5$ (,, ammeter)

$9{\cdot}975 \times S = {\cdot}025 \times 5$

Hence $S = \dfrac{{\cdot}025 \times 5}{9{\cdot}975} = {\cdot}0125$ ohm.

30. To convert an ammeter into a direct reading voltmeter, the shunt of the ammeter is to be removed and then a suitable resistance R′ is to be used in series with the **galvanometer**.

Let the ammeter indicate i amperes for a full scale deflection. Then the current through the galvanometer is given by

$$i_g = \frac{S}{S+G} \cdot i$$

where i_g is the current through the galvanometer, S, the shunt resistance and G, the Galvanometer resistance.

When the shunt is removed and the extra resistance R′ is connected in series with the galvanometer, the current through the galvanometer is given by $i_g = \dfrac{V_1 - V_2}{R' + G} = \dfrac{i}{R' + G} \therefore \dfrac{S}{S+G}$,

since $V_1 - V_2$ is numerically equal to the current i indicated by the ammeter. Here $V_1 - V_2$ is potential difference between the points across which the galvanometer with the resistance R′ is connected.

Therefore, from $\dfrac{i}{R'+G} = \dfrac{S}{S+G}$,

we have $R' + G = \dfrac{S+G}{S}$ or $R' = 1 + \dfrac{G}{S} - G$ (1)

Thus, the extra resistance R′ which is to be **connected in series** with the galvanometer after removal of the shunt is determined by the expression (1).

30(a). Example: An ammeter of ·1 ampere range is to be converted into a voltmeter reading up to 20 volts (Fig. 35), the resistance of the ammeter being 4 ohms.

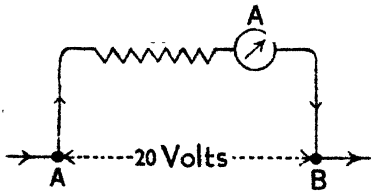


Fig. 35

Here, P D between A and B is to be made 20 volts. A resistance R is to be joined in series with the ammeter.

Then total resistance in ammeter path $= R + 4$.

Maximum current that passes through ammeter $=$ ·1 amp.

$\therefore\ 20 = \cdot 1(R+4)$ or, $R + 4 = 200$ $\therefore\ R = 196$ ohms.

31. Voltmeter: It is an instrument for indicating the difference of potential between two points across which it is connected.

Various forms of instruments are in use and different principles have been adopted in their construction.

Two types of the instrument need a special consideration.

The *electro-magnetic type* (Fig. 36) is a high resistance galvanometer having a fixed graduated scale to read volts, and the *electrostatic type* is a modified form of the electrometer described in Statical Electricity.

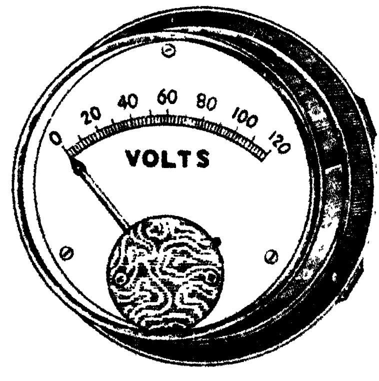


Fig. 36

The main point which is common to these types and which is a necessary requisite is that the voltmeters must have very high resistances so that the current taken up by them is inappreciable otherwise the potential difference between the points across which they are to be placed may change, and that the heat produced in them is negligible.

31(a). Moving Coil Voltmeter: In this electro-magnetic type of the instrument the coil pivoted between two needle points

is situated between the two curved pole-pieces N, S of a horse-shoe magnet MM (Fig. 37) and controlled by two hair-springs. The description of this voltmeter is exactly the same as the moving coil ammeter except in the fact that it is provided with a high resistance R in series with the coil of the instrument and that its scale is calibrated in volts.

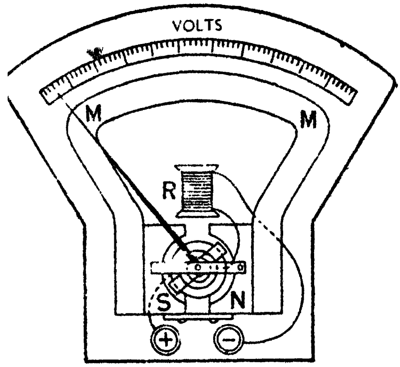


Fig. 37

Of all forms of voltmeters the electro-static voltmeter is the most satisfactory form.

Voltmeter is a high-resistance galvanometer or a galvanometer with a high resistance placed in series.

32. To increase the range of a voltmeter : To increase the range of the scale n times, the resistance of the voltmeter should be increased n times, such that the current passing through the voltmeter remains the same.

Let R be resistance of the voltmeter and let i be the current in it producing a particular deflection.

Then $\dfrac{V}{R}$ where V is the voltmeter reading.

When the voltmeter reading is multiplied n times, R must be increased n times so that i remains the same.

Let R′ be a high resistance placed in series with a voltmeter of resistance R so as to increase the range to n times. In order that the current passing through the voltmeter remains the same

$$i = \frac{nV}{R+R'} = \frac{V}{R} \qquad nR = R + R' \quad \text{or} \quad R' = (n-1)R.$$

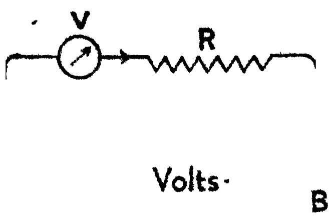


Fig. 38

32(a). Example : A voltmeter of 10 volts range is to be made to read up to 200 volts (Fig. 38), the resistance of the voltmeter being 20 ohms.

A resistance R is to be connected in series with the voltmeter.

The maximum current that can pass through the voltmeter $= \frac{10}{20} = \frac{1}{2}$ amp. After connecting R in

series, same current $\frac{1}{2}$ amp. must pass through the voltmeter when 200 volts are applied across the new terminals A ane B.

Therefore, $200 = (R + 20) \times \frac{1}{2}$ or, $R + 20 = 400$

$\therefore \quad R = 380$ ohms.

33. To convert a voltmeter into an ammeter : A voltmeter has high resistance whereas an ammeter is a low resistance galvanometer. Hence, for conversion, a shunt of suitable resistance is to be joined in parallel with the original instrument.

33(a). Example : A voltmeter reading up to 200 volts is to be converted into an ammeter of 10 amps. range, the resistance of the voltmeter being 250 ohms.

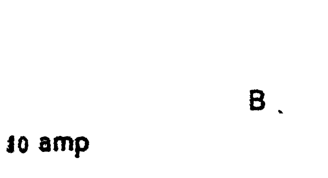


Fig. 39

The maximum current that can pass through the voltmeter $= \frac{200}{250} = {\cdot}8$ amp.

Since the instrument is to read up to 10 amps, current through the shunt S which is to be connected in parallel, $= 10 - {\cdot}8 = 9{\cdot}2$ amperes. If A and B (Fig 39) be common terminals of the converted ammeter and the shunt, then

$V_A - V_B = 9{\cdot}2 \times S = {\cdot}8 \times 250$

$$S = \frac{{\cdot}8 \times 250}{9{\cdot}2} = 21{\cdot}71 \text{ ohms.}$$

34. Difference between an Ammeter and a Voltmeter :

(A). **Ammeter :** It is a low resistance galvanometer and must be placed in series with the circuit through which the current to be measured passes.

If the resistance of the ammeter R be very high, (1) it will reduce the strength of the current and result in a decided drop of volts in itself ($E = i.R$) and (2) it would waste considerable power ($P = i^2R$).

If the resistance of the ammeter be not low, the resistance of the instrument will affect the current and the recorded current will not indicate the 'true' current. So the resistance of the ammeter as a current-measuring instrument must be low so that the current to be measured is not much affected.

(B) **Voltmeter :** It is a high resistance galvanometer and is connected *in parallel* with that portion of the circuit the potential difference for which is required.

Let R be the resistance of the voltmeter which is joined as a shunt to a coil of wire having a resistance r and through which a current of strength i is passing.

If R be high, the difference of potential (P. D.) between the ends of the coil remains unaltered by connecting the voltmeter in parallel with the coil and is equal to *ir*.

Again, if the resistance of the instrument is small, the P.D. is greatly reduced.

• If R be high, the power wasted is very small since the power $\frac{E^2}{R}$ is small.

35. Diagramatic representation of a circuit connecting an ammeter and a voltmeter : As shown in Figure 40, the ammeter having a low resistance in parallel, *i.e.*, as a shunt, is connected in series with the main circuit. But the voltmeter having a high resistance in series is connected in parallel with the system (here a lamp) whose terminal P.D. is to be ascertained.

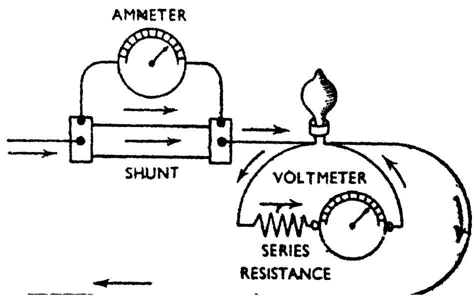


Fig. 40

36. Ballistic Galvanometer : This galvanometer is generally used to measure the quantity of electricity which passes when a current of extremely small duration traverses the coil of the instrument. Ballistic galvanometers are of two different types— (1) **Suspended needle type** and (2) **Suspended coil type.** In either of these forms the needle or the coil, as the case may be, should be so constructed that the duration of the current is small compared with the time of swing of the needle and that the damping is slight. As a momentary current passes through the galvanometer the *throw* or the first swing of the needle is observed and the quantity of electricity Q is determined in the **needle type** galvanometer by the expression.

$$Q=\frac{2H}{G}\sqrt{\frac{I}{MH}}\sin \quad \frac{HT}{\pi G}\sin\frac{\theta}{2}, \text{ Since } T=2\pi\sqrt{\frac{I}{MH}}$$

$$=\frac{HT}{\pi G}\cdot\frac{d}{4L}$$

where d is the deflection in scale division and L, the distance in the same units between the mirror and the scale.

Here, H = the horizontal intensity of earth's magnetic field.
G = Galvanometer constant.
I = Moment of Inertia of the needle.
M = magnetic moment of the needle.
θ = angle of throw.
T = the time period.

In the **suspended coil type**, we may consider all the charge as flowing while the coil is in its zero position. When a current of small duration is passing through the coil, a couple acts on it, giving it an impulse which can be calculated in terms of the amplitude of the resultant swing or ballistic throw.

When the impulse is known, the quantity of charge can be obtained from the ballastic throw. We know that when a current i is passing through a coil having n turns of area A, the moment of the couple acting on the coil at any instant, $G = niAH$, where H is the radial controlling field.

The impulse of this couple G.

$$= \int_0^t G.\ dt = \int_0^t nAHi dt = nAHQ, \text{ where } i.\ dt = Q.$$

Here the initial Kinetic energy is expended in rotating the coil against the torsional force in the suspension.

The kinetic energy acquired by the coil is $\frac{1}{2}I\omega^2$, while I is the moment of inertia of the coil and ω, the initial angular velocity.

$$\text{Hence, } \frac{1}{2} I\omega^2 = \int^{\theta} c\theta d\theta = \frac{1}{2}c\theta^2 \text{ or, } \omega = \sqrt{\frac{c}{I}}.\theta \qquad (1)$$

where θ is the angle of throw and c the couple for unit twist.

By the laws of motion, the momentum of the impulse is equal to the initial angular momentum of the coil.

$$\text{Therefore, } nAHQ = I\omega = I\sqrt{\frac{c}{I}}.\theta \qquad \text{from (1)}$$

$$Q = \frac{c}{nAH}\sqrt{\frac{c}{I}}.\theta$$

But we know that the period of the swing T is given by

$$T = 2\pi\sqrt{\frac{I}{c}} \qquad Q = \frac{c}{nAH}.\frac{T}{2\pi}\theta = K.\frac{T}{2\pi}.\theta \quad \text{where } K = \frac{c}{nAH}$$

$$Q = K'\theta, \text{ where } K' = \frac{KT}{2\pi}$$

where K is the *current reduction factor* for the galvanometer and K′ is the *quantity called ballistic reduction factor.*

The constant K may be obtained by observing the deflection θ produced by a steady current i.

37. Difference between (A) a Suspended Coil Galvanometer and (B) a Ballistic Galvanometer.

(A). Suspended Coil Galvanometer :

(1) It is intented for the measurement of a steady current and not suitable for the measurement of quantity of electricity.

(2) The coil of the galvanometer is wound on a frame of copper sheet and so damping of oscillation is quick.

(B). Ballistic Galvanometer :

(1) It is intended for the measurement of quantity of electricity passing through the coil.

(2) The coil of the galvanometer is wound on a frame of non-conducting sheet and so the damping is not quick and as low as possible.

In this instrument the duration of the current is small compared with the time of swing of the coil.

QUESTIONS

1. Discuss the several forces or moments which act on the needle of a tangent galvanometer when deflected by the action of a current passing through the coil of the galvanometer and deduce the law of action of the instrument. [C. U. 1942]

2. Write a short note on different types of galvanometers. [C. U. 1944]

3. Describe the construction of a D'Arsonval type of moving coil galvanometer and deduce an expression for the deflecting torque acting on its moving system when a current of i amperes flows through the coil. [C. U. 1945, '48, '53, '56, '58]

4. How should you determine the figure of merit of a suspended coil galvanometer experimentally ? [C. U. 1948]

In what why is a ballistic galvanometer different from an ordinary suspended coil galvanometer ? [C. U. 1948]

5. Describe the construction and mode of operation of any form of ammeter with which you are acquainted. Why does an ammeter generally have a low resistance ? What modification would you introduce to make the instrument a direct reading voltmeter and why ? [C. U. 1946, '50]

6. *Show how an ammeter and a voltmeter can be shunted so as to multiply their range n times.* [C. U. 1938]

7. Describe the principle of Ballistic Galvanometer and deduce the working formula.

8. Describe Helmholtz double-coil galvanometer and deduce the working formula. [C. U. 1950]

9. Discuss the difference between an ammeter and a voltmeter. For what purposes are they used?

EXAMPLES

1. You are given a millivoltmeter of range 1 to 30 millivolts and of internal resistance 25 ohms. Shew how you will use the instrument to measure (*a*) potential between 1 and 30 volts (*b*) currents between 1 and 3 amperes. [C. U. 1930]

(*a*) Since the instrument reads 30 millivolts the current C passing through it is given by $i=\dfrac{30\times10^{-3}}{25}$

To make the instrument read 30 volts the current is kept constant by using an additional resistance R in series with the instrument or by changing the resistance of the instrument to the required amount.

Now we have $i(R+25)=30$ or, $R=\dfrac{30}{i}-25=\dfrac{30\times25}{30\times10^{-3}}-25=24975$ ohms.

(*b*) To make the instrument read 3 amperes it is shunted with a resistance S.

Since the instrument reads 30 millivolts the p. d. at the ends of the shunt resistance S is equal to 30×10^{-3} volts and since the instrument has a high resistance in comparison with S, the current of 3 amp. may be supposed to be passing through S.

We have $S\times3=30\times10^{-3}$ or $S=10^{-2}=\cdot01$ ohm.

2. A certain ammeter has a resistance of one ohm and the full scale deflection is obtained when a current of 0·05 ampere flows through it. Find what shunt must be connected with it in order that the ammeter may read up to 100 amperes. [C. U. 1921]

We know that $i_g=\dfrac{S}{S+G}i$ $\therefore$ $\cdot05=\dfrac{S}{S+1}\times100$ $\therefore$ $S=\dfrac{\cdot05}{99\cdot95}=\cdot0005$ ohm.

3. A portable galvanometer, whose needle deflects 5 scale divisions per milli-ampere, is to be used as an ammeter. Its resistance is 238 ohms.

What should be the resistance of the shunt in order that the needle may deflect 10 divisions per ampere? [C. U. 1919]

We know that $i_g=\dfrac{S}{S+G}$

Again 1 m. a. produces a deflection of 5 divisions

2 m. a " " " 10 "

So $i_g=\cdot002$ amp., $G=238$ ohms. $C=1$ amp.

$\therefore$ $\cdot002=\dfrac{S}{S+238}\times1$ or $S=\cdot478$ ohms.

4. *Calculate the strength of the current in amps. from the following data :—*

Radius of the coil $=12$ *cm.*
Number of turns $=50$
Deflection $=45^\circ$
H $=0{\cdot}37$ gauss. [C. U. 1942]

We know $i=\dfrac{10rH}{2\pi n}\tan\theta=\dfrac{10\times12\times{\cdot}37}{2\times\frac{22}{7}\times50}\times1={\cdot}1415$ *amp.*

5. How may a galvanometer of resistance 10 ohms which gives a full-scale deflection when a current of 1 milli-ampere passes through it, be used as (*a*) an ammeter reading to 5 amperes and (*b*) a voltmeter reading up to 10 volts ?

(*a*) The value of shunt resistance must be such that when a current of 5 amperes flow through the galvanometer, the ammeter reading will be 1 milli-amp. *i.e.* 1×10^{-3} amp.

Therefore from the relation $i_g=\dfrac{\quad}{S+G}\,i$

We have $1\times10^{-3}=\dfrac{S}{S+10}\times5$, or $S=\dfrac{10}{4999}={\cdot}002$ ohms.

(*b*) To convert the galvanometer into a range of 5 volts, a resistance of R ohms is to be connected in series with the galvanometer across a source of 10 volts so that the current through the combination is 1 milli-ampere or 1×10^{-3} amp.

$$i=1=10^{-3}\text{ amp.}=\frac{10}{R+G}\quad\frac{13}{R+10}\text{ or } R=9990\text{ ohms.}$$

6. A moving coil galvanometer has a resistance of 10 ohms, and gives a full-scale deflection when the current through it is one milli-ampere. What will you do to convert it into an ammeter reading up to ten amperes ? [C. U. 1956]

To convert the galvanometer into an ammeter reading up to 10 amperes, a suitable shunt should be used in parallel with the galvanometer coil.

The value of the shunt resistance must be such that when a current of 10 amps. flow through the galvanometer, the ammeter reading will be 1 milli-amps., *i. e.* 1×10^{-3} amps.

$\therefore$ from the realation $i_g=\dfrac{s}{s+10}\,i$, where i_g is the current in the galvanometer; s, the shunt resistence, G, the resistance of the galvanometer coil and i, the main current.

$$\therefore\quad 1\times10^{-3}=\frac{s}{s+10}\times10\quad\text{or } s=\frac{10}{9999}={\cdot}001\text{ ohm.}$$

7. The coil of a galvanometer has 60 turns, a width of 2 cm. and a depth of 3 cm. It hangs in a uniform radial field of 500 C. G. S. units. Find the turning moment of the coil in dyne-centimeters when it is carrying a current of 1 m.amp.

Since the coil hangs in a uniform radial field H, the turning moment $=nHiA$, where n is the number of turns ; A, the area of the coil H the field and i the current in C. G. S. units.

$\therefore$ The turning moment $=60\times500\times10^{-4}\times2\times3=18$ C.G.S. units.

Since 1 m.a. $=1\times10^{-3}$ amp. $=1\times10^{-4}$ C.G.S. units current.

CHAPTER IV

AMPERE BALANCE AND WATTMETER

38. Action of current on current : Let two like currents i_1 and i_2 be travelling along two parallel wries AB and CD (Fig. 41) at a distance r apart. The intensity of the field due to i_1 at a distance r is $\frac{2i_1}{r}$ per unit length of the wire. By Maxwell's Cork-screw rule, the direction of the field in the case shown in the figure is perpendicular to the plane of the paper and is directed towards the paper. The current i_2 is situated in this field, so that the force on the current i_2 per unit length of the wire CD is $\frac{2i_1i_2}{r}$ [compare with Hil ; here $H=2i_1/r$, $i=i_1$ and $l=1$ cm.]. The force according to Fleming's Left-hand rule is directed towards the current i_1, *i. e.*, the current i_2 is attracted by current i_1. Considering the force acting on current i_1 per unit length due to the field produced by current i_2 (per unit length at a point on AB at a distance r we can show similarly that current i_1 is attracted by current i_2. Thus current i_1 and i_2 attract each other. When the currents are unlike, they repel each other. Hence we have the rule 'Like currents attract, unlike current repel."

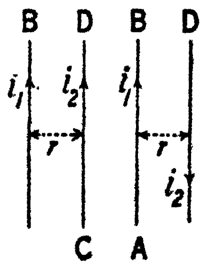


Fig. 41

When the currents are oblique, there will be mutual attraction if both the currents move towards or away from actual or apparent point of intersection as in (c, a) of Fig. 42.

If one of the currents flows away [Fig. 42 (b)] and the other towards the point of intersection, there is mutual repulsion.

39. Kelvin's Ampere Balance : The apparatus (Fig. 44) consists of four fixed coils A, B, C and D and two moveable coils E and F, the latter being held by a lever which can turn about a fulcrum O. A horizontal graduated arm GH fixed to the lever carries a rider of known weight W. Since the six coils are connected in series, same current passes through all of them. The direction of windings of the coils is such that on passing current, the middle coils E on the left side is repelled by the upper coil A and attracted by the lower coil C. It, therefore, tends to move down.

Similarly, the middle coil on the right side tends to move up. The lever carrying the coils E, F thus tends to turn about O in anti-clockwise direction. This effect is prevented by properly adjusting the position of the rider weight W. As the arm GH is previously calibrated by passing known currents through the coils, any unknown current may be ascertained by noting the position of W which corresponds to the balance point for the unknown current. The connections of different coils with source of current is shown in Figure 43.

Fig. 42

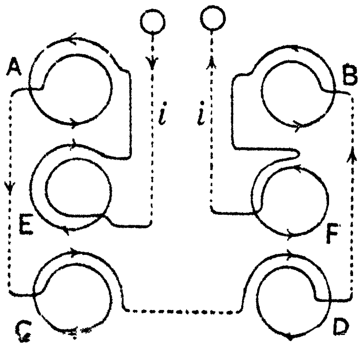


Fig. 43

Note : In this apparatus, the force between neighbouring coils is proportional to the square of current and hence it is independent o : direction of the current. Thus the apparatus is also suitable for measurement of alternating current.

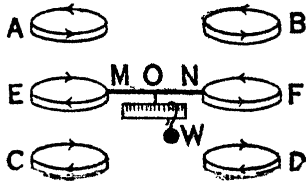


Fig. 44

40. Siemen's Electro-dynamometer : It consists of two vertical coils ABCD (Fig. 45) of low resistance and EFGH of high resistance, of which ABCD is fixed, and EFGH is moveable having its ends dipped in mercury. The coil EFGH is suspended by a silk thread and controlled by a steel or phosphor-bronze spirals, the upper end of which is attached to a torsion head T. The coils are joined in series if the instrument is to be used as an *Ammeter*. The coil EFGH is at first set at right angles to the coil ABCD by turning T.

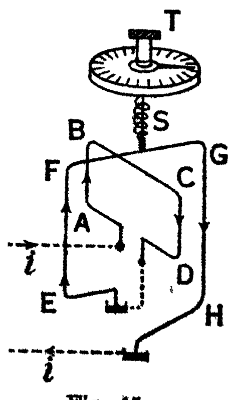


Fig. 45

When a current i flows through the coils, the arm EF of the moveable coil is attracted by the arm AB and repelled by the arm CD of the fixed coil. Similarly the arm GH is attracted by the arm CD and repelled by the arm AB. The coil

EFGH, therefore, rotates under a couple. As shown in the figure this rotation is in anti-clockwise direction. The coil is again brought back to its original position by turning the torsion head T at the top of the instrument. The angle θ through which the torsion head is turned and which is a measure of torsion, is measured by the pointer P rotating over a fixed graduated circle. The couple tending to produce rotation is proportional to the attractive or repulsive forces between the arms and hence is proportional to the squa of the current.

Therefore $i^2 \propto \theta$ or $i^2 = c.\theta$, c = constant

$$\therefore \quad i = \sqrt{c\theta}$$

The constant c can be found by noting θ after passing a known current.

41. Wattmeter : Siemen's electro-dynamometer can be use as a wattmeter by which electric power absorbed in an electr machine or a bulb is measured. Suppose we are to measure pow absorbed by electric heater H. To do it, the low resistance co ABCD is joined in series and high resistan coil EFGH in parallel with the heater (Fi 46). Let i be the current through th heater and potential difference between it terminals, E. Then current through the coil ABCD is i, and if R be the resistance EFGH, current through it is E/R. Then t couple producing rotation of the coil EFGH will be proportional to Ei/R.

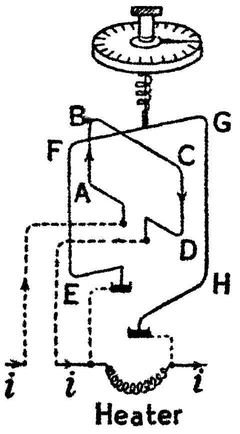


Fig. 46

Hence if θ be the angle of torsion requir to bring the coil EFGH to its origin position.

$$\text{We have } \frac{Ei}{R} \propto \theta \text{ or, } Ei \propto R\theta$$

But Power = Ei ; $\therefore$ power $\propto \theta$ $\because$ R i constant, or power = Kθ where K is constant.

The value of K is obtained by noting θ when known power employed, *i.e.*, in which E, i are known separately.

42. Energy Meters : In house electric supply meters, the function of the instrument is not simply to indicate power or rate of supply of electrical energy, but also to consider the interval of time for which energy is supplied. These meters are called energy meters and they indicate at once the product of voltage, current and length of time of current supply. The readings of the

graduated dials of energy meters are expressed in B.O.T. units or kilowatt-hours and their submultiples.

QUESTIONS

1. How will you prove that like currents attract and unlike currents repel ?
2. Describe Kelvin's Ampere Balance and explain its action.
3. What is an Wattmeter ? Describe Simen's Electro-dynamometer and show how it can be used as an Wattmeter.
4. Write short notes on energy meter.

CHAPTER V

OHM'S LAW : RESISTANCE

43. Electromotive Force of a Cell : The electromotive force of a cell is that which drives electric charge through the circuit and is caused by the chemical changes occuring inside the cell. It depends on the nature but not upon the size of the plates used inside the cell. The greater the distance between the two metals used as plates in the electromotive series, the greater becomes the difference of the potential and the E.M.F. of the cell.

44. Electromotive Force and Potential Difference : The two terms can be better explained by considering the chemical action which takes place when two plates of different metals, such as copper and zinc, are immersed in a suitable solution.

As soon as the plates are put in the solution, positive electricity begins to move by some driving force known as the E. M. F. of the cell, from the zinc to the copper plate through the liquid and then reaches the copper plate whose potential is thereby raised above that of zinc which is left negatively charged.

Thus a positive charge inside the cell is acted on by two forces in opposite directions, the E. M. F. of the cell causing it to move from zinc to copper and the potential difference between the plates from copper to zinc inside the solution.

When the plates are not connected externally by any conducting wire the motion of electricity will continue until the potential difference between the plates will rise to such a value due to the accumulation of electricity that the tendency of the positive electricity to flow from zinc to copper under the E. M. F. of the cell

is just balanced by its tendency to flow from copper to zinc due to the potential difference. At this stage all action ceases and there is no movement of electricity in either direction through the cell.

Thus the potential difference between the plates is only a result of the action of the E. M. F. and these two, the potential difference and the E. M. F. of the cell act in opposition and become equal to one another when the cell is in open circuit, *i.e.*, when plates are not connected externally.

Again when the plates of the cell are connected externally by a wire of resistance R, the potential difference between the plates will gradually diminish owing to the passage of electricity from copper to zinc through the wire and a certain state will be reached at which the potential difference will fall to some value, say V such that current i_1 inside the cell from zinc to copper driven by the net E. M. F., E − V (where E is the E. M. F.), is equal to the current i_2 as flowing outside the cell from copper to zinc by the potential difference V.

Then, by Ohm's law $i_2 = \frac{V}{R}$, $i_1 = \frac{E-V}{r}$

where r is the internal resistance of the cell.

But since $i_2 = i_1$, we have, $\frac{V}{R} = \frac{E-V}{r}$ or $ER = V(R+r)$...(1)

If the cell is on *open circuit*, *i.e.*, if R be infinitely large, r is negligibly small in comparison with R.

therefore $ER = VR$, *i.e.*, $E = V$

That is, the E. M. F. of the cell = the potential difference between the terminals of the cell.

Thus *the E. M. F. may be defined as the p. d. between the plates when no current flows.*

An electromotive force has always the same direction in the circuit, while the potential difference has a direction depending on that of the current.

The electromotive force corresponds to a motive mechanical force, whereas the potential difference corresponds to a frictional force which depends for its direction on the direction of the motion of current and is a measure of heat produced per second between the two points.

45. Electromotive force and Power: When a cell maintains a current it supplies energy which does work when a charge passes along a conductor connected to its terminals.

The E. M. F. of a cell in volts may be defined as the number of joules of energy supplied to drive one coulomb of electricity to circulate completely through the cell and round an external circuit.

Thus if the cell sends a current of i amperes in t seconds the total work done by the cell is Eit, where E is the E. M. F. of the cell in volts. Of this total work a part i^2rt is expended in sending the current through the cell and the rest i^2Rt in maintaining the current through the external circuit, where r is the internal resistance of the cell and R, the resistance of the external circuit.

The power delivered by the cell is Ei and so the E. M, F. of a cell may be defined as the *total power per unit current.*

We know that E joules of energy is provided by the cell for each coulomb of charge passing round the whole circuit.

We have therefore, $Eit = i^2Rt + i^2rt.$

or $E = iR + ir = i(R + r)$

Now $iR = V$, p. d. across the external resistance R.

$\therefore \quad E = V + ir \quad \text{or} \quad V = E - ir$

ir is sometimes called the **lost volt** in the cell.

Thus the p. d., across the external resistance is always less than the E. M. F. of the cell except when $r = 0$.

46. Ohm's Law : Dr. C. G. S. Ohm deduced a certain law between the three quantities—current strength, electromotive force and resistance. His law states that "in any part of an electric circuit at uniform temperature, the strength of the current varies directly as the potential difference (P. D.) between them." If the whole circuit is considered, the law states that the current strength varies directly as the E. M. F. of the cell and inversely as the resistance of the whole circuit.

Thus we have $i = \dfrac{V_A - V_B}{R}$ (P.D.) (1) for part of the circuit

and $= \dfrac{E \text{ (E. M. F.)}}{R + r}$ for the whole circuit

where i is the current passing through a wire, say AB, of resistance R, the two ends of which being at potentials V_A and V_B and r, the internal resistance of the cell of E. M. F., E.

From equation (1) we notice that the resistance of a conductor is the ratio of potential difference $V_A - V_B$ between the ends of the conductor to the current i flowing through it.

The value of the resistance is constant so long as the temperature remains constant. It increases as a rule with the rise of temperature and decreases with the fall of temperature.

Thus we conclude that Ohm's Law is true so long as temperature remains constant.

47. Grouping of cells : Voltaic cells can be arranged in three different ways to get a suitable current in the external circuit.

They can be arranged—(1) *in series*, (2) *in parallel* and (3) *in series-parallel*, *i. e.*, in mixed circuit.

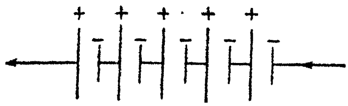


Fig. 47

(1) **Cells in Series** : The cells are connected in series (Fig. 47) when the positive terminal of one is connected to the negative of the next and so on.

If n number of cells, each having an E. M. F. of E volts and internal resistance r ohms are arranged in series, the total E. M. F. and internal resistance of n cells will be respectively nE and nr. If the circuit be completed through an external resistance of R ohms, then by Ohm's Law, current i in the circuit and through each cell is given by

$$i = \frac{nE}{R + nr} \quad \ldots \quad \ldots (1)$$

If **R** be very **large** compared with nr, $i = \frac{nE}{R}$, *i. e.*, the current is n times that due to a single cell.

(2) **Cells in Parallel** : In this arrangement the positive terminals of all the cells (Fig. 48) are connected to one another and form a positive pole and all the negative poles connected together to form a negative pole.

The E. M. F. of the system is the same as that (E) of a single cell, and the total internal resistance $\frac{1}{n}$th of that (r) of one cell.

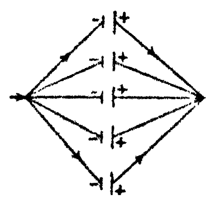


Fig. 48

Then current i in the external resistance R is given by

$$i = \frac{E}{R + \frac{r}{n}} = \frac{nE}{nR + r} \quad \ldots \quad \ldots (2)$$

If **R** be very **small** $i = \frac{nE}{r}$, *i.e.*, the current is n times that due to one cell.

Note : It can be easily shown from relations (1) and (2) in Art. 47, that when the external resistance R is great compared with r, *series* arrangement is advantageous for a strong current. But when r is great compared with R *parallel* arrangement is advantageous.

(3) Cells in Series-Parallel grouping. (Mixed Circuit):

The cells are arranged in a mixed circuit when they are divided into several sets or rows arranged in parallel, each set containing a certain number of cells in series (Fig. 49).

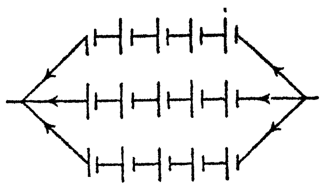


Fig. 49

Let n be the number of cells arranged in *series* in each row and m the number of rows and let the circuit be completed through a resistance R. In this case total effective E. M. F. is nE and total internal resistance nr/m.

$$\text{Then, external current } i = \frac{nE}{R + \frac{nr}{m}} = \frac{mnE}{mR + nr} = \frac{NE}{mR + nr}$$

where N is equal to mn which is the total number of cells.

The value of the current i is maximum when $mR + nr$ is minimum.

That is, when $(\sqrt{mR} - \sqrt{nr})^2 + 2\sqrt{mnRr}$ is minimum since this quantity is equal to $mR + nr$.

That is, when $\sqrt{mR} - \sqrt{nr} = 0$; $mR = nr$; $R = \dfrac{nr}{m}$

That is, the current is maximum when the *external resistance* of the system is equal to the total *internal resistance.* This arrangement is most *efficient* but not the most *economical*, for if the external and internal resistances are equal just as much energy is wasted inside the cells as is utilised in the external circuit.

48. Verification of Ohm's Law: To verify this law, a current is made to pass from a storage cell through a tangent galvanometer and a variable resistance box, the circuit being completed by copper wires. The resistance and the galvanometer are connected in series through a commutator by means of which the current is reversed.

A resistance R is inserted in the box and a deflection θ is noted in the galvanometer. The resistance is altered to R_2 and the new deflection θ_2 is again noted.

$$\text{Thus, according to Ohm's Law, current } i = \frac{E}{R + x}$$

where E is the E. M. F. of the cell and x the resistance of the

portion of the circuit including that of the battery and the galvanometer other than that of the resistance box.

But we know the value of i from the principle of the tangent galvanometer as equal to K $\tan\theta$ where K is the reduction factor of the galvanometer.

$$\text{Therefore } K\tan\theta = \frac{E}{R+x} \quad \ldots \quad \ldots(1)$$

The variable resistance is then altered to value R_1 so that the deflection becomes θ_1. Then,

$$K\tan\theta_1 = \frac{E}{R_1+x} \quad \ldots \quad \ldots(2)$$

Thus from (1) and (2) we have

$$K\tan\theta\,(R+x) = K\tan\theta_1\,(R_1+x) = E \quad \ldots \quad \ldots(3)$$

Thus by inserting different values of resistance in the box if $(R+x)\tan\theta = (R_1+x)\tan\theta_1 = (R_2+x)\tan\theta_2$ and so on, be found to be constant and equal to E, the E. M. F. of the cell, Ohm's Law is verified.

The value of x is determined from (3) and is equal to

$$\frac{R_1\tan\theta_1 - R\tan\theta}{\tan\theta - \tan\theta_1}$$

49. Different Units: All the quantities involved in the expression for Ohm's Law are measured in practical units.

There are generally two kinds of units :—(1) Absolute or C.G.S. Electromagnetic unit and (2) Practical unit.

C. G. S. Unit Current: It is the current which flowing along a wire of length one centimetre bent into a circular arc of radius one centimetre exerts a force of one dyne on a unit north megnetic pole placed at the centre of the circle. The practical unit of current is Ampere (A).

Ampere is that steady current which flowing through a solution of Copper Sulphate deposits ·0003293 gramme of copper on the Cathode in one second.

1 Ampere = $\frac{1}{10}$ C.G.S. unit of current.

C. G. S. Unit Quantity of charge: It is the amount of electricity conveyed in one second by a unit current. The practical unit of quantity of electricity is the **coulomb.**

Coulomb is that quantity of electricity which liberates ·0003293 gramme of copper from a solution of copper sulphate or ·001118 gramme of silver from a solution of silver nitrate.

An *ampere-hour* is the quantity of electricity conveyed by a steady current of one ampere flowing for one hour.

One ampere-hour = 3600 coulombs.

The smallest quantity or natural unit of electricity is $1{\cdot}55 \times 10^{-19}$ coulomb $= 1{\cdot}55 \times 10^{-20}$ C. G. S units.

1 Coulomb $= \frac{1}{10}$ C.G.S. unit of quantity.

C.G.S. Unit of Resistance : A wire has a resistance of one C.G.S electromagnetic unit if a P. D. of one electromagnetic unit applied to its ends causes a current of one electromagnetic unit to flow through it. The practical unit of resistance is **Ohm**. denoted by sign (Ω).

1 ohm. $= 10^9$ C G.S. unit of resistance.

C. G. S. Unit E. M. F. : A unit E. M. F. is that which is required to drive unit current through a unit of resistance. The practical unit of E. M. F. is **Volt.**

1 volt $= 10^8$ C.G.S. unit of E. M. F.

1 milli-ampere (mA) $= \frac{1}{1000}$ A $= 10^{-3}$ A

1 micro-ampere (μA) $= \frac{1}{10^6}$ A $= 10^{-6}$ A

kilo-ohm ($k\Omega$) $= 1000$ ohms $= 10^{-3}$ ohms
mega-ohm ($M\Omega$) $= 10^6$ ohms

1 micro-ohm ($\mu\Omega$) $= \frac{1}{10^6}$ $= 10^{-6}\,\Omega$

1 milli-volt (mV) $= \frac{1}{1000}$ Volt $= 10^{-3}$ Volt.

1 micro-volt (μV) $= \frac{1}{10^6}$ Volt $= 10^{-6}$ Volt.

50. Resistance: It is the property of a substance of opposing the flow of current. In a wire *it varies directly as the length of the wire when the cross-section is constant and inversely as the area of cross-section when the length is constant.* The two may be grouped into a single relation.

That is, $R \propto l$, when s (area of cross-section) is constant

$$R \propto \frac{1}{s}, \text{ when } l \text{ (length) is constant}$$

$$\therefore \quad R \propto \frac{l}{s}, \text{ when both } l \text{ and } s \text{ vary.}$$

$R = \rho \frac{l}{s}$, where ρ is a constant, called the **specific resistance** of the material of the wire. If $l = 1$ cm., $s = 1$ sq. cm., then $\rho = R$.

50(a). Specific Resistance or Resistivity : (ρ) It is the resistance of unit length of a wire of unit area of cross-section.

It is also defined as the resistance of unit cube of a given material. In the metric system ρ may be expressed in ohms per centimetre cube (not cubic centimetre) or simply as **ohm-cm.**

The value of ρ is generally expressed in micro-ohms per cm, cube.

ρ for manganin = 44 micro-ohms per cm. cube
ρ ,, eureka = 49 ,, ,, ,,
ρ ,, Nichrome = 110 ,, ,, ,,

The value of ρ for nichrome is very high which is, therefore, used for making high resistances. But its temperature coefficient is also high.

50(b). Conductance : The reciprocal of the resistance is called the conductance.

51. Conductivity : (σ) The reciprocal of the resistivity or specific resistance is called conductivity.

Thus $\sigma = \frac{1}{\rho}$ and is sometimes expressed in 'mhos' per cm.-cube.

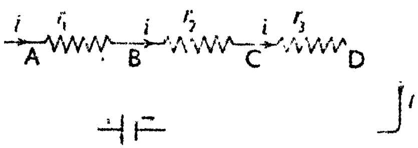


Fig. 50

52. Conductors in series : Conductors AB, BC and CD are said to be arranged in series when the same current traverses each conductor. If r_1, r_2, r_3 etc. be the resistances of the conductors arranged in series, the effective or the total resistance R of the conductors is given by $R = r_1 + r_2 + r_3$.

Proof :—$V_A - V_B = i.r_1$; $V_B - V_C = i.r_2$; $V_C - V_D = i.r_3$

Adding $V_A - V_B + V_B - V_C + V_C - V_D = i(r_1 + r_2 + r_3)$

Or, $V_A - V_D = i(r_1 + r_2 + r_3)$. But $V_A - V_D = i.r$ where r is equivalent resistance.

$\therefore \quad V = r_1 + r_2 + r_3.$

53. Conductors in parallel : When several conductors are joined between two points so that the current divides between them they

are said to be in parallel. The total current i entering at A divides into three parts i_1, i_2 and i_3 which unite again at B (Fig. 51).

Then $i = i_1 + i_2 + i_3$ ... (1)

Let $(V_A - V_B)$ be the potential difference between the points A and B. Let r_1, r_2 and r_3 be the resistances of the separate conductors and R be the equivalent resistance of the conductors between A and B so that same current will travel in the circuit when the component resistances are replaced by it.

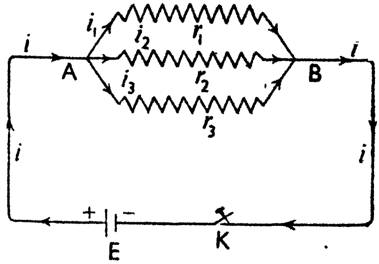


Fig. 51

Then, considering the conductors separately we have

$$i_1 = \frac{V_A - V_B}{r_1} \qquad i_2 = \frac{V_A - V_B}{r_2} \qquad i_3 = \frac{V_A - V_B}{r_3};$$

$$i = \frac{V_A - V_B}{R}, \text{ where R = equivalent resistance.}$$

Therefore from (1) $\dfrac{V_A - V_B}{R} = (V_A - V_B)\left(\dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3}\right)$

$$\text{or} \qquad \frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

Thus when the conductors are arranged in parallel the reciprocal of the equivalent resistance is equal to the sum of the reciprocals of the resistances of the separate conductors.

$$\frac{1}{R} \text{ is } > \frac{1}{r_1} \text{ or } \frac{1}{r_2} \text{ or } \qquad R \text{ is } < r_1 \text{ or } r_2 \text{ or } r_3$$

This equivalent resistance is less than any of the component resistances.

54. Temperature Coefficient of Resistance: It has been found that the resistance of metal increases with the rise of temperature while for most non-metals including india-rubber, carbon, mica, ebonite, etc. the resistance falls rapidly with the rise of temperature.

For metals, if R_0 be the resistance at 0°C, R_t the resistance at t°c is represented by the equation

$$R_t = R_0(1 + \alpha t + \beta t^2)$$

for a wide range of temperature where α and β are constants.

For a moderate range of temperature say, between 0°C to 190°C, β may be neglected and the equation becomes

$$R_t = R_0 \ (1+\alpha t)$$

where α is the *temperature coefficient*

Values of α for **manganin** and **eureka** are extremely **low** and so these alloys are very suitable for constructing standard resistances.

Carbon and non-metals have a negative temperature coefficient as the resistances of these substances decrease as their temperature rises.

The variation of resistance with temperature for pure platinum is utilised in the measurement of temperatures by an instrument known as Platinum Resistance Thermometer. (Consult Article on Platinum Resistance Thermometer).

In the case of *electrolytes* the resistance falls rapidly with the rise of temperature and its negative temperature coefficient varies from 5% to 2% per degree centrigrade.

With alloy resistance, increase of resistance with temperature is not very perceptible.

Again if the temperature of conductor gradually falls and approaches the **absolute zero** its resistance will at first decrease normally but it suddenly falls **to an** almost zero value **at a** temperature in the neighbourhood of 5° absolute known as the **critical temperature.**

Kamerlingh Onnes while carrying out of researches at low temperatures with the aid of liquid helium, demonstrated that mercury, lead and some alloys at their respective *critical temperatures* offer no measurable resistance to the passage of current.

Mercury at 4°·2 absolute (−268·8°C) and lead at 7°·3 absolute exhibit this phenomenon. This is known as *supra conductivity.*

We know that the conductivity of a conductor is the reciprocal of its resistivity. So when the resistance of a conductor is minimum at the critical temperature, its *conductivity* at this temperature is maximum.

If a metal at a temperature corresponding to supra-conductivity be placed in a changing magnetic field, the current-induced in it will flow for months without appreciable diminution.

55. Effect of temperature on metals and electrolytes: It is found that the specific resistance or resistivity of most substances varies with temperature.

For pure metals it is almost proportional to the absolute temperature and the temperature coefficient of resistance per degree centigrade rise is about $\frac{1}{273}$.

The specific resistance of alloys is usually greater than that of pure metals and the temperature coefficient is much smaller. For alloys such as *manganin* and *constantine* it may be regarded as negligible. So these alloys are very suitable for constructing

standard resistances. The value of specific resistance is very high for *nichrome* which is, therefore, used for making high resistances.

The temperature coefficients for carbon and all electrolytes are negative. The same phenomenon is shown by most semi-conductors and non-metals.

From the kinetic theory, we should expect the specific resistance to become zero when there is no thermal agitation of the atoms, *i.e.*, at 0° Abs. Kamerlingh Onnes shewed that in the region of 0° – 10° Abs., the specific resistance of several pure metals such as mercury (4·22°), tin (3·6°), lead (7·2°), thallium, etc., drops to a value which is too small to be measured. This effect is called **supra-conductivity**. The transition temperatures for these metals are in absolute scale and indicated in the bracket.

In a *metal* the electrical resistance increases with rise in temperature as the more active thermal motion of the molecules interfere strongly with motion of the electrons.

For a *semi-condutcor*, the electrical resistance diminishes with the rise of temperature since more and more electrons succeed in entering the upper energy states and come out as free electrons (which convey the current).

In *electrolytes*, the viscosity of water decreases very rapidly as the temperature rises and so it is to be expected that the resistance offered to the passages of ions through it will also be reduced.

56. Shunt : It is a resistance which is used in parallel with the resistance coil of a galvanometer so as to allow only a small fraction of the main current to pass through the instrument. It is used to protect the galvanometer from being damaged by the passage of a high current.

57. To determine the current flowing through the galvanometer when it is shunted by a wire : The cell of E. M. F., E and internal resistance r is connected to the terminals of the galvanometer and thus the circuit is completed. A shunt resistance S is connected in parallel with the galvanometer having a resistance of G ohms.

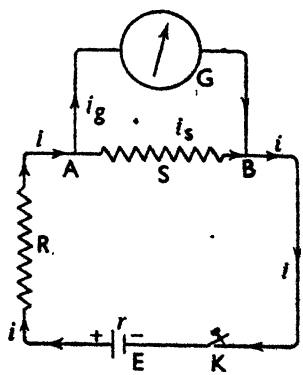


Fig. 52

The main current i is divided into two portions i_g and i_s, one flowing through G and the other through the shunt S.

As evident from Figure 52, the difference of potential between the terminals of the galvanometer is the same as the difference of potential between the ends A and B of the shunt, hence we have $V_A - V_B = i_g.G = i_s.S$

$$\therefore \quad i_gG = i_s S \text{ or } \frac{i_s}{i_g} = \frac{G}{S} \text{ or } \frac{i_s + i_g}{i_g} = \frac{G+S}{S}$$

$$\therefore \quad i_g = \frac{S}{G+S} i \qquad \text{since } i = i_s + i_g$$

Similarly we can show $i_s = \dfrac{G}{S+G} i$

57(a). To find the main current :

By Ohm's Law $i = \dfrac{E}{R'+r} = \dfrac{E}{\dfrac{GS}{G+S}+r}$

where R′ is the total equivalent resistance of G and S arranged in parallel and is equal to $\dfrac{GS}{G+S}$

$$\text{Thus} \qquad i_g = \frac{S}{G+S} \cdot \frac{E}{\dfrac{GS}{G+S}+r} = \frac{E.S}{r(G+S)+GS}$$

$$i_s = \frac{G}{G+S} \cdot \frac{E}{\dfrac{GS}{G+S}+r} = \frac{E.G}{r(G+S)+GS}$$

In the above, external resistance in series with battery is taken as zero. If this be R

$$i = \frac{E}{R+R'+r} = \frac{E}{(R+r)+\dfrac{SG}{S+G}} = \frac{E(S+G)}{(R+r)(S+G)+SG}$$

The expressions for i_g and i_s may then be obtained as before.

58. Resistance of $\frac{1}{n}$th Shunt : The $\frac{1}{n}$th shunt is a shunt which allows $\frac{1}{n}$th of the total main current to pass through the galvanometer.

If a portion i_g of the total current i flows through the galvanometer of resistance G connected in parallel with a shunt of resistance S, we have $i_g = \dfrac{S}{G+S} i$

If $\dfrac{i_g}{i} = \dfrac{1}{n}$, then $\dfrac{1}{n} = \dfrac{S}{S+G}$ or $S = \dfrac{1}{n-1} G$.

That is, if $\dfrac{1}{n} = \dfrac{1}{10}$. $\therefore$ $S = \frac{1}{9} G$.

That is, the resistance of the shunt should be $\frac{1}{9}$th of the resistance of the galvanometer in order that $\frac{1}{10}$th of the total current will pass through the galvanometer.

59. A Universal Shunt: The shunt consists of a high resistance S connected between the points A, A in parallel with the galvanometer of resistance G.

Tappings of certain fractions of S such as $\frac{1}{10}$, $\frac{1}{100}$, etc., are taken to the points marked $\frac{1}{10}$, $\frac{1}{100}$, etc. A rotating conductor P connected to the knob at K makes contact between the shunted galvanometer and the negative (−) terminal. The current enters at one end of S through the positive (+) terminal in the upward direction (not in the downward direction as shown in the Figure 53) and leaves at the other end through the negative (−) terminal.

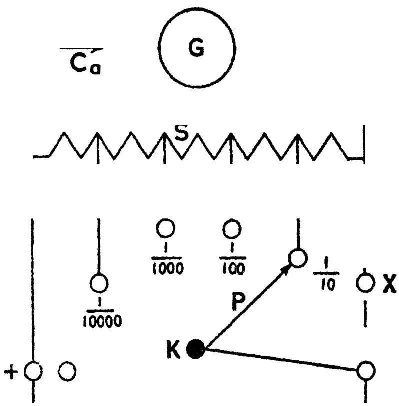


Fig 53

If the rotating conductor P makes contact with X, the whole of the shunt S is in parallel with the galvanometer of resistance G, and the current through the galvanometer i_g is given by

$$i_g = \frac{S}{G+S} \cdot i \qquad (1)$$

If the conductor P makes contact with a point on the shunt which gives $\frac{1}{n}$S from the left-hand terminal where n is an integer 10, 100, etc. S in equation (1) is replaced by $\frac{S}{n}$ and G by $G+S-\frac{S}{n}$.

Then the current through the galvanometer i'_g is given by

$$i'_g = \frac{\frac{S}{n}}{G+S-\frac{S}{n}+\frac{S}{n}} \cdot i = \frac{1}{n} \cdot \frac{S}{G+S} \cdot i = \frac{i_g}{n}$$

Thus the current through the galvanometer is reduced to $\frac{1}{n}$th of its value by changing the point of contact on the shunt whose resistance is independent of that of the galvanometer.

59(a). Example: A universal shunt has several steps AB, BC, CD, DE of resistances 10, 90, 900 and 9000 ohms respectively. A handle H turning about O may be joined to any of these steps. The ends A and E are joined to the terminals of the galvanometer, while O and A are joined to the leads for the main current. Find to what fractions, the galvanometer current may be reduced when this universal shunt is used.

When the handle touches E, all the four steps are used as shunt so that total resistance of the shunt, $S = 10000$ ohms.

Then if main current be i, galvanometer current

$$i_g = \frac{S}{S+G} \cdot i = \frac{1000}{G+10000}\, i \qquad (1)$$

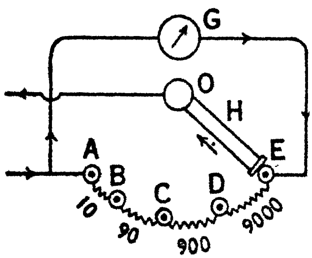


Fig. 54

If the handle be turned to touch D, steps AB, BC and CD are used as shunt, the step DE being joined in series with galvanometer.

Then total resistance in galvanometer arm $= G + 9000 = G'$ and total resistance used as shunt $= 1000 = S'$

$\therefore$ Galvanometer current i_g becomes $= \dfrac{S'}{G'+S'}\, i$

$$= \frac{1000}{G+9000+1000}\, i = \frac{1000}{G+10000}\, i$$

$= \frac{1}{10}$ of the previous current in (1).

When the handle touches C, galvanometer arm resistance $= G + 9900 = G''$,

total resistance used as shunt $= 100 = S''$

$\therefore$ Galvanometer current i_g becomes $= \dfrac{S''}{G''+S''} \cdot i$

$$= \frac{100}{G+9900+100} \cdot i = \frac{100}{G+10000}\, i = \tfrac{1}{100} \text{ of current in (1)}$$

Note: By increasing number of steps, or suitably choosing other resistances for different steps other fractions of the main current may be passed through a galvanometer.

QUESTIONS

1. What is the E. M. F. of a battery ? Distinguish it from potential difference. [C. U. 1945]

2. State Ohm's Law and discuss its dependence upon the temperature of the conductor. [C. U. 1939, '41]

3. Describe the most efficient arrangement of a large number of similar cells that will give the maximum current through an external resistance. [C. U. 1942]

4. Define specific resistance and conductivity.

Temperature co-efficient of resistance of a metal wire is generally positive while that of an electrolyte is negative. How do you account for this difference in behaviour ? [C. U. 1943]

What is supra-conductivity ? [C. U. 1941]

5. What is a shunt ? Explain its use in an actual electrical experiment. Explain the use of a universal shunt. [C. U. 1938, '49]

EXAMPLES

1. A certain modern influence machine produces a difference of potential between the electrodes of 135000 volts, the experimental conditions are such as to allow a current of 600 microamperes to pass ; calculate the resistance of the machine, if the external resistance requires a voltage of 3000 for the current to pass. [C. U. 1910]

Let r be the resistance of the machine.

The potential difference required by the current to establish itself through the machine $=(135000-3000)$ volts $=132000$ volts. But according to Ohm's Law we have, Potential difference = current × resistance

$\therefore\ 132000=600\times10^{-6}\times r$ or $r=22\times10^{7}$ ohms.

2. With a given cell of small internal resistance and a tangent galvanometer of unknown resistance, I get a deflection of 45°, the connection being made by stout wires. If a wire of 3 ohms resistance be substituted for one of the stout wires, the deflection is 30°. Find the resistance of the galvanometer. [C. U. 1914]

Let E and r be the E. M. F. and internal resistance of the cell respectively and let R be the external resistance of the circuit and G, the galvanometer resistance.

By Ohm's Law we have, the current $i=\dfrac{E}{R+r+G}=\dfrac{E}{R+G}$, $\because r=0$

But from the principle of a tangent galvanometer the current $i=10k\tan\theta$, where the current is expressed in amperes.

Therefore, we have $10\,\tan 45^\circ=\dfrac{E}{G}$, in the first case

$10k\tan 30^\circ=\dfrac{E}{G+3}$, in the second case. $\therefore\ \dfrac{\tan 45^\circ}{\tan 30^\circ}=\dfrac{G+3}{G}$

or $\dfrac{G+3}{G}=\dfrac{}{\sqrt{3}}$ or $G=\dfrac{3}{\sqrt{3}-1}=\frac{3}{2}(\sqrt{3}+1)=4{\cdot}09$ ohms.

3. The potential difference between the poles of a battery (of 1·2 ohms resistance) is 6·0 volts when the poles are insulated, and 4·5 volts when they are joined by a wire. What is the resistance of the wire? [C. U. 1915]

Let R be the resistance of the wire and let i be the current passing through the wire.

Since the potential difference between the terminals of the cell, *i.e.*, between the ends of the wire is 4·5 volts, then, by Ohm's Law, we have

$$4{\cdot}5 = i \times \mathrm{R} \qquad (1)$$

Again, for the whole circuit $6 = i(\mathrm{R}+1{\cdot}2)$ ... (2)

Dividing (2) by (1) we have $\frac{6}{4{\cdot}5} = \frac{\mathrm{R}+1{\cdot}2}{\mathrm{R}}$, whence R = 3·6 ohms.

4. The E. M. F. of a battery as found by an electrostatic voltmeter connected to its two poles is 8 volts. The reading changes to 6 volts when the circuit of the cell is closed through a resistance of two ohms. Calculate the internal resistance of the battery. [C. U. 1923]

(*Ans.* $r = \frac{2}{3}$ ohms)

5. A cell of E. M. F. 1·5 volts and resistance 2 ohms maintains a current in an external resistance of 6·5 ohms. Find the potential difference between the terminals of the cell. [C. U. 1925]

(*Ans.* 1·147 volts)

6. You have a circular loop of resistance 20 ohms and at two points at a quarter of the circumference apart, a battery of internal resistance 0·5 ohm and of E. M. F. 4 volts is connected by two wires of resistance 1 ohm each. Find the current at different parts of the circuit. [C. U. 1940]

Let A and B be the two points on the circular loop at a quarter of the circumference apart and so the resistance of the portion AB is 5 ohms and that of the remainder is 15 ohms.

Let i_1 be the current through AB and i_2 through the remainder and let i, the total current, then $i = i_1 + i_2$.

We have, $\frac{1}{\mathrm{R}} = \frac{1}{5} + \frac{1}{15} = \frac{4}{15}$

i.e., $\mathrm{R} = \frac{15}{4}$ ohms, where R is the equivalent resistance.

We have again, $i = \frac{4}{1+1+{\cdot}5+\frac{15}{4}} = \frac{4}{\frac{5}{2}+\frac{15}{4}} = \frac{16}{25}$ amp.

Again $\frac{i_1}{i_2} = \frac{15}{5} = \frac{3}{1}$ or $\frac{i_1+i_2}{i_2} = 4$ or $i_2 = \frac{1}{4}.i = \frac{1}{4}.\frac{16}{25} = \frac{4}{25} = {\cdot}16$ amp.

Again $\frac{i_2}{i_1} = \frac{1}{3}$, $\frac{i_2+i_1}{i_1} = \frac{4}{3}$, $i_1 = \frac{3}{4}.i = \frac{3}{4}.\frac{16}{25} = \frac{12}{25} = {\cdot}48$ amp.

7. Shew that the least number of Leclanche cells (E. M. F. = 1·55 Volts, internal resistance = 0·7 ohms), by which an incandescent lamp requiring a current of 2 amperes and a potential difference of 10 volts can be worked is 24. [C. U. 1912]

Resistance of the lamp $= \frac{10}{2} = 5$ ohms. Let m be the number of rows in parallel, each row containing n cells in series.

$\therefore\ 2 = \frac{n \times 1{\cdot}55}{5+\frac{7 \times n}{m}}$ (No. of cells $\mathrm{N} = mn$) $\therefore\ 10 + \frac{1{\cdot}4n}{m} = 1{\cdot}55n$

If	$m=1$	$n=67$	$mn=67$
	$m=2$	$n=12$ (nearly)	$mn=24$
	$m=3$		$mn=27$

∴ The minimum number is 24 (2 rows of 12 cells each)

8. The resistance of a given D' Arsonval type of galvanometer is 1560 ohms. The maximum current which may be sent through the galvanometer is 5 microamperes. What shunt will have to be used, if the current in the main circuit is 40 milliamperes? [C. U. 1911]

5 microamperes $=5\times10^{-6}$ amp., and 40 milliamperes $=40\times10^{-3}=4\times10^{-2}$ amp.

Now $i_g=i\cdot\frac{S}{G+S}$ where i, i_g = main current and current through the galvanometer, and S = resistance of the shunt.

$$5\times10^{-6}=4\times10^{-2}\frac{S}{1560+S} \qquad S=\frac{1560}{7999}\text{ ohm.} =\cdot195 \text{ ohm nearly.}$$

9. A cell of E. M. F. 2 volts is joined through a resistance 10,000 ohms to a galvanometer of resistance 200 ohms shunted by a resistance of 10 ohms. Calculate the current through the galvanometer. Find also the sensitiveness of the galvanometer, if a deflection of 30 mm is obtained in the above case, on a scale placed at a distance of 150 mm. from the mirror. [C. U. 1922]

Let R = equivalent resistance of the galvanometer and the shunt, then

$$\frac{1}{R}=\frac{1}{200}+\frac{1}{10}=\frac{21}{200} \qquad R=\frac{200}{21}\text{ ohms.}$$

By Ohm's Law the total current $i=\frac{2}{10000+\frac{200}{21}}$ amp. $=\frac{2\times21}{210200}$ amp.

∴ Current through the galvanometer $=i.\frac{S}{G+S}=\frac{21\times2}{210200}\times\frac{10}{200+10}$ amp.

$=\frac{1}{1051000}=9{\cdot}5\times10^{-6}$ amp. nearly $=9{\cdot}5$ microamperes nearly.

Again, sensitiveness of a galvanometer is measured by the current which would deflect the light spot through 1 mm. on a scale placed 1 meter from the mirror of the galvanometer

A deflection of 30 mm. at a distance of 150 cm, is equivalent to a deflection of $20/150\times100$ mm., *i.e.*, 20 mm. at a distance of 100 cm. from the mirror; and this is caused by a current of 9·5 microamperes.

∴ Sensitiveness of the galvanometer $=\frac{9{\cdot}5}{20}$ microamperes $=\cdot475$ microamperes.

$=475$milliamperes

[Note : The above definition of sensitiveness has been used in Hadley's Electricity and Magnetism for students. The correct definition of sensitiveness is expressed as the deflection produced by one microampere and denoted by $\frac{\theta}{i}$, where θ is the deflection produced by a current of strength i.

Therefore sensitiveness $=\frac{\theta}{i}=\frac{20}{9{\cdot}5}=2{\cdot}1$ **mm.]**

10. The terminals of a galvanometer of resistance 300 ohms shunted with a resistance of 100 ohms are connected through a resistance of 2000 ohms to the poles of a cell of E. M. F. 2 volts. Calculate the current flowing through the galvanometer. [C. U. 1917]

(*Ans.* ·106 milli-amp.)

11. It is proposed to utilise the 220 volts supply line to work an electric bell which is ordinarily run from a 6 volt battery. Describe carefully giving a diagram how would you proceed to connect up the bell with the line. You are provided with a glow lamp (428 ohms), bell push, constantan wire (resistance 12 ohms per metre) to make additional resistance coils if necessary and insulated copper wire for making connections. [C. U. 1929]

12. A current of 1·5 ampere is passed through a wire of which the length is one metre and the diameter is 2 mm. If the specific resistance of the meterial of the wire is $2·42\times10^{-6}$ ohm, what will be the difference of potential between the ends of the wire? [C. U. 1949]

$$=\rho\frac{1}{s}\therefore\frac{2·42\times10^{-6}\times100}{\pi(·1)^2}=·77\times10^{-2}\text{ ohm.}$$

$V=i\times R=1·5\times·77\times10^{-2}=1·155\times10^{-2}$ volts.

13. Find the specific resistance of a column mercury if the resistance of a quantity of mercury 1 mm. diameter and 1 metre long is 1·2012 ohms.

The resistance of a uniform conductor is directly proportional to its length and inversely to its sectional area. If R be the resistance of a conductor of length l and sectional area s

$$\text{then } R\propto\frac{l}{s} \qquad R=\rho\frac{l}{s}$$

Then the constant ρ is called the specific resistance of the material of the conductor.

$$\text{Here,}\quad \rho=\frac{Rs}{l}=\frac{1·2012\times\frac{22}{7}\times(0·05)^2}{100}=\frac{1·2012\times22\times·0025}{100\times7}=94·34\times10^{-6}\text{ ohms.}$$

14. Find the resistance of a cubic centimetre of copper (*a*) when drawn out into a wire of diameter 0·32 mm., and (*b*) when hammered into a flat sheet of thickness 1·2 mm., the current flowing perpendicularly through the sheet from one face to the other (sp. resistance of copper $1·59\times10^{-6}$). [C. U. 1944]

(*a*) We know that $R=\rho\frac{l}{s}=\frac{\rho}{s^2}$ since $l.s=1$ or $l=\frac{1}{s}$.

$$\therefore\quad R=\frac{1·59\times10^{-6}}{(256.\pi)^2\times10^{-12}},\text{ since } s=\pi r^2\text{ cm}^2=\pi(·016)^2\text{ cm}^2=256\pi\times10^{-6}$$

$$=2·458\text{ ohms}$$

(*b*) Again $R=\rho.\frac{l}{s}=l^2\rho\left(\text{since } s=\frac{1}{l}\right)=(·12)^2\times1·59\times10^{-6}$ ohms

$=·0229\times10^{-6}$ ohms. $\because$ $l=·12$ cm.

15. If the specific resistance of platinum at o°c is $8·95\times10^{-6}$ ohms and its temperature coefficient 32×10^{-4}, find the length of wire of diameter 0·0274 cm. has a resistance of 7 ohms at 50°c. [C. U. 1936]

We have $R_t = R_0(1+\alpha t)$, and $R_0 = \rho . \frac{l}{s}$

$$\therefore \quad R_0 = 8{\cdot}96 \times 10^{-6} \frac{l}{\pi \left(\frac{{\cdot}0274}{2}\right)^2} = \frac{8{\cdot}96 \times 10^{-6} . l \times 7}{22 \times ({\cdot}0137)}$$

Thus $R_t = \dfrac{8{\cdot}96 \times 10^{-6} \times 7 \times l}{22 \times ({\cdot}0137)^2} (1 + 32 \times 10^{-4} \times 50)$

$$\therefore \quad l = \frac{4 \times 22 \times ({\cdot}0137)^2}{8{\cdot}96 \times 10^{-6} \times 7 \times (1 + 32 \times 10^{-4} \times 50)} = 227 \text{ cms.}$$

CHAPTER VI

DISTRIBUTION OF CURRENT IN A NETWORK OF CONDUCTORS

60. Introductory : In simple cases of electric circuits as discussed in last chapter, problems can be easily solved by Ohm's law. Kirchhoff formulated two laws by means of which the distribution of a steady current in a complicated circuit consisting of a network of conductors can be easily determined.

61. Kirchhoff's Laws : First Law : *In an electric circuit, the algebraic sum of currents meeting at a point is zero.*

The word algebraic is significant ; it implies that in adding up the currents, care must be taken as regards signs of the currents. The convention by which the signs are determined is that currents coming towards a point have signs opposite to the currents moving away from the point. If the former be taken with positive signs, the latter are to be taken with negative or *vice versa*. In our discussion that follows currents coming to a point will be taken as positive, and currents going away from the point as negative.

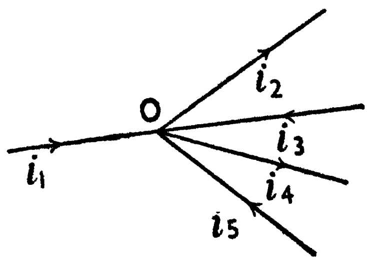


Fig. 55

The first law expresses that electricity does not accumulate at any point in a network *i.e.*, as much electricity or current flows towards a point, equal amount flows away from it if the currents in the conductors are in a steady state.

In Fig. 55, currents i_1, i_2, i_3, i_4 and i_5 meet at point O. By Kirchhoff's first law and applying proper sign

$$i_1+i_3+i_5-i_2-i_4=0.$$

Second Law : *In any closed circuit, the algebraic sum of the products of current and resistance of each part of the circuit is equal to the total electromotive force in that circuit.*

The law means that the total fall of potential round any simple circuit is equal to the total E. M. F. of that circuit.

In applying the second law, it is important to note that once the directions of the current in different parts of mesh are fixed, positive sign is to be given to a fall of potential ir (product of current and resistance) along the direction of current, and negative sign to ir against the direction of the current.

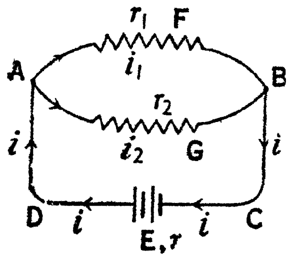


Fig. 56

Consider the circuit as shown in Figure 56 in which i, i_1 and i_2 are currents flowing along the conductors BDA, AFB and AGB of resistances r, r_1 and r_2 respectively, the E. M. F. of the battery being E, r being external resistance inclusive of internal resistance of battery.

At point A, $i=i_1+i_2$ (By the First law) ... ... (1)
In the circuit or mesh AFBCEDA, by IInd. law $i_1r_1+ir=E$ ... (2)
In the circuit or mesh AGBCEDA by IInd. law $i_2r_2+ir=E$ ... (3)

In the circuit or mesh AFBGA $i_1r_1-i_2r_2=0$, since no e. m. f. is included in this circuit or mesh.

or $i_1r_1=i_2r_2$ ... ... (4)

This result also follows from relations (2) and (3), above.

To find i, i_1 and i_2

Applying Kirchhoff's second law as above $i_1r_1=i_2.r_2$

or $i_2=\frac{i_1r_1}{r_2}$. From first law $i=i_1+i_2$; $\therefore\ i=i_1+\frac{i_1r_1}{r_2}$

$=i_1\left(1+\frac{r_1}{r_2}\right)$. Hence $i_1=\frac{r_2}{r_1+r_2}i$; similarly,

we have $\frac{i_2}{i_1}=\frac{r_1}{r_2}$, or $\frac{i_2}{i_1+i_2}=\frac{r_1}{r_1+r_2}$, or $\frac{i_2}{i}=\frac{r_1}{r_1+r_2}$.

The same values for i_1 and i_2 can also be deduced by applying the Ohm's Law to the above network of conductors.

62. Applications of Kirchhoff's Laws: In applying Kirchhoff's laws to a complicated circuit, the first thing to do is to indicate currents by some specific values in different parts of a mesh or network, with proper directions, according to first law. It may be remembered that the same current that leaves a battery from the +ve pole also enters the battery through −ve pole. In case of two or more batteries in a circuit, the above rules are to be followed. In case, the direction of current is reversed due to opposition of two or more batteries, it will be indicated by the result from calculation. The number of unknown symbols for currents should be made as minimum as possible.

62(a). Example: The terminals of two cells E_1 and E_2 having E. M. Fs. e_1 and e_2 and internal resistances r_1 and r_2 respectively, are connected to the points A and B as shown in Figure 57. Let C_1 and C_2 be the currents through r_1 and r_2 of the corresponding cells E_1 and E_2 and through the portions of the conductors joining the cells to the points A and B. The points A and B are also connected externally by a resistance R through which the resultant or main current C is passing.

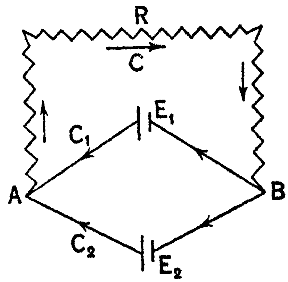


Fig. 57

Applying Kirchhoff's second law to the closed circuits each containing a cell and formed by the external resistance R we have,

$$\left.\begin{aligned} e_1 &= C_1 r_1 + CR \quad (1) \\ e_2 &= C_2 r_2 + CR \quad (2) \end{aligned}\right\} \text{By Law II}$$

At the point A, $C = C_1 + C_2$ (3) By Law I

Then multiplying (1) by r_2 and (2) by r_1 and adding we have

$$e_1 r_2 + e_2 r_1 = (C_1 + C_2) r_1 r_2 + CR(r_1 + r_2)$$

$$\text{or } e_1 r_2 + e_2 r_1 = C r_1 r_2 + CR(r_1 + r_2) = C\{r_1 r_2 + R(r_1 + r_2)\}$$

$$\therefore \quad C = \frac{e_1 r_2 + e_2 r_1}{r_1 r_2 + R(r_1 + r_2)}$$

Again from (1), (2) and (3) $e_1 = C_1 r_1 + CR$

$$e_2 = (C - C_1) r_2 + CR$$

$$\text{or } e_1 - e_2 = C_1(r_1 + r_2) - C r_2.$$

$$\therefore \quad C_1 = \frac{e_1 - e_2}{r_1 + r_2} + \frac{C r_2}{r_1 + r_2}. \text{ Similarly } C_2 = \frac{e_2 - e_1}{r_1 + r_2} + \frac{C r_1}{r_1 + r_2}$$

For three cells of E. M. F. e_1, e_2 and e_3 and internal resistances r_1, r_2 and r_3 respectively the resultant current can be similarly determined.

$$C = \frac{e_1 r_2 r_3 + e_2 r_1 r_3 + e_3 r_1 r_2}{r_1 r_2 r_3 + R(r_1 r_2 + r_2 r_3 + r_3 r_1)}.$$

62(b). To determine the equivalent E. M. F. of the combination : Let E be the potential difference between the points A and B and let C be the resultant current flowing in the direction of the arrow (Fig. 57).

We have, since $e_1 > e_2$

$$e_1 = E + Cr_1$$
$$e_2 = E - Cr_2$$

Solving these two equations, we have $E = \dfrac{e_1 r_2 + e_2 r_1}{r_1 + r_2}$

In the case of three cells having E. M. F.s e_1, e_2 and e_3 and internal resistances r_1, r_2 and r_3 respectively.

$$E = \frac{e_1 r_2 r_3 + e_2 r_1 r_3 + e_3 r_1 r_2}{r_1 r_2 + r_2 r_3 + r_3 r_1}.$$

If $e_1 = e_2 = e_3 = e$; E = (same as that of a single cell)

If $r_1 = r_2 = r_3 = r$, $E = \dfrac{e_1 + e_2 + e_3}{3}$

(Arithmetic mean of separate E.M.Fs.)

62(c). To determine the equivalent resistance of the Combination : We know that when the cell is closed the difference of potential between the terminals is less than the E.M.F. of the cell by an amount equal to Cr where C is the current flowing through the internal resistance r of the cell.

If E be the electromotive force and $(V_1 - V_2)$ or F, the difference of potential between the terminals of the cell when they are connected by a resistance R, we have

$$F = E - Cr = CR.$$

Again for each closed circuit with a single cell we have

$$F = e_1 - C_1 r_1 = e_2 - C_2 r_2 = e_3 - C_3 r_3 = CR$$
$$C = C_1 + C_2 + C_3.$$

Considering all these equations and eliminating C_1, C_2, C_3, we have

$$C = \frac{e_1 r_2 r_3 + e_2 r_1 r_3 + e_3 r_1 r_2}{r_1 r_2 r_3 + R(r_1 r_2 + r_2 r_3 + r_3 r_1)}$$

$$F = \frac{e_1 r_2 r_3 R + e_2 r_1 r_3 R + e_3 r_1 r_2 R}{r_1 r_2 r_3 + R(r_1 r_2 + r_2 r_3 + r_3 r_1)}$$

We have $Cr = E - F$ or $r = \frac{E-F}{C} = \frac{E-CR}{C} = \frac{E}{C} - R$, or

$$r = \frac{e_1 r_2 r_3 + e_2 r_1 r_3 + e_3 r_1 r_2}{r_1 r_2 + r_2 r_3 + r_3 r_1} \times \frac{r_1 r_2 r_3 + R(r_1 r_2 + r_2 r_3 + r_3 r_1)}{(e_1 r_2 r_3 + e_2 r_3 r_1 + e_3 r_1 r_2)} - R$$

$$= \frac{r_1 r_2 r_3}{r_1 r_2 + r_2 r_3 + r_3 r_1}.$$

Thus we see that the equivalent resistance of a number of conductors or cells arranged in parallel is always obtained from the rule that the reciprocal of the equivalent resistance is equal to the sum of the reciprocals of the component resistances.

The equivalent E. M. F., E for the three cells can be obtained from the knowledge of r and the resultant current C in the expression $E = C(R + r)$.

62(d). Wheatstone's Bridge or Network: The Wheatstone's Bridge or Network consists of four resistances P, Q, R, S, arranged as in Fig. 58. The points A and C are connected through a battery and the points B and D through a galvanometer of resistance G.

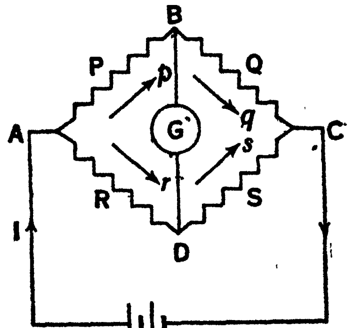


Fig. 58

Let I be the current in the battery circuit and let p, q, r, s and g be the currents in P, Q, R, S and G respectively.

Applying Kirchhoff's 1st law,

at A, $I = p + r \quad \therefore \quad r = I - p$

at B, $p = g + q \quad \therefore \quad q = p - g$

at C, $I = q + s = p - g + s$

$\therefore \quad s = I - p + g = p + r - p + g$
$= r + g.$

Applying Kirchhoff's 2nd law to the circuit ABDA,

$$pP + gG - rR = 0 \quad \therefore \quad pP + gG - (I - p)R = 0$$

$$p(P + R) + gG - IR = 0 \quad \ldots \quad \ldots \quad (1)$$

For circuit BCDB, $qQ - sS - gG = 0$

$$\therefore \quad (p - g)Q - (I - p + g)S - gG = 0$$

$$\text{or} \quad p(Q + S) - gG - g(Q + S) - IS = 0 \quad \ldots \quad (2)$$

To eliminate p multiplying (1) by $(Q + S)$,

$$p(P + R)(Q + S) + gG(Q + S) - IR(Q + S) = 0.$$

Again multiplying (2) by $(P + R)$,

$$p(P + R)(Q + S) - gG(P + R) - g(Q + S)(P + R) - IS(P + R) = 0$$

Subtracting,

$$gG(P + Q + R + S) + g(Q + S)(P + R) - I(QR - PS) = 0.$$

Hence $g = \dfrac{QR - PS}{G(P+Q+R+S)+(Q+S)(P+R)} \cdot I$

For a balanced bridge, $g=0 \quad \therefore \quad QR = PS$

or $\dfrac{P}{Q} = \dfrac{R}{S}$.

Alternative Proof :

Applying Kirchhoff's 2nd law to the circuit ABDA, $pP+gG-rR=0$ (1)
to circuit BCDB, $qQ-gG-sS=0$ or $(p-g)Q-gG-(r+g)S=0$ ... (2)
For $g=0$, from (1) $pP-rR=0$ *i.e.*, $pP=rR$ (3)
from (2) $pQ-rS=0$ *i.e.*, $pQ=rS$ (4)

or from (3) $\dfrac{p}{r}=\dfrac{R}{P}$, from (4) $\dfrac{p}{r}=\dfrac{S}{Q}$ $\quad \dfrac{R}{P}=\dfrac{S}{Q}$ or $\dfrac{P}{Q}=\dfrac{R}{S}$

62(*e*). Example : Let three cells A, B and C of E. M. F.'s, e_1, e_2 and e_3 and of internal resistances r_1, r_2 and r_3 respectively be joined in parallel so that a current i flows through external resistance R. Let the currents through the different cells be i_1, i_2 and i_3.

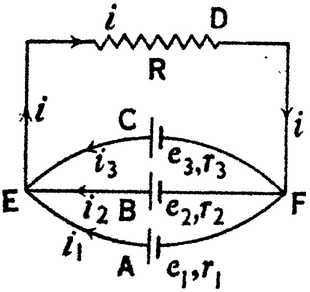


Fig. 59

By Kirchhoff's first law

$$i = i_1 + i_2 + i_3 \quad .. \quad (1)$$

Applying second law to the mesh EAFDE,

$$i_1 r_1 + iR = e_1 \quad ... \quad (2)$$

" " EBFDE,

$$i_2 r_2 + iR = e_2 \quad ... \quad (3)$$

" " ECFDE,

$$i_3 r_3 + iR = e_3 \quad ... \quad (4)$$

Dividing (2), (3) and (4) relations by r_1, r_2 and r_3 respectively then adding,

$$(i_1+i_2+i_3)+i\left(\frac{R}{r_1}+\frac{R}{r_2}+\frac{R}{r_3}\right)=\frac{e_1}{r_1}+\frac{e_2}{r_2}+\frac{e_3}{r_3} \quad ... \quad (5)$$

This relation is independent of the current in the battery circuit.

But from (1) $i = i_1 + i_2 + i_3$

$\therefore$ Relation (5) becomes, $i\left(1+\dfrac{R}{r_1}+\dfrac{R}{r_2}+\dfrac{R}{r_3}\right)=\dfrac{e_1}{r_1}+\dfrac{e_2}{r_2}+\dfrac{e_3}{r_3}$

$$\text{or } i=\left(\frac{e_1}{r_1}+\frac{e_2}{r_2}+\frac{e_3}{r_3}\right)\Big/\left(1+\frac{R}{r_1}+\frac{R}{r_2}+\frac{R}{r_3}\right)$$

whence the value of i can be found out. Then knowing i, the values of i_1, i_2 and i_3 can also be found from (2), (3) and (4).

Note : In the above problem the currents i, i_1, i_2 and i_3 are four unknown quantities, since other quantities are supposed to be known. Now, four equations are required to find four currents. One equation is obtained by Kirchhoff's first law. Kirchhoff's second law applied to three meshes will give three equations. There are altogether six circuits or meshes in the problem. But only those meshes or circuits have been taken which are independent of each other. If we choose other circuits or meshes, the same result would have been found, but calculation would then have been not easy.

62(f). Example : Comparison of E.M.F.'s of two cells :

Let e_1 and e_2 be the E. M. F.s of the two cells, r_1 and r_2 their internal resistances respectively, and g the resistance of the galvanometer, as shown in the given circuit (Fig. 60)

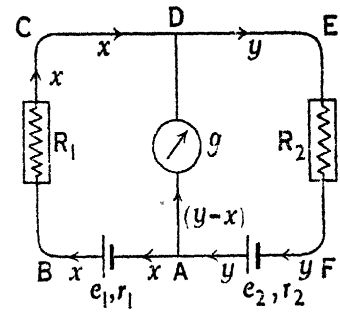


Fig. 60

In circuit ABCD and ADEF, by 2nd law,

$$xr_1 + R_1x - (y-x)g = e_1 \quad (1)$$

$$yr_2 + R_2.y + (y-x)g = e_2 \quad (2)$$

If R_1 and R_2 are so adjusted that there is no deflection of galvanometer, then $y - x = 0$ or $y = x$

$\therefore$ From (1) and (2) $x.r_1 + R_1.x = e_1$

$$x.r_2 + R_2\,x = e_2$$

$$\therefore \frac{e_1}{e_2} = \frac{R_1 + r_1}{R_2 + r_2} \quad (3)$$

To eliminate r_1 and r_2 take another observation with different values of R_1 and R_2, for no deflection of galvanometer, so that

$$\frac{e_1}{e_2} = \frac{R_1' + r_1}{R_2' + r_2}$$

From (3) and (4) $\dfrac{e_1}{e_2} = \dfrac{R_1 + r_1}{R_2 + r_2} = \dfrac{R_1' + r_1}{R_2' + r_2}$

$$= \frac{(R_1 + r_1) - (R_1' + r_1)}{(R_2 + r_2) - (R_2' + r_2)} = \frac{R_1 - R_1'}{R_2 - R_2'}$$

Thus e. m. fs. of two cells may be compared. The method is due to Lumsden.

QUESTIONS

1. State and explain Kirchhoff's Laws for the distribution of currents in a network of conductors. [C. U. 1935, '40, '44, '47, '51, '59]

Or, State the laws governing the distribution of current in a network of conductors.

2. Discuss the theory of Wheatstone's network of conductors for measuring a resistance in the laboratory. [C. U. 1946]

EXAMPLES

1. A battery consists of three cells arranged in mixed circuit so that there are two rows containing 1 and 2 cells respectively and the terminals are connected by a wire of 10 ohms resistance. If the E.M.F. of each cell be 2 volts and the internal resistance of each cell is half an ohm, find the current strength. [C. U. 1912]

Let the two cells contained in one of the rows be formed into a single cell of E. M. F. e_2 *i.e.*, 4 volts and internal resistance r_2, *i.e.*, 1 ohm and let the E. M. F. of the other cell be denoted by e_1 or 2 volts and the resistance by r_1 *i.e.*, ·5 ohms.

Let the two cells be arranged in parallel circuit and let the terminals to the cell be connected by a wire of resistance 10 ohms.

Let C_1 and C_2 be the currents through the batteries of E. M. F. e_1 and e_2 respectively.

Then, as we have according to the two laws of Kirchhoff,

$$e_1 = C_1 r_1 + CR \qquad \ldots \qquad (1)$$
$$e_2 = C_2 r_2 + CR \qquad \ldots \qquad (2)$$
$$C = C_1 + C_2 \qquad \ldots \qquad (3)$$

Then from (1), (2) and (3) we have by substituting the values of e_1, e_2, r_1, r_2 and R,

$$2 = \cdot 5C_1 + 10C$$
$$4 = C_2 + 10C$$
or $$C_1 + 20C = 4$$
$$C_2 + 10C = 4$$
or $$(C_1 + C_2) + 30C = 8$$
or $$C + 30C = 8$$
or $$C = \tfrac{8}{31} = \cdot 26 \text{ amp.}$$

Similarly, $C_1 = -1\cdot 16$ amp. and $C_2 = 1\cdot 42$ amp.

2. The positive and negative poles of a battery of 2 Grove cells are connected respectively to the positive and negative poles of a battery of 2 Daniell cells, by wires of resistance 10 and 6 ohms. The middle points of the wires are connected by another wire of resistance 4 ohms. Calculate the current flowing through each of the batteries. [C. U. 1917]

E. M. F. of a Grove cell	$= 1\cdot 9$ Volts.
E. M. F. of a Daniell cell	$= 1\cdot 1$ Volts.
Int. resistance of a Grove cell	= 1 ohm.
Int. resistance of a Daniell cell	= 2 ohms.

Let the two Grove cells be formed into a single cell of E. M. F. 3·8 volts and internal resistances 2 ohms and let the Daniell cell be also combined to form a single cell of E. M. F. 2·2 volts and internal resistance 4 ohms.

Let the positive poles of these newly formed cells be connected together by a conductor of resistance 10 ohms, and let the negative poles of these cells be connected by a wire of 6 ohms resistance.

The middle points of these wires are connected by a wire of resistance 4 ohms.

Then applying Kirchhoff's laws to the middle points of the wires and to the closed circuits each containing a cell and formed by the external resistance, we have

$$C = C_1 + C_2 \qquad \ldots \qquad (1)$$
$$3\cdot 8 = C_1(5+2+3) + 4C \qquad \ldots \qquad (2)$$
$$2\cdot 2 = C_2(5+4+3) + 4C \qquad \ldots \qquad (3)$$

or from ($3\cdot 8 = 10C_1 + 4(C_1 + C_2) = 14C_1 + 4C_2$

and from (, $2\cdot 2 = 12C_2 + 4(C_1 + C_2) = 4C_1 + 16C_2$

Solving these two equations, we have

$C_1 = \cdot 25$ amp. $C_2 = \cdot 075$ amp. C = amp.

3. Twelve wires each of resistance r are connected in the form of a cube. What is the effective resistance of the cube when current enters the cube at one end and goes out from the diagonally opposite end?

The main current i entering at A is by symmetry divided into three parts, each equal to $i/3$ along AA_1, AD and AB. Each of these parts is again equally divided along corresponding two wires, as shown in Fig. 61.

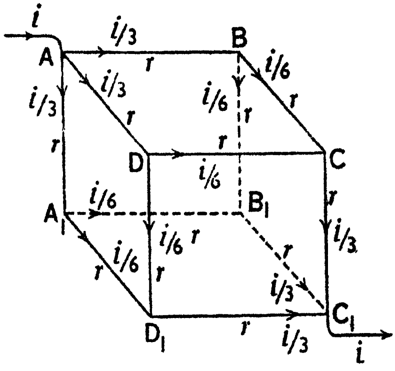


Fig. 61

Then, P. D. between A and C_1 is given by,

$V_A - V_{C_1} = (V_A - V_{A_1}) +$
$(V_{A_1} - V_{D_1}) + (V_{D_1} - V_{C_1})$

$+ \,_{6}\, r + \,_{3}\, r \quad 6$

If the cube be replaced by a single wire of resistance R between A and C_1, so that for same current i, P. D between A and C_1 remains the same,

then $V_A - V_{C_1} = iR \quad \therefore \quad iR = \frac{5}{6} ir \quad$ or $\quad R = \frac{5r}{6}$

4. Two batteries of 4 and 6 Daniell cells respectively are connected in parallel by wires of resistances 2 ohms each. The middle points of the connecting wires are joined by a conductor of resistance of 10 ohms.

Find the currents in the different parts of the circuit. E. M. F. of each cell =1 volt. [C. U. 1919]

Internal resistance of each cell = 1 ohm.

Proceed as before.

[*Ans.* C_1 = ·0638 amp. C_2 = ·298 amp. C = ·3618 amp.]

5. A Grove cell of E. M. F. 2 volts and resistance 1 ohm is joined in parallel to a Daniell cell of E. M. F. 1 volt and resistance 1 ohm by two wires of resistance 2 ohms each. The electrical middle points of the connection wires are joined by a wire of resistance 9 ohms. Calculate the current through the Grove cell. [C. U. 1922]

Proceed as before. [*Ans.* C = ·24 amp.]

6. A battery of E. M. F., 12 volts and of internal resistance 6 ohms is connected in parallel to another of E. M. F., 8 volts and of internal resistance 4 ohms. The terminals of the composite battery so formed are joined by a wire of resistance 12 ohms. Calculate the current passing through the different parts of the circuit. [C. U. 1932]

[*Ans.* C_1 = C = ·67 amp. C_2 = 0]

7. A battery of E. M. F. 6 volts and 0·5 ohm internal resistance is joined in parallel with another battery of 10 volts E. M. F. and internal resistance 1 ohm and the combination is used to send current through an external resistance of 12 ohms. Calculate the current through the resistance and through each battery. [C. U. 1935, '59]

[*Ans.* C_1 = −2·29 amp., C_2 = 2·86, C = ·59]

8. In a Wheatstone's network the four resistances are equal and the galvanometer is replaced by a battery of the same E. M. F. as the one already present. If the resistances in the battery circuit are each the same as those in the other four arms, find the current in the various branches. [C. U. 1941

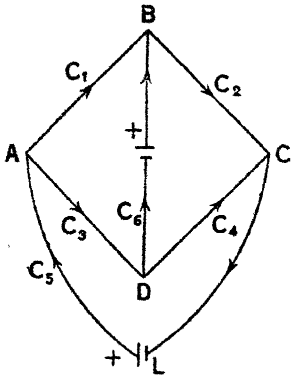


Fig. 62

Applying Kirchhoff's Laws to the network shown in Fig. 62.

At the point

$$\left.\begin{array}{ll} A,\ C_5 = C_1 + C_3 & (a) \\ B,\ C_2 = C_1 + C_6 & (b) \\ D,\ C_3 = C_4 + C_6 & (c) \end{array}\right\} \quad \text{Law I}$$

By law II for the meshes,

ACL, $C_5.r + C_3.r + C_4.r = E$
or $C_5 + C_3 + C_4 = E/r$ (1)

... ... ABD, $C_3\,r + C_6.r - C_1.r = E$
or $C_3 + C_6 - C_1 = E/r$ (2)

... ... BCD, $C_2\,r + C_6\,r - C_4\,r = E$
or $C_2 + C_6 - C_4 = E/r$ (3)

Here E is the E. M. F. of the cells and are r, the resistance of the battery circuit as well as those of the branches.

Substituting in (2) and (3) values of C_3 and C_2 from (c) and (b)
we have, $C_4 + 2C_6 - C_1 = C_1 + 2C_6 - C_4$
$\therefore\ C_1 = C_4$

Again we have, $C_2 = C_1 + C_6 = C_4 + C_6 = C_3$
i.e., $C_3 = C_2$

Again substituting in (1) and (2) values C_5 and C_3 from (a) and (c)

$$C_1 + 2C_3 + C_4 = 2C_1 + 2C_3 = \frac{E}{r} \quad \because\ C_4 = C_1 \text{ and } C_3 = C_2$$

and $C_4 + 2C_6 - C_1 = 2C_3 - 2C_1 = \frac{E}{r}$ $\quad \because\ C_6 = C_3 - C_1$

$$\therefore\ C_1 + C_3 = C_3 - C_1 = \frac{E}{2r} \qquad C_1 + C_3 = C_3 - C_1,\quad 2C_1 = 0$$

$$C_1 = 0 = C_4,\ C_2 = C_3 = C_5 = C_6 = \frac{E}{2r}$$

Alternative Method :

No current flows through the arms AB and CD since the two batteries have equal E. M. F. and all the branches have equal resistances and the arm AB is connected to the two positive poles and the arm CD to the negative poles of the two batteries.

There is current only in the circuit LADBCL in series. Then according to Kirchhoff's 2nd law we have,

$$Cr + Cr + Cr + Cr = 2E \text{ or } C.4r = 2E.$$

$$\text{Or}\quad C = \frac{2E}{4r} = \frac{E}{2r}$$

where C is the current flowing through the other arms except AB and CD, E the E. M. F. of each battery and r, resistance of each arm.

9. Two cells of E. M. Fs. 1·5 and 1·1 volts and of internal resistances 0·1 ohm and 0·2 ohm respectively are joined in opposition by two wires of resistance ·3 and ·4 ohms. Calculate the E. M. F. of the combination. [C. U. 1946]

Use the relation $E=\dfrac{e_1r_2+e_2r_1}{r_1+r_2}$ [*Ans.* 1·31 volts]

CHAPTER VII

ELECTRICAL MEASUREMENTS

63. Measurement of Resistance : The method of Wheatstone's net affords the most accurate and convenient method of comparing resistances and makes use of the principle of the divided circuit.

AECD is a network of conductors. The arms AE, EC, CD and DA have resistances equal to P, Q, X and R respectively. A galvanometer is connected between the points E and D through a Key K_1 and a battery is connected to A and C through a key K_2. The main current C is divided at A into two parts, one C_1 flowing along AEC and the other C_2 along ADC and these two currents reunite at C and flow back to the battery. This is possible only when the points E and D to which the galvanometer is connected are at the same potential. If the points E and D are not at the same potential, a current will flow through the galvanometer and cause a deflection. The experiment is performed and the measurement made by adjusting the resistances of the arms until the galvanometer shews no deflection, *i.e.*, the points E and D are at the same potential. Let V_1 be the potential of A, V, that of D and E and V_2 that of C.

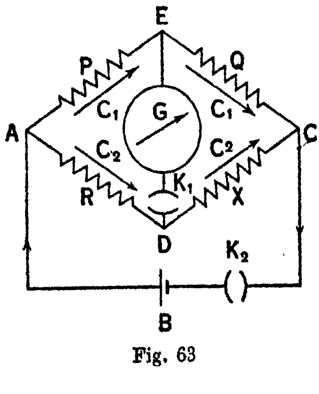


Fig. 63

Then by Ohm's Law,

$$C_1=\frac{V_1-V}{P}=\frac{V-V_2}{Q}$$

C_2, $\frac{V_1 - V}{}$ $\qquad\qquad$ $\frac{V_1 - V}{V - V_2} = \frac{P}{Q} = \frac{R}{X}$

$\therefore \quad X = \frac{Q}{P} \times R$. If Q, P and R are known, X can be found o

The superiority of this method of measuring resistance is due the fact that it is far more easy to detect a null point than to re a deflection.

Note: Whichever (galvanometer or battery) has the greater resistance m be placed between the junction of the two arms having greater resistances a the junction of the two arms having smaller resistances.

64. Practical arrangement of the experiment: Vario forms of Wheatstone's bridge have been constructed for measuri resistance, of which metre bridge is the commonest form.

64(a). Metre-Bridge Method: The metre-bridge consists a long (1 metre) bare wire (brass or german silver) of uniform c os section stretched on a wooden board by the side of a metre sc fixed to the board. The ends of the wire are soldered to two thi copper plates at AT and CF. On the board there is another copp plate MN with binding screws at M, D and N (Fig. 64).

The plates TL and JF have screws at T and L, and at J and respectively.

A known resistance R is connected to the screws at L and M in the gap LM while the unknown resistance X to be measure connects the gap NJ at screws N and J.

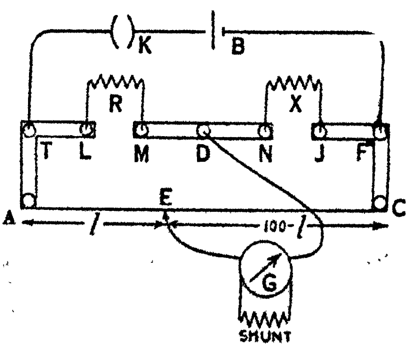


Fig. 64

The screw D and the movable **slider** E called the **jockey** are connected through a galvanometer* provided with a shunt. The two poles of a battery are connected through key K to the screws at T and F. The key is closed and then by moving the slider E a position is found on the wire, at which when the slider is pressed the galvanometer shows no deflection.

*Note: The galvanometer is to be shunted if it is of suspended coil type. No shunt is required if it be ordinary unipivot type with graduated scale and pointer.

Then lengths of the portions AE and EC are measured by the metre scale and let them be l and $100-l$ respectively.

The resistance of the length $l = l\rho$ and that of length $100-l = (100-l)\rho$, where ρ is the resistance per unit length of the wire.

Then from the principle of Wheatstone's Bridge, we have

$$\frac{X}{R} = \frac{(100-l)\rho}{l\rho} = \frac{100-l}{l} \quad \text{or} \quad X = \frac{(100-l)}{l} \times R. = \frac{l_2}{l_1} \times R$$

If R be known, the unknown resistance X is determined from the knowledge of the lengths l_2 (*i.e.*, $100-l$) and l_1 (*i.e.*, l), which are proportional to their resistances.

64(b). The **Specific Resistance** of the material of a wire can be determined from the expression $\rho = R\frac{s}{l}$ in which the resistance R is determined by the metre-bridge method and the length and the area of cross-section of the wire obtained by direct measurements with a metre scale and a screw-gauge respectively.

65. Sources of Errors :

(1) If the bridge wire is soldered imperfectly at its ends to the two copper plates, the joints and the copper plates introduce appreciable resistance to their respective arms and the formula

$$X = \frac{100-l}{l}.R = \frac{l_2}{l_1}.R$$

becomes equal to $X = \frac{l_2\rho + \alpha}{l_1\rho + \beta}.R$

where ρ is the resistance of 1 cm. of bridge wire and α, β are resistances of joints and copper plates at the ends of the wire called the **end corrections** and are expressed as equivalent lengths of the two corresponding parts of the wire.

For method of eliminating them a book on Practical Physics may be consulted.

(2) When a current is passed for a long time through the bridge wire and the junctions of different metals in the metre-bridge, besides the production of heat due to resistances of the arms, thermo-currents are generated which cause a shift in the balance point.

To avoid the effects of thermo-currents, the readings for the balance points are taken with direct and reversed currents and in all cases it is better to connect the cell in the tapping circuit, *i.e.*, between E and D and the galvanometer between T and F.

(3) If the conductors in the arms of the bridge are in the forms of coils, due to self-induction an extra current, at the time of make or break in the battery circuit will shift the balance point and so to avoid it, it is always necessary to close the battery circuit before the galvanometer circuit.

(4) The Wheatstone's bridge is unsuitable for the comparison of very low resistances for two reasons, (1) due to the want of sensitiveness of the bridge, (2) due to the fact that the connecting wires and the contacts at the terminals have resistances which are no longer negligible.

Note: The bridge becomes insensitive because the current flowing through the galvanometer before the balance is reached depends on the numerator $QR-PS$ in the expression (1) for the current g in Art. 62(*d*).

$$g=\frac{QR-PS}{G(P+Q+R+S)(Q+S)(P+R)}\cdot I \qquad (1)$$

where P, Q, R and S are the resistances of the arms of the bridge and I, the current from the battery.

If R and S are low, the current in the galvanometer becomes very small and the bridge becomes insensitive. Similarly if P and Q are large the current becomes small as the denominator of the expression (1) increases.

Large resistances can not be measured accurately with this bridge as the current in the galvanometer will decrease with the increase of the denominator.

66. Post Office Box. (P. O. Box): It is a compact form of Wheatstone's Bridge and consists of three sets of coils of manganin wire (Fig. 65) arranged in three rows in a box. The ends of the coils are soldered to brass blocks with holes between them. The blocks are short circuited by inserting brass plugs between them. The first row consists of two parts AE and EC each containing three coils of resistances 10, 100 and 1000 ohms. These parts represent P and Q of the ratio arms of the Wheatstone's Bridge. The other two rows contain resistances varying from 1 ohm to 5000 ohms. These two rows of resistances are as a whole known as the third arm R of the box.

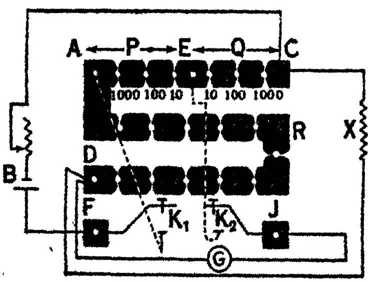


Fig. 65

There are binding screws at A, C and D. The screws at F and J are provided with two tapping keys K_1 and K_2. The points A

and E are joined as indicated by white lines to F and J through the keys K_1, K_2 respectively.

The battery B is connected to A and C through the key K_1 and the galvanometer to D and E through the key K_2. The unknown resistance X is connected to D and C.

The internal connections are shewn by dotted lines.

66(a). Determination of the unknown resistance by P. O. Box : After making connections as shown in Fig. 65 and fixing all the plugs tight, take away the plugs from the 10 ohms coils from the arms P and Q. Taking zero resistance in 3rd arm, close the battery key K_1 and then the galvanometer key K_2. A deflection is observed in the galvanometer in a certain direction. Repeat the experiment taking an infinite, *i.e.*, high resistance in 3rd arm. The deflection should be in opposite direction. Then adjust the resistance in the third arm R until the galvanometer shews no deflection (null point).

At this stage, according to the principle of Wheatstone's Bridge,

$$\frac{P}{Q}=\frac{R}{X}\ i.e.,\ X=\frac{Q}{P}\cdot R=\frac{10}{10}\cdot R=R.$$

If however, the null point is not obtained, the values of resistances in the arms P and Q are altered to (100 : 10) or (1000 : 10) so that there is no deflection in the galvanometer, the unknown resistance is determined to the first and second places of decimal respectively. (For more detailed procedures, consult any treatise on Practical Physics.)

67. Measurement of High Resistance : It is rather by a convention that we classify resistances into three groups—**high, low** and **ordinary**. No hard line of demarcation can be set up between any two of these groups. In general, resistances between 1 ohm and 1000 ohms are regarded as ordinary, those under 1 ohm, low, and those above 1000 ohms as high.

Wheatstone's bridge arrangement is highly sensitive when the resistances of the four arms are all of the same order. When two of them are abnormally large, the bridge arrangement is found to be insensitive. The usual method of P. O. box, therefore, is unsuitable for the measurement of high resistance. A method known as method of substitution which is very simple, is considered below. This method involves the use of a moving coil galvanometer.

67(a). High Resistance by Method of Substitution : A known resistance R_1 and an unknown resistance, say R_2 are in turn connected in series with a battery and a moving coil galvanometer

G. For this a two-way key K as shown in Figure 66, may be used with advantage. For reversal of current a commutator C, preferably of Pohl's type is usually used. The shunt resistance S, which is connected in parallel with galvanometer G is adjusted in each case, so that the deflections in two cases are nearly of the same order and within the limit of the scale.

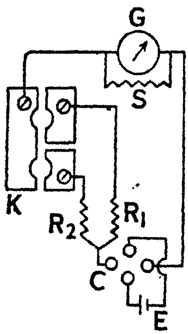


Fig. 66

Let S_1 be the shunt resistance when known resistance R_1 is used, E the *e. m. f.* of the battery, g the resistance of the galvanometer and θ_1, the angle of deflection, then current in the galvanometer Cg_1 is given by,

$Cg_1 = C . \dfrac{S_1}{S_1+g}$, where C = main current, But

$C = \dfrac{E}{R_1 + \dfrac{S_1 g}{S_1+g}}$, since $S_1g/(S_1+g)$ is the equivalent resistance of the shunted galvanometer.

$$Cg_1 = \frac{E(S_1+g)}{R_1(S_1+g)+S_1g} \cdot \frac{S_1}{S_1+g} = \frac{ES_1}{R_1(S_1+g)+S_1g} = K\theta_1 \ldots(1)$$

$$[\because \quad Cg_1 = K\theta_1]$$

If then, S_2 be the shunt resistance in case of unknown resistance R_2 and θ_2 be the corresponding deflection,

$$Cg_2 = \frac{ES_2}{R_2(S_2+g)+S_2g} = K\theta_2 \qquad (2)$$

From (1) and (2) we have.

$$\frac{ES_1}{R_1(S_1+g)+S_1g} = K\theta_1 \ldots (1a) \; ; \quad \frac{ES_2}{R_2(S_2+g)+S_2g} = K\theta_2 \; \ldots (2a)$$

Dividing (1*a*) by (2*a*) $\dfrac{R_2(S_2+g)+S_2g}{R_1(S_1+g)+S_1g} \cdot \dfrac{S_1}{S_2} = \dfrac{\theta_1}{\theta_2} \quad \ldots \quad \ldots \quad (3)$

Knowing all other quantities except R_2, the latter can be found out.

Note 1. If any of R_1, R_2—say R_2, be so large that no shunt is required to reduce the current in galvanometer to obtain desired

deflection, the value of S_2 becomes infinite. Then the expression (2a) is given by

$$K\theta_2 = \frac{E}{R_2\left(1+\frac{g}{S_2}\right)+g} \text{ becomes equal to } \frac{E}{R_2+g} = K\theta_2 \quad \dots (4)$$

Then from (1a) and (4) $\dfrac{S_1(R_2+g)}{R_1(S_1+g)+S_1g} = \dfrac{\theta_1}{\theta_2}$ ... ... (5)

Note 2. If the deflections θ_1, and θ_2 are not nearly equal, the constant K of galvanometer will not strictly remain same in the two cases.

68. Resistance of Galvanometer : Resistance of a galvanometer can be easily obtained by clamping the galvanometer coil and then employing either metre-bridge or P. O. Box to find its resistance. But for this, another galvanometer is necessary. The use of a second galvanometer is not required in the following method.

68(a). Half-deflection Method : In Figure 67, R and R′ are two resistance boxes and S, low resistance box. R′ and S are connected to a battery through a commutator C. The low resistance box S is again connected in parallel with a galvanometer through resistance Box R. R being kept equal to zero and S being given a small value r, the battery circuit is completed. The value of R′ is adjusted so that deflection θ is not very large.

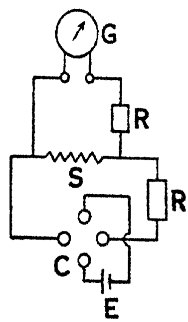


Fig. 67

Neglecting S in comparison to R′ current in battery circuit $= \frac{E}{R'}$, where E is *e. m. f.* of the battery. Then P. D. between terminals of low resistance $S = \frac{E}{R'} \times r$

$\therefore$ Current through galvanometer $= \frac{Er}{GR'} = K\theta_1$ (1), where G = galvanometer resistance.

Now keeping R′ same and hence P. D. across S constant, R is adjusted so that deflection becomes half the previous value.

Current through the galvanometer $\frac{Er}{R'(R+G)} \cdot K.\frac{\theta}{2}$ (2)

Dividing (1) by (2) $\frac{R+G}{G}=2$ or $R+G=2G$ $G=R$.

Therefore resistance used in Box R gives the resistance of the galvanometer.

68(b). Thomson's method for finding the Galvanometer Resistance : As it is Wheatstone's bridge method, a P. O. box may be used. The diagram of connections is shown in Figure 68. The galvanometer is placed in the fourth arm and its usual place is taken by a key. When battery key is closed, a deflection of the galvanometer will occur for all values of P, Q and R, and on closing the key K, there will in general be a redistribution of current in the network and, hence, galvanometer deflection will change. If the terminals of the key, *i. e.*, the junction of P and Q, and that of R and G be at the same potential, then there will be no change in galvanometer deflection, whether key K is closed or opened. Therefore keeping resistances P, Q as 10 − 10, R is adjusted, till on opening or closing the key K the galvanometer deflection remains unchanged. Then from the relation $P/Q=R/G$, which holds good here, the value of G can be found out.

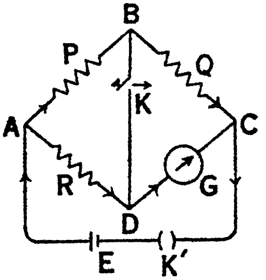


Fig. 68

69. Measurement of battery Resistance by Mance's Method : A P. O. box may be used for this. As shown in Figure 69, battery is placed in the fourth arm, key K and a galvanometer G are placed in the two diagonal arms of Wheatstone's bridge. Then for all values of P, Q, R there will be a steady galvanometer deflection. The values of resistances P, Q being 10 each, R is adjusted so that no change in galvanometer deflection occurs on closing or opening the key K. Under this conditions $P/Q=R/r$, whence r the battery resistance can be determined.

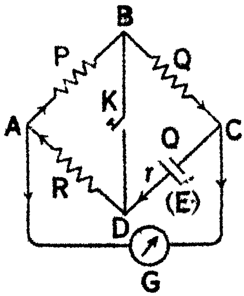


Fig. 69

70. Measurement of Low Resistance : In using a metre-bridge or P. O. Box to measure resistance, the resistances of copper strips and connecting wires are neglected in comparison with the resistance to be

measured which is usually fairly large. But this can not be done when resistance to be measured is low. Mathiessen and Hopkinson's 'Fall of Potential' method which is avery accurate one, is, therefore, used to measure low resistance.

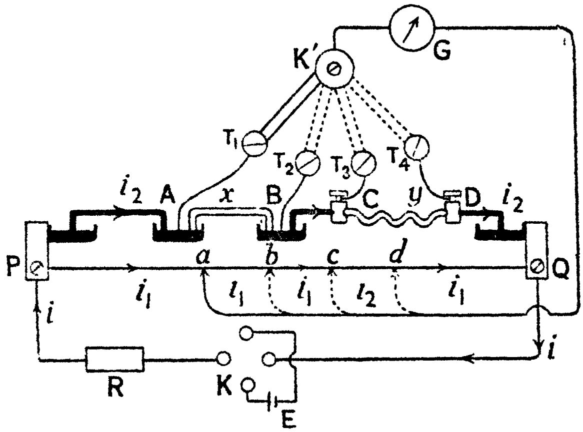


Fig. 70

The connections are shown in Figure 70. Two low resistances and y are placed in two gaps of a metre-bridge the terminals of the bridge wire are connected to a battery through a commutator K, a resistance being included in the battery circuit to minimise current strength. One terminal of the galvanometer is connected to the common terminal K' of a four way key. By rotating the connector rod in turn this terminal of galvanometer may in turn be connected to the terminals A, B, C and D of two low resistances. The other terminal of galvanometer is connected to the jockey. The other connections along the path of low resistances are made by thick rods dipped in mercury.

The current i from the battery divides itself into two parts, one part i_2 goes along low resistances and the other part i_1 along the bridge wire. If a, b, c, d be the null points on the bridge wire when the galvanometer is connected to A, B, C and D in succession through T_1, T_2, T_3 and T_4 respectively, then for condition of no deflection $V_a = V_A$, $V_b = V_B$, $V_c = V_C$, $V_d = V_D$

Hence, $V_A - V_B = V_a - V_b$ or $i_2.x = i_1 l_1 \sigma$ ... ... (1)

and $V_C - V_D = V_c - V_d$ or $i_2.y = i_1 l_2.\sigma$ ... ... (2)

where $l_1 = b - a$; $l_2 = d - c$, and σ = resistance per unit length of the bridge wire.

Then dividing (1) by (2) $\frac{l_1}{l_2}$; knowing l_1, l_2, x and y can be compared. If y be known x can be found out. Usually a standard low resistance equal to ·01 ohm may be used in place of y.

71. Construction of Standard ohm : Carey Foster's Bridge :

The Carey Foster's Bridge differs slightly from Wheatstone's Bridge in the fact that it contains two additional gaps in which resistances to be compared can be placed, the two extreme gaps being used for increasing the effective length of the bridge wire.

To construct a standard ohm, two nearly equal resistances R_1 and R_2 are connected across the middle gaps and in the extreme gaps two resistances one, the resistance X to be standardised and the other, the standard ohm Y, are placed.

The galvanometer and the battery are connected as shewn in Figure 71.

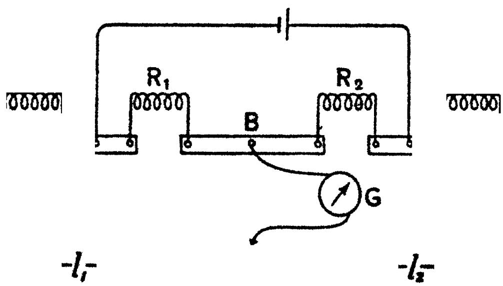


Fig .71

As in Wheatstone's Bridge we have for no galvanometer deflection

$$\frac{R_1}{R_2} = \frac{X+(l_1+\alpha)\rho}{Y+(l_2+\beta)\rho} \qquad \ldots \qquad \ldots \qquad (1)$$

where l_1 and l_2 are the lengths of bridge wire from the ends as determined by the first balance point and α and β are the corresponding end correction lengths and ρ, the resistance of the bridge wire per cm.

The resistances X and Y are interchanged and a new balance point is found at a point whose distances are l_3 and l_4 from the ends of the wire. Then we have,

$$\frac{R_1}{R_2} = \frac{Y+(l_3+\alpha)\rho}{X+(l_4+\beta)\rho}. \qquad \ldots \qquad \ldots \qquad (2)$$

1 (1) $$\frac{R_1}{R_1+R_2} = \frac{X+(l_1+\alpha)\rho}{X+Y+(l_1+l_2)\rho+(\alpha+\beta)\rho} \qquad (3)$$

From (2) $$\frac{R_1}{R_1+R_2}=\frac{Y+(l_3+\alpha)\rho}{X+Y+(l_3+l_4)\rho+(\alpha+\beta)\rho} \qquad (4)$$

Since $l_1+l_2=l_3+l_4=$ length of bridge wire, the denominators of (3) and (4) are equal. Then since ratios are equal, their numerators are also equal

$$\therefore\quad X+(l_1+\alpha)\rho=Y+(l_3+\alpha)\rho$$
$$\text{or}\quad X=Y+(l_3-l_1)\rho \quad \ldots \quad \ldots \quad (5)$$

Thus the value of the resistance X is obtained in terms of the standard ohm provided ρ is determined before.

To construct a standard ohm a piece of manganin wire is taken and connected to two binding screws fitted on the top of a wooden bobbin. The length of the wire should be so chosen that its resistance as determined by the above method is slightly greater than one ohm. Then to adjust the resistance to exactly one ohm the wire is shortened by twisting the middle part of the loop together and soldering the twisted part of the wire when the adjustment is correct.

71(a). To determine ρ, the resistance per unit length of the bridge wire: The above experiment is to be done using a low resistance box for Y and using a metal strip in place of X so that $X=0$. The relation (5) above for a value of $Y=r$ then becomes $0=r+(l_3-l_1)\rho$

$$\text{or}\quad \rho=\frac{r}{l_1-l_3}.$$

knowing l_1, l_3 and r, ρ can be found out. If $r=1\Omega$, ρ becomes $=1/(l_1-l_3)$.

72. Calibration of a Bridge Wire: To *calibrate* a *metre-bridge, i.e.*, to divide the bridge wire into a number of parts having equal resistances a Carey Foster's bridge is employed and the principle stated above is utilised.

From (5) $X-Y=(l_3-l_1)\rho$

That is the difference between the resistance X and Y is equal to the resistance of the bridge wire between the two points of balance. If $Y=0$, $X=(l_3-l_1)\rho$

Thus the resistance of a certain length of wire is determined.

For the purpose of calibration, Y is a thick copper strip and X, a short piece of manganin wire.

By interchanging X and Y for different balance points, the whole length of the bridge wire is divided into lengths of equal resistances. The wire is thus calibrated.

73. Platinum Resistance Thermometer: For measuring high temperatures mercury thermometers are quite unsuited and although a gas thermometer may be a necessary substitute, yet its use is very inconvenient and, therefore, not very frequently adopted.

The accuracy with which the variation in resistance of a wire due to changes of temperature is measured gives us a basis for constructing a thermometer for measurements of both high and low temperatures. The material required for the thermometer should be such as to shew no change of temperature without undergoing any other change. All these conditions are fulfilled by platinum if it is wound in the form of wire on a mica frame and enclosed in a glass tube. But for measuring high temperatures the frame should be made of biscuit porcelain and should be enclosed in a porcelain tube.

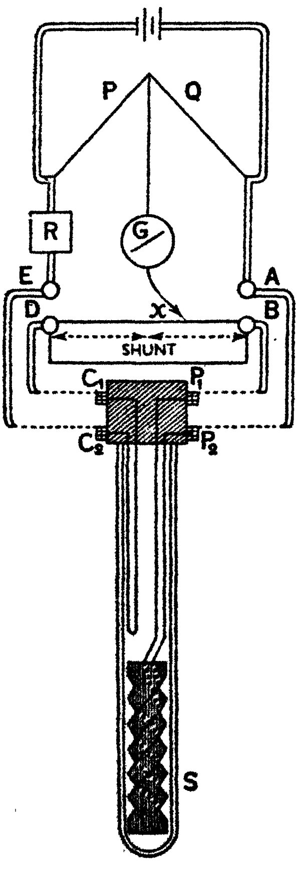


Fig. 72

The ends of the platinum wire are welded to two thick platinum leads joined to the screws P_1 and P_2. A second pair of leads of exactly the same resistance as the other, is connected to the screws C_1 and C_2 and placed close to the other pair inside the same tube. Since the change in resistance due to the changes in temperature of the coil is compensated by the second pair, the second pair is, therefore, known as a *Compensator*.

The terminals P_1, P_2 and C_1, C_2 of the coil and the compensator respectively are connected by equal lengths of copper wires to the two opposite arms of the Wheatstone's net as shewn in Fig. 72, *i.e.*, P_1 and P_2 are connected to the end B of the bridge wire BD and to A connected to the ratio arm Q, and C_1 and C_2 are connected to the end D of the bridge wire and to E connected to the third arm of a P.O. box.

Inserting equal resistances in the arms P and Q (10, 10) a balance point is obtained at a distance x from the middle point of

the wire DB. Thus if ρ be the resistance per centimetre length of the wire, then since $P = Q$ we have,

$R + C + (m + \rho x) = X + C + (m - \rho x)$ where R is the resistance of the third arm of the P. O. box, C the resistance of the leads, m, the resistance of half of the wire BD and X, the unknown platinum resistance.

Then $X = R + 2\rho x$

Thus, at any desired temperature the resistance of platinum wire can be accurately determined.

Note : With the help of the Platinum Resistance thermometer it can be easily seen that the resistance of the wire increases or the conductivity decreases with the rise of temperature.

73(a). To measure temperatures by Platinum Thermometer : The relation between the resistance of a platinum coil and temperature is according to Prof. Callendor, fairly represented by the parabolic equation

$$R_t = R_0 \ (1 + \alpha t + \beta t^2) \qquad \ldots \qquad \ldots \qquad (1)$$

over a very wide range of temperatures, where R_t, α and β are constants which may be found by measuring Rt at three different temperatures.

Note : The expression is true for a range of temperature $0^\circ C - 500^\circ C$ and not for all temperatures, for α and β are opposite in signs.

Since β is very small and equal to $-\cdot 0000088$, it may be neglected. So the platinum temperature is obtained from the linear expression

$$R_t = R_0 \ (1 + \alpha t_p) \qquad \ldots \qquad \ldots \qquad (2)$$

where R_t is the resistance of platinum at the platinum temperature t_p. Similarly R_{100} is the value of R_t at $t_p = 100^\circ C$.

Then $\qquad R_{100} = R_0 \ (1 + 100\alpha) \qquad \ldots \qquad \ldots \qquad (3)$

From (2) and (3) we have,

$$t_p = \frac{R_t - R_0}{R_{100} - R_0} . 100 \qquad \ldots \qquad \ldots \qquad (4)$$

This is Prof. Callender's expression for t_p.

The difference $(t - t_p)$ between t_p and the temperature t on the gas thermometer scale is given by the relation

$$t - t_p = \delta\left(\frac{t}{100} - 1\right)\frac{t}{100} = \delta \times 10^{-4} \times t(t - 100) \qquad \ldots \qquad (5)$$

Note : The constant δ is determined for any particular thermometer from the knowledge of the resistance of the platinum coil at three known temperatures such as the melting point of ice, boiling point of water and the boiling point of sulphur.

73(b). Discussion : To standardise the instrument a correction curve is drawn by plotting *correction* $(t - t_p)$ against t_p and the unknown temperature is measured from the knowledge of t_p and its corresponding correction from the curve.

In the case of fairly low temperatures the correction $(t - t_p)$ is calculated by replacing t in the right-hand side of (5) by t_p and taking the value of δ as equal to 1·5.

To deduce t when t_p has been calculated, the method of successive approximation may be used. Suppose $t_p = 300°C$,

then $\quad t - 300 = 1·5 \times 10^{-4} \times 300 \times 200 = 9 \quad \therefore \quad t = 309°C$

Using this value of t we have,

$$t - 300 = 1·5 \times 10^{-4} \times 309 \times 209 = 9·7 \quad \therefore \quad t = 309·7°C.$$

Following this procedure t, the temperature on the gas thermometer scale, can be found to any required degree of accuracy.

For temperatures up to 300°C the thermometer consists of a platinum wire wound upon a mica frame and enclosed in a glass tube but for higher temperatures the tube must be of glazed porcelain.

Graphical study : Let the curve ABCD represent the relation between temperature and resistance for platinum and let B and C be the points on the curve corresponding to 0° and 100°C. Draw the straight line BCE through the points B and C.

To determine an unknown temperature of boiling of Aniline, let the platinum thermometer be immersed in Aniline contained in a vessel and its resistance determined at this temperature t.

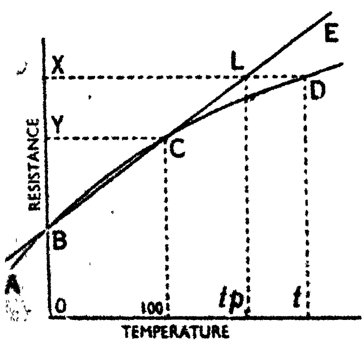


Fig. 73

Let OX represent the resistance of a thermometer at the temperature t. Draw XL parallel to the axis of temperature and Lt_p parallel to the axis of resistance.

Then, from the two similar triangles CBY and BXL (Fig. 73) we have

$$\frac{XL}{YC} = \frac{XB}{YB}, \quad \frac{tp}{100} = \frac{Rt - R_0}{R_{100} - R_0},$$

$$tp = 100\frac{Rt - R_0}{R_{100} - R_0}. \qquad (2)$$

Rt, R_{100}, R_0 are the resistances of the thermometer at $t°$, 100° and 0° respectively.

In arriving at the relation (2) we have incorrectly assumed that the straight line BLE represents the relation between temperature and resistance. So from the Figure 73 we see that the true temperature in the air scale is really represented [illegible] which is slightly greater that XL, *i.e.*, tp. The difference between the

readings on the air thermometer scale and the platinum scale is calculated from the equations (1) and (2) as follows,

$$tp = 100 \times \frac{R_0(1+\alpha t+\beta t^2)-R_0}{R_0(1+100\alpha+100^2\beta)-R_0}, \qquad tp = \frac{\alpha t+\beta t^2}{\alpha+10^2\beta}$$

That is $t-tp = t-\dfrac{\alpha t+\beta t^2}{\alpha+10^2\beta} = \dfrac{-\beta t(t-100)}{\alpha+10^2\beta}$

$$= \frac{-\beta(100)^2}{\alpha+100\beta}\left[\left(\frac{t}{100}\right)^2-\frac{t}{100}\right]$$

$$i.e., \quad t-tp = \delta\left(\frac{t}{100}-1\right)\frac{t}{100}, \text{ where } \delta = \frac{-\beta(100)^2}{\alpha+100\beta} \quad \ldots \quad (3)$$

$$= \delta \times 10^{-4} \times t(t-100)$$

74. Potentiometer: It consists usually of five or ten wires (Eureka or manganin) of uniform cross-section, each one metre in length stretched parallel to each other on a wooden board on which a graduated scale is fitted parallel to the wires. They are joined in series by thick copper or brass strips.

There is a tapping key attached to a triangular jockey which slides over the wires in a groove and can make contact with any point on the wire.

The instrument is used for the measurement of *potential, current* and *resistance* of cells.

75. Comparison of E.M.F. of cells by a potentiometer: The best method of comparing the E. M. F.s of two cells is known as the potentiometer method and it is based on the fact that the E. M. F.s are balanced in turn over a known length of a conductor traversed by a current.

The potentiometer wire AB of uniform cross-section is joined in series with a battery E of constant E. M. F. such as a secondary cell through a variable resistance R. The positive pole of one of the cells E_1 and E_2 whose E. M. F.s are to be compared is connected to the point A and its negative pole is connected through a galvanometer G to the sliding contact K resting on the wire AB. The positive pole of the secondary cell E is also connected to the point A so that the potential falls

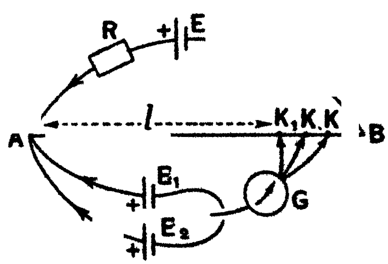


Fig. 74

uniformly from A to B. By sliding the key K, a point K_1 on the

wire is reached at which when the key K is pressed the galvanometer shews no deflection, *i.e.*, no current flows through the galvanometer G.

Since no current flows through the galvanometer, the potential at the point K_1 is the same as that of the negative terminal of the cell E_1 and therefore the potential difference between the terminals of the cell or rather the E. M. F. of the cell E_1 on open circuit is equal to the fall of potential along AB between the points A and K_1.

If a second cell of E. M. F., E_2 be substituted in place of the cell having E. M. F., E_2, second null point K_2 will be found on the wire AB such that the difference of potential between the points A and K_2 becomes equal to the E. M. F., E_2 of the the second cell.

Thus

$$\frac{E_1}{E_2} = \frac{\text{difference of potential between A and } K_1}{\text{difference of potential between A and } K_2} = \frac{l_1 . \rho . i}{l_2 . \rho . i}$$

where l_1 and l_2 are lengths of AK_1 and AK_2, ρ is the resistance per unit length of wire AB, and i is the current flowing along AB.

$$\therefore \quad \frac{E_1}{E_2} = \frac{l_1}{l_2} \quad \ldots \quad (1)$$

the value of E_1 is determined if E_2 be known. In determining the E. M. F. of a cell, a standard cell, such as a cadmium cell is used and from the knowledge of the E. M. F. of the standard cell, the E. M. F. of the first cell is determined.

The E. M. F.s of the cells to be compared should have E. M. F.s smaller than that of the secondary cell used in the main circuit and the resistance used in series with the secondary cell to prevent any heating effect in the wire AB, should not be very large. If it is not so, the potential difference between the points A and B of the wire AB will be smaller than the E. M. F.s of the cells to be compared and the key cannot be adjusted to produce no deflection in the galvanometer which will in this case show deflection in one direction only.

Note : The relation (1) can be obtained by applying the second law of Kirchhoff to the closed circuit AE_1GK_1.

Thus we have $E_1 = ir_1 + i_1R \quad \ldots \quad \ldots \quad (2)$

where i is the current flowing in the direction AB and i_1 the current in the galvanometer circuit AGK_1 of resistance R and resistance of $AK_1 = r_1$.

When the key is pressed at the point K_1 such that no current flows through the galvanometer, i_1 in the expression. (2) becomes zero and ir_1, becomes equal to E_1, the E. M. F., of the cell in the branch circuit. With a second cell of E. M. F., E_2 the same relation $ir_2 = E_2$ will hold under similar conditions, since i will remain the same in both the cases.

Thus we have,

$$\frac{E_1}{E_2} = \frac{r_1}{r_2} = \frac{AK_1 . \rho}{AK_2 . \rho} = \frac{l_1}{l_2}$$

76. Measurement of a very small E. M. F. as in the case of a thermo-couple: In order to measure the E. M. F. of the order of millivolts or micro-volts, the potentiometer is made sensitive by reducing the current in it and thereby diminishing the fall of potential over the whole wire to a small fraction of a volt. This is done by placing a resistance in series with the potentiometer in the manner described below.

In determining the E. M. F. of a thermo-couple which is in the order of micro-volts, a known resistance R is used in series with the potentiometer wire, the circuit being completed through a secondary cell B as shown in Figure 75.

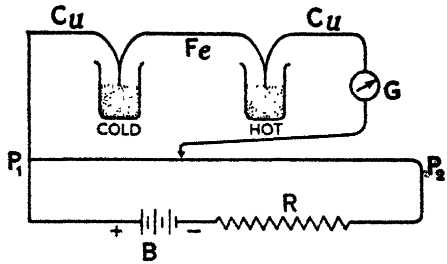


Fig. 75

To determine the E. M. F. of a thermo-electric couple say, *Fe* and *Cu*, one of the junctions is placed in the ice contained in a beaker to keep its temperature constant and the other junction is raised to various temperatures by immersing it in water placed in a vessel.

One of the leads from the couple is connected to the end P_1 of the potentiometer wire and the other to the tapping key through a galvanometer G and the reading for the length of the wire corresponding to the null point from the end P_1 of the wire at a certain temperature of the hot junction is obtained. Let l be this length.

If P be the resistance of the potentiometer wire, of length L, then the resistance per unit length of the wire $=\dfrac{P}{L}$.

And the current flowing through the potentiometer circuit $=\dfrac{E}{R+P}$, where E is the E. M. F. of the secondary cell.

The fall of potential per unit length of the potential wire = current flowing through the circuit × resistance of the unit length of the wire $=\dfrac{E}{R+P}\cdot\dfrac{P}{L}$

∴ the fall of potential e for length l, which is the E. M. F. of the thermo-couple is given by,

$$e=\frac{E.P}{(R+P)L}\cdot l$$

The value of e is determined from the knowledge of E, R, L, l and P. In the experiment P is determined by using a P. O. box and E, the E. M. F. of the secondary cell by a voltmeter.

The E. M. F. of the couple is obtained in micro-volts.

Note : A very small E. M. F. can also be measured by a *milli-voltmeter* by connecting it in series with the couple.

A milli-voltmeter is a voltmeter whose scale is calibrated in millivolts (one-thousandth part of a volt).

77. Measurement of current : A current of strength i is made to pass through a variable resistance R, a standard low resistance r of known magnitude and through an ammeter Am.

The fall of potential through the standard low resistance r through which the current i passes is compared with the E. M. F., E of a standard cell such as a Clerk or Cadmium cell E_1 by means of a potentiometer.

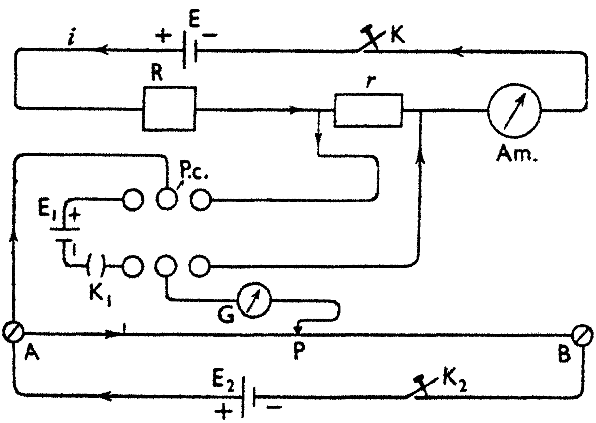


Fig. 76

Let l_1 and l_2 be respectively the lengths of the potentiometer wire to adjust the galvanometer to no deflection when the standard cell and the potential fall $(V_1 - V_2)$ over the resistance r, are used in the branch circuit of the potentiometer.

Then $\dfrac{V_1 - V_2}{E} = \dfrac{l_2}{l_1}$ $\qquad V_1 - V_2 = E\dfrac{l_2}{l_1}$.

But $V_1 - V_2 = ir$, we have $i = \dfrac{V_1 - V_2}{r} = \dfrac{E}{r} \cdot \dfrac{l_2}{l_1}$

Note: To **calibrate an ammeter**, an adjustable resistance R and an ammeter Am. are connected in series with the standard resistance r. By varying the adjustable resistance, the current i passing through the ammeter is altered and the values of the current for different resistances are calculated by the above formula and compared with the readings of the ammeter.

78. Measurement of resistance of a cell : The E. M. F., E of the cell whose internal resistance is to be determined, is at first balanced against the potential difference between two points, at a distance l_1 apart, on the potentiometer wire AB conveying a current. The terminals of the cell are then connected by a conductor of resistance R and the fall of potential between the ends of R is again balanced on the wire as usual—by closing the key K. Let l_2 be the corresponding length in this case,

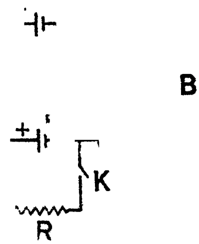


Fig. 77

Then, if $(V_1 - V_2)$ be the potential difference between the ends of R we have

$$\frac{V_1 - V_2}{E} = \frac{l_2}{l_1}$$

But since $V_1 - V_2 = iR = \frac{E}{R+r}.R$, where i is the current flowing through R, and r the internal resistance of the cell, we have again

$\frac{l_1}{l_2} = \frac{R+r}{R}$ $\therefore$ $r = \frac{l_1 - l_2}{l_2}.R$, when the value of r can be found out.

QUESTIONS

1. Describe a method for comparing low resistances.
What difficulties are experienced in measuring a very small resistance by the Wheatstone's bridge method? [C. U. 1941, '44, '51]
2. Summarise various methods of measuring resistances of different orders, namely, *ordinary*, *low* and *high* and explain why ordinary Wheatstone's bridge is not suitable in all cases. [C. U. 1948]
What is the dimension of resistance in electro-magnetic units? [C. U. 1948 ; See, Appendix]
3. Describe an experimental arrangement to show that the conductivity of a metal wire decreases with the rise of temperature (temperature range $0° - 100°C$). [C. U. 1938, '49]
Is the statement valid at all temperatures? [C. U. 1949]
4. Write a short note on a resistance thermometer. [C. U. 1940, '41, '42, '50, '59]
5. **Explain the potentiometer method of comparing two electromotive forces. Shew how it can be used to measure an electric current.**
6. **The galvanometer, in a potentiometer experiment, is found to give deflections in the same direction when connection is made at any point on the wire. Explain two causes which would produce this effect.**

EXAMPLES

1. A potentiometer wire is 100 cms. long and has 5 ohms resistance and a 2 volt accumulator is used with it. What resistance is required in series with the wire if a P. D. of 1600 microvolts is to be balanced by the whole length of the wire. What P. D. then corresponds to a balance at 12·5 cm. along the wire ?

[*Ans.* ·6245 ohms. 200 microvolts]

CHAPTER VIII

THERMAL EFFECTS OF ELECTRIC CURRENT

79. Energy transformation in an electric circuit: Whenever a potential difference exists between two points in an electric circuit an energy transformation occurs between them.

If the poles of a battery are joined by an ordinary conductor, the whole of the energy supplied by the battery is transformed into heat, partly in the battery and partly in the conductor.

If the circuit contains a voltmeter or a motor, a certain amount of energy is utilised in chemical or mechanical work, the balance again appearing as heat in the circuit.

It has been assumed that in a conductor, besides the electrons lodged within the atom there are free electrons which move in the inter-atomic spaces and frequently collide with the atoms.

When an electromotive force acts at the ends of a conductor the free electrons tend to move forward in the direction in which the E. M. F. acts. The drift of electrons is not a smooth affair. The electrons during the course of this motion collide with the atoms. As a result of this collision, the atoms as a whole vibrate increasing the kinetic energy. So according to kinetic theory the rise of temperature is due to the increased vibration of the atoms.

The greater the value of E. M. F., the greater will be the velocity of the moving electrons and consequently more violent will be the collisions and as a result greater heating will be produced.

80. Relation between work and heat: When a current of electricity passes through a wire of some appreciable resistance, the wire becomes heated and the amount of heat developed depends on the amount of work done in driving the current through it. We know that when work is transformed into heat or heat into

work, the quantity of work is mechanically equivalent to the quantity of heat. This is known as Joule's Law.

Thus we have Work $\propto$ Heat

Work = a constant × Heat or $W = J.H.$ or $H = \frac{W}{J}$

The constant J known as Joule's constant is defined as the work done (or energy transferred) between two points in a circuit when one coulomb of electricity passes between them differing in potential by one volt. It is equal to 10^7 ergs or 778 ft. lbs.

80 (a). Expression for heat produced in a conductor due to flow of current through it: Let a conducting wire AB traversed by current i have potential difference E between two ends A and B. Then by definition of potential difference, we have,

work done to carry unit charge from any end to the other $= E$. Then, if Q amount of charge has passed from A to B in t seconds, total work done in t secs. $= EQ$.

But since current means rate of flow of charge, we have, $i = \frac{Q}{t}$ or $Q = it$.

Therefore total work done in t seconds, $W = Eit$; but heat developed H is equal to $\frac{W}{J}$ where J is the mechanical equivalent of heat.

$$H = \frac{W}{J} = \frac{Eit}{J} = \frac{i^2Rt}{J},$$ since $E = iR$, where R is resistance of AB.

If i be in ampere, E in volt, and R in ohm, *i.e.*, all in practical units, then i amp $= i \times 10^{-1}$ *e.m.u*; E volt $= E \times 10^8$ *e.m.u* and R ohm $= R \times 10^9$ *e.m.u*, so that work

$$W = E \times 10^8 \times i \times 10^{-1} \times t = Eit \times 10^7 \text{ ergs}$$
$$= i^2(10^{-1})^2 \times R \times 10^9 \times t = i^2Rt \times 10^7 \text{ ergs.}$$

Therefore heat developed is given by

$$H = \frac{Eit \times 10^7}{J} \text{ calories} = \frac{C^2Rt \times 10^7}{J} \text{ calories} \quad \dots \quad (1)$$

Putting $J = 4{\cdot}2 \times 10^7$ ergs per calorie

$$H = \frac{Eit \times 10^7}{4{\cdot}2 \times 10^7} = \frac{Eit}{4{\cdot}2} \text{ calories} = {\cdot}24 \; Eit \text{ calories}$$ in terms of i and E ... (2)

Again, $H = \frac{i^2Rt \times 10^7}{4{\cdot}2 \times 10^7} = \frac{i^2Rt}{4{\cdot}2}$ calories $= {\cdot}24 i^2Rt$ calories. .(3

From Relation (1) $J = \frac{Eit \times 10^7}{H}$ ergs per calorie

$= \frac{i^2Rt \times 10^7}{H}$ ergs per calorie.

Note : 1 Calorie = 4·2 Joules ; 1 Joule = ·24 calories.

81. Joule's Experiment : A current of strength i is made to pass through a coil of wire W of resistance R dipped in oil contained in a calorimeter, the circuit being completed through an ammeter A which directly reads the strength of the current passing

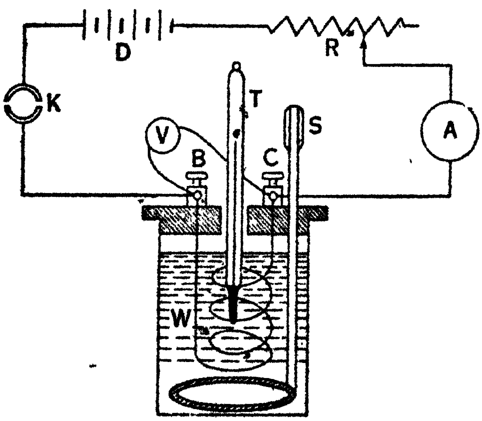


Fig. 78

through the coil. The circuit contains also a battery D, a rheostat R, and a plug Key K in series with the heating coil W. The voltmeter V is connected (in parallel) to the terminal screws B and C of the coil for reading the potential difference between the ends of the coil.

The resistance of the coil is measured by a Wheatstone's bridge. When the current passes through the wire for t secs. heat is generated which is taken up by the oil and the calorimeter and in consequence of which the temperature of the oil rises. The total amount of heat thus developed is equal to $(ms+w)(\theta_2-\theta_1)$, where m is the mass of oil, s its sp. ht., w, the water-equivalent of the calorimeter and $(\theta_2-\theta_1)$ the rise of temperature.

Thus we have,

$$J = \frac{i^2Rt \times 10^7}{(ms+w)(\theta_2-\theta_1)} \text{ ergs per calorie}$$

and from the known value of the quantities in the right hand side of this relation, the value of J is calculated and found to be equal to $4{\cdot}2 \times 10^7$ ergs.

The value of J can also be determined by the formula $J = Eit \times 10^7/(ms+w)(\theta_2-\theta_1)$ ergs per calorie so that E and not R is required.

Here E, the E.M.F. is obtained from the voltmeter reading.

Note : To minimise loss of heat by radiation, current should be passed for a time in which the rise of temperature is between 4 and 5°c.

81(a). Joule's Law : The result expressed in the equation $H = \frac{i^2Rt}{J}$ is known as Joule's Law.

The law states that

The rate of heat generated in a circuit is proportional (1) to the square of the current for a given resistance (2) to the resistance for a given current and for a given resistance, and (3) to the time in seconds during which the current passes.

It is to be noted that for constant current heat produced is directly proportional to the resistance through which current is passing. Thus when two resistances are connected *in series* the heat produced in the higher resistance is greater than that in the smaller resistance since $H \propto R$.

But when the two resistances are connected *in parallel* as in the case of lamps across the main, more heat is developed in the lamps whose filaments have a smaller resistance : since in this case $H \propto \frac{1}{R}$

Note : Joule heating effect is proportional to the square of the current and is therefore independent of the direction of the current. It is an irreversible phenomenon.

82. Electric Energy or Work : If Q units of charge be conveyed between two points differing in potential by E, the work done is $Q \times E$, and if these quantities are expressed in C.G.S. units, the work done, *i.e.*, $Q \times E$ is expressed in ergs. But if Q and E are expressed respectively in coulombs and volts the unit of work is in Joule.

Since 1 *coulomb* $= 10^{-1}$ C.G.S. unit
1 *volt* $= 10^8$ C.G.S unit
$\therefore$ 1 *Joule* $= 10^{-1} \times 10^8 = 10^7$ C.G.S. units of work (ergs).

83. Power : It is the rate at which the work is done.

$$\text{Work} = Q \times E$$

Power $= \dfrac{Q \times E}{t} = iE$, where t is the time during which work is performed and i, the current strength.

Thus power consumed in an electric circuit is Ei, or i^2R or E^2/R where E, i and R have usual meanings.

The practical unit of power is called watt. Therefore when i, E and R are given in practical units (*i.e.* ampere, volt and ohm respectively), the power consumed is given by Ei, i^2R or E^2/R—all expressed in Watts.

From above, Power $=$ *e.m.f.* $\times$ current
Watt $=$ volt $\times$ amp.

Again 1 Watt $= 1$ amp. $\times$ 1 volt $= 10^{-1}$ *e.m.u.* $\times 10^8$ *e.m.u.*
$= 10^7$ ergs per sec. $= 1$ Joule per sec.

83(a) Definition of Watt : If a steady current of one *ampere* flows between two points differing in potential by one volt the rate of consumption of energy, *i.e.*, of doing work is one Joule or one watt.

The practical unit of power is 1 kilowatt $=$ 1000 watts.

83(b). Horse-power and Watt :

One horse power (H.P) $=$ 550 ft. *lbs* per sec.
$= 550 \times 453.6 \times 12 \times 2.54$ gm-cm/sec.
$\because$ 1 lb $= 453.6$ gm and 1 foot $= 12 \times 2.54$ cm.
$= 550 \times 453.6 \times 12 \times 2.54 \times 991$ ergs per sec.
$= 746 \times 10^7$ ergs per sec. $=$ 746 Joules per sec.
$=$ 746 watts.

83(c). Kilowatt-hour : Energy consumed in a given time is the product of power into time. When power of one watt is consumed for an interval of one hour the energy consumed
$=$ one watt $\times$ one hour $= 1$ watt-hour.
$=$ ·001 kilowatt-hour, since 1000 watt-hours $= 1$ kilowatt-hour.
Board of Trade unit (B.O.T. unit) of electric power is a kilowatt-hour by which the consumption of electric energy is measured and charged by the power supply authorities.

Suppose an electric bulb marked '220 volts – 60 watts' is lighted for a month (30 days) at the rate of 5 hours per day. The total

energy consumed in one month $= 60 \times 5 \times 30 = 9000$ watt-hours $= 9000/1000 = 9$ kilowatt-hours or 9 B. O. T. units. If the cost of unit be 3 *as*, the cost of lighting the bulb in the above case is 9×3 *as* or rupee one and annas eleven only.

84. Rule for calculation of cost : Add up the wattages consumed by different systems (electric lamps, fans, heaters, irons or motor pumps, etc.), multiply it by the time in hours for which wattages are consumed. Then dividing this product by thousand obtain number of B.O.T. units. Finally multiply this number by the rate of charge per unit, to obtain the total cost for the given time.

If in a problem instead of giving wattage of a lamp, any two factors of the lamp—its resistance R, current i and P.D.,E for its operation be given, then wattage of the lamp is equal to Ei or i^2R or E^2/R. If an electric motor pump of 1 horse-power operates with full efficiency, it takes 746 watts for 1 H.P. Then proceed as in previous paragraph.

85. Industrial applications :

(A). Electric Lamps : The construction of incandescent electric lamps or bulbs is the most important application of heating effect of current. The first electric lamp made by Edison in 1878 consisted of a very fine platinum wire. Subsequently he made carbon filaments by carbonising (heating in insufficient air) bamboo fibres or threads and used them in glass bulbs. The bulbs were evacuated to the greatest possible degree to prevent chemical combination of carbon with oxygen at higher temperature. The melting point of carbon is very high being about 4200°C. But the carbon filament can be heated upto 1865°C, above which carbon filament rapidly disintegrates and darkens the walls of the bulb. More-over the resistance of carbon filament decreases with increase of temperature so that it becomes sensitive to small changes in the applied voltage.

For all these drawbacks, carbon filaments have been replaced by metal filaments. Tungsten which was found to be most suitable after numerous experiments and trials, is now a days widely used, either in an evacuated bulb, forming vacuum electric bulb, or in a gas filled bulb. Tungsten melts at 3400°C and it can bear a running temperature of about 2100°C.

A long and thin filament of tungsten is to be used, as tungsten has low specific resistance. As the filament of an evacuated electric bulb quickly disintegrates at rather high temperature, gas filled lamps are now in general use. The gases used in places of air are inactive, such as argon and nitrogen. The presence of gases

again produces convection currents to reduce which the filament is taken in a coiled form.

The efficiency of gas filled lamps, which is given by watts per candle power, is about double that of an ordinary evacuated lamp. A gas filled lamp consumes about ·8 watts per candle power whereas a vacuum electric lamp consumes about 1·6 watts per candle power. Therefore the former are called *'Half watt'* lamps. Electric lamps of all types are rated in Wattages. By a 100 watt Lamp we mean that it consumes 1 kilowatt-hour or 1 B.O.T. unit of energy in 10 hours ; since 1 unit or rather B.O.T. unit = 100 × 10 kilowatt-hours.

No definite relation between the candle power of the light emitted by a bulb and the corresponding power in wattage consumed by it can be set up. For an evacuated bulb, also called hard bulb, the number of watts consumed is somewhat greater than the number expressing its candle power. Thus a bulb consuming 36 watts emits light of candle power 30. In case of gas filled lamps, it is claimed by the makers that the number of wattage of a lamp is half the number giving its candle power. As for example, a gas filled lamp consuming 50 watts, emits light of candle power 100. These lamps are, therefore, as already mentioned elsewhere are often termed half-watt lamps.

The electric lamps described above are **glow lamps**, and only a very small fraction of the electric energy spent appears as light energy, the greater part of energy is converted into and dissipated as heat. Thus efficiency of these lamps is limited, since temperature of the filament can not be raised beyond a certain limit. Hence lamps depending on discharge of electricity through an evacuated tube or bulb are also now widely used. These lamps known as **'discharge lamps'** have been discussed in subsequent chapter on conduction of elctricity through gases.

(B). Electric heaters : Electric heaters, such as electric furnace or stove, electric kettle and electric irons, are appliances depending on heating effect of current. The conducting materials used are thin wires of platinum, irridium, iron etc. For ordinary furnaces nichrome wire, and for high temp. furnaces, molybdenum wires are used.

The **efficiency** of any form of heater is defined as the ratio of the amount of heat energy obtained to the amount of electric energy expended. As some heat energy is lost by conduction, convection, etc., the efficiency of a heater is always less than unity.

(C). Electric fuse : It consists of a wire of tin or an alloy of tin and lead or copper, having low melting points. The fuse wire

is included in an electric circuit as a safety device. The thickness of the wire is so selected that it can burn and blow out at a specified value of current, which is slightly below the rated current circulating in the wirings of the house. This protects the house wire and electrical appliances from damage by overheating.

86. General plan of electric installation in a house: There are generally two different systems of wiring employed in any installation.

(1) Tree System. (2) Distribution System.

In the tree system, a pair of mains runs through the building and branch circuits are tapped off from them at any points which seem convenient. As the arrangement resembles the trunk and branches of a tree it is called the *Tree system.* This system is now obsolete for various practical difficulties and the distribution system is universally employed in modern work.

In the **Distribution system,** the current is led into one or more distribution centres by means of main cables. At these points sub-circuits are led off by means of distribution boards D or boxes.

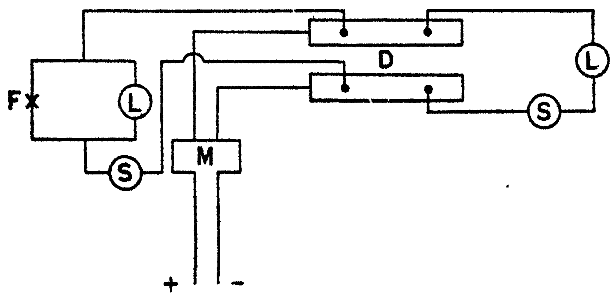


Fig. 79

The adjoining Figure 79 shows the general plan of the system, with distribution box, sub-circuits, lamps and fans.

The main cables are led into the distribution box D through the meter M. From the distribution box two sub-circuits are taken and in which lamps L, fans F, and switches S are inserted.

QUESTIONS

1. State the laws relating to the heating effects of a current through a conductor. [C. U. 1945, '53]

2. Shew that the heat produced by the current i amperes flowing along a conducting path of resistance 'r' ohms is i^2r watts. Can you suggest any explanation as to the cause of generation of heat? [C. U. 1956]

3. Define Watt, kilowatt and kilowatt-hour. [C. U. 1945]

4. Define electric power and efficiency and express the former in C.G.S. units. [C. U. 1937, '40]

5. From the definition of difference of electric potential deduce an expression for the heat developed in a conductor due to the passage of a steady current. [C. U. 1956]

What is joule and a watt? [C. U. 1957]

EXAMPLES

1. A coil of resistance 2 ohms is immersed in a kilogram of water and is connected to the terminals of the battery of internal resistance 4 ohms and E. M. F. 6 volts for 3 minutes. Find the elevation of temperature of the water assuming that the heat produced is wholly absorbed by the water.
[$J = 4{\cdot}2 \times 10^7$ C.G.S. units] [C. U. 1913]

Let t°C be the rise of temperature of water.
The strength of current flowing through the wire is given by

$$i = \frac{E}{R+r} = \frac{6}{2+4} = 1 \text{ amp.}$$

Heat developed in the coil

$$= \frac{i^2 RT \times 10^7}{J} = \frac{1 \times 2 \times 3 \times 60 \times 10^7}{4{\cdot}2 \times 10^7} \text{ cal.} = \frac{6 \times 60}{4{\cdot}2} \text{Cal.}$$

Heat absorbed by water $= 1000 \times t$ cal.

$$1000t = \frac{6 \times 60}{4{\cdot}2} \qquad \therefore \quad t = \frac{6 \times 60}{4{\cdot}2 \times 1000} = {\cdot}085^\circ\text{C nearly.}$$

2. A Daniell cell has an E. M. F. of 1·08 volts and ½ ohm internal resistance. If its terminals are connected successively by wires whose resistances are 2 and 3 ohms, compare the amounts of heat developed in the external circuits in the two cases in a given time. [C. U. 1915, '47]

Let i_1 and i_2 be the strengths of current when the poles are connected by 2 and 3 ohms resistance respectively.

Then

$$i_1 = \frac{1{\cdot}08}{2+\frac{1}{2}} \text{ amp.} = \frac{1{\cdot}08}{2{\cdot}5} \text{ amp. and } i_2 = \frac{1{\cdot}08}{3+\frac{1}{2}} \text{ amp.} = \frac{1{\cdot}08}{3{\cdot}5} \text{ amp.}$$

Heat developed in the first wire : heat developed in the second wire.

$$= \frac{i_1^2 R_1 T \times 10^7}{J} : \frac{i_2^2 R_2 T \times 10^7}{J} = i_1^2 R_1 : i_2^2 R_2$$

$$\left(\frac{1{\cdot}08}{2{\cdot}5}\right)^2 \times 2 : \left(\frac{1{\cdot}08}{3{\cdot}5}\right)^2 \times 3 = 2 \times (3{\cdot}5)^2 : 3 \times (2{\cdot}5)^2 = 1 : {\cdot}0917$$

3. A current of 1·2 amperes passes for one minute through a wire whose resistance is 25 ohms. Find the amount of heat developed [$J = 4{\cdot}2 \times 10^7$ C.G.S. units] and the difference of potential between the ends of the wire. [C. U. 1916]

$$\text{Heat developed } \frac{i^2 RT \times 10^7}{J} = \frac{(1{\cdot}2)^2 \times 25 \times 60 \times 10^7}{4{\cdot}2 \times 10^7} \text{ cal.}$$

$$= 514{\cdot}28 \text{ cal. nearly.}$$

The potential difference between the ends of the wire = current strength × resistance = $1{\cdot}2 \times 25 = 30$ volts.

4. A current of 2 amperes is sent for 10 minutes through a coil of resistance 10 ohms, immersed in 500 grammes of water in a calorimeter of water-equivalent 10 grams. The rise of temperature of the water after correcting for radiation is 11°8. Calculate the value of J. [C. U. 1918]

Heat developed in the wire

$$\frac{i^2RT\times10^7}{J}=\frac{2\ \times10\times10\times60\times10^7}{}\text{Cal.}\qquad 4\times6\times10^{10}\ \text{Cal.}$$

Heat absorbed by the calorimeter and water being heated through 11° 8C= $(10+500)\times11{\cdot}8$ Cal.

$$\therefore \frac{4\times6\times10^{10}}{J}=510\times11\ 8\ ;\quad \therefore\ J=\frac{4\times6\times10^{10}}{51\times118}=4\times10^7 \text{ ergs per calorie.}$$

5. A current of one ampere passes for one minute through a wire whose resistance is 40 ohms. Find the amount of heat developed. [$J=4{\cdot}2\times10^7$ C. G. S. units.]

[*Ans.* 571·43 Cal.] [C. U. 1913]

6. For tungsten wire, the average temperature coefficient above 20°c is $5{\cdot}1\times10^{-3}$. The filament of an electric lamp has a "cold resistance of 9·7 ohms at 20°c. When glowing it has a resistance of 121 ohms. Calculate its temperature. If it is a 100 watt lamp, what current it consumes? [C. U. 1943]

$R_t=R_{20}(1+\alpha t)$

$121=9{\cdot}7(1+5\ 1\times10^{-3}t)$. Here t is the difference of temperature above 20°c.

$\therefore\ t=2250°c$; $\therefore$ the temperature of glowing is $2250°+20°=2270°$

Again Watts = volts × amperes ; $\therefore\ 100=V.i.=\frac{V^2}{R}=\frac{V^2}{121}$;

$V=110$ volts ; $\quad =\frac{110}{121}=\cdot909$ amp.

7. If a cell has an E. M. F. of 1·08 volts and ·5 ohm internal resistance and if the terminals are connected by two wires in parallel, 1 ohm and 2 ohm respectively, what is the current in each and what is the ratio of the heat developed in each? [C. U. 1947]

[$i_2=\cdot617$ amp. ; $i_1=\cdot308$ amp ; $\frac{H_1}{H_2}=\frac{i_1^2R_1t}{i_2^2R_2t}=\frac{1}{2}$

Here i_1 and i_2 are the currents and H_1 and H_2 are the heats developed.]

8. Two resistances r_1 and r_2 are connected in series and an E. M. F. of 10 volts maintained continuously between the ends. It is found that the heating of the resistances r_2 is three times that of r_1. If $r_1=2$ ohms, find the current flowing in the circuit. [C. U. 1936]

We know that the heat H_2 developed in $r_2=\frac{i^2r_2t\times10^7}{J}$ therms.

H_1

$\frac{H_2}{H_1}=\frac{r_2}{r_1}=3$; $\qquad r_2=3r_1=6$ ohms.

Therefore the current i in the circuit $=\frac{E}{R}=\frac{10}{2+6}=1{\cdot}25$ amp.

9. An electric kettle taking 3 amps. at 220 volts bring one litre of water from 18°c to the boiling point in 11 minutes. Find its efficiency.

We know that efficiency $=\frac{\text{Total power consumed}}{\text{Total power generated}}=\frac{\text{Work performed}}{\text{Energy consumed}}$

Energy consumed by the kettle in 11 minutes $=3\times220\times10^7\times11\times60$ ergs.

Since energy consumed per sec. $=3\times220\times10^7$ ergs.

Again heat developed in heating 1000 grams. of water from 18°C to 100°C $=1000\times1\times(100-18)$ Cal.

∴ Energy developed $=$ J. H. $=1000\times82\times4{\cdot}2\times10^7$ ergs.

where J = Joule's equivalent

$$\text{Efficiency}=\frac{1000\times82\times4{\cdot}2\times10^7}{3\times220\times10^7\times11\times60}=\frac{287}{363}.\ i.e.\ \frac{287}{363}\times100=79{\cdot}06\%$$

10. The lighting of a room requires 300 candle power and the lamps supplied have an efficiency of 1·5 watts per c. p. What is the cost of lighting the room for 24 hours if the cost of supply is 3*d*. per Board of Trade Unit ?

[*Ans*. 2*s*. 8·4*d*.]

11. A dwelling house is installed with 15 lamps (1000 ohms resistance each) and 4 ceiling fans each driven by ⅛th horse-power motor. If the lamps and fans are run on an average of 6 hours daily, calculate the cost of electric power consumed in a month of 31 days. (The supply pressure is 220 volts and one kilowatt-hour costs four annas.) [C. U. 1926]

Energy consumed by each lamp $=i\times E=\frac{E^2}{R}=\frac{220\times220}{1000}=48{\cdot}4$ watts.

∴ The total energy consumed by 15 lamps running on an average of 6 hours daily $=15\times48{\cdot}4\times6$ watts

The energy consumed in 31 days $=15\times48{\cdot}4\times6\times31$ watts

No. of kilowatt-hours consumed $=\frac{15\times48{\cdot}4\times6\times31}{1000}$

Again the energy consumed by each fan in 31 days running for 6 hours daily $=\frac{1}{8}\times746\times6\times31$ watts.

The energy consumed by 4 fans in 31 days and running 6 hours daily $=\frac{1}{8}\times746\times4\times6\times31$ watts

No. of kilowatt-hours consumed $=\frac{1}{8}\times\frac{746\times4\times6\times31}{1000}$

Since one kilowatt-hour costs 4 annas *i.e.*, ¼th of a Rupee the total cost for electric power consumed

$$=\text{Rs. }\frac{1}{4}\times\frac{1}{1000}\times31\times6\times(15\times48{\cdot}4+\frac{746}{8}\times4)=\text{Rs. }51\text{ (approximately).}$$

12. A village is to be supplied with current at 200 volts. The following list shews the uses to which the current will be put. What must be the horse power of the driving engine ? 600 lamps each having a resistance of 1000 ohms, 1800 lamps each having a resistance of 3000 ohms, 20 motors each using 2 amps. (One horse-power = 746 watts.)

Watts consumed by a lamp having a resistance of 1000 ohms

$$=E.i=\frac{E^2}{R}=\frac{200\times200}{1000}=40\text{ watts.}$$

Total watts consumed by 600 such lamps $= 600 \times 40 = 24000$ watts.

Watts consumed by a lamp of 3000 ohms resistance $= \frac{200 \times 200}{3000} = \frac{40}{3}$ watts.

Total watts consumed by 1800 such lamps $= \frac{40}{3} \times 1800 = 24000$ watts.

Watts consumed by each motor $= i.E = 2 \times 200 = 400$ watts.
Total watts consumed by 20 motors $= 20 \times 400 = 8000$ watts.
Total watts consumed by both kinds of lamps and motors
$= (24000 + 24000 + 8000) = 56000$ watts.

$$\text{Horse Power consumed} = \frac{56000}{746} = 75{\cdot}06 \text{ H. P.}$$

The H. P. of the driving engine can not be less than 75·06 H. P. considering some losses during the transformation of mechanical energy into electrical energy, the H.P. should at least be 76 or somewhat more.

13. A railway carriage is lit up by thirteen 9 candle-power lamps each taking 1·22 amps. at 15 volts. What is the resistance of each lamp, how much heat in calories, is generated per sec. in each lamp and what is the total power in watts used in lighting the compartment? [C. U. 1933]

Resistance in each lamp $= \frac{15}{1{\cdot}22} = 12{\cdot}29$ ohms.

Heat generated in each lamp per second

$$= \frac{i^2 Rt}{4{\cdot}2} = \frac{(1{\cdot}22)^2 \times 12{\cdot}29 \times 1}{4{\cdot}2} = 4{\cdot}35 \text{ calories.}$$

Power consumed by each lamp $= 1{\cdot}22 \times 15 = 18{\cdot}3$ watts.
$\therefore$ Total power consumed by 13 lamps $= 18{\cdot}3 \times 13 = 237{\cdot}9$ watts.

14. A high tension line carrying electrical energy is 150 kilometers in length and the fall of potential along the line is 8 volts per km. The line is of copper and has a section 0·8 sq. cm If the loss of energy in the line is 27 per cent., what is the current in the line and also the original output in kilowatts? (The specific resistance of copper $= 1{\cdot}6 \times 10^{-6}$ ohm-cm.) [C. U. 1940]

Let R be the resistance of the line per km. *i.e.*, of 2 km., since direct and return lines are considered.

Then $iR = 8$ where i is the current strength.

But $R = \frac{2000 \times 100 \times 1{\cdot}6 \times 10^{-6}}{{\cdot}8}$ $\quad \frac{}{{\cdot}4} = 20$ amps.

Loss in the line $= i^2 R \times 150$ watts $= 400 \times {\cdot}4 \times 150 = 24$ k. w.

But this 24 k. w. = 27 p.c. of the original output.

$\therefore$ The original output $= \frac{24 \times 100}{27} = 88{\cdot}8$ k. w.

15 A 10-ohm coil of wire is used to heat 1000 grams of water from 15°C to 100°C in 20 minutes. How much current must be used? What is the amount of power consumed? If one B.O.T. unit costs 4 as., how much will you pay? [C. U. 1945]

To *determine the current*, use the formula $JH = i^2 Rt$

or $i^2 = \frac{J.H}{R \times t \times 10^7}$ in practical units.

$H = 1000 \times 1(100-15) = 85000$ cal. $J = 4.2 \times 10^7$ ergs. per cal.

$$\therefore \quad i^2 = \frac{85000 \times 4.2}{10 \times 60 \times 20} = 29.75 \quad \text{or} \quad i = 5.454 \text{ amps.}$$

To *determine the power consumed*, use the formula,
$P = iE = i^2R = 29.75 \times 10 = 297.5$ watts.
No. of B. T. units consumed use the relation

For No. of B.T. units $= \frac{\text{Watts} \times \text{hours}}{1000} = \frac{297.5 \times \frac{1}{3}}{1000} = .09916.$

Cost at 4 as. per unit $= .09916 \times 4 \times 12 = 4.76$ pies $= 5$ pies (approx)

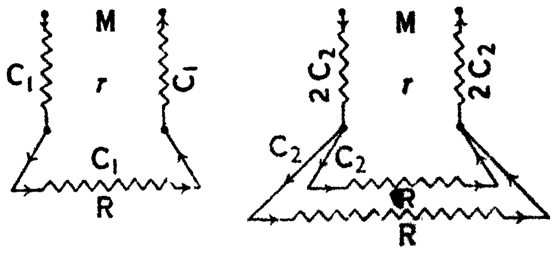
16. It takes 20 minutes to heat 200 gms. of water at 30° to 100°C with the help of an electric kettle. The heating kettle has a resistance of 10 ohms. How much current is taken by the kettle? What is the amount of power consumed by the kettle? If one B.O.T. unit cost 6 pies how much will you pay? (Water equivalent of the empty kettle is neglected for calculation) [*Ans.* ·98 *pies* = 1 *pie* approx.] [C. U. 1954]

Proceed as above.

17. A 200 volt kettle boils a certain quantity of water in 15 minutes when connected alone to a socket. When two such kettles each containing the same quantity of water as was used in the first experiment, are connected in parallel to the socket, they take 16 minutes to boil. What is the resistance of the wiring from the mains to the socket?

Energy consumed by the kettle $= iV = \frac{V^2}{R}$, where i is the current, V the voltage and R the resistance of the coil inside the kettle.

Here $V = 200$ volts $\therefore \frac{200 \times 200}{R} = 500$ or $R = 80$ ohms.



(a) Fig. 80 (b)

From fig. 80 (a) and fig. 80 (b) in the two cases we have

$$H = \frac{i_1^2 R \times 15 \times 60}{J} = \frac{i_2^2 R \times 16 \times 60}{J}$$

since the heat developed in the two cases is the same.

Here H is the heat developed in the two cases and J, Joule's equivalent and i_1 and i_2 are respectively the currents flowing through the coil of wire in the two cases.

$\therefore \quad \frac{i_1^2}{i_2^2}=\frac{16}{15}$ or $\frac{i_1}{i_2}=\frac{4}{\sqrt{15}}=\frac{4}{3\cdot 87}$ But from fig. 72 (a) $i_1=\frac{E}{R+r}=\frac{E}{80+r}$

from fig. 72 (b) $2i_2=\frac{E}{R^1+r}=\frac{E}{\frac{R}{2}+r}=\frac{E}{40+r}$

or $i_2=\frac{E}{80+2r} \quad \therefore \quad \frac{i_1}{i_2}=\frac{80+2r}{80+r}=\frac{4}{3\cdot 87} \quad \therefore \quad r=2\cdot 78$ ohms

Here E is the voltage across the mains M, r the resistance of the wiring rom the mains to the socket and r' the equivalent resistance in the second case.

18. Two resistances 20 and 40 ohms respectively can be connected to a source f *e.m.f.*; compare the rate of heat production in the two resistances when they re (a) connected in series and (b) connected in parallel. [C. U. 1957]

Case (a). When the resistances are connected in series, the total resistance $=20+40=60$ ohms.

Heat in joules per sec $H_1=\frac{E^2}{60}$ where E is the applied E.M.F.

Case (b). When the resistances are connected in parallel, the equivalent sistance R′ is given by $\frac{1}{R'}=\frac{1}{20}+\frac{1}{40}$, whence $R'=\frac{20\times 40}{20+40}=\frac{20\times 40}{60}=\frac{40}{3}$ ohms.

Heat in joules per sec. $H_2=\frac{E^2\times 3}{40}$

$$\therefore \quad \frac{H_1}{H_2}=\frac{40}{60\times 3}=\frac{2}{9}$$

Again let i be the current through the resistances when connected in series.
Heat in joules per sec. in 20 ohms, $H'_1=i^2\times 20$
... ... 40 ohms $H'_2=i^2\times 40$

$$\therefore \quad \frac{H'_1}{H'_2}=\frac{20}{40}=\frac{1}{2}$$

Case (c). Let V be the pot.difference between the ends of the resistances when onnected in parallel.

Heat in joules per sec. H''_1 in 20 ohms $=\frac{V^2}{20}$

... ... 40 ohms $=\frac{V^2}{40}$

$$\therefore \quad \frac{H''_1}{H''_2}=\frac{40}{20}=2:1$$

CHAPTER IX

THERMO-ELECTRICITY

87. The Seebeck Effect: If any two dissimilar metals, say copper and Iron are soldered together at both ends to form a couple, known as the thermo-couple and if one of the junctions is heated and the other kept cold, an electric current called thermo-electric current will be seen to flow round the circuit. This phenomenon was discovered in 1821 by Seebeck and so it is known as Seebeck Effect.

In the following list called Seebeck or thermo-electric series, a current flows across the cold junction from one occurring later in the series than the other.

87(a). Thermo-electric or Seebeck Series :—

Bismuth	Silver
Copper	Iron
Lead	Antimony

Thus in a thermo-couple made of iron and copper current flows from Iron to Copper through the cold junction B (Fig. 81) when the junction A is heated. If junction B be heated and A kept cold, a current flows in the opposite direction. Therefore a current will flow as long as there is a difference in temperature between the two junctions. The electromotive force developed in the circuit, which produces thermo-electric currents is known as thermo-electric E. M. F. or thermo-electric force.

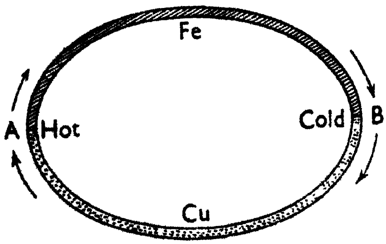


Fig. 81

87(b). Origin of Thermo-electric E. M. F.: The electromotive force producing the thermo-electric current may be supposed to exist at the junctions of the two metals. If the temperatures of the junctions are the same, the E.M.F.s acting at the junctions are equal and act in opposite directions and consequently the resultant E.M.F. is zero and no current is produced. But if one of the junctions is at a higher temperature, the E.M.F. at the junction whose temperature is higher will have a higher E. M. F. and consequently there is a resultant E. M. F. acting in the circuit and due to which a current is produced.

The existence of thermo-electric $e.m.f.$ is also explained by the movement of the 'free' electrons inside a conductor. If the junctions of the couple are at different temperatures the rates of diffusion of the electrons across the junctions differ with the result that one metal becomes negatively charged and the other positively charged when a state of dynamic equilibrium is reached.

The result is that the junction potentials will not be equal (except when the temperatures of the junctions are equal) so that there will be a resultant e.m.f., in the circuit equal to the difference between the junction potentials.

88. Seebeck's Experiment : In one of Seebeck's early experiments the ends of a copper bar Cu were bent as shown in Fg. 82 and soldered to a bar of bismuth Bi. The plane of this thermocouple was placed in the magnetic meridian, and a pivoted magnetic needle was supported near to the middle of the circuit. When the junction A was heated, B remaining cold, the n pole of the needle was deflected towards the east indicating that an electric current flowed from copper to bismuth across the cold junction which was in accordance with Ampere's swimming rule.

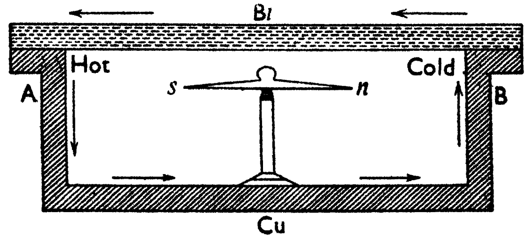


Fig. 82

89. Some Fundamental Laws :

(1) *Laws of Intermediate Metals* : The thermo-electric E.M.F., E across a junction i.e., the junction potential, for metals A and D at any given temperature is the sum of the E.M.F's across junctions A-B, B-C and C-D at the same temperature.

Thus we have $E_A^D = E_A^B + E_B^C + E_C^D$

(2) *Law of Intermediate temperatures* :—The E.M.F. for a couple with junctions at temperatures T_1 and T_3 is the sum of the $e.m.f's$ of two couples of the same metals, one with junctions at T_1 and T_2, and the other junctions at T_2 and T_3.

Thus we have $E_1^3 = E_1^2 + E_2^3$

These laws may be verified experimentally or may be shown to follow from the laws of thermodynamics.

90. Neutral Temperature and Thermo-Electric Inversion : If the temperature of one of the junctions of a thermo-electric couple be kept at 0°C and that of the other be gradually raised, the resultant E. M. F. of the couple will increase and attain a

maximum value at a particular temperature t_n of the hot junction. The temperature t_n of the hot junction at which the E. M. F. is maximum is called the **Neutral Temperature**. If the temperature of the hot junction be still further increased, the E.M.F. will begin to diminish at first slowly, then rapidly and finally vanish at t_4 (Fig. 83). If now the temperature of the hot junction be raised above the temperature t_4 for which the E. M. F. is zero, the E.M.F. is reversed at t_5 and the current round the circuit flows from Iron to Copper through the hot junction. This reversal of the E.M.F. and consequently of the direction of current is what is known as **Thermo-electric Inversion.**

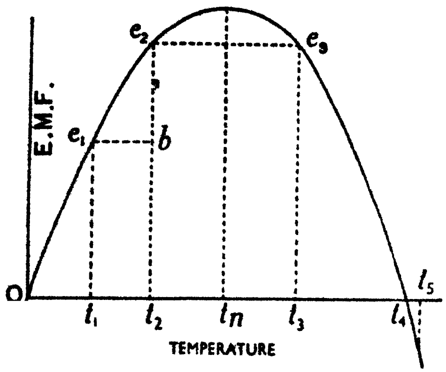


Fig. 83

If the temperatures of the junctions of the couple are equally distant on either side of the neutral temperature, *e.g.*, t_2 and t_3, the E. M. F. is zero.

The temperature t_4 of the hot junction at which the E. M. F vanishes is also called the **temperature of inversion** for the resulting E.M.F. changes sign as the temperature is further raised.

91. Thermo-electric Diagram : The variation of the E. M. F. of a couple with temperature can be graphically represented by a curve in which the ordinates represent the E. M. F., and the abscissae, the temperature of the hotter junction, the cold junction being kept uniformly at 0°C. The resulting curve is a parabola and is known as the thermo-electric curve or diagram (Fig. 83). The E.M.F. of the couple is expressed by $E=bt+ct^2$, where t is the temperature of the hotter junction, the other junction being kept at 0°C.

If the two junctions are at t_2°C and t_1°C($t_2>t_1$) respectively,

$$E=(t_2-t_1)\left[b+2c\left(\frac{t_2+t_1}{2}\right)\right], \text{ where } b \text{ and } c \text{ are constants.}$$

The E. M. F. is zero when either (1) $t_2=t_1$ or (2) when $\frac{t_2+t_1}{2}=-\frac{b}{2c}$(neutral temperature).

Proof : By law of successive temperatures

$$E_{o}^{t_2} = E_{t_1}^{t_2} + E_{o}^{t_1} \quad \text{or} \quad E_{t_1}^{t_2} = E_{o}^{t_2} - E_{o}^{t_1}\text{; Hence in above case}$$

$$E = (bt_2 + ct_2^2) - (bt_1 + ct_1^2) = b(t_2 - t_1) + c(t_2^2 - t_1^2)$$

$$= (t_2 - t_1)\left[b + c(t_2 + t_1)\right] = (t_2 - t_1)\left[b + 2c\left(\frac{t_2 + t_1}{2}\right)\right]$$

where b and c are constants.

91(a). Alternative Method : Another method is generally adopted for shewing the relationship between the E. M. F. and temperature of the junctions of the couple.

Instead of E. M. F. against the temperature of the hotter junction, the rate of change of E. M. F. with temperature or rather the thermo-electric power of all the metals using *lead* as one element of the couple is plotted against temperature, the curve for any pair of metals will be a straight line. The curve thus obtained is known as the **thermo-electric diagram,** the ordinate of each point on the line being the **thermo-electric power** o the metal at each temperature.

92. Thermo-electric Power : The thermo-electric power for two metals is defined as the rate of change of the electromotive force acting round a couple formed of the two metals with the change of the temperature of one junction.

The value of $\frac{dE}{dt}$ at any temperature is given by the slope of the tangent to the thermo-electric curve at the temperature and is usually expressed in microvolts.

At the neutral temperatures t_n, $\frac{dE}{dt} = 0$. We have $E = bt_n + ct_n^2$,

Hence $\frac{dE}{dt} = b + 2ct_n$ or $o = b + 2ct_n$ *i.e.,* $tn = -\frac{b}{2c}$

the temperature of inversion $2t_n = -\frac{2b}{2c} = -\frac{b}{c}$

If *lead* is taken as the standard metal in thermo-electric work (Thompson effect being zero in *lead*), the thermo-electric power of any metal forming a couple with *lead* is the E.M.F. per degree centigrade at a certain temperature of one of the junctions, the other junction being kept uniformly at 0°C.

The variation of the thermo-electric power p with temperature t is a straight line ; it is consequently more practical to classify thermo-electric properties by reference to $p-t$ values than by reference to $E-t$ values.

Note : Since the relation between $\frac{dE}{dt}$ and t is linear we may write

$$\frac{dE}{dt}=b+c't$$

where c' is the slope of thermo-electric line and b the intercept of the line on the ordinate. Here b and c' are constants of the couple.

The *e.m.f.* generated in the couple with junctions at $t°c$ and $0°c$ is given by

$$\int_0^t \frac{dE}{dt}\cdot dt=\int_0^t (b+c't)dt= \quad \therefore \quad E=bt+\frac{c'\times ct^2}{2}=bt+ct^2. \text{ Here } c \quad \frac{c'}{2}$$

The relation $E=mT^n$ is very often used for the E. M. F. in a thermo-couple where T is the absolute temperature of the hot junction and m and n are constants.

93. Calculation of the E. M. F. due to a couple : Consider a couple consisting of lead and another metal with junctions $t°C$ and $0°C$. The E. M. F. due to the couple is expressed by $E=bt+ct^2$

By differentiation, we get $\frac{dE}{dt}$ or p which is called the thermo-electric power of the metal with respect to *lead* at the temperature $t°C$ and is expressed in microvolts per $1°C$.

Now if the values of $\frac{dE}{dt}$ at different temperatures be plotted on a squared paper, a straight line is obtained. Let it be represented by AB.

To calculate the E. M. F. due to the couple when the junctions are at temperatures t_1 and t_2 consider a point A, on the line AB and draw an ordinate At_1 representing the thermo-electric power at the temperature t_1. If another ordinate be drawn at temperature t' greater than t_1 by a very small quantity dt then the area of the strip At'

$$=\frac{dE}{dt}.dt = dE$$

where $\frac{dE}{dt}$ is the average thermo-electric power between the temperatures t_1 and t' (Fig. 84).

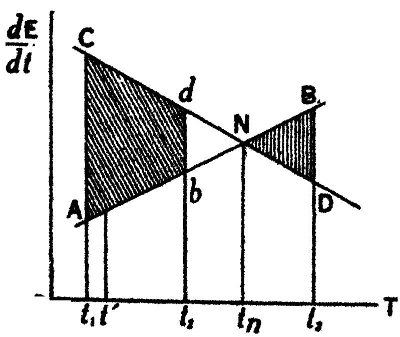


Fig. 84

Thus the area of the portion At_2 is equal to sum of all such smaller strips between At_1 and bt_2 and represents the total E.M.F.

due to the couple when the junctions are at temperatures t_1 and t_2, which is equal to the product of the average thermo-electric power and the difference in temperature, *i e.*,

$=\frac{1}{2}(At_1+bt_2)(t_2-t_1)=\frac{1}{2}(p_1+p_2)(t_2-t_1)$ where p_1 and p_2 are the thermo-electric powers at t_1 and t_2 respectively.

Now if another couple be used (**lead** being one of its elements) with junctions at t_1 and t_2 and if values of $\frac{dE}{dt}$ and temperatures are plotted as before on the same squared paper, the thermo-electric line CD is obtained. Then according to the Law of Intermediate Metals, the E. M. F. of the couple of the two Metals AB and CD with junctions at t_1 and t_2 is represented by the shaded area $AbdC$ and is equal to $\frac{1}{2}\{(q_1-p_1)+(q_2-p_2)\}(t_2-t_1)$ where p_1 and p_2 are the thermo-electric powers of the metal AB and q_1 and q_2 are the thermo-electric of the metal CD at temperatures t_1 and t_2 respectively.

If the temperature of the hotter junction gradually increases, the E. M. F. of the couple as represented by the shaded portion gradually increases until the neutral temperature t_n is reached.

At a higher temperature, say t_3, the total E. M. F. is equal to the difference of the shaded areas on the left and the right of N. Thus when the temperature of one junction passes the neutral temperature, the E. M. F. begins to decrease. If the temperatures of the junctions are equidistant from N, the E. M. F. of the couple becomes zero.

94. Peltier Effect : In Seebeck effect we have observed that when the junctions of a Copper-Iron couple are at different temperatures, a current flows from Cu to Fe through the hot junction (Fig. 81). Now if the junctions are kept at the same temperature and if a current be made to pass round the circuit (Fig. 85) in the same direction as the current produced in Seebeck effect, the junction which was at a higher temperature in Seebeck experiment will now appear to be cooled and the other junction heated. Thus due to the passage of a current round the circuit heat may be supposed to be transferred from the hot to the cold junction.

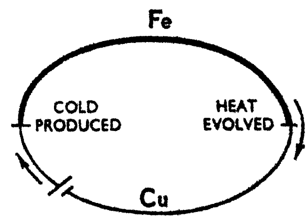


Fig. 85

If the direction of the current in the couple is reversed, opposite thermal effects are produced. This **reversible thermal**

effect at the junction of two different metals when current is passed through it, is called **Peltier Effect.**

Thus the effect of heating and cooling of the junctions of two dissimilar metals due to the passage of a voltaic current round the couple is known as the **Peltier Effect** and the energy generated or absorbed at the junction of two metals traversed by a unit current in unit time is called **Peltier coefficient**.

In a thermo-couple due to the combined action of both the Seebeck and Peltier effects the junctions will gradually be brought to a uniform temperature.

The heat absorbed or generated at a junction by the passage of a current i in t secs $= Pit$ where P is the Peltier coefficient.

95. Peltier and Joule's effects compared: In **Joule's effect** the heat developed in a circuit of resistance R when traversed by a current i in t secs. is equal to i^2Rt. The amount of heat developed is proportional to the square of the current and therefore independent of the direction of the current. In Joule's effect the circuit is always heated but never cooled. It is an *irreversible* process.

But in Peltier effect, the junction is heated or cooled according to the direction of the current since the amount of heat developed at any junction is merely proportional to the strength of of the current. Hence it is a *reversible* process.

In a circuit containing a single thermo-junction the total energy appearing as heat due to Joule and Peltier effects is equal to JH. That is $JH = i^2Rt + Pit$, where $J = 4{\cdot}2 \times 10^7$ ergs.

Similarly when the current is reversed, $JH_1 = i^2Rt - Pit$

Therefore $\quad J(H - H_1) = 2Pit \quad$ or $\quad P = \dfrac{J(H - H_1)}{2it}$.

95(a). Experiment: A current of known strength i is made to pass through a couple of which the junction at which the heat is developed is immersed in water contained in a calorimeter of which the water-equivalent is known. The heat H developed when the current is passed in one direction for t secs is calculated and also the heat H_1 is calculated when the current is sent in the reversed direction. Thus from the knowledge of H, H_1, i, t and J the value of the Peltier coefficient P can be found out.

96. Seebeck and Peltier Effects—Explanation: In Seebeck effect a current flows round a couple when its junctions are at different temperatures. To maintain the current, some energy must

be required and it is the heat energy which when applied to the junction supplies the energy necessary for the maintenance of the current.

At the junction heat energy is converted into electrical energy and consequently the existence of E. M. F. at the junction has been suggested.

Peltier discovered this E. M. F. at the junction and so it is known as Peltier co-efficient. If the current flows round the couple in the direction in which the E. M. F. tends to drive it heat will be absorbed from external source and converted into electrical energy. But if it flows in the opposite direction the electrical energy will be converted into heat. Now if the current be made to flow round the circuit in the directions in which the E. M. F. tends to drive it, by a cell heat will be absorbed from the external source and in the absence of any external source heat will be absorbed from the metal itself and consequently the junctions will be cooled.

Thus the production of a thermo-electric current is accompanied by a transfer of heat from the hot to the cold junction and the thermo-electric circuit may be regarded as heat engine in which the heat taken in at the higher temperature is not wholly given out to the condenser at the lower temperature, for there is a loss of heat due to Joule's effect and conduction along the wire which may be minimised by making the resistance of the circuit as small as possible and the wire very long.

Thus the Peltier effect is the only heat effect in the circuit and is *reversible*.

If we compare the circuit in which the heat effect is completely reversible to a perfectly reversible heat engine the resultant E. M. F. in the couple would be proportional to the difference of temperature of the junctions, which is contrary to experimental results.

To explain this discrepancy, Thomson was led to infer that in a thermo-electric circuit there must be other reversible thermal effects due to the current besides Peltier effect. He concluded from his experiments that the E. M. F. in the circuit is not confined to the junction as suggested by Peltier but exists between parts of a metal at different temperatures.

97. Demonstration of Peltier Effect: Rods of Antimony and Bismuth are joined as shewn in the Figure 86. The two junctions are completely surrounded by bulbs of the same size and connected together by a short capillary tube containing a thread

of coloured alcohol. On passing a current through the junctions by a battery, the Joule effect is the same in each bulb and produces no effect on the position of the index, but the Peltier effect causes the generation of heat in one bulb and absorption of heat in the other and the effect is shown by the motion of the index from the bulb in which a greater quantity of heat is liberated. Reversal of the current causes the index to move in the opposite direction.

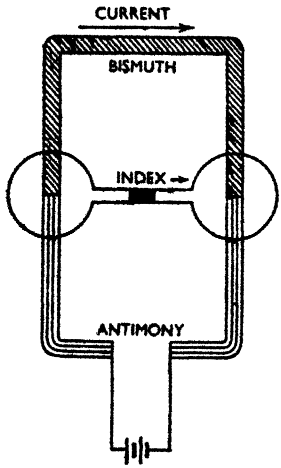


Fig. 86

98. Application of Thermodynamics: Since Peltier effect is *reversible*, Lord Kelvin applied the laws of a reversible heat engine to thermoelectric circuit.

The thermo-electric circuit is in reality a heat engine with source at one temperature T_2 and refriegerator at lower temperature T_1 and that the ratio of the heat absorbed at T_2 to that given up at T_1 should be the same as that of T_2 to T_1 where T_2 and T_1 are the absolute temperatures.

In deriving reversible conditions for a thermo-couple, which are essential in the cycle of operations, the Joule heating effect, an irreversible energy change, is made negligible by considering a small flow of current.

Let π_2 be the Peltier coefficient at the hot junction and π_1 at the cold one; the *e.m.f.* from B to A at the hot junction is π_2 and from B to A at the cold junction is π_1. The resulting *e. m. f.* $\pi_2 - \pi_1$ volts tend to send the current in the direction of the big arrow from A to B at the cold junction.

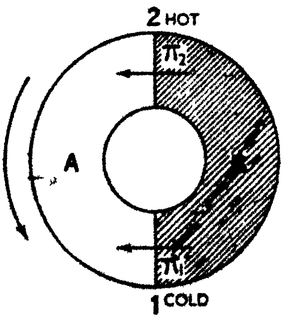


Fig. 87

On carrying a charge q round the circuit, the heat absorbed at the hot junction T_2 is $\pi_2 q$ measured in absolute units, and that given up at cold junction T_1 is $\pi_1 q$, where π_2 and π_1 are the Peltier coefficients for the junctions of the couple at T_2 and T_1 respectively.

Hence $\frac{\pi_2 q}{\pi_1 q}=\frac{T_2}{T_1}$ or $\frac{\pi_2}{\pi_1}=\frac{T_2}{T_1}$. Therefore, $\frac{\pi_2-\pi_1}{\pi_1}=\frac{T_2-T_1}{T_1}$

If $\pi_2-\pi_1=e$, the total electromotive force in the circuit, then

$$e=\pi_1\left(\frac{T_2-T_1}{T_1}\right). \quad \therefore \quad e \propto (T_2-T_1) \quad \ldots \quad \ldots(1)$$

assuming the temperature T_1 of the cold junction to be kept constant so that π_1 is also constant.

The exp. (1) suggests that the total E. M. F. in the circuit is directly proportional to the temperature difference of the junctions, which is contrary to experimental evidence.

Obviously then, e is not proportional to T_2-T_1. So there must be another reversible effect which was suggested by Lord Kelvin.

According to him, Peltier effect is not the only source of E. M. F. in the circuit but there is another source of E. M. F. known as **Kelvin** or **Thomson effect** existing between the different parts of metal at different temperatures.

99. Thomson's Effect : When a current traverses a conductor of which the temperature varies from point to point, heat is absorbed at some part of it when the current flows in a certain direction and liberated at the same part when it flows in the opposite direction. In the case of a copper rod unequally heated, heat will be absorbed if the current flows from the colder to the hotter parts and given out from the same part of the rod if the current flows from hotter to colder parts. The effect is called *Thomson's positive effect* (Fig. 88).

The opposite effect is observed in Iron. In the case of iron, the current gives out heat as it passes from colder to hotter parts and absorbs heat as it flows from hotter to colder parts. This effect is called *Thomson's negative effect.*

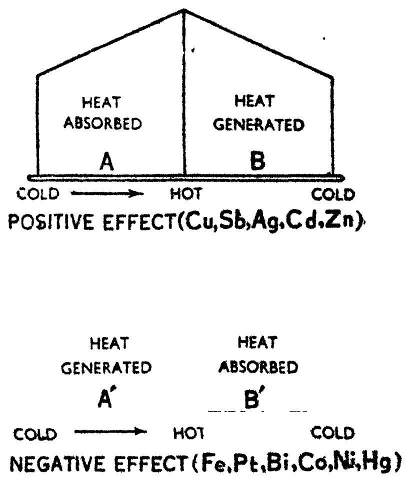


Fig. 88

This reversible absorption or production of heat in an unequally heated conductor due to flow of current is Thomson's Effect.

These two effects, both positive and negative have been explained by Thomson by considering the existence of an electromotive force between different parts of the same metal when at different temperatures.

It has also been found that the resultant E. M. F. in a thermo-couple is the algebraic sum of the Peltier and Thomson E. M. F.s and that the variation of the E. M. F. with temperature has been explained by considering these two effects.

100. Demonstration of Thomson Effect: The adjoining Figure 89 illustrates the arrangement for demonstrating the Thomson effect.

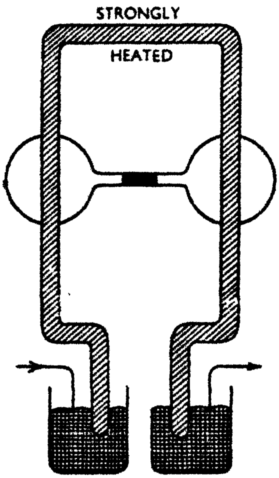


Fig. 89

A metal rod is bent so that it may be strongly heated in the middle while the ends are kept at a low temperature by immersion in mercury bath.

The middle portions of the two limbs of the bent rod are surrounded by bulbs of the same size and connected together by a short capillary tube containing coloured alcohol.

On passing a current through the rod from the low to high temperature in one half and from high to low temperature in the other half, the Thomson effect causes the absorption of heat in one section and liberation in the other, the Joule's heating effect remaining the same in each section.

The difference of the heat generated in the two parts is detected by the movement of the liquid index.

101. Thomson Coefficient: The **Thomson coefficient** σ is defined as the heat absorbed in Joules when a coulomb of electricity moves from one point to another point in a metal 1°C higher in temperature.

The coefficient σ is thus the electromotive force due to unit difference of temperature between two points of the substance and is expressed in microvolts per degree centigrade.

The total e.m.f., e between points in the substance at temperature T_1 and T_2 is expressed as $e = \int_{T_1}^{T_2} \sigma dT$. The value of σ for

is practically zero and for copper, antimony etc. it is positive and for metals such as iron, bismuth etc. it is negative.

Since the Thomson coefficient for lead is zero, the metal lead is taken as the reference or standard in thermo-electric diagrams.

102. Thermodynamics of Thermo-Electric circuits :

(A). Expressions for Thermo-electric Power : Let us consider a couple formed of two metals A and B (Fig. 90) having positive Thomson coefficient σ_a and σ_b with $\sigma_a > \sigma_b$ and with junction at absolute temperatures T and $(T - dT)$. or T_2 and T_1 respectively.

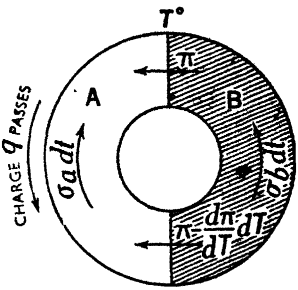


Fig. 90

Let π and $(\pi - d\pi)$ be the Peltier coefficients at the hot and cold junctions at T_2 and T_1 respectively. Neglecting Joule heating effect the net *e. m. f.* in the circuit is given by

$$e = \pi - (\pi - d\pi) - \int_{T_1}^{T_2} (\sigma_a - \sigma_b) dT$$

This may written in the form

$$e = \int_{T_1}^{T_2} \frac{d\pi}{dT} . dT - \int_{T_1}^{T_2} (\sigma_a - \sigma_b) dT$$

Differentiating both sides, we have $\dfrac{de}{dT} = \dfrac{d\pi}{dT} - (\sigma_a - \sigma_b)$ (1)

This is the expression for the Thomson's thermo-electric power.

(B). Expression for Peltier and Thomson Coefficients : Let us suppose that q coulombs of electricity flow round the couple consisting of metals A and B, having junctions at T_2 (T°) and T_1 $(T - dT)$ in the direction shewn in the Figure (90).

Then according to Peltier effect,

heat absorbed at the hot junction $= \pi q$ joules or $\pi_2 q$ joules ; heat rejected at the cold junction $= (\pi - d\pi) q$ joules. or $\pi_1 q$ joules, where π_2 and π_1 are respectively the Peltier co-efficients at the temperatures of the hot and cold junctions.

According to Thomson effect,

heat liberated along A $= \sigma_a dT q$ joules

heat absorbed along B $= \sigma_b dT q$ joules.

Since the energy changes are all reversible, according to 2nd Law of Thermodynamics and the principle to **entropy** $\int \frac{dQ}{T}$ for all parts of the cycle must be zero and hence, after cancelling q

$$\frac{\pi_2}{T_2} - \frac{\pi_1}{T_1} - \int_{T_1}^{T_2} \frac{\sigma_a - \sigma_b}{T}.dT = 0$$

Differentiating $\frac{d}{dT}\left(\frac{\pi}{T}\right) = \frac{\sigma_a - \sigma_b}{T}.$ (3)

Hence $\sigma_a - \sigma_b = T. \frac{d}{dT}\left(\frac{\pi}{T}\right).$

Substituting this value of $\sigma_a - \sigma_b$ in (1)

$$\frac{de}{dT} = \frac{d\pi}{dT} - T\frac{d}{dT}\left(\frac{\pi}{T}\right) = \frac{\pi}{T}$$

$$\therefore \pi = T.\frac{de}{dT} = T(b + 2ct) = (273 + t)(b + 2ct)$$

since $e = b + 2ct^2$ or $\frac{de}{dT} = b + 2ct$ and $\frac{d^2e}{dt^2} = 2c$

Substituting this value of $\frac{\pi}{T}$ in (3) we have $\frac{d^2e}{dT^2} = \frac{\sigma_a - \sigma_b}{T}$

or $\sigma_v - \sigma_a = T.\frac{d^2e}{dT^2} = T.2c = (273 + t).2c$

103. Experimental measurement of Thermal E. M. F.s.: A method based on simple use of potentiometer has been already described in Article 76. More accurate measurement of thermal *e.m.f.* can be made by using a standard cell. Figure 91 shows the necessary electrical circuit. Two adjustable resistances R and R_1 are used in series with battery E and potentiometer wire AB. The sum of R and R_1 is such that potential drop along AB is comparable with that of the thermo-couple. The standard cell is placed in series with a high resistance R′ across R through key K_2 to avoid large current from it. Keeping K_1 and K_2 closed R and R_1 are so

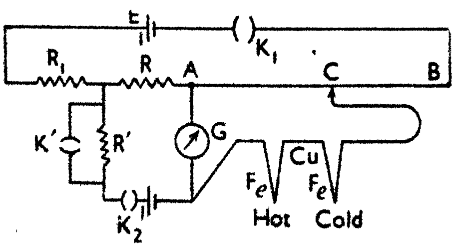


Fig. 91

adjusted that the galvanometer G gives no deflection. The potential drop across R is, therefore, equal to *e.m.f.* of standard cell in open circuit, as this cell supplies no current. At balance point, the key K′ may be closed to short-circuit R′. Then keeping K_2 and K′ open, a point C is found on the potentiometer wire for no deflection of the galvanometer. Then *e.m.f.* of the thermo-couple is equal to potential difference between A and C.

Let E_s be *e.m.f.* of the standard cell, the fall of potential across R is E_s from first observation, so that current in the main circuit is E_s/R. If r be resistance of potentiometer wire, then potential drop across it is $\frac{E_s}{R} \times r$, and potential drop per unit length is equal to $\frac{E_s r}{R \times AB}$. Hence *e.m.f.* of the thermo-couple, required

$$= \text{Potential difference across AC} = \frac{E_s . r}{R} \cdot \frac{AC}{AB}.$$

104. Thermo-couple Instruments : Of all the thermo-electric effects Seebeck effect is utilised in the measurement of temperature by the following instruments.

(A). Thermopile : The thermo-electric effect is used for measurement of a very feeble rise of temperature produced by radiant heat. A thermo-couple consisting of antimony and bismuth can produce a comparatively large *e.m.f.* for a small difference of temperatures between their junctions. For the increase of sensitiveness, a number of such couples are joined in series and the whole combination is called a **thermo-pile.** The two dissimilar metals antimony and bismuth are taken in the form of thick bars (Fig. 92) to reduce the resistance of the system. The free ends of the thermopile are joined to a galvanometer whose resistance should be equal to that of the thermopile. One set of junctions, J_2 at one side is covered by a brass cap to keep the temperature of these junctions at constant temperature. The other set of junctions, J_1 at the other side is exposed to heat radiation to be measured. The *e. m. f.* and therefore the galvanometer deflection produced is nearly proportional to the radiant heat energy incident on the thermopile. A difference of temperature of ·001°C can be easily measured.

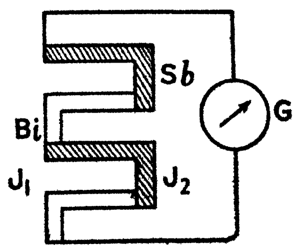


Fig: 92

(B). Thermo-ammeter : Several forms of thermo-galvanometers are in use. The form used by Fleming resembles two glass test

tubes one inside the other with their rims fused together. The space between the two tubes is exhausted and contains a thin constantan wire the ends of which are connected to two thick wires passing up through the bottom of the inner tube A couple consisting of Bismuth and Tellurium has one junction soldered to the middle point of the constantan wire, the other ends being connected to the galvanometer by two other wires passing up through the inner tube.

The galvanometer scale is calibrated by sending known currents through the constantan wire and heating the junction and the instrument is used for measuring a very weak direct or alternating current. It is very sensitive.

(C) Boy's Radio-micrometer : In this instrument the thermo-electric couple and the galvanometer are combined to give the desired effect. Two small bars, one of Antimony (A) and the other of Bismuth (B) [Fig. 93] are attached at their lower ends to a copper disc covered with soot, the upper ends of the bars being connected through a loop of wire which is suspended in the field of a powerful magnet by a quartz fibre.

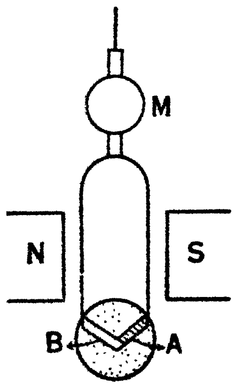


Fig. 93

The radiation to be detected falls on the copper disc, heats the junction and produces a current in the coil which is turned through an angle observed by the deflection of a spot of light in the mirror M attached to the coil.

This instrument may be made highly sensitive so that heat radiations from a candle flame at good distance can produce large deflection of the spot of light.

105. Measurement of temperature by a Thermo-couple : The most satisfactory thermo-couple for the measurement of temperatures consists of pure platinum and an alloy of platinum and iridium. One of the junctions of the couple is maintained at a constant temperature, say 0°C and the other exposed to the temperature, to be determined.

The E. M. F. of the couple at a certain temperature of the hot junction is determined by a potentiometer and the temperature is then calculated from the expression, $E=b(t_2-t_1)+c(t_2{}^2-t_1{}^2)$,

where E is the E. M F. of the couple, the junctions being at temperatures t_2 and t_1, and b and c are constants.

In our experiment, $E=bt+ct^2$ Since $t_2=t$ and $t_1=0$

The constants are determined from the above formula by measuring the E. M. Fs. at two different pairs of known temperatures by means of the potentiometer.

The E. M. F. in a thermo-couple is given by the relation $E = mT^n$, where T is the temperature of the hot junction in absolute degrees, the cold junction being kept at 0°C and m and n are constants for the particular couple chosen.

To determine the melting point of a metal by the formula given above a suitable couple is chosen whose neutral temperature is higher than the melting point to be determined. A milli-voltmeter is arranged in series with the couple to determine the E. M. F. at the temperature of the hot junction the cold junction being kept at 0°C.

In the first experiment, one of the junctions of the couple is immersed in **ice**, the other junction being exposed to **steam**. The E.M.F. is observed in the milli-voltmeter at the temperature of steam.

The same experiment is repeated with the hot junction in **boiling sulphur** whose temperature is known, and the E. M. F. is similarly observed at the temperature of bo ling sulphur.

From the knowledge of E. M. F. and temperature T in the above two cases (when the hot junction is at the temperatures of steam and boiling sulphur) two equations are obtained from the expression $E = mT^n$ and from which the constants m and n are determined.

The hot junction of the couple is then put in the metal contained in a crucible which is heated till the metal melts and the E. M. F. at the melting point is observed in the milli-voltmeter.

Thus with the knowledge of m, n and the E. M. F., E. the temperature T is determined.

106. Electric Pyrometer : The principle on which the action of this instrument depends is the measurement of temperature from the electromotive force generated at the junctions of a thermo-electric couple when they are at different temperatures.

It has been noticed that for each E. M. F. of a thermo-couple we have two temperatures. But if the metals of the thermo-couple are so chosen that their thermo-electric lines are parallel, each E. M. F. would correspond to one temperature.

Such a couple is chosen and used in pyrometers for measurement of high temperatures.

The couple for this purpose consists of platinum and rhodium or of platinum and an alloy of platinum and rhodium are placed inside an iron or porcelain tube.

The couple is placed in series with a galvanometer or a milli-ammeter and one of the junctions is kept at constant temperature and the other at the temperature to be measured. The scale of the galvanometer is calibrated to indicate the temperature of the hotter junction up to about 2000°C. Such an instrument is usually employed for measuring furnace temperature and is known as thermo-electric *pyrometer*.

QUESTIONS

1. What is a thermo-couple ? [C. U. 1938, '44, '50

By what reasoning was Lord Kelvin led to assume that Peltier effec is not the only reversible effect in a thermo-electric circuit ?

Describe an experimental arrangement to measure the thermo *e. m. f* of a thermo-couple. [C. U. 1941, '50

2. What is Peltier effect and how can it be measured ? Two wires one o copper and the other of iron are twisted together at one end, the other end being connected to a low resistance galvanometer. Describe and explain the indications of the galvanometer, as the iron-copper junction is gradually heated to redness. [C. U. 1939

3. What is Peltier effect and how is it distinguished from Joule hea production ? [C. U. 1941, '43

How can Peltier effect be demonstrated before a large audience ? [C. U. 1936, '49

4. What are meant by Peltier and Thomson effects in thermo-electricity ? How would you demonstrate experimentally the Peltier and Thomson effects ? [C. U. 1958

5. The E. M. F. of a thermo-electric current is given by the relation $E = mT^n$, where the cold junction is kept at 0°C and T is the temperature of the hot junction in absolute degrees, m and n being constants.

Describe an experimental arrangement that will enable you to determine the melting point of some metal by the help of the thermo-couple and the above relation. [C. U. 1941

6. What do you mean by the neutral temperature for a thermo-couple ?

The *e. m. f.* in a simple thermo-electric circuit, one junction of which is heated when the other is kept at 0°C, is given by the expression, $E = bt + ct^2$ where t is the temperature of the hot junction. Determine the neutral temperature for the couple and the Peltier and Thomson effects in the circuits. [C. U. 1952

7. Give the theory of thermo-electromotive force and show that the Peltie effect is $T. \dfrac{dE}{dt}$, T being the absolute temperature of the hot junction and E, the total E. M. F. acting in the circuit. [C. U. 1948

7. Write short notes on (i) Thomson effect. [C. U. 1948
(ii) Thermo-galvanometer. [C. U. 1941, '42
(iii) Pyrometer. [C. U. 1958

EXAMPLES

1. If the thermo-electric power of iron is $1734 - 4{\cdot}87t$, and that of copper is $136 + 0{\cdot}95t$ (where t is the temperature on the centigrade scale), shew that the E. M. F. of an iron-copper couple, the terminals of which are at 0°C and 100°C = 130700. (C. U. 1916

As in Art. 93 the E. M. F. of a thermo-couple consisting of two metals, iron and copper $= \frac{1}{2}[(p_1 - q_1) + (p_2 - q_2)]\ (t_2 - t_1)$, where p_1 and p_2 are the thermo-electric powers of iron and q_1 and q_2 the thermo-electric powers of copper at temperatures t_1 and t_2 respectively.

Here p_1 at 0°C $=1734-4{\cdot}87\times0 \quad =1734$
p_2 at 100°C$=1734-4{\cdot}87\times100=1247$
q_1 at 0°C $=136 \quad +0{\cdot}95\times0 \quad =136$
q_2 at 100°C$=136 \quad +0{\cdot}95\times100=231$

the E. M. F. of the couple between 0°C and 100°C

$=\frac{1}{2}((1734-136)+(1247-231))100=\frac{1}{2}(1598+1016)100=130700.$

2. The thermo-electric power $\frac{dE}{dT}$ against lead for iron at any temperature t°C is given by $1734-4{\cdot}87t$ and that for copper by $136+0{\cdot}95t$, find the *e.m.f.* of a copper-iron couple with junctions at 20°C and 100°C. (Ans. 0·999 millivolt).

CHAPTER X

CHEMICAL EFFECTS OF CURRENT : ELECTROLYSIS

107. Introductory : The conduction of electric current in a solid conductor, say metal wire, is caused by the drift of free electrons through the conductor. When current passes through a wire, some heat is produced in the wire, and a magnetic field is also created near about it, but no change in the composition of the wire or any solid conductor is produced. This type of conduction is known as **metallic conduction.** There is, however, a class of conducting substances such as fused salts or solutions of salts, acid or alkalies in water, which permit flow of electric current through them, but the substances are decomposed by the passage of the current. The mechanism of this conduction, known as electrolytic conduction is quite different from metallic conduction.

108. Electrolysis : The process of splitting up or decomposing a liquid by passing an electric current through it, is known as electrolysis and the compounds whether fused or in solution which undergo decomposition by the current, are called **electrolytes.**

The metal terminals or plates by means of which the current is conveyed into the electrolyte are called the electrodes, of which the one leading the current into the electrolyte is called the **anode** and the other which leads the current out of the electrolyte is called the **cathode.** The substances given off at the electrodes are called **ions,** that appearing at the **anode** is called **anion,** and that at the cathode is called the **cation.**

109. Mechanism of Electrolysis : In electrolysis, the electricity is supposed to be carried through the solution by charged atoms,

called *ions.* When substances such as acids or salts get dissolved in water they become dissociated into oppositely charged ions.

Each monovalent atom of any element in the ionic state in the electrolyte carries a definite charge e equal to the charge on an electron (1.51×10^{-19} coulomb), a diavalent atom twice that charge and so on.

When a substance, say H_2SO_4 is diluted with water, some of the H_2SO_4 molecules are dissociated into H_2 and SO_4 ions. H_2 carries the positive charge and SO_4, the negative charge.

The dissociation of a molecule of H_2SO_4 may be represented by the equation,

$$H_2SO_4 = 2H^{++} + SO_4^{--}$$

where each plus and minus sign indicates respectively a positive and a negative charge of magnitude e (numerical value of the charge of an electron)

The phenomenon of separation of a molecule into oppositely charged ions is termed *ionisation.* The degree of ionisation depends largely on the concentration of the solution. It is more marked in dilute than in strong solutions.

When a current is made to pass through the electrodes dipped into the solution, positive ions H_2 are driven towards the cathode and the SO_4 ions charged negatively towards the anode.

The ions, both positive and negative on arriving at the electrodes give up their respective charges and resume their atomic condition. These streams of ions in opposite directions constitute the current.

In an electrolytic cell, charges conveyed by both cations and anions are equal in quantity. The ionic charge is in close agreement with that of an electron.

109(a). Electrolysis of water : If a current is passed through water containing a little sulphuric acid (when platinum electrodes are used) hydrogen is given off at the cathode and oxygen at the anode. But when the current is stopped by breaking the circuit the total amount of acid in the solution remains unaltered but some water has disappeared.

To explain this effect, it may be stated that sulphuric acid is at first decomposed into hydrogen and SO_4 radicals which appear at the electrodes. H_2 escapes at the cathode and the SO_4 radical being unable to exist in the free state and having no effect on the platinum electrode reacts with water to form more sulphuric acid and liberates oxygen at the anode which then escapes.

But if copper electrodes are used instead of platinum electrodes, hydrogen appears at the cathode and SO_4 radicals act on the copper anode and dissolve it with the formation of copper sulphate.

This amount of electricity is equal to 96540 coulombs.

If one gram-atom of a monovalent substance contains N atoms and if e be the quantity of charge carried by each atom through the electrolyte, then $Ne=96540$.

According to Avogadro, Perrin and others, the mean value of $N=6{\cdot}02\times10^{23}$, therefore

$$e=\frac{96540}{N}=\frac{96540}{6{\cdot}02\times10^{23}}=1{\cdot}59^{-19}\text{coulomb}=1\ 59\times10^{-20}\ e.m.u.\text{ of charge}$$

$$=4{\cdot}78\times10^{-10}\text{e. s. u. of charge.}$$

Again the charge e' carried by each atom of a diavalent substance is given by

$$Ne'=2\times96540 \quad \therefore \quad \frac{2\times965400}{N}=2e$$

Similarly for a trivalent substance the charge carried by each atom would be $3e$.

Thus the charge carried by all monovalent atoms is the same as the gramme-equivalents of all monovalent substances contain the same number of atoms. Again the charge of all divalent atoms is the same and twice the charge on monovalent atom. Similarly the charge on a trivalent atom is three times that of a monovalent atom.

These facts suggest that atoms are carriers of charges and that the smallest charge met with in electrolysis is that carried by a monovalent ion and other ionic charges are integral multiples of it.

116. Measurement of Reduction Factor of a Tangent Galvanometer : To determine the *reduction factor*, the tangent galvanometer T. G. (Fig. 96) is included in an electric circuit containing a battery B, a variable resistance R and a copper voltameter V. A current of moderate strength i is passed through the voltameter for t seconds and a smooth deposit of copper is taken on the cathode K which has been previously cleansed and washed with dilute acids, caustic soda and distilled water. The weight of the deposit is thus obtained by a balance and the reduction factor K of the tangent galvanometor which has been adjusted during the experiment to give a deflection equal to 45°, is calculated from the expression, $W=zit$, the known value of E. C. E. for copper being assumed.

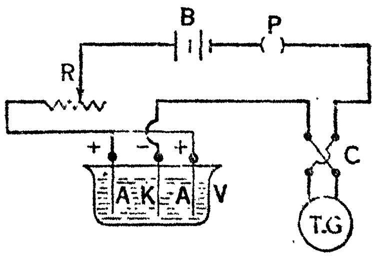


Fig. 96

Thus, $W = z.\ 10K.\ \tan\theta.t$ since $i = 10K.\tan\theta$

or $K = \dfrac{W}{10z \tan 45^\circ.t} = \dfrac{W}{10zt}$ whence K can be found out.

117. Comparison of the actions of a Voltameter and a Tangent Galvanometer : In a voltameter we do not obtain the strength of the current at any given moment, for in order to get a measurable quantity of any liberated substance, the current must be continued for some time and in this time if there be any change in the intensity, we can not record it but we can get the mean intensity. Moreover it cannot be used for weak currents as it offers a great resistance and consequently yields quantities too small for accurate measurement. The indications of the voltameter depends not only on the intensity of the current but also on the distance and size of the electrodes and the acidity of the solution.

The *tangent galvanometer* is more delicate and offers less resistance and gives the intensity at any moment. Again the indications of the tangent galvanometer are true for one instrument and for one place only, for they depend on the number of turns in the coil, diameter of the coil and on the horizontal intensity of the earth's magnetic force.

118. Electrode potential ; Nernst's Theory : When a metal, say zinc be placed in a solution containing ions of the metal, say zinc sulphate solution, a junction potential difference is set up at each metal-electrolyte boundary and is known as the *electrode potential* of the metal with respect to the electrolyte.

According to Nernst, the positive zinc ions at the surface of the metal have a tendency to pass into the solution. The force which produces this tendency of the positive ions to pass into the solution is called the solution pressure and depends upon the nature of the metal and the solution.

Again the positive ions of the solution tend to deposit themselves on the metal due to the *osmotic* pressure of the metallic ions in the solution.

If the solution pressure of the metal exactly equals the osmotic pressure of the ions in the solution, then solution will not at all take place.

But if the solution pressure of the metal zinc be greater than the osmotic pressure of the zinc sulphate solution, positively charged zinc ions will go into solution forming a layer of positive charge close to the surface of the metal, which is itself left with a layer of negative charge on its surface. This layer is known

as **electric double layer**. Across this double layer there is a potential difference which increases until it prevents any further transfer of ions and equilibrium is established. The metal is left at a lower potential than the solution.

On the other hand if the osmotic pressure exceeds the solution pressure as in the case of copper immersed in the solution of copper salt, ions will be driven to the metal and there will be double layer with positive charge on the metal and negative charge in the solution. Equilibrium is established when the metal is at a higher potential than the solution.

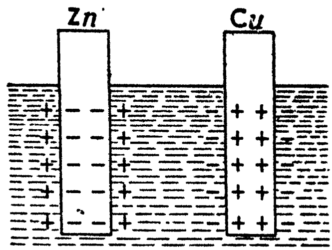


Fig. 97

When Copper and Zinc are placed inside dilute sulphuric acid (H_2SO_4) electric double layer is formed around each in no time and the p.d. between Cu and the solution is $+0{\cdot}46$ and that between the solution and Zn is $-0{\cdot}62$. Consequently the p. d. between Cu and Zn in the acid is $0{\cdot}46-(-0{\cdot}62)=1{\cdot}08$ volts.

119. Contact Difference of Potential: According to electron theory, in every conductor there is a number of free electrons which move about freely in inter-atomic spaces like the molecules of a gas. Like gas molecules the free electrons due to their motion develop electronic pressure. Electronic pressure is different for different conductors due to the difference in the number of free electrons per unit volume of the conductor.

If two different conductors are in contact, electrons of the conductor having higher electronic pressure pass into the conductor of lower electronic pressure, the former being thereby positively charged and the latter negatively charged.

The contact difference of potential between two solid conductors is thus explained.

120. Source of energy and the E. M. F. of a cell: The energy of the current maintained by a volatic cell is derived from the chemical changes occurring inside it. Let us consider the case of the Daniell cell. In this cell, the chemical changes are (1) the action of Zn on sulphuric acid with the liberation of Hydrogen (2) the action of Hydrogen on copper sulphate with the liberation of copper.

(1) Zinc acts on H_2SO_4 and forms $ZnSO_4$ and H_2 according to the following equation $Zn + H_2SO_4 = ZnSO_4 + H_2$ and a certain amount of heat is generated.

(2) Again H_2 acting on $CuSO_4$ forms H_2SO_4 according to the following equation : $H_2 + CuSO_4 = H_2SO_4 + Cu$, and generates a certain amount of heat.

Thus the total amount of heat generated by the solution of 65 grams of zinc in H_2SO_4 is obtained by adding the different amounts of heat developed in the above two chemical actions and has been found to be equal to 50100 calories.

Therefore the heat generated by the solution of one electro-chemical equivalent of zinc.

$$= \frac{50100 \times 32{\cdot}44 \times 1{\cdot}044 \times 10^{-5}}{65} = {\cdot}261$$

since E.C.E. for $Zn = 32{\cdot}44 \times 1{\cdot}044 \times 10^{-5}$

As the energy of the current is supplied by the chemical changes occuring inside the cell we have,

Heat developed in ergs. = Electric energy of the cell = Eq in C.G.S. unit where E is the E.M.F. and q, the quantity of electricity.

But since the solution of one electro-chemical equivalent of Zinc is accompanied by the transferance of 1 coulomb or ·1 C. G. S. unit of electricity we have

$${\cdot}261 \times 4{\cdot}2 \times 10^7 = E \times 10^{-1} \text{ ergs.}$$

$$E = \frac{{\cdot}261 \times 4{\cdot}2 \times 10^7}{10^{-1}} = 1{\cdot}09 \times 10^8 \text{C.G.S. unit} = 1{\cdot}09 \text{ volt.}$$

121. Primary cells and Secondary cells : In all primary cells such as Daniell cell, electric energy is obtained from the chemical actions occurring inside them. Some of the cells are reversible and others irreversible.

A cell is said to be reversible when the chemical action inside the cell is reversed and the cell brought back to the original condition by sending a current against the electromotive force of the cell by some external means. Daniell cells may or may not be reversible but secondary cells must be reversible.

A reversible primary cell may be used as a secondary cell but in practice no primary cell is used as a secondary cell. But special types of secondary cell are used for obtaining a strong current of which lead secondary cell and Edison's accumulator need a special mention.

The disadvantages of primary cells, such as the Daniell and Leclanche's cells are that some of the materials inside them are being used up and have to be replaced after a time.

122. Secondary cells or Accumulators : We know that in a simple cell in which two plates, one of copper and the other of zinc are immersed in dilute H_2SO_4 the copper plate becomes polarised, *i.e.* coated with a film of Hydrogen as the circuit is completed. Now this polarised plate will send an opposite current and can be detected by a galvanometer if the zinc plate be replaced by a fresh copper plate. The principle of secondary cells depends on this phenomenon.

In the original type of this cell as devised by M. Plante two lead plates are immersed in a solution of sulphuric acid (10% to 15%) and a current is passed through the cell. Oxygen is liberated at the anode and oxidises the surface of lead and forms PbO_2, while Hydrogen bubbles away at the cathode. Now the current is stopped and the two lead plates are connected externally by a conducting wire, a current will then pass from the oxidised plate to the unoxidised plate through the external circuit. This current is called the **discharge current.**

Now, when the discharge takes place, the oxidised plate becomes the cathode and H_2 coming in contact with it reduces it to PbO which with H_2SO_4 forms $PbSO_4$. Oxygen going to the other plate oxidises it and forms $PbSO_4$ with H_2SO_4. The current will cease when both the plates have reached the same condition.

If now, a current is passed through the cell, one plate, *i.e.* the anode is further, oxidised and the other *i.e.*, the cathode is reduced to metallic lead in the spongy form. If charging and discharging be repeated several times, the spongy lead gets thicker and thicker and the capacity of the cell increases. This process of repeated reversals of current to increase the storage capacity of the cell is called the *forming* of the cell.

Reactions (after forming process) :—

During charging :—

Positive plates—$PbSO_4 + SO_4 + 2H_2O = PbO_2 + 2H_2SO_4$

Negative plates—$PbSO_4 + H_2 = Pb + H_2SO_4$

During discharging :—

Positive plate—$PbO_2 + H_2SO_4 + H_2 = PbSO_4 + 2H_2O$

Negative plate—$Pb + H_2SO_4 + O = PbSO_4 + H_2O$

During discharge H_2SO_4 disappears and water is formed and so the density of the electrolyte slowly falls. When density falls below 1·17 the cell is said to be discharged.

In the modern form of secondary cells, such as Faure cells the lead plates have been replaced by lead grids, the spaces of which

are filled with a pasty mixture of red lead (Pb_3O_4), or litherge (PbO), dipped in dilute sulphuric acid. In some cells red lead (Pb_3O_4) is packed in the grids of the positive plate and litherge (PbO) in those of the negative plate (Fig. 98). The chemical actions are practically the same as above. The E.M.F. of the cell when fully charged is about 2·2 volts and is fairly constant throughout the greater part of the discharge. It is detrimental to allow the voltage of the cell to fall below 1·8. The resistance of the cell is small, being about ·01 ohm.

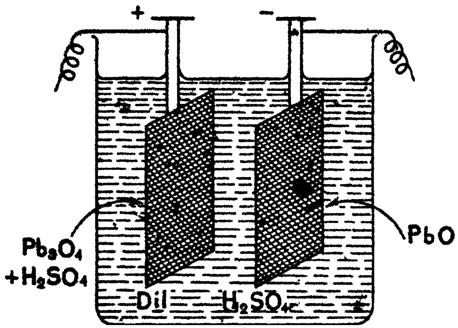


Fig. 98

123. Edison's Accumulator (Non-acid) : In this accumulator the anode is of nickel hydroxide $Ni(OH)_3$ and the cathode consists of finely divided iron mixed with a percentage of cadmium. They are supported in grids or frames consisting of perforated nickelled steel pockets. The plates are immersed in a 20% solution of caustic potash contained in a nickel-plated steel box. The E.M.F. of the cell is fairly constant and is about 1·2 vlots.

Reactions :—

During discharging :—

Positive plate—$KNi(OH)_3 = Ni(OH)_2 + KOH$

Negative plate—$Fe + 2OH = Fe(OH)_2$

During charging :—

Positive plate—$Ni(OH)_2 + OH = Ni(OH)_3$

Negative plate—$Fe(OH)^2 + 2K = Fe + 2KOH$

This cell is known commercially as a Nife Cell from the (Ni) Nickel iron (Fe) constituents.

It has several advantages over the lead accumulator. Unlike the acid in the lead accumulator, the electrolyte remains unaltered in concentration.

It takes less time to charge, is capable of maintaining a large current and has a longer life. The disadvantages of this cell are that it has a lower efficiency and has a larger drop of e.m.f. during discharge than the lead accumulator.

124. Capacity of the cell : It is the product of the total discharge current which may be safely taken from the cell and its

duration in hours. It is expressed in *ampere hours* and depends on the number and size of the plates.

Energy-capacity is expressed in *kilowatt hours* which involves both the current and the voltage.

Efficiency: It is the ratio of the discharging capacity to the charging capacity.

If the discharge takes place slowly the maximum available energy is 80% of that spent in charging.

If the cell be accidentally **short-circuited,** the discharge takes place suddenly and the cell is spoiled.

Note: The secondary cell is called a *storage cell.* It does not really store electricity but it stores chemical energy.

125. Standard Cells: Cells which have a fairly constant electromotive force and in which no polarisation occurs, are known as standard cells. **Latimer Clark Cell** and **Cadmium** or **Weston Cell** may be taken to be such reliable standards.

125(a). Latimer Clark Cell: It has been already described in Article 2 in connection with primary cells.

125(b). Standard Cadmium or Weston Cell: The cadmium cell resembles the Clark cell except that the zinc rod is replaced by cadmium amalgum and cadmium sulphate is used instead of zinc sulphate. A slight modification in the construction of the outer vessel of the cell is necessary.

Instead of using a single test tube a H-shaped glass tube (Fig. 99) is used. Into one of the limbs a certain quantity of mercury C is placed with a paste of mercurous sulphate D on it and into the other a layer of cadmium amalgam A is also kept and the quantity of them is so adjusted as to cover the tips of the platinum wires sealed into the lower ends of the two limbs. Upon mercurous sulphate D and also the amalgam A rests a saturated solution of cadmium sulphate S and to ensure its saturation, a few crystals of cadmium sulphate B are placed upon mercurous sulphate and the amalgam in the two limbs.

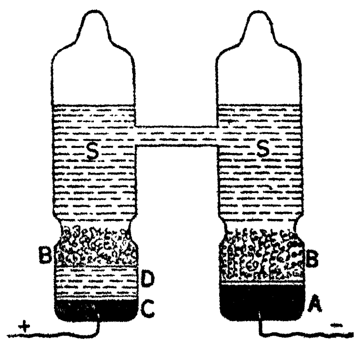


Fig. 99

The E.M.F. of this cell varies slightly with temperature and has consequently been adopted as the international standard of electromotive force.

It is expressed by

$$\text{E.M.F.} = 1{\cdot}0183 - {\cdot}0000406\,(t - 20) \text{ volt.}$$

Note : In using these standard cells great care must be taken not to allow more than a very minute current to pass through it and for this reason the cells should either be used in series with a high resistance or in a circuit when necessary.

The above cells are used as standards of E. M. F. and so to keep the E. M. F. of the cells constant no current should be taken from the cell. The cells should always be kept inside a room whose temperature is constant.

126. How to charge a battery of accumulators : The accumulators B are at first connected together in series and the two poles of the battery are connected to the similar poles of the mains through a lamp resistance and an ammeter A as shewn in Figure 100.

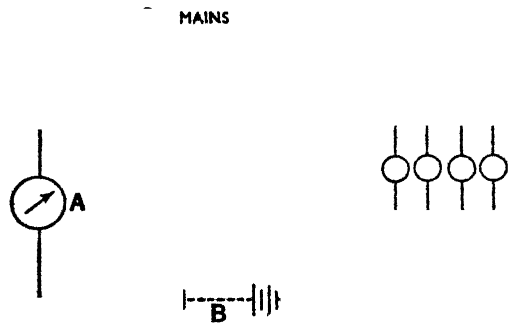


Fig. 100

The resistance of the circuit is adjusted by increasing or decreasing the number of lamps in the lamp resistance board so that the required charging current passes through the cells.

Let us consider a typical case in this connection. A 20 volt accumulator is to be charged by a current of 4 amperes from the 100 volt mains,

Since the 20 volt cells oppose the 100 volts of the main the effective voltage neglecting the resistance of the cell $= 100 - 20 = 80$ volts.

To keep the current through the cells equal to 4 amperes the resistance to be included in the circuit $= \frac{80}{4} = 20$ ohms.

If each of the lamps in the resistance board be 100 volt – 40 watts then the resistance R of each lamp is obtained from

$$i \times E = 40 \text{ watts}, \frac{E^2}{R} = 40 \text{ watts, since } i = \frac{E}{R}$$

$$i.e. \quad \frac{E^2}{R} = \frac{100 \times 100}{R} = 40 \quad \text{or } R = \frac{100 \times 100}{40} = 250 \text{ ohms.}$$

Here i is the current, E, the voltage at the terminals of a lamp and R the resistance of each lamp.

To reduce the resistance of the circuit to 20 ohms, the number of lamps n, to be used in parallel is given by

$$\frac{1}{20} = \frac{n}{R} = \frac{n}{250} \quad \text{or} \quad n = 12\cdot5$$

So the number of lamps required should be 13.

127. Polarisation in a Simple Cell : As already stated in connection with simple voltaic cell, when the copper and zinc plates of the simple cell are connected through a galvanometer, it will be found that the deflection gradually diminishes and ultimately reaches a low steady value.

This diminution in the deflection is caused by the decrease of the strength of the current in the cell.

It will also be observed that bubbles of Hydrogen accumulate on the copper plate.

This hydrogen has two effects. In the first place it reduces the effective area of the copper plate by covering a greater portion of the exposed surface of the plate. Since these bubbles of Hydrogen are bad conductors, they offer a great resistance to the passage of the current and therefore the main current is diminished to some extent.

In the second place it is, like zinc, a readily oxidisable element and behaves in a manner similar to zinc and produces an electromotive force tending to send a current through the cell in a direction opposite to that in the cell.

This E. M. F. thus set up in the cell due to deposition of Hydrogen on the copper plate is known as the **back E.M.F.** or polarisation E.M.F. and the entire phenomenon is known as **polarisation.**

To prevent polarisation, accumulation of Hydrogen is to be avoided or Hydrogen is to be removed by an oxidising agent which will oxidise Hydrogen to water.

This oxidising agents or substances used to remove polarisation are generally called **depolarisers.**

QUESTIONS

1. Explain how from the phenomenon of electrolysis, we get an idea of the atomicity of electricity. [C. U. 1936, '44, '49, '58]
Why can not a single Daniell cell decompose water continuously? [C. U. 1949]

2. State Faraday's laws of electrolysis and explain how they may be verified experimentally.
What is the reason for believing that each monovolent ion during electrolysis carries one electric charge? [C. U. 1958]

3. Write a short note on Polarisation in an electric cell. [C. U. 1939]

4. What is a "Storage cell"? And why is it so called?
Describe the working of any form of storage battery with which you have worked. [C. U. 1943]

5. How would you connect up a set of apparatus for charging the accumulator from the mains? [C. U. 1951]

6. What is an electrode potential? How does it account for the electromotive force of a cell?

7. Write a short note on "electro-chemical equivalent." [C. U. 1938]

EXAMPLES

1. To deposit one gramme equivalent of silver from a solution of silver nitrate requires a passage of 96550 coulombs. Define coulombs. Find the electro-chemical equivalent of bivalent copper ($Ag = 107{\cdot}96$, $Cu = 63{\cdot}3$). How much copper is deposited from a solution of cupric sulphate in 25 minutes by a current of 650 milliamperes? [C. U. 1910]

1 gram equivalent of silver = equivalent weight of silver in grammes
= 107·96 grammes.

$$\therefore \text{ electro-chemical equivalent of silver} = \frac{107{\cdot}96}{96550}$$

$$\frac{\text{E.C.E. of bivalent copper}}{\text{E.C.E. of silver}} \quad \frac{\text{Chemical equivalent of copper}}{\text{Chemical equivalent of silver}}$$

$$= \frac{\frac{63{\cdot}3}{2}}{107{\cdot}96} = \frac{63{\cdot}3}{2 \times 107{\cdot}96}$$

$$\text{E.C.E. of bivalent copper} \quad \frac{63{\cdot}3}{2 \times 107{\cdot}96} \times \frac{107{\cdot}96}{96550} = {\cdot}000328.$$

Since $W = izt$ $\therefore$ Mass of copper deposited

$$= \frac{650}{1000} \times {\cdot}000328 \times 25 \times 60 = {\cdot}3199 \text{ grammes nearly.}$$

2. What is meant by electro-chemical equivalent of a substance? How it is related to atomic weight and valency? To deposit one gramme equivalent of silver (from a solution of silver nitrate) requires a passage of 96000 coulombs. How much copper ($Ag = 108$, $Cu = 63$) is deposited from a solution of cupric sulphate in 10 minutes by a current of 636 milliamperes.
[Valency of copper = 2] [C. U. 1912]

E.C.E. of substance is proportional to its chemical equivalent, *i.e.*, to the ratio $\dfrac{\text{atomic weight}}{\text{valency}}$.

1 gramme equivalent of bivalent copper, *i.e.* $\frac{63}{2}$ grammes of copper is also deposited by 96000 coulombs since 1 gramme equivalent of silver is deposited by 96000 coulombs.

Hence, E.C.E. of bivalent copper $= \frac{63}{2 \times 96000}$

Now, $W = izt$ $\therefore$ Mass of copper deposited

$$= \frac{636}{1000} \times \frac{63}{96000 \times 2} \times 10 \times 60 \text{ gms.} = \cdot 1252 \text{ grammes nearly.}$$

3. Calculate how much copper would be deposited in the copper voltameter if 0·1 amp. pass through the voltameter for 1 hour.

(1 amp. per sec., liberates 0·00001035 gm of Hydrogen. Atomic weight of copper = 63). [C. U. 1928] [*Ans.* ·1173 gms. nearly]

4. A zinc and chromic acid primary cell gives about 2·25 volts. From this calculate the rise of temperature produced when one gram of zinc filings is stirred into 500 grams of dilute sulphuric acid and chromic and mixed. ($J = 4·2 \times 10^7$ ergs). [C. U. 1931]

(The c.g.s. unit of electricity deposits 0·0001038 gramme of Hydrogen and the atomic weight of zinc is 65·4. The sp. heat of the dilute acid may be taken as unity).

Electro-chemical Eq. of $Zn = \frac{65·4}{2} \times \cdot 0001038$ per C. G. S. unit quantity.

Again we know that the electric energy of the cell in C. G. S. unit = Heat generated in ergs.

Let H be the amount of heat in calories developed when one electro-chemical equivalent of *Zn* is dissolved, then

$H \times 4·2 \times 10^7 = Eq = 2·25 \times 10^8$ Here q = C.G S. unit quantity.

E = E.M.F. in C.G.S. unit. $H = \frac{225}{42}$ Cal.

Let h be the heat in calories developed when one gram of zinc is dissolved,

then $h = \frac{2 \times 225}{65·4 \times \cdot 0001038 \times 42}$ Cal.

If t be the rise of temperature we have $500 \times s \times t = h$

Here s, the Sp. heat of the acid = 1.

$$t = \frac{h}{500} = \frac{450}{500 \times 65·4 \times \cdot 0001038 \times 42} = 3°·15C.$$

5. An E. M. F. of 3 volts is required to force a current of 1 amp. through a voltameter containing acidulated water. If the work required to separate 1 gm. of hydrogen is 142 kilowatt-seconds and the electro-chemical equivalent of hydrogen is 0·00001035, find the resistance of the voltameter. [C. U. 1939]

Energy required for separating 1 gm. of H_2

$= 142 \times 10^3$ watts per sec. $= 142 \times 1000 \times 10^7$ ergs

$\therefore$ Energy for separating ·00001035 gms. of H_2

$= 142 \times 10^{10} \times \cdot 00001035$ ergs = the work done when 1 amp. passes through the voltameter for 1 sec. $= i^2Rt \times 10^7$ ergs, where i is the current in amps. passing through the resistance R in ohms of the voltameter for t secs. $= (1)^2 R \times 1 \times 10^7$ ergs

$$R = \frac{142 \times 10^3 \times 1035}{10^7} \text{ ohms} = 1·469 \text{ ohms}$$

6. A Current of 2 amperes is passed through copper sulphate solution. The area of the cathode surface is 1·5 sq. metres. Calculate the average increase in the thickness of the copper deposit per minute. (E.C.E. of copper $= \cdot 0003294$. Density of copper $= 8 \cdot 9$.) [C. U. 1936]

We have $W = izt = 2 \times \cdot 0003294 \times 60 = \cdot 039528$ gm.

But we know that $W = V.\ \rho$ where V is the volume and ρ, the density of the substance deposited.

But $V = \text{area} \times \text{thickness} = 1 \cdot 5 \times (100)^2 \times t$ (t is the thickness)

$\therefore \quad \cdot 039528 = 1 \cdot 5 \times (100)^2 \times t \times 8 \cdot 9$

$$t = \frac{\cdot 039528}{1 \cdot 5 \times (100)^2 \times 8 \cdot 9} = \cdot 2960 \times 10^{-6}$$

7. Calculate the *minimum e.m.f.* necessary, in order to decompose water given the electro-chemical equivalent of hydrogen $= \cdot 0000105$ gm./ampere. sec., the heat yield of 1 gm. of hydrogen in combining to form water $= 34500$ calories, $J = 4 \cdot 2 \times 10^7$ ergs and 1 watt $= 10^7$ ergs/sec. [C. U. 1949]

When one gram of Hydrogen combines with oxygen to form water 34500 calories are evolved or $34500 \times 4 \cdot 2 \times 10^7$ ergs of energy are liberated.

For the decomposition of water into 1 gm. of hydrogen energy must be supplied.

We know that the work done when Q coulombs of electricity pass between two points differing in potential by V volts is $QV \times 10^7$ ergs.

E.C.E. of hydrogen $= \cdot 0000105$ gm. amp. sec., *i.e.* for the passage of one coulomb of electricity $\cdot 0000105$ gm. of H_2 is deposited. Then for the production of 1 gm. of H_2, Q coulombs or $\frac{1}{\cdot 0000105} = 95238$ coulombs are required.

$\therefore$ The energy required $= 95238 \times V \times 10^7$ ergs

Hence the minimum *e.m.f.* necessary to decompose water is given by

$$95238 \times V \times 10^7 = 34500 \times 4 \cdot 2 \times 10^7$$

$$\frac{34500 \times 4 \cdot 2}{95238} = 1 \cdot 5 \text{ Volts.}$$

8. A battery of lead accumulators of *e.m.f.* 50 volts and internal resistance 2 ohms is charged on a 100 volt direct current mains. What series resistance will be required to give a charging current of 2 amperes? If the price of electrical energy is 1*d* per kilowatt-hour, what will it cost to charge the battery for 8 hours and what percentage of energy supplied will be wasted in the form of heat. [*Ans.* 23 ohms, 1·6*d*. 50%]

CHAPTER XI

ELECTROMAGNETIC INDUCTION

128. Induced Current: Oersted's experiments led to the discovery of the production of a magnetic field due to the passage of an electric current in a conducting wire. Ampere showed that a closed wire traversed by a current behaves as a magnetic shell and also deduced an expression for the intensity of the magnetic field produced by current in a wire. In 1818 Faraday discovered the converse effect to that observed by Oersted, *i.e.*, production of

current in a circuit by means of magnetic field. He found that if a magnet be moved towards a closed coil of wire (connected with a galvanometer) an instantaneous current is found to be generated in the coil, and that if the magnet be moved away from the same coil, an instantaneous current in opposite direction is produced in the coil, the momentary deflection of the galvanometer needle indicating the transient current in each case. The current disappears when the magnet is stopped and kept stationary; it exists only as long as the magnet is in motion. The E. M. F. or the current thus set up is temporary and depends on the rate of motion of the magnet. The E. M. F. is an induced E. M. F. and this temporary current is called an induced current. The phenomenon of the production of induced current is said to be due to **electromagnetic induction.**

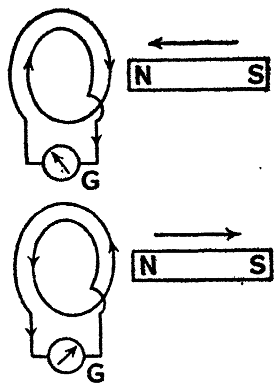


Fig. 101

If the polarity of the magnet near the coil be opposite to that in the above case, then with approaching or receding away of the magnet, the current set up in the coil flows in direction opposite to that in the above case.

Again, keeping the magnet fixed, if the coil be moved towards or away from the magnet, same effects are produced. Hence the production of current in the coil depends on the relative motion of the coil and the magnet.

Faraday also found similar effects by placing a coil of wire (called primary coil) connected with a key and battery near another coil (called secondary coil) forming a closed circuit with a galvanometer. As soon as a current is started in the primary coil, a current is found to be induced in the secondary in a direction opposite to the direction of the current in the primary coil. Again if the current be stopped in the primary coil, another current will be induced in the secondary in the same direction as that in the primary. These induced currents are also momentary.

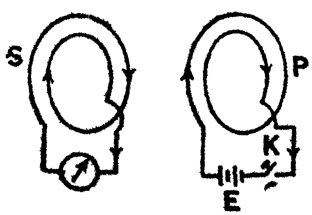


Fig. 102

129. Laws of Electromagnetic Induction: When a magnet is placed at a short distance from a coil, some of the lines of

force due to the magnet cut the coil, *i.e.*, pass through the area bounded by the coil. These lines of force or flux as they are called, are said to be linked up with the coil. As the magnet is moved towards the coil or the coil moved towards the magnet, it is obvious that the number of lines of force cutting or embracing the coil increases. When the magnet or the coil is moved away from the other, the number is decreased. In each case, an E.M.F. is induced in the coil.

If the magnet be moved slowly towards or away from the coil, the flux linked up with the coil varies by a small number, and the induced E.M.F. and hence current is found to be weak. If however, the relative motion be quick, the rate of variation of the flux is also large and the induced *e. m. f.* and hence current is found to be large.

From the above we have the following two laws of electro-magnetic induction.

First Law: Whenever the number of magnetic lines of force cutting or linked up with a coil varies, an instantaneous E.M.F. is induced in the coil.

Second Law: The magnitude of the E. M. F. induced in a coil is directly proportional to the rate of variation of the lines of force cutting the coil. Thus if N be the number of lines of force linked up with the coil at any instant, then

$$\text{Induced E. M. F.} = \frac{dN}{dt} \qquad \ldots \quad (1)$$

Note: Total number of lines of force linked with a coil of n turns is equal to the number of lines of force cutting the area enclosed by the coil multiplied by n, the number of turns of the coil.

Third Law: The induced current is always in such a direction that it tends to resist the relative motion to which the induced current is due.

Thus in the magnet and coil experiment in Article 128, when the N pole of the magnet is moved towards the coil, the induced current developed in the coil produces a magnetic field which acts on the magnet. The face of the coil towards the magnet must behave as a north pole so that it can repel and, therefore, resist the approach of the magnet, which is the cause of induction. The *current in the coil, therefore, appears anti-clockwise* to an observer looking from the side of the magnet. If the N pole be moving away from the coil, the coil must behave as a South Pole so that it can attract and thus resist the receding motion of the magnet. In this case, current will appear clock-wise to the same observer.

If the South Pole of the magnet be moved towards or away from the magnet, reverse effects will occur.

If the primary circuit carrying a current approaches the secondary circuit, then according to Lenz's Law, the direction of the induced current in the secondary will be opposite to that in the primary circuit, since by taking up such a direction, the action between the current in the two circuits will be to repel one another, *i.e.*, to resist the motion of approach of the primary circuit. Again if the primary circuit moves away from the secondary circuit the induced current will be in the same direction as that in the primary, and the action between the two currents will be to attract one another, *i.e.*, to resist the backward motion of the primary current.

130. Lenz's Law from Principle of Conservation of Energy: If the magnet with its N Pole facing the coil be slightly pushed towards the coil, a small induced current is produced in the coil. If owing to this current, the magnet is attracted the magnet should continue to move towards the coil. Hence by expenditure of very small energy in moving the magnet slightly towards the coil, the magnet is enabled to move through a large distance in reaching up to the coil. This is in clear violation of the principle of conservation of energy. Hence it is obvious that when the magnet is moved towards the coil, the induced current in the coil should be such that the magnet is repelled by the action of the current and the motion of the magnet towards the coil is resisted. The effect will be reverse if the magnet be moved away from the coil.

Consider the case of two circuits of which the primary contains a battery and key, and the secondary a galvanometer. When the secondary circuit is moved near to the primary circuit an induced current is generated in it and for the production of this current some electrical energy must have been used up. This energy is supplied by the work done by the secondary circuit in moving towards the primary circuit against the force, which is here the force of repulsion between the two circuits. From the laws of parallel currents we know that currents in opposite directions repel one another and consequently when there is repulsion between the two circuits, the direction of the induced current must be in a direction opposite to that in the primary circuit.

Again if the secondary circuit be moved away from the primary circuit, the same reasoning may be applied for determining the direction of the induced current.

130(a). Fleming's right-hand rule: The right-hand rule is a convenient method of deducing the directions of the induced

current in a conductor which is moving across the lines of force of a magnetic field. It is stated as follows :—Let the thumb and the first two fingers of the right-hand be extended out so that they are at right angles to one another. If the first finger gives the direction of the magnetic field, the thumb that of motion, then the second finger will indicate the direction of the induced current.

131. Mathematical expression for Third Law : The third law may be expressed by introducing a negative sign before dN/dt in equation (1) of Article 129, so that we have

Induced E.M.F., $E = -\frac{dN}{dt}$ (2), where negative sign indicates that the direction of the induced E.M.F. is such that it opposes the change of flux to which the E.M.F. is due.

From (2) $E = -K.\frac{dN}{dt}$, where K is a constant. suitable choice of units the value of K may be made equal to one, so that

$$\text{Induced E. M. F.} = -\frac{dN}{dt} \qquad (3)$$

132. Mathematical Proof of the relation $E = -\frac{dN}{dt}$

The value of the electromotive force due to a change of magnetic induction in a circuit may be deduced from the principle of conservation of energy.

Let AB a metal rod of length l which can slide on two straight and parallel conductors CX and DY, and let C and D be connected to a battery. Let a magnetic field of intensity H act at right angles to the plane of the closed portion CABD of the circuit, and away from it. The force acting on the rod in a direction at right angles to its length and also to the magnetic field H, is equal to Hil, where i is the strength of the current. Let the rod move through a distance δx, from AB to A_1B_1 in time δt, due to this force.

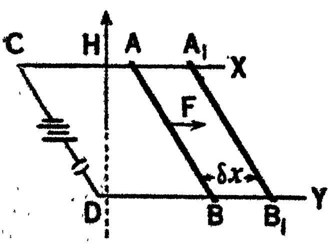


Fig. 103

Then the work done due the displacement $Hil\delta x$.

Now if E be the electromotive force of the battery maintaining the current i in the circuit energy spent in time δt is $E.i.\delta t$.

This energy is partly spent in overcoming the resistance of the circuit, the remainder being used in moving the rod.

The work done in overcoming the resistance R of the circuit is equal to $i^2R\delta t$ which is ultimately converted into heat.

Therefore we have, according to the principle of conservation of energy $Ei\delta t = i^2R\delta t + Hil\delta x$

$$i = \frac{E\delta t - Hl.\delta x}{R\delta t} \quad \frac{E - \dfrac{H \ . \ \delta A}{\delta t}}{R} \qquad \ldots(1)$$

Since $l\delta x$ is the increase in area of the circuit it is written as δA.

Again $H\delta A$ is the increase in the number of lines of force in the circuit or the change in the magnetic flux and is written as δN.

Therefore the expression (1) becomes $i = \dfrac{E - \dfrac{\delta N}{\delta t}}{R}$

Comparing this relation with Ohm's law relation for a steady circuit, namely $i = \dfrac{E}{R}$, we find that an extra E. M. F. of value $-\dfrac{\delta N}{\delta t}$ is induced in the circuit due to the variation of the lines of force linked with the closed circuit caused by the movement of the rod AB.

In the limit when δt is infinitesimally small, we have induced E. M. F., $E = -\dfrac{dN}{dt}$

133. Induced E. M. F. in a coil: Consider a coil of n turns each of area A placed normally to a field of intensity H. Then the induction or the number of lines passing normally through each turn of the coil is μAH, where μ is the permeability of the medium surrounding the coil. This is the magnetic flux through one turn. For n turns the total flux $N = \mu H.An$ lines.

If the *flux density*, *i.e.*, the number of lines passing per unit area of each turn of the coil ($B = \mu H$) varies with time t, the induced *e.m.f.* at the instant is given by

$$\frac{d}{dt}(nA.B) = e = -\frac{d}{dt}(nA.\mu H) = -\frac{dN}{dt} \text{ E. M. U.}$$

If the plane of the coil rotates through an angle θ the total flux through the coil $= nA.\mu H \cos\theta$.

133(a). Induced Current : If R be the resistance of the whole circuit in absolute units, the current in E.M.U. is given by

$$i=\frac{e}{R}=-\frac{1}{R}\cdot\frac{dN}{dt}\text{ E. M. U.}$$

133(b). Induced Charge : If the magnetic flux through a circuit of resistance R changes from N_1 to N_2, the value of the induced current at any instant is given by

$$i=\frac{e}{R}=-\frac{1}{R}\cdot\frac{dN}{dt}$$

But current is the rate of change q and hence

$$\frac{dq}{dt}=i=-\frac{1}{R}\cdot\frac{dN}{dt}\text{ or }dq=-\frac{1}{R}\,dN$$

$$q=-\frac{1}{R}\int_{N_1}^{N_2}dN=-\frac{1}{R}(N_2-N_1)$$

If R is one ohm, $q=\frac{N_1-N_2}{R}\times10^{-8}$ coulombs.

It is evident from the above expression for q, that the total quantity of electric charge induced is independent of the rate at which the effective flux changes, but is inversely proportional to the resistance of the circuit.

134. Illustrative examples of Induction ; Arago's disc : Arago's disc (Fig. 104) consists of a copper disc C, situated below a magnetised needle NS, which could be rotated rapidly about a vertical axis through its centre. The disc was mounted horizontally inside a box so that the needle was screened from air currents caused by the motion of the disc. The needle was supported on a pivot fixed

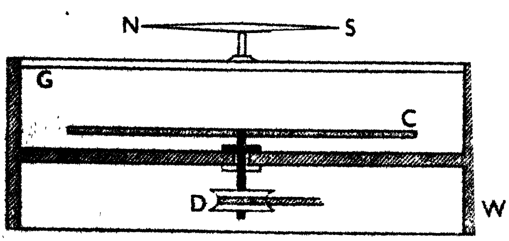


Fig. 104

glass lid of the box. On rotating the disc the magnetic needle deflected and it tended to move in the direction of rotation of If the speed of the latter were considerably increased,

the needle was found to rotate continuously. The motion of the needle was caused by induced current produced in the disc. Arago performed the above experiment before Faraday discovered how to produce induced currents. Faraday gave correct explanation of Arago's experiment.

135. Eddy Currents : Currents can not only be induced in a closed wire circuits when the magnetic flux threading them change, but also in any conducting material placed in changing magnetic field. These are called Eddy Currents.

When a mass of metal is situated near a changing current or a varying magnetic field, an electromotive force and therefore a current will be generated in it and the metal will act like a secondary circuit of a very low resistance. The E. M. F. will be very great if the rate of change of magnetic flux through the metal is considerable and the generated current will depend on the resistance of the metal. Thus the induced current in the metal due to a change in the magnetic flux is considerable and circulates round the metal in a direction determined by Lenz's Law.

135(a). Use of Eddy Currents : The action of Eddy currents is illustrated in the **damping of the oscillations** of a needle or a coil in a galvanometer, which is often a source of considerable loss of time, for the needle or the coil takes an appreciable time to come back to its position of rest after it has been deflected.

In a suspended needle galvanometer the oscillating needle is generally enclosed in a solid metal chamber and in the suspended coil galvanometer, the coil is generally wound on a sheet of metal or enclosed in a metal cylinder. The oscillations of the needle or the coil in the magnetic field create induced currents known as Foucault currents in the metal chamber or cylinder, which check the motion of the oscillating system according to Lenz's law and thus the oscillations are damped.

If the coil be not provided with a damping metal sheet or cylinder, the oscillations are damped by short—circuiting the galvanometer terminals, the induced current being generated in the coil itself.

In recent times eddy currents have been utilised to melt metals and prepare alloys. The generation of high temperature can be shown by a simple experiment. An aluminium ring is placed over a solenoid through which an alternating current is flowing. If the ring is fixed, it becomes exceedingly heated, if it is free to move it will be thrown off violently.

135(b). Eddy Currents as source of troubles : Eddy currents are often a source of great trouble in metal apparatus kept in a changing magnetic field. They cause the metal to become much heated. This

may be minimised considerably by building up the apparatus with thin metal strips insulated from each other so that currents are reduced. As for example if an iron rod forms the core of an electro-magnet carrying alternating current the iron is strongly heated. If the core is made of sheets of iron insulated from each other by paper, heating effect is greatly decreased.

136. Self-Induction : When a current passes round a coil of many turns, a number of its own lines of force passes through the coil and so a current is induced in it whose direction will be opposite to that of the steady current in the coil. The effect of this opposing current will be to weaken the original current.

Again if the current through the coil be suddenly stopped, an induced current in the same direction will flow round the coil and will tend to prolong the original current.

Such a circuit containing the coil is called an **Inductive circuit.**

The property possessed by a coil to oppose the rise or fall of current in it is known as its *self-inductance* or briefly its *inductance.*

The above phenomenon is called *Self-Induction* and the induced current set up is called an *Extra-current*. It has been found that the E. M. F. due to self-induction at *break* may far exceed the E. M. F. of the battery which produces the original current and may cause sparks.

A wire from one end of the poles of a battery is placed in contact with a file and another wire from the other pole of the battery is drawn across the file, no effect is visible. But if an electromagnet or a self-induction coil be inserted in the circuit, a series of bright sparks will be obtained as the circuit is broken at each rib of the file.

If N be the number of lines of force passing through any circuit having self-induction L when traversed by a current i c.g.s. units, then $N = Li$, when $i = 1$, $L = N$.

Hence self-inductance of a coil is defined as the lines of force linked up with the coil due to unit current flowing through it.

The induced E.M.F. due to a change in flux through the circuit is given by $e = -\dfrac{dN}{dt} = -L\dfrac{di}{dt}$.

The direction of the induced E. M. F. is negative when the current is increasing, *i.e.*, when $\dfrac{di}{dt}$ is negative. If $\dfrac{di}{dt}$ is unity L is numerically equal to e.

Thus the coeff. of self-induction of a coil may also be defined as the E. M. F. induced in the coil due to unit rate of change of current in it.

The practical unit of inductance is called the **henry** which is the inductance of the circuit in which an e.m.f. of 1 volt is set up by a change of current at the rate of 1 ampere per second.

$$1 \text{ henry} = 10^9 \text{ C. G. S. units.}$$

If e be the induced $e. m. f.$ and i the current, at any instant, then

$$\text{work done in time } dt = ei.dt = -Li\frac{di}{dt}.\ dt$$

$$\text{Total work done W} = -\int^{i_m} Li\frac{di}{dt}.\ dt = -L\int^{i_m} idi = -\tfrac{1}{2}Li_m{}^2$$

i_m = steady final current. If $i_m = 1$, then $L = 2W$

Hence coefficient of self inductance is twice the work done in establishing magnetic flux by unit current in the circuit.

Note: Self-induction is a property of the coil and it depends upon the area enclosed, shape, number of turns and the manner of winding *i.e.* on the geometry of the coil.

137. Non-inductive winding: A straight wire has far less inductance than when wound in the form of a solenoid. A coil of wire which has both resistance and self-inductance is represented as shown in Figure 105(a). A coil may also be wound such that its self-inductance is zero. Such winding called non-inductive winding is shown in Figure 105(b). To have non-inductive resistance (as in a resistance box) in the form of a coil of insulated wire the wire is doubled upon itself and then wound over a bobbin as shown in Figure 105(b). In such a coil, the current flows in opposite direction in two halves of the coil. Hence E.M.F. induced in one half is cancelled by the E.M.F. induced in the other half. Fig. 105(c) shows a pure ohmic resistance without appreciable self-inductance.

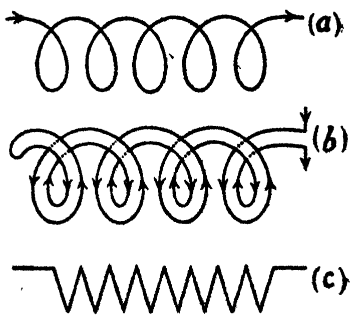


Fig. 105

138. Inductance of a Solenoid: We know that the field inside a solenoid is $4\pi ni$ where n is the number of turns of wire per unit length of the solenoid and i the current passing through it in C.G.S. unit.

If a be its area of cross-section, the magnetic flux $N = 4\pi n a i$. But each turn of wire embraces this number of lines of force and therefore the inductance in the solenoid when traversed by a C.G.S. unit of current is given by

$$L = 4\pi n a . \ nl = 4\pi n^2 al \text{ E.M.U.} = \frac{4\pi n^2 al}{10^9} \text{ henries.}$$

where l is the length of the solenoid.

Again, if a bar of soft iron of length l, cross-section a' and permeability μ be inserted in the solenoid, the flux due to unit current through the bar of iron $= 4\pi n . \mu a'$.

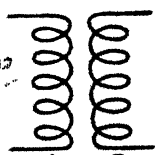
The flux through the solenoid due to unit current $= 4\pi n(a - a')$

$\therefore$ The total flux through the solenoid containing the iron bar

$$= 4\pi n(a - a' + \mu a').$$

$\therefore$ The inductance $= 4\pi n(a - a' + \mu a') . \ nl = 4\pi n^2 l(a - a' + \mu a')$.

139. Mutual Induction: The coefficient of mutual induction of two coils (Fig. 106) may be defined as the total magnetic flux which passes through one of the coils (A) when the other (B) is traversed by the C. G. S. unit of current. We have $N = Mi$,


A B
Fig. 106

where N is the magnetic flux linked with one coil due to C. G. S. current i in the other and M, the coefficient of mutual induction

$$e = -\frac{dN}{dt} = -\frac{d}{dt}(Mi) = -M . \frac{di}{dt} \quad \text{If } \frac{di}{dt}$$

is unity $e = -M$.

That is, the coeff. of mutual induction of two circuits is numerically equal to the induced E. M. F. round one circuit due to unit rate of change of current in the other.

It is also measured in **henrys.**

140. Coefficient of Mutual Induction of two Solenoids: Let us consider two co-axial solenoids of wire. The inner one known as the primary has n_1 turns of wire per unit length and is of cross-section a. The outer one, known as the secondary has n_2 turns of wire in all.

The magnetic field near the middle of the primary is

$$H = 4\pi n_1 i \quad \therefore \quad N = 4\pi n_1 i . a$$

where i is the current in C.G.S. unit traversing the primary and N, the flux linked with each turn of the secondary.

$\therefore$ Linkage for secondary $=$ Flux $\times$ Turns $= 4\pi n_1 n_2 i a$.

$$e = -n_2 \cdot \frac{d}{dt}(4\pi n_1 a i) \text{ and } e = -4\pi n_1 n_2 a \frac{di}{dt}$$

But $e = -M. \dfrac{di}{dt}$ $\qquad M = 4\pi n_1 n_2 a.$

Again if a bar of cross-section a' and permeabilty μ be introduced inside the primary coil the total flux is $4\pi n_1 i(a - a' + \mu a')$. Linkage for the secondary = Flux × Turns

$$\therefore \quad M = 4\pi n_1 n_2 (a - a' + \mu a')$$

Note: If i and e are expressed in amperes and volts M will be expressed in henries.

141. The Rhumkorff Induction Coil: In this instrument, the principle of mutual induction between two coils is utilised for transforming a low potential difference between the ends of a primary coil to a high potential difference between those of a secondary coil by rapidly making and breaking the primary circuit.

The apparatus consists of a primary coil PP′ of a few layers of closely wound thick insulated copper wire wound round a soft iron core. The iron core consists of a bundle of soft iron wires and is used in preference to solid iron so as to minimise the induced Foucault current in the core. One end of the primary coil is

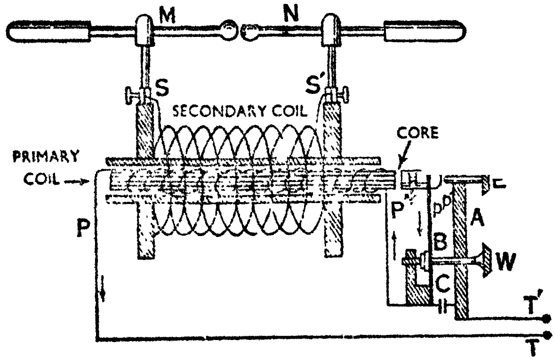


Fig. 107

connected to the brass spring B which carries an iron hammer H and is provided at the back with a platinum contact piece p while the other end of it is connected to one of the battery terminals at T. The remaining terminal T′ of the battery is connected to the stout brass pillar A which carries at the top an adjustable screw E whose point p' is just opposite to p and in line with the hammar H.

The secondary coil SS′ consisting of several thousands of turns of a very thin insulated copper wire is wound round an ebonite tube which entirely covers the primary coil and is provided with ebonite flanges and discs so as to allow the winding of the coil in separate sections and thereby avoid sparking between contiguous layers by the high potential difference set up in the secondary.

Action : Now, when the current passes through the primary coil, the circuit being completed through contact-pieces p' and p, the iron core becomes magnetised and attracts the iron hammar H attached to the spring B. As soon as H is attracted, the primary circuit is broken at pp', the iron core is demagnetised and the hammar is drawn back by the screw W until p and p' touch. Now the circuit being again completed, the core is magnetised and the iron hammer is again attracted and consequently the current in the primary stopped. These series of operations proceed with great rapidity and the spring is vibrated to and fro, the circuit being made and broken every time. Thus an induced E.M F. is set up during each make and break at the extremities of the secondary coil.

Although an electromotive force is produced in the secondary coil at every make and break of the primary circuit the latter is by far the greater, since the primary current dies away much more rapidly than it grows.

Use of iron core : The use of the iron core inside the primary is to increase the induction and thereby add to the number of lines of force generated in the primary by the current passing through it. The result would be to increase the self-induction of the circuit. So at make, the growth of the current in the primary will be retarded and consequently the E. M. F. depending on the rapidity with which the lines of force are cut $\left(e = -\dfrac{dN}{dt} \right)$ will be less and so the secondary E.M.F. will be reduced.

The use of the iron core in the form of a bundle of wires instead of a solid rod is to prevent the formation of eddy currents in the mass of iron as much as possible, for these currents would not only waste the electrical energy but, being in opposite directions to those in the primary, would by their reaction oppose the growth or decay of the primary current. So to prevent the generation of the eddy currents insulated iron wires, instead of a solid rod are used. Even if eddy current is generated, it will be very weak due to the resistance offered by each individual wire.

The iron used must be soft iron as it has the least coercivity. The effect of the smallest amount of coercivity in soft iron is to

demagnetise the iron as quickly as possible and therefore at the time of break the rate of change of lines of force will be rapid and consequently the E.M.F. at break would be greater. Soft iron is chosen for it possesses very small hysteresis and consequently the loss of electrical energy due to this is very small.

Use of condenser : The use of a condenser in parallel with the contact pieces pp' very greatly increases the efficiency of the coil. The high self-induction in the primary coil creates an induced E.M.F. in pp' when the circuit is broken, which if the condenser be absent will produce intense sparking at pp' and thus retard the cessation of the current. The contact-pieces are generally faced with platinum to prevent undue sparking and wearing away. When the condenser is arranged in parallel with the spark gap, the induced E.M.F. instead of causing spark at the contact-pieces will charge the condenser and the charge instead of remaining in the condenser will immediately discharge itself sending a current through the primary in the opposite direction and thus increase the rapidity of the break. So the induced E.M.F. at break is very high and greater than that at make and except when the spark-discharge is short, a secondary discharge is an intermittent current in one direction only and not an alternating discharge.

142. Varying Current : Consider a circuit (Fig. 108) containing an inductance (L), a resistance (R), a battery (B) and a key (K). If the key is closed, the current does not at once attain steady value but increases from zero to steady maximum value. Again, after attainment of the maximum steady value, if the battery be removed but the circuit kept complete, the maximum current does not at once fall to zero, but decreases gradually from maximum to zero [Fig. 110 curve (A)]. Such current in an inductive circuit is called varying current.

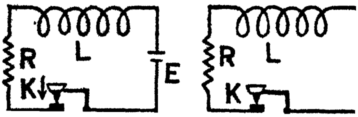
B

Fig. 108

The variation of current from zero to maximum value on completing the circuit is spoken of as *growth of the current*, while the fall of current from maximum to zero on removal of the battery after the attainment of the maximum current, is spoken of as *decay of current*.

143. Growth of Current : Let us consider a circuit containing a battery E of E.M.F., E and of resistance R and self inductance L.

When the current is growing *i.e.*, increasing (Fig. 109*a*), the magnetic flux linked with the circuit also increases and consequently, according to Faraday's Law of Induction, an electromotive force is generated in the circuit whose value is given by $E = -\frac{dN}{dt}$, where N is the number of tubes of induction passing at any time t through the circuit.



(*a*) Fig. 109 (*b*)

So the resultant E.M.F. overcoming the resistance of the circuit at any instant is equal to $E - \frac{dN}{dt}$

Hence equation for the instantaneous current in the circuit is given by,

$$\frac{E - \frac{dN}{dt}}{R} = \frac{E - L\frac{di}{dt}}{R} \quad \ldots(1) \qquad \because N = Li \text{ so that } \frac{dN}{dt} = L\frac{di}{dt}$$

where L = self-inductance of the circuit.

$$L\frac{di}{dt} = E - iR \quad \text{or} \quad \frac{di}{E - iR} = \frac{1}{L}\,.\,dt \quad \text{or} \quad \frac{di}{\frac{E}{R} - i} = \frac{R}{L}\,.\,dt. \quad \ldots (2)$$

$$\text{Integrating (2)} \quad \int_0^i \frac{di}{\frac{E}{R} - i} = \frac{R}{L}\int_0^t dt.$$

$$\text{or} \quad \left[-\log\left(\frac{E}{R} - i\right)\right]_0^i = \frac{R}{L}\left[\,t\,\right]_0^t$$

$$\text{or} \quad -\log\left(\frac{E}{R} - i\right) + \log\frac{E}{R} = \frac{Rt}{L}$$

$$\text{or} \quad \log\left(\frac{E}{R} - i\right) - \log\frac{E}{R} = -\frac{Rt}{L}$$

$$\text{or} \quad \log\frac{\frac{E}{R} - i}{\frac{E}{R}} = \log e^{-Rt/L} \qquad \text{or} \quad \log\frac{E - Ri}{E} = \log e^{-Rt/L}$$

$$\text{or} \quad \frac{E - Ri}{E} = e^{-Rt/L} \qquad \text{or} \quad 1 - \frac{R}{E}\,.\,i = e^{-Rt/L}$$

or $\frac{R}{E}i = 1 - e^{-Rt/L}$

$$\therefore \quad i = \frac{E}{R}\left(1 - e^{-Rt/L}\right) = \frac{E}{R}\left(1 - e^{-t/\lambda}\right) \ldots(3), \text{ where } \lambda = \frac{L}{R}.$$

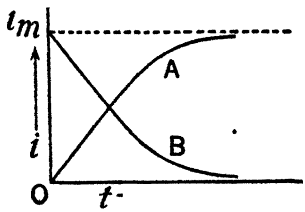
$$= i_m \left(1 - e^{-t/\lambda}\right] \ldots (3a), \text{ where } \frac{E}{R} = i_m, \text{ the maximum}$$

steady value.

From (3a) it is evident that instantaneous current is an exponential function of R, t and L or in other words current grows exponentially,

In (3a) if $t = \propto$ (infinity), $i = i_m$; thus theoretically, the current grows to the maximum value i_m at infinite time ; in fact for all practical purposes the maximum value is attained in a short interval of time.

The curve (Fig. 110A) representing relation between i and t starts from the origin corresponding to $i = o$ at $t = o$, and is found to meet the line $i = i_m$ asymptotically as shown in the figure.



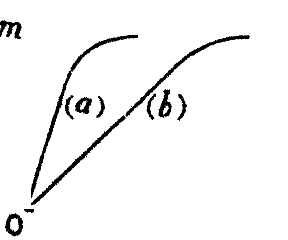
A-GROWTH ; B-DECAY

Fig. 110

The rate of growth depends on the ratio L/R and not on the separate values of R and L. The ratio L/R or λ is called the time constant, *i.e.*, the time in which the current attains roughly $\frac{2}{3}$rd of its maximum value. [For when $t = L/R$,

$i = i_m \ (1 - e^{-1})$ from 3a ; *i.e,*

$i = i_m \ (1 - 1/e) = i_m \ (1 - \cdot37) = \cdot63 i_m$]



GROWTH OF CURRENT

Fig. 111

The rate of growth of current is rapid if time constant $\frac{L}{R}$ is small (Fig. 111a) and it is slow if $\frac{R}{L}$ large (Fig. 111b). In other words, if L be small and R large, rate of growth is rapid, while if L be large and R small, rate is slow.

143(a). Alternative method : From (2) in previous method, we have

$$\frac{L}{R}\frac{di}{dt} + i - \frac{E}{R} = 0 \quad .. (1)$$

Let $i = x + E/R$

Differentiating, $\frac{di}{dt}=\frac{dx}{dt}$, substituting in (1), $\frac{L}{R}\cdot\frac{dx}{dt}+x=o$

or $\frac{dx}{dt}=-\frac{R}{L}.x$ or $\frac{dx}{x}=-\frac{R}{L}\,dt$

Integrating, $\log x=\frac{-Rt}{L}+C$, C = Integration constant

or $\log\left(i-\frac{E}{R}\right)=\frac{-Rt}{L}+C$ ... (2) $\because$ $x=i-E/R$.

To find C, we apply initial condition, *i. e.*, at $t=o$, $i=o$.

$\therefore$ $C=\log\left(-\frac{E}{R}\right)$

Hence from (2), $\log\left(i-\frac{E}{R}\right)=\log e^{-Rt/L}+\log\left(-\frac{E}{R}\right)$

or $i-\frac{E}{R}=-\frac{E}{R}e^{-Rt/L}$.

or $i=\frac{E}{R}\left(1-e^{-Rt/L}\right)=\frac{E}{R}\left(1-e^{-t/\lambda}\right)$

144 Decay of Current: If when the steady value i_m of the current has reached, the battery is out of action but the circuit kept closed, (Fig. 109*b*) the rate of decay of current is determined by the relation,

$\frac{Ldi}{dt}+Ri=0$ or $\frac{di}{i}=-\frac{R}{L}dt$.

Integrating, $\log i=\frac{-Rt}{L}+c$ ... (1), where c = a constant.

Here, when $t=o$, $i=i_m$, or E/R $\therefore$ $c=\log i_m$

Then, $\log i=\log e^{-Rt/L}+\log i_m$,

or $i=i_m.e^{-Rt/L}=i_m.e^{-t/\lambda}$ ... (2), where $\lambda=\frac{L}{R}$.

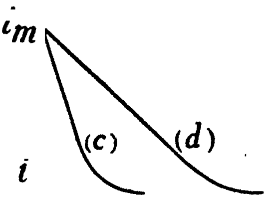
Thus current decays exponentially. From (2), at $t=\infty$, $i=o$, nce theoretically, current falls to zero value at infinite time. In fact however, it practically vanishes after a very short time. A curve (Fig. 110B) between i and t starts from the point

corresponding to i i_m at $t=o$, and meets the time axis asymptotically.

The rate of decay depends upon L/R or λ. If λ be small, rate of decay is quick, as in Figure 112 (c), if λ be large and t is also large the rate of decay is slow [Fig. 112(d)].

If the battery be removed and the circuit kept open, R will be infinitely large so that $\lambda = o$. Hence current falls at once to zero along vertical line i_m.O.

Note : The rate of growth or decay of current is shown in single Figure 110. It cannot be observed by ordinary galvanometer but can be demonstrated by using a vibration galvanometer with moving parts having a high frequency.



BECAY OF CURRENT
Fig. 112

144(a). Energy stored up in a circuit : The equation (1) in Article 143(a) may be written in the form $i^2\mathrm{R} + \mathrm{L}i.\dfrac{di}{dt} = \mathrm{E}i,$

The factors Ei and i^2R represent respectively the rate of supply of energy by the battery and the rate of dissipation of energy as heat in the circuit.

The other factor $\mathrm{L}i\dfrac{di}{dt}$, represents the rate at which energy is stored up in the medium surrounding the conductor of self-inductance L.

The total energy stored up when the current increases from 0 to i is given by.

$$\int^{i} \mathrm{L}i.\frac{di}{dt} = \tfrac{1}{2}\,\mathrm{L}i^2.$$

QUESTIONS

1. What do you understand by an inductive circuit? Define the coefficient of self-inductance of such a circuit. [C. U. 1938, '49]

2. Enunciate and explain on a quantitative basis the laws of induction of currents in a closed circuit by means of external influences. [C. U. 1937]

3. Explain what you mean by (1) Coefficient of self-inductance and (2) Co-efficient of mutual induction. [C. U. 1941, '42, '43, '48, '52, '55]

Describe suitable experiments to illustrate their presence in an electric circuit. [C. U. 1948]

4. Show that in a circuit with inductance and resistance and subject to a constant E. M. F. the current rises exponentially. [C. U. 1938, '39]

5. Show that in a circuit with a steady E. M. F. and an inductive resistance, the current decays exponentially with time when the E. M. F. is suddenly withdrawn. [C. U. 1941, '57]

Deduce expressions for the growth and decay of current in a circuit containing an induction L and a resistance R in series with a battery of e. m. f., E. What is the time constant? [C. U. 1957]

6. What is an eddy current? How is the effect of this current utilised in a class of electrical measuring instruments? [C. U. 1937]

7. Calculate the value of the self-induction per unit length near the centre of a solenoid.

8. Explain briefly the working of an ordinary induction coil. [C. U. 1948]

9. Calculate the inductance of a solenoid with an iron core. [C. U. 1955]

EXAMPLES

1. A solenoid produces flux of 30,000 Maxwells in an iron core when a current of two amperes flow through it. If it has 400 turns, calculate its coefficient of self-induction. [C. U. 1942]

Flux per *amp.* through each turn $= \frac{30000}{2} = 15000$ Max.

E. M. F. induced in each turn when the current changes by 1 *amp. per sec.* $= \frac{15000}{10^8}$ volt $= 15 \times 10^{-5}$ volt.

$\therefore$ Total E. M. F. induced $= 15 \times 10^{-5} \times 400$ volts $= {\cdot}06$ volt

$\therefore$ L $= {\cdot}06$ henry.

2. Find the inductance of a coil of 100 turns, wound on a paper tube 25 cms. long and 4 cms. radius. [C. U. 1949]

Use the formula $L = \frac{4\pi n^2 al}{10^9}$ henries; Here $n = \frac{100}{25} = 4$

Ans. $L = 2{\cdot}527 \times 10^{-4}$ henries

3. A coil of 100 turns of area 20 sq. cm. is wound on an iron core for which $\mu = 1000$ and held at right angles to a field of 50 oersteds. It is removed in $\frac{1}{10}$ sec. The resistance of the coil is 4 ohms and it is in series with a galvanometer of resistance 6 ohms. Find the average current induced in the circuit.

Flux through the coil $= 1000 \times 50 \times 20 \times 100 = 10^8$ lines
Rate of change of flux $= 10^8 \div \frac{1}{10} = 10^9$ E. M. U.
Resistance of the circuit $= 10$ ohms $= 10^{10}$ E. M. U.
Average Current $= 10^9 \div 10^{10} = 10^{-1}$ E. M. U. $= 1$ amp.

CHAPTER XII

ALTERNATING CURRENT

145. Alternating Current: We know that the current derived from an ordinary cell or battery is known as a *continuous current* as it flows continuously in one direction so long as the external circuit is closed.

But if the current does not flow in this way but periodically reverses, flowing for one time in one direction and then reversing, flowing for another equal time in the opposite direction, this current in the external circuit is called an *alternating current.*

If a coil be rotated in a uniform magnetic field, the E.M.F. induced in the coil is given by the formula $E = E_m \sin wt$, where E is the E.M.F. induced at any instant t, E_m the maximum E.M.F. and ω the angular velocity of the coil or a measure of the frequency (n) of alternation being equal to $2\pi n$.

The E.M.F. and therefore the current in the coil alternately flows in the opposite directions and the intensity varies from maximum to zero and *vice versa*.

Such a current is called an alternating current. The number of times the current attains a maximum value in each second is called the number of cycles. The strength of the current in the alternating circuit does not depend on the E.M.F. and the resistance does not follow Ohm's Law.

As the current is variable, the electromotive force arises in the circuit on account of its self-induction and capacity. For the same reason the current and the E.M.F. **are not in the same phase,** *i.e.*, the current is not maximum when the E.M.F. is maximum.

146. The Earth Inductor: The earth inductor is a circular or a rectangular coil C of a single or many turns of insulated wire capable of rotation about a vertical axis AB in a uniform magnetic field. If the plane of the coil be at first at right angles to the direction of the lines of force in the field and rotated through 90°, a temporary induced current will be set up in a direction determined by Lenz's Law due to the decrease in the lines of force passing through the coil. If again the coil be rotated from this position of 90° to 180°, the lines of force passing through the coil will increase to its maximum value and therefore the induced current will be in the same direction as before.

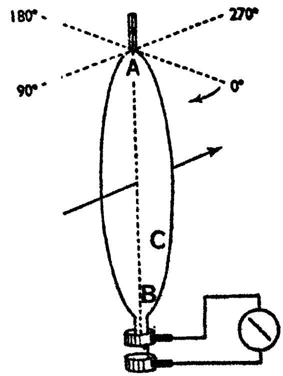


Fig. 113

If the coil be again rotated from 180° through 270° to 360°, *i.e.*, 0° the induced current will be produced in the coil but its direction will be opposite to that when the coil is rotated from 0° to 180°.

So for a complete revolution of the coil, from the position in which its plane is at right angles to the lines of force in the field, the E.M.F. and the current gradually increase and become maximum when the plane coincides with the direction of the lines of force

and then they are reversed in direction and attain the same values when rotated from 180° to 0° again.

The induced current set up in the circuit may be conveyed to a galvanometer by means of metal brushes in contact with the ring terminals mounted on one of the axles and joined to the ends of the coil and the quantity of electricity set in motion is measured by means of a ballistic galvanometer.

146(a). Theory of its use: Let A be the area of each turn of the coil, F, the intensity of a uniform magnetic field and R, the total resistance of the coil.

The number of lines of force cut by the coil when it rotates from 0° to 90° = AF.

The induced current being in the same direction when the coil rotates from 0° to 180°, the total quantity of electricity set in motion is $Q = \dfrac{2AF}{R}$

If the inductor consists of n turns $Q = \dfrac{2nAF}{R}$.

The same quantity of electricity will be set in motion when the coil is rotated from 180° to 360° but its direction will be opposite to the previous one.

Weber used this inductor to determine the **magnetic dip** of a place.

We know that the dip δ of a place is given by $\tan \delta = \dfrac{V}{H}$ ... (1),

where H and V are respectively the horizontal and vertical components of the intensity of the earth's magnetic field.

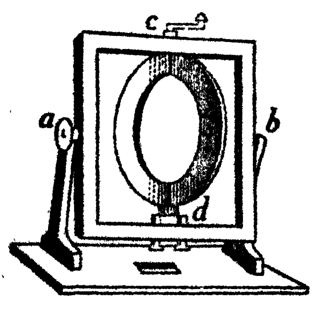


Fig. 114

In Figure 114 is shown an earth inductor in which the coil is so mounted in a wooden frame that it can be rotated about a vertical axis *cd* and also about a horizontal axis *ab*.

The component V is determined by rotating the coil round a horizontal axis and since the vertical component V is effective here, the quantity of electricity set in motion in the coil is given by

$$Q_1 = \frac{2nAV}{R} = K\theta_1 \quad ...(2) \quad \therefore \quad V = \frac{RK\theta_1}{2nA}$$

where θ_1 is the throw in the ballistic galvanometer connected in series with the coil and K, the constant of the galvanometer.

Again the inductor is rotated round a vertical axis, the earth's horizontal component H is now effective and we have

$Q_2 = \frac{2nAH}{R} = K\theta_2$ where θ_2 is the throw in this case.

$$H = \frac{RK\theta_2}{2nA} \qquad \dots (3)$$

Hence resultant Intensity of earth's magnetic field $= \sqrt{H^2 + V^2}$

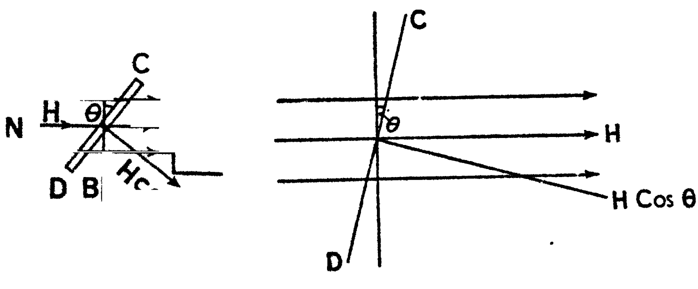
$$= \frac{RK}{2nA}\sqrt{\theta_1{}^2 + \theta_2}$$

Then from (1), (2) and (3) $\tan\delta = \frac{V}{H} = \frac{RK\theta_1}{2nA} \Big/ \frac{RK\theta_2}{2nA} = \frac{\theta_1}{\theta_2}$

Thus knowing θ_1 and θ_2, $\tan\delta$, δ the angle of dip can be found out.

Note : Although used for finding δ, an earth inductor serves as an appliance for generating alternating current.

147. Calculation of the E.M.F. and the current at any instant in a rotating coil : Consider a coil CD revolving in a uniform magnetic field provided (*a*) by a magnet or (*b*) by earth as in Figure 115.



(*a*) Fig. 115 (*b*)

Let A be the area of the coil CD, then at its initial position when its plane is at right angles to the lines of force, the total number of lines of force N passing through it is given by $N = AH$, where H is the intensity of the field.

Then, at any instant, when the coil is rotated through θ, the component of the intensity H acting at right angles to the plane of the coil is equal to $H \cos\theta$.

Therefore, the induction or the total number of lines of force, N passing through the coil at this position is $AH \cos\theta$.

Since the induced E.M.F., E at any instant is given by

$E = -\frac{dN}{dt}$, the induced E.M.F., E at the instant when the angle of rotation is $\theta = -\frac{d(\mathrm{AH}\cos\theta)}{dt} = -\frac{d}{dt}(\mathrm{AH}\cos wt)$, since $\theta = \omega$

That is $E = \omega \mathrm{AH} \sin \omega t = E_m \sin \omega t$ (1), where $E_m = \omega \mathrm{AH}$ the maximum e.m.f.

or $E = \omega \mathrm{AH} \sin \frac{2\pi}{T} t = \omega \mathrm{AH} \sin 2\pi n t$ since $\omega = \frac{2\pi}{T} = 2\pi n$,

where T and n are period and frequency of alternation respectively.

If R be the resistance of the coil, the current i at the instant when the angle of rotation is θ is given by

$$\frac{\omega \mathrm{AH}}{R} \sin \theta = i_m \sin \omega t \ (2) \text{ where } i_m = \text{maximum current.}$$

Hence the E.M.F. or the current at any instant is proportional to the sine of angle described by the coil from its initial position. It is zero when $\theta = 0^\circ$ or 180°, *i.e.*, when the plane of the coil is perpendicular to the lines of force. It has a maximum value equal to ωAH when the angle $\theta = 90^\circ$, and maximum negative value when $\theta = 270^\circ$. It is again equal to zero when $\theta = 360^\circ$.

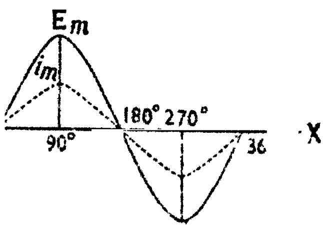


Fig. 116

Thus in one complete revolution, the *e.m.f.* and the current change from 0 to 0 through positive maximum, and then from 0 to 0 through negative maximum of same value.

The E. M. F. and current vary harmonically as is evident from the equations (1) and (2) and from Figure 116.

In a non-inductive circuit, *i.e.*, where the self-inductance has no effect, the *e.m.f* and the current are in the same phase.

148. Average Value of E.M.F.: The average value of E.M.F. during a complete cycle, *i e.*, during one complete rotation, is zero, since during one half of a complete rotation, if the E.M.F., be positive then E. M. F. is negative in the other half. Hence when we use the term average E. M. F., we refer to the average M. F. over half a complete cycle.

148(a). Calculation of Average E.M.F : Let the instantaneous e.m.f., E be given, by the relation $E = E_m \sin\theta$, where E_m = maximum *e.m.f.*, θ = angle of inclination of the coil to its initial position.

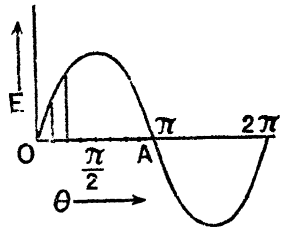


Fig. 117

The adjoining sine curve (Fig. 117) shows the relation between E and θ. Divide OA which corresponds to half cycle, into n equal parts each being equal to $d\theta$; then $nd\theta = \pi$. If n ordinates be drawn from OA, they will represent instantaneous E. M. F,s during half cycle.

$$\text{Therefore, average E. M. F} = \frac{\Sigma E}{n} = \int_0^{\pi} E_m \frac{\sin\theta\, d\theta}{nd\theta}$$

$$= \frac{E_m}{\pi}\int_0^{\pi} \sin\theta . d\theta = \frac{E_m}{\pi}\Big[-\cos\theta\Big]_0^{\pi}$$

$$\text{Hence, average } e.m.f. = \frac{E_m}{\pi}\Big[-\cos\pi - (-\cos o)\Big] = \frac{E_m}{\pi}\Big[1+1\Big] = \frac{2}{\pi}E_m$$

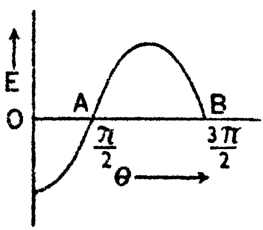


Fig. 118

Thus, mean or average E. M. F. is

$$= \frac{2}{\pi} \times \text{maximum E. M. F.}$$

Similarly using relation $i = i_m \sin\theta$ in place of $E = E_m \sin\theta$, we can similarly show that average current $= \frac{2}{\pi} \times$ maximum current.

Note : If in Figure 115, θ be the angle between the coil and the field direction at any instant, $N = AH \sin\theta = AH \sin wt$. The instantaneous *e. m. f.*

$$\therefore E = \frac{-dN}{dt} = -wAH\cos wt = -E_m\cos\theta.$$

Considering half cycle from A to B (Fig. 118) and proceeding as above,

$$\text{Average } e.\ m.\ f. = \frac{\Sigma E}{\pi} = -\int_{\pi/2}^{3\pi/2} \frac{E_m \cos\theta\, d\theta}{\pi}$$

$$= -\frac{E_m}{\pi}\Big[\sin\theta\Big]_{\pi/2}^{3\pi/2} = -\frac{E_m}{\pi}\Big[-1-(1)\Big] = \frac{2}{\pi}E_m$$

Thus for both sine and cosine functions average E. M. F. $= \frac{2}{\pi}.E_m$

Note: Without dividing OA into n parts as above we can directly write,

average e. m. f. for half cycle $= \int_0^{\pi} \frac{E_m \sin\theta\, d\theta}{\pi} = \int_0^{T/2} \frac{E_m \sin wt\, dt}{T/2}$ where

T = period of alternation.

Figure 117 represents a curve shewing the variation of induced E.M.F. with the position of the coil during one complete revolution.

If successive positions of the coil are taken as abscissae and the corresponding E.M.Fs as proportional to the sines of the angles in each case, are taken as ordinates, a curve is obtained which is known as the **sine curve.** It is to be noted that the ordinates are drawn above and below the axis of θ according as the values of the sines are positive or negative.

From the nature of the curve it is understood that the E.M.F. in the case of a rotating coil varies harmonically and the current in the absence or self-induction varies in the same manner.

This average value of E.M.F. or current is alternately positive and negative for the two halves of the cycle. So a suspended needle or coil galvanometer can not be used for measurement of alternating current as they will receive equal and opposite impulses and the reading will be zero.

For this reason to measure alternating current, instruments have been devised in which deflections are all in one direction whatever may be direction of the electromotive force or the current.

This is possible only when the deflection is proportional to the square of the E.M.F. or the current as in the case of a *hot-wire voltmeter or ammeter.*

149. Mean or Average Value of Alternating Current : In an alternating current circuit, the current varies harmonically, *i.e.*, if a graph be plotted with current as ordinate OY and time as abscissa OX it will be found that in the positive half of the curve the current as represented by the ordinate gradually increases to its maximum value i_m (peak value) then decreases to zero value and in the negative half of the curve it then increases to its negative maximum value and then decreases to zero value.

The mean or average value of the current during a complete cycle is zero since during one half of the cycle the current is positive and in the other half it is negative and of the same value. Hence when we refer to the average current we means average current over a half complete cycle.

149(a). Peak value of alternating current : In an alternating current circuit the current attains a maximum value in course of its positive or negative swing. This maximum value of the current is called the *peak value* of the current.

150. Virtual Current : The average current or rather the *mean current* does not express the strength of steady current which would have the same effect as the alternating current. So a current known as the **virtual current** is the steady continuous current which would have the same heating effect as the alternating current.

When a simple harmonic current is made to flow through a piece of wire heat is generated and its temperature attains a constant value.

According to Joule's Law, heat generated is proportional to the square of the current strength at any instant.

A curve (Fig. 119) is constructed with the squares of the instantaneous currents as ordinates and successive positions of the inductor as the abscissa and it represents the variation in the generation of heat.

The **square root** of the average ordinate will be the magnitude of the steady current which will generate the same heat in the same time as the given alternating current.

This *constant current* is known as the *virtual current.*

150(a). Calculation of virtual or root mean square (R.M.S.) value of current : Let the current at any instant be given by $i = i_m \sin\theta$, where i_m = maximum current. The adjoining sine curve (Fig. 119) for one cycle shows relation between squares of the instantaneous currents and corresponding values of θ. Divide OA which corresponds to half cycle, into n equal parts each being equal to $d\theta$. Then $nd\theta = \pi$.

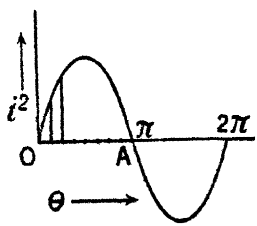


Fig. 119

Then, mean of square of currents $= \dfrac{\Sigma i^2}{n}$,

where n is the number of ordinates indicating different instantaneous values of i^2 over half cycle.

Mean of square of currents $= \dfrac{\Sigma i^2}{n} = \dfrac{\Sigma i^2.d\theta}{nd\theta} = \dfrac{\Sigma i^2.d\theta}{\pi}$

$$= \int_0^{\pi} \frac{i_m^2 \sin^2\theta d\theta}{\pi} \qquad [\because\ i^2 = i_m^2 \sin^2 \theta]$$

$$= \frac{i_m^2}{2\pi}\int_0^{\pi} 2\sin^2\theta.d\theta = \frac{i_m^2}{2\pi}\int_0^{\pi}(1-\cos 2\theta)d\theta$$

or Mean of square of currents

$$= \frac{i_m^2}{2\pi}\int_0^{\pi} d\theta - \frac{i_m^2}{2\pi}\int_0^{\pi}\cos 2\theta.d\theta = \frac{i_m^2}{2\pi}\Big[\ \theta\ \Big]_0^{\pi} - \frac{i_m^2}{2\pi}\left[\frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{i_m^2}{2\pi}\,\pi - \frac{i_m^2}{2\pi}\Big[0-0\Big] = \frac{i_m^2}{2}$$

Root of mean of square of currents $= \dfrac{i_m}{\sqrt{2}}$

Thus **Virtual or Root mean square current** $= \dfrac{1}{\sqrt{2}} \times$ *max.* current.

It has the same value for the other half of the cycle.

The same value will be obtained if instantaneous current is given by $i = i_m\cos\theta$, taking care to use proper limits of integration.

Note I: If instantaneous *e. m. f.* be given by $E = E_m \sin \theta$, then proceeding as in Article 150 (a), **Virtual or R. M. S. E. M. F.**

$$= \left\{\frac{E_m^2}{2\pi}\int_0^{\pi} 2\sin^2\theta.\,d\theta\right\}^{\frac{1}{2}} = \frac{E_m}{\sqrt{2}}$$

Note II: If an alternating current be varying between +ve maximum of 20 amperes and $-ve$ maximum of 20 amperes, then the virtual current

$$= \frac{1}{\sqrt{2}} \times 20 = \frac{1}{1\cdot 4} \times 20 = \cdot 7 \times 20 = 14 \text{ amperes.}$$

151. Virtual Volt or Effective Volt. It is that volt which when applied to the ends of a resistance produces the same heating effect as that produced by a steady potential difference of 1 volt for the same time.

Thus virtual volt $= \dfrac{1}{\sqrt{2}}$**. maximum volt = ·7 maximum volt.**

152. Relation between average, virtual and maximum value of alternating current :

Average or mean value of current $= \frac{2}{\pi} \times$ maximum current

$= \frac{2\sqrt{2}}{\pi} \times$ virtual current

Virtual or Root Mean Square (R.M.S) value of current

$\frac{}{\sqrt{2}} \times$ maximum current

The alternating E.M.F. may be measured by an electrostatic voltmeter which indicates the virtual E.M.F.

An alternating current is usually measured by Duddle's Thermo-galvanometer and Fleming's Thermo-ammeter. In these two instruments the heating effect of the current has been utilised in measuring the virtual current from the deflection.

153. Duddle's Thermo-galvanometer : In this instrument a loop of fine silver wire with a thermo-couple (Sb-Bi) at the bottom is suspended between the poles N. S. of a permanent magnet by means of a glass rod and quartz fibre F with a mirror M for observing deflections. Just below the thermo-couple is stretched a very fine wire H through which the current to be measured is passed.

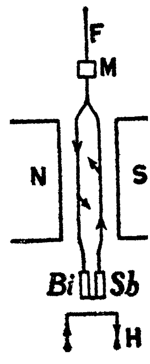


Fig. 120

The wire becomes heated, and by radiation and convection the junction is heated giving rise to a current in the loop. The loop is then rotated and deflections are observed.

Since the deflection is proportional to the square of the current, it is independent of the direction of the current. So both direct and alternating currents can be measured with the instrument.

154. Power in a Steady Current Circuit : In the case of steady direct current flowing in a circuit the power or the rate of working is $= E.i$, where E is the steady voltage and i the steady current.

Power in A. C. Current :—

For a circuit in which an alternating current is flowing we will have to consider two cases.

154(a). Case I—Non-inductive Circuit :

As there is no inductance, there will be no phase difference between current and *e.m.f.* at any instant.

Hence *e.m.f.* at any instant t is given by $E = E_m \sin \omega t$

and current at that ... ... $i = i_m \sin \omega t$

where E_m and i_m are the maximum values of E and i, ω the angular frequency or 2π times the number of revolutions of the rotating coil in the magnetic field, or number of alternations of current per second.

Then mean power consumed in the circuit at instant t

$= Ei = E_m.\sin\omega t.i_m \sin \omega t = E_m i_m \sin^2\omega t$

$= E_m.i_m.\sin^2\theta$ [where $\omega t = \theta$, angle of inclination of coil to its initial position, at instant t].

$\therefore$ The mean power for a cycle

$$= \int^{2\pi} \frac{E_m i_m \sin^2\theta \, d\theta}{2\pi} = \frac{E_m.i_m}{2\pi} \int^{2\pi} \sin^2\theta.d\theta.$$

$$= \frac{E_m.i_m}{2\pi \times 2} \int^{2\pi} (1 - \cos 2\theta) d\theta = \frac{E_m i_m}{4\pi} \left[\int^{2\pi} d\theta - \int^{2\pi} \cos 2\theta.d\theta \right]$$

$$= \frac{E_m.i_m}{4\pi} \left[(2\pi - 0) - \left(\frac{\sin 2\theta}{2} \right)^{2\pi} \right]$$

$$\frac{E_m.i_m}{4\pi} \Big(2\pi - (0 - 0) \Big) = \frac{E_m.i}{2}$$

Mean power $\frac{E_m}{\sqrt{2}} . \frac{i_m}{\sqrt{2}}$ Virtual volt × Virtual current.

Case II—When Circuit is Inductive :

In this case, if the *e. m. f.* at any instant be $E = E_m \sin \omega t$ then current at that instant is $i = i_m \sin(\omega t - \alpha)$, where α is the phase lag of current behind the electromotive force. The power consumed at the instant t

$= E.\,i = E_m.\sin \omega t.\, i_m \sin(\omega t - \alpha)$

$= E_m.i_m.\sin\theta.\sin(\theta - \alpha)$ [Putting $\omega t = \theta$]

$= E_m.i_m.\sin\theta\,(\sin\theta\cos\alpha - \cos\theta\sin\alpha)$

$= E_m.i_m\sin^2\theta.\cos\alpha - E_m.i_m.\sin\theta.\cos\theta.\sin\alpha$

$= \frac{E_m.i_m}{2}(1 - \cos 2\theta).\cos\alpha - \frac{E_m.i_m}{2}.\sin 2\theta.\sin\alpha$

The mean power for a cycle

$$=\cos\alpha\ \frac{E_m.i_m}{2}\int_0^{2\pi}\frac{(1-\cos 2\theta)d\theta}{2\pi}-\frac{E_m.i_m}{2}\sin\alpha\int_0^{2\pi}\frac{\sin 2\theta\ d\theta}{2\pi}$$

$$=\frac{E_m.i_m}{2\times 2\pi}\cos\alpha\left[\left\{\theta\right\}_0^{2\pi}-\left\{\frac{\sin 2\theta}{2}\right\}_0^{2\pi}\right]-\frac{E_m.i_m}{2\times 2\pi}\sin\alpha$$

$$\times\left[-\frac{\cos 2\theta}{2}\right]_0^{2\pi}$$

$$=\frac{E_m.i_m}{2}\cos\alpha-\frac{E_m.i_m}{4\pi\times 2}\sin\alpha\Big(-1-(-1)\Big)=\frac{E_m.i_m}{2}\cos\alpha-0$$

Hence mean power $=\frac{E_m.i_m}{2}.\ \cos\alpha=\frac{E_m}{\sqrt{2}}\times\frac{i_m}{\sqrt{2}}.\ \cos\alpha$

$=$ Virtual volt $\times$ Virtual current $\times\cos\alpha$,

The quantity $\cos\alpha$ is called the **power factor.**

155(a). Expression for Instantaneous current in a circuit containing resistance, inductance and source of alternating or sinusoidal *e. m. f.* **given by** $E=E_m\sin\omega t$.

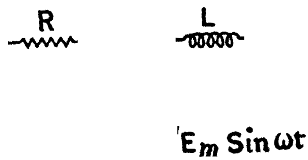


Fig. 121

Let the resistance and inductance in the circuit be R and L respectively. If i be the current at any insatnt, the *e. m. f.* due to self inductance is $-\frac{Ldi}{dt}$, then

$$i=\frac{E-\frac{Ldi}{dt}}{R}=\frac{1}{R}\left(E_m\sin\omega t-\frac{Ldi}{dt}\right) \qquad (1)$$

where $E=$ *e. m. f.* at instant t.

$$\text{Rearranging (1)}\ \frac{Ldi}{dt}+Ri=E_m\sin\omega t \qquad (2)$$

where E_m is the maximum *e. m. f.* and $\omega=2\pi/T=2\pi n$, T, n being the period of alternation and frequency of alternation respectively.

From (2) $\frac{di}{dt}+\frac{R}{L}.\ i = \frac{E_m}{L}\sin\omega t.$

Multiplying both sides by $e^{Rt/L}$

$$e^{Rt/L}\frac{di}{dt}+\frac{R}{L}.\ i.e^{Rt/L}=\frac{E_m}{L}.\ e^{Rt/L}.\ \sin\omega t$$

or $$\frac{d}{dt}\left(i.e^{Rt/L}\right)=\frac{E_m}{L}.\ e.^{Rt/L}\sin\omega t$$

Integrating, we have

$$i.\ e^{Rt/L}=\int\frac{E_m}{L}e^{Rt/L.}\sin\omega t.\ dt=\frac{E_m}{L}\int e^{Rt/L}\sin\omega t.\ dt \qquad (3)$$

$$=\frac{E_m.e^{Rt/L}}{L}.\ \frac{\sin\ (\omega t-\tan^{-1}L\omega/R)}{\sqrt{\frac{R^2}{L^2}+\omega^2}}+C$$

where C = a constant

$$i=\frac{E_m}{L}L.\frac{\sin\ (\omega t-\theta)}{\sqrt{L^2\omega^2+R^2}}+C.\ e^{-Rt/L} \quad \text{where } \theta=\tan^{-1}L\omega/R.$$

or $$i=\frac{E_m}{\sqrt{L^2\omega^2+R^2}}\sin\ (\omega t-\theta) \qquad (4)$$

Since the term $e^{-Rt/L}$ reduces to zero with increase of time. The relation (4) gives the expression for the instantaneous current.

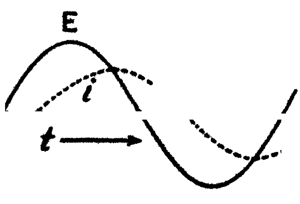


Fig. 122

There is, as evident from (4) a phase difference θ between the e.m.f. and the current, *i.e.*, the current is not in step with the *e. m. f.* (Fig. 122). The negative sign of θ shows that current lags behind the *e.m.f.*, E. In other words maximum and minimum currents are reached a short interval after the corresponding values of E. M. F. The angle θ, showing the phase difference is called the *angle of lag*.

[Note : The term $\int e^{Rt/L}\sin\omega t\ dt$ may be compared to standard form $\int e^{ax}\sin bx.dx$, where $a=R/L$, $b=\omega$ and $x=t$.]

From (4) the maximum value of current is given by

$$i_m = \frac{E_m}{\sqrt{L^2\omega^2+R^2}} \quad \ldots \quad \ldots \quad (5)$$

whence $\sqrt{L^2\omega^2+R^2} = \dfrac{E_m}{i_m}$ (5a)

The quantity $\sqrt{L^2\omega^2+R^2}$ is called *Impedance* which is the ratio of the maximum E. M. F. and maximum current. The quantity $L\omega$ is called *Reactance*, and R is simply ohmic resistance.

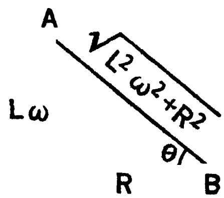


Fig. 123

Again $\tan\theta = \dfrac{L\omega}{R}$

or tangent of the angle of lag $= \dfrac{\text{Reactance}}{\text{Resistance}}$.

Impedance has the same function in A. C. circuits as resistance in D. C. circuits. Impedance has for its limiting value the resistance for a very low frequency and the reactance for a very high frequency. The four quantities impedance, reactance, resistance and angle of lag can be shown by a right-angled triangle as in Figure 123.

When the electromotive force oscillates so rapidly that $L\omega$ is large compared to R, we have approximately $i = \dfrac{E_m}{L\omega}$ $sin\omega t$ and thus the current through the circuit is approximately indepenent of the resistance and depends on the coefficient of self-induction and the frequency of the electromotive force.

Thus a very rapidly alternating current sends more current through a short circuit with a small self-induction, though made of a badly conducting material than through a circuit of large self-induction but made of an excellent conductor. The effect is opposite in the case of a steady electromotive force.

155 (b). Alternative Method for deduction of Intantaneous Current :

Proceeding as in Article 155 (a), $\dfrac{Ldi}{dt} + Ri = E_m.\sin\omega t$ (1)

Let $D = \dfrac{d}{dt}$, where D is known as an operator. Then $D(\sin\omega t) = \omega\cos\omega t$, $D^2(\sin\omega t) = D(\omega\cos\omega t) = (-\omega^2).\sin\omega t$.

$\therefore\ D^2 = -\omega^2$.

The above expression (1) can be written as

$$(LD+R)i = E_m \cdot \sin \omega t \qquad \text{or} \qquad i = \frac{E_m \cdot \sin \omega t}{LD+R}.$$

Multiplying both numerator and denominator by R − LD.

$$\cdot E_m \cdot \frac{(R-LD)\sin \omega t}{R^2 - L^2D^2} = E_m \cdot \frac{R \sin\omega t - L.D(\sin \omega t)}{R^2 + L^2\omega^2} \qquad D^2$$

$$\text{or} \quad i = E_m \frac{R \sin \omega t - L\omega \cos \omega t}{L^2\omega^2 + R^2}$$

$$\cdot E_m \frac{A \cos \theta . \sin \omega t - A \sin \theta \cos \omega t}{L^2\omega^2 + R^2}$$

Putting $R = A \cos \theta$
and $L\omega = A \sin \theta$
so that $\tan\theta = L\omega/R$
and $A^2 = L^2\omega^2 + R^2$
or $A = \sqrt{L^2\omega^2 + R^2}$.

$$i = E_m \cdot \frac{A . \sin(\omega t - \theta)}{L^2\omega^2 + R^2} = E_m \frac{\sqrt{L^2\omega^2 + + R^2} . \sin(\omega t - \theta)}{L^2\omega^2 + R^2}$$

$$\therefore \quad i = \frac{E_m . \sin(\omega t - \theta)}{\sqrt{L^2\omega^2 + R^2}} = i_m \sin(\omega t - \theta), \ (\theta = \tan^{-1} L\omega/R)$$

This is same as in (4) in Article 155 (a).

156. Idle or Wattless Current: When the inductance of a circuit is so great in comparison with the resistance, that the latter may be neglected, the current is entirely wattless and is, therefore called *wattless current.*

157. Skin Effect: When a steady voltage acts at the ends of a uniform wire, the density of the current is uniform over any cross-section. But when the voltage acting is alternating and has a high frequency, the current is not uniformly distributed over the cross-section but entirely confined to the surface layer of the wire. This phenomenon is known as the *skin effect* and on account of it the effective resistance of the wire is increased.

158. Rheostat and Choke Coils: Rheostat is a variable resistance and is used in a direct current circuit for regulating the strength of the current by altering the resistance.

In this case, a good deal of energy is wasted in the form of heat ($JH = C^2Rt$) in overcoming the resistance.

Choke coil consists of a coil of thick wire, the reactance of which can be varied by means of an adjustable soft-iron core. It is used instead of a rheostat, for regulating the current in the alternating current circuit.

In this case the alteration of reactance diminishes the current by setting up an opposite E.M.F.

Hence reactance diminishes the current without any waste of energy as heat.

Note: In a straight wire the inductance is small; it is greater in a coil of many turns especially if the coil is wound on iron.

The inductance of a circuit is an important factor if the current in the circuit is constantly varying in strength or direction or if it is an alternating current,

Choke coils, transformers and tuning coils in wireless work depend for their action on their inductances.

159. Properties of an Electric Circuit: In an electric circuit, the three things, *viz.*, **resistance, capacitry** and **inductance** are to be known for determining the current produced in it.

With a *constant* current *resistance* is only to be considered but with a *varying current* both the *capacity* and *inductance* are to be taken into consideration.

160. Electro-magnets: If a coil of wire be wound round a piece of soft iron, either straight or bent in the shape of a horse-shoe and if a current be made to pass through the coil the iron is magnetised and the piece becomes a magnet with opposite poles at the extremities. This combination is called an electro-magnet. The end round which the current flows in anti-clockwise direction becomes the north pole and the end round which the current flows in clockwise direction becomes the south pole.

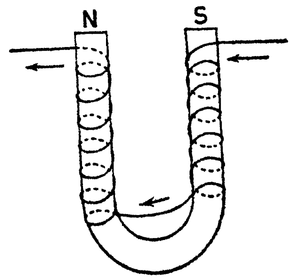


Fig. 124

Soft iron is the most suitable material for the core of the electro-magnet for it is magnetised as soon as the current is started and demagnetised as soon as it is stopped, *i.e.*, the soft iron possesses a large susceptibility and a small retentivity.

For practical purposes electro-magnent is used in preference to permanent magnets for the following reasons.

(1) The magnetic strength between the poles may be made very strong, maintain at a constant value and increased or diminished by adjusting the current in the coil of the electro-magnet.

In permanent magnets such an alteration in the strength of the magnetic field is not possible.

(2) The polarity of an electromagnet can be altered by altering the direction of the current, but in permanent magnets the polarity may be altered but with a great difficulty.

(3) The electromagnet may be demagnetised completely and can be used as a magnet when necessary, but this advantage can not be obtained in the case of permanent magnents.

161. The Magnetic Circuit: We know that a magnetic field may be completely represented by lines of induction. Since the lines of induction are closed curves they will enclose a tube when produced in both directions and for every section of the tube the quantity B. S. is the same. Such a tube of induction is called the ***magnetic circuit.***

Here B is the induction, *i.e.*, the number of lines of force per unit area of section of the tube and S is the area of the cross-section of the tube.

The quantity B. S. is called the magnetic flux or the total number of lines of induction.

Thus the magnetic flux proceeds in a closed path (circuit) just as does the electric current. Hence the name magnetic circuit.

Electric current needs an agent known as E. M. F. for its production and its value is given by the relation.

$$\text{Current} = \frac{\text{Electromotive force (E.M.F.)}}{\text{Resistance (R)}}$$

Similarly the flux when produced in a magnetic circuit needs an agent known as magneto-motive force (M.M.F.).

$$\text{So magnetic flux} = \frac{\text{Magnetomotive force (M.M.F.)}}{\text{Reluctance}}$$

The **Reluctance** is known as the magnetic resistance and is analogous to resistance in the electric circuit.

We know that the electrical resistance of a circuit is expressed as $R = \rho . \dfrac{l}{s}$

Similarly the *magnetic resistance* or *reluctance* is expressed as $R = \dfrac{1}{\mu} . \dfrac{l}{s}$, where μ is the permeability of the magnetic circuit. The reciprocal $\dfrac{1}{\mu}$ is called the *reluctivity* of the magnetic substance.

161(a). *Ring wound with endless solenoid* :

In this case, Magnetic flux $= \dfrac{\text{M. M. F.}}{\text{Magnetic resistance}}$

Magnetic flux $N = \dfrac{4\pi ni\mu s}{10 \times 2\pi r}$

Induction $B = \dfrac{N}{s} = \dfrac{4\pi ni\mu}{10 \times 2\pi r}$

Intensity $= \dfrac{B}{\mu} = \dfrac{4\pi ni}{10 \times 2\pi r}$

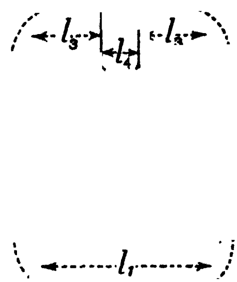


Fig. 125

161(b). To determine the magnetic field in the gap of an electro-magnet we have to consider the core of the electro-magnet to be made up of different cross-sections and different permeabilities.

Let the yoke have an effective length l_1 and cross-section s_1 and be made of iron of permeability μ_1.

The magnetic resistance of this part is $\dfrac{l_1}{\mu_1 s_1}$, so similarly the magnetic resistance of each limb is $\dfrac{l_2}{\mu_2 s_2}$, and of each pole piece is $\dfrac{l_3}{\mu_3 s_3}$, and of the gap between the pole pieces $\dfrac{l_4}{s_4}$

Therefore the magnetic resistance of the whole magnetic circuit is the sum of the magnetic resistance of its separate parts and is equal to

$$\frac{l_1}{\mu_1 s_1} + \frac{2l_2}{\mu_2 s_2} + \frac{2l_3}{\mu_3 s_3} + \frac{l_4}{s_4}$$

But the product of magnetic induction or magnetic flux and the magnetic resistance is known as the magneto-motive force or M. M. F.

In the electromagnet, the magneto-motive force depending on the current strength i and the number of turns n is expressed as $\dfrac{4\pi ni}{10}$.

Therefore the magnetic flux $N = \dfrac{4\pi ni}{10 \times (\text{Magnetic Resistance})}$

$$= \frac{4\pi ni}{10\left(\dfrac{l_1}{\mu_1 s_1} + \dfrac{2l_2}{\mu_2 s_2} + \dfrac{2l_3}{\mu_3 s_3} + \dfrac{l_4}{s_4}\right)}$$

Therefore the magnetic field in the air gap is obtained by dividing the flux N by the area of the gap.

QUESTIONS

1. Describe the means by which coils rotating in a magnetic field may be arranged to furnish (*a*) alternating current, (*b*) continuous current. [C. U. 1944]

2. Write short notes on
   (*a*) Alternating current. [C. U. 1942]
   (*b*) Earth Inductor—Its uses. [C. U. 1941, '43, '50, '55]

3. Show that for a coil rotating uniformly in earth's magnetic field *e. m. f.* is sinusoidal. [C. U. 1954, '55]

4. Show how the average, virtual and maximum values of the alternating current are related to one another. [C. U. 1945, '55]

5. What is meant by electric impedance?

An alternating *e. m. f.*, E *sin pt* is applied to a circuit containing resistance and self-inductance. Write down the differential equation for the current and obtain an expression for the current and impedance when the steady state is reached. [C. U. 1946, '47, '52]

6. Distinguish between the mean value and the root mean square value of an alternating current and find the relation between them. [C. U. 1956, '57]

Deduce an expression for the power consumed in a circuit carrying alternating current. [C. U. 1953]

7. What is an electro-magnet and why is it used in preference to permanent magnets? Explain on what depends the field strength giving quantitative relation as far as possible. What precaution is to be taken to maintain the field of a strong electro-magnet. [C. U. 1932]

8. What is meant by "electric impedance"?

Deduce an expression for the impedance of a coil of self inductance "L" and resistance "R" when a sinusoidal *e. m. f.* of frequency "f" acts on it. [C. U. 1959]

EXAMPLES

1. The primary of an induction coil is taking a current of 10 amps. at 16 volts. The secondary is delivering a current of 2·5 milliamperes in an X-ray tube. Assuming that the whole of the energy in the circuit is utilised by the X-ray tube, find the resistance of the latter. [C. U. 1926]

... of the primary $= 16 \times 10 = 160$ watts.

... of the secondary $= 2{\cdot}5 \times 10^{-3} \times E$, where E is the voltage in the secondary.

We have, $2{\cdot}5 \times 10^{-8} \times E = 160$. $\quad E = \dfrac{160}{2{\cdot}5 \times 10^{-8}} = 64 \times 10^{8}$ volts; since this voltage is utilised in the X-ray tube we have the resistance R of the tube as equal to $\dfrac{E}{i} = \dfrac{64 \times 10^{8}}{2{\cdot}5 \times 10^{-3}} = 256 \times 10^{5}$ ohms

As the whole amount of energy is used by the X-ray tube, the value of R gives the resistance of the tube, otherwise R should also include the resistance of the secondary of the coil.

2. An alternating pressure of 100 volts (virtual) is applied to a circuit of resistance 0·5 ohm and self-inductance 0·01 henry, the frequency being 50 cycles per second. What will be the reading of an ammeter included in the circuit? [C. U. 1945]

We have i (virtual current) $= \dfrac{E \text{ (virtual volt)}}{\sqrt{L^2\omega^2 + R^2}} = \dfrac{E \text{ (virtual volt)}}{\sqrt{L^2(2\pi n)^2 + R^2}}$

$$= \frac{100}{\sqrt{4\pi^2(50)^2(\cdot 01)^2 + (\cdot 5)^2}} = 31{\cdot}4 \text{ amps. (nearly).}$$

3. A disc of radius 8 cm. is rotated inside a long solenoid of 50 turns per cm. the axis of rotation of the disc coinciding with the axis of the solenoid.

If the disc makes 600 rev. per min. and the current in the solenoid is 1 amp., find the potential difference between the centre and the circumference of the disc. [C. U. 1954]

The magnetic field H inside the solenoid $= 4\pi ni = \cdot 4 \times 3{\cdot}142 \times 50 \times 1$.

The radius of the disc cuts a flux $\pi a^2 H$ in each revolution

$\therefore$ the $p.d.$ between the centre and the circumference of the disc $= \pi a^2$. N.H where a is the radius of the disc and N, the number of revolutions of the disc per minute.

$\therefore$ $p.d.$ between the centre and the circumference

$$= \frac{\cdot 4\pi^2 a^2 nNi}{10^8} \text{ volts} = \frac{\cdot 4 \times (3{\cdot}142)^2 \times 8 \times 50 \times 10 \times 1}{10^8} \text{ volts} \qquad N = \frac{600}{60} = 10$$

$$= \cdot 001263 \text{ volts.}$$

4. The equation of an alternating current is $I = 50 \sin 400\pi t$ What is the frequency and peak value of the current? What is the R M.S. value?

From the expression $I = 50 \sin 400\pi t = 50 \sin 2\pi nt$

Frequency $n = \dfrac{400}{2} = 200$. Here $2n = 400$

Peak value of the current = 50 amp.

R. M. S. value $= \dfrac{50}{\sqrt{2}} = 35{\cdot}36$ amp.

CHAPTER XIII

ELETRO-MAGNETIC MACHINES

162. Direct Current Dynamo : It is a machine for the production of electric current by means of mechanical work. The working of all dynamos is based on the principle of electro-magnetic induction. We know that when a closed coil is made to move rapidly in a magnetic field so as to cut magnetic lines of force a current is generated in the coil and the induced E. M. F. causing the current depends on the rate at which the lines are cut. So either a magnet may be moved near a coil or a coil may be moved near a magnet for this purpose.

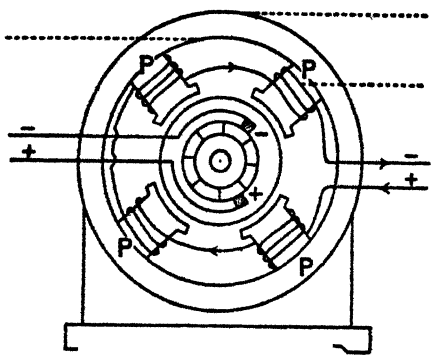


Fig. 126

A direct current dynamo consists of the following parts :

(1) Field magnets with curved pole-pieces P, P ; (2) Yoke ; (3) Armature ; (4) Commutator and Brushes ; (5) Frame ; (6) Shaft and bearings.

(1) Field magnet : In the ordinary type of a D. C. dynamo there are two pairs of curved pole-pieces P, P at right angles to one another. The magnets and the poles are surrounded by coils of wire known as *field coils* which carry the current to excite the magnet.

(2) Yoke : It is a part which joins the opposite pole-pieces and from which the poles projects radially.

(3) Armature : It consists of a number of turns of wire held in position in grooves by a soft-iron core in series, arranged at various angles to each other and they lie in slots cut in the cylindrical surface of a soft-iron core known as the *armature core.*

As the armature revolves inside the pole-pieces, lines of force are cut by the coil and E.M.F. is generated.

The iron core offers an easy path from a pole to the opposite one and helps the rotation of the coil.

(4) Commutators and Brushes : The commutators are ats made of copper and are insulated from each other by

insulating materials. They are fitted on the armature shaft and insulated from it by means of mica. These segments are suitably connected to the different positions of the armature coils.

(5) Brushes : They are generally made of copper or brass gauze compressed into a block and they are also of hard carbon. They are connected to the armature coil through the commutator and are the terminals of the external circuit.

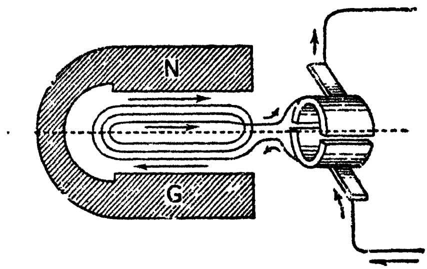


Fig. 127

In the earliest dynamo, *i.e.*, in **magneto-electric machines** a coil or a system of coils called the **armature** is made to rotate in the magnetic field produced by *permanent magnets* but in modern machines, *i.e.*, **dynamo-electric machines** the armature is made to rotate in the magnetic field produced by powerful *electro-magnets* with curved pole-pieces, known as *field magnets* the current required to excite the field magnet being taken from the armature round the field coils. Before the armature is rotated the residual magnetism of the field magnet is sufficient to start the generation of the current which again strengthens the magnetic field as the speed increases.

The armature having a very large number of turns of wire is fitted on a horizontal axle and revolves inside the pole-pieces and a current is generated in the coil which changes its direction after half revolution. By a special *commutator* (*split-rings*) *and brushes* the opposing currents are made to flow in one direction along the line wire and utilised for driving machines.

***Technical Facts** : If the armature and the field coils are joined in series the machine is said to be '*series wound*'.

If they are joined in parallel, the machines is said to be '*parallel wound*' or '*shunt wound*'.

If there are two sets of field coils, one of which is in series with the armature and the other parallel to it, the machine is said to be *Compound wound.*'

In a series-wound machine the E. M. F. increases with the load.

In a shunt-wound machine the E. M. F. decreases with the increase of the load.

In a compound-wound machine the E.M.F. is practically constant for all loads and the mechanical energy which is converted into electric energy is supplied by a prime mover usually a steam or oil engine and sometimes a water-wheel or turbine.

* These may be omitted by students.

Note : The core of a dynamo is subjected to fluctuating magnetising forces and this act involves an expenditure of energy. The loss of energy due to hysteresis is estimated by the area of the loop of hysteresis curve drawn with the intensity of magnetisation I as ordinate and the magneting force H as abscissae. The narrower the loop the less is the work wasted due to hysteresis. If the hysteresis loop is large, the core will be heated to a very high temperature. Iron core gives a narrow loop. So it is preferably used in the dynamo or a transformer as the core.

The loss of energy in ferro-magnetic substances, in the form of heat is due to two distinct causes, one due to hysteresis and the other due to eddy current developed in the mass of the substance.

The loss of energy in the core due to hysteresis is avoided by using soft iron cylinder as the core of the armature.

The loss of energy in the form of heat due to eddy currents (internal currents circulated locally in the mass of the metal) is reduced as much as possible by making the core, not of solid iron but of thin iron discs. The discs are all varnished and the surfaces of separation between the discs offer considerable resistance to the current but offer no appreciable resistance to the magnetic flux passing through them.

The *efficiency* of the dynamo is the ratio of the electric power generated to the mechanical power supplied.

Dynamos are used for generating electricity in Power House for supplying electric energy in large cities and towns while magneto-electric machines are used for igniting gas or oil in petrol engines and for medical purposes.

Since the magnet in a magneto-electric machine is a permanent magnet, it suffers certain disadvantages in comparison with a dynamo.

For large E.M.F. or currents, the magneto-electric machines are heavier, and the magnets used in these are more liable to lose their magnetism than the dynamos.

163. Principle of a D. C. Dynamo : To understand the principle of a **Direct Current Dynamo (D. C.)** let us first of all try to consider how the alternating current in the external circuit is rectified, *i.e.*, made to flow always in the same direction.

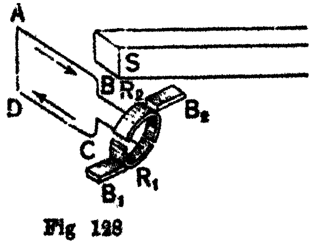


Fig 128

—In Figure 128 let the ends of the coil be connected to the two halves R_1 and R_2 of the copper ring fitted on a non-conducting axle or shaft (not shown in the figure) on which the coil turns and split diametrically. Two copper springs called ***Brushes*** **B_1 B_2 rest against the split copper rings and are connected to the ends of the external circuit.**

The position of the two *brushes* are arranged in such a way that they are diametrically opposite instead of being side by side. As the coil revolves, the two brushes cross the gaps in the ring just as the coil ABCD is passing through the position in which the plane of the coil is perpendicular to the lines of force of the magnetic field and that the induced current is zero.

In the same Figure 128 the end of the coil connected with R_2 is at a higher potential and the current flows through the external circuit from B_2 to B_1.

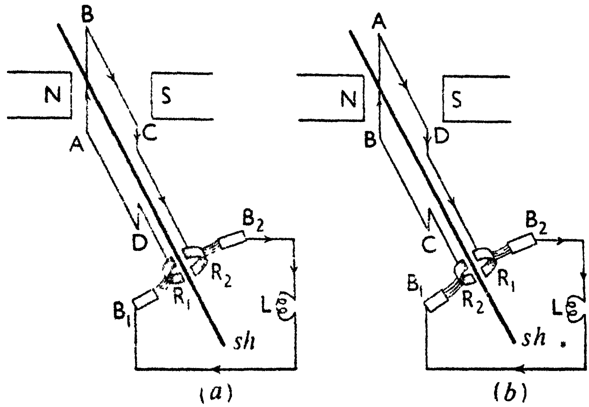


Fig. 129

Now when the coil has passed the position at which the induced current is zero, the direction of the E.M.F. is reversed and R_1 is now at a higher potential and touches the *brush* B_2. Thus the current in the external circuit still flows from B_2 to B_1.

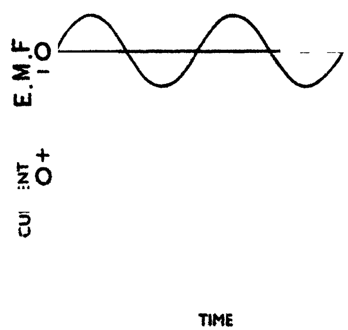


Fig. 130

From Figures 129(*a*) and 129(*b*) it will be evident that current flows in the same direction in the external circuit, during one complete rotation of the coil.

It must be remembered that no contrivance prevents the current in the coil or armature reversing.

In Figure 130, A represents the alternating current in the coil varying with time before rectification. B, the current after rectification which although flows in the external circuit in the same direction

is not a constant current but twice in each revolution it attains a maximum value and twice in each revolution it attains a zero value.

In Figure 130, C the dotted curves denote the currents due to two coils separately, the coils being of the same dimensions and fitted to the same axle or shaft in such a way that their planes are at right angles to each other and supplied with their own commutators.

The full-line curve represents the actual current due to the combined action of the two.

If the number of coils be sufficiently increased a uniform current may be obtained in the external circuit.

164. Dynamo and Ruhmkorff's coil: The principle of **working** of both **a dynamo and a Ruhmkorff's coil** is the same and depends on the production of induced current caused by the rapid change of lines of force in a circuit.

In the dynamo the coil in which the induced current is produced rotates in a fixed magnetic field, whereas the secondary of the Ruhmkorff's coil in which the induced current is produced is fixed but the change in the lines of force in the secondary causing the production of the induced current is due to the make and break of the primary circuit.

The dynamo may be used for generating either an alternating current or a direct current, but in Ruhmkorff's coil, the current between the ends of the secondary may be unidirectional if the spark gap is very large but ordinarily the current generated at a very high voltage is alternating, though at make the current is much weaker than at break.

In comparing the action of Ruhmkorff's coil and dynamo we notice that in Ruhmkorff's coil a large primary current supplied at low voltage from a few secondary cells is transformed into a small secondary current at a very high voltage.

It is like a step-up transformer in which the voltage is changed from low to high value. Ruhmkorff's coil may be said to be an appliance for converting electrical energy at low voltage into electrical energy at high voltage.

In dinamo the mechanical energy supplied is converted into electric energy in the form of current.

165. Efficiency of dynamo:

$$\text{Efficiency} = \frac{\text{Power delivered}}{\text{Power supplied}} \times 100 \text{ per cent.}$$

166. D. C. Motor: An electric motor may be called a reversible dynamo and it converts electric energy into mechanical work. Barlow's wheel is the simplest form of electric motor.

The modern motor consists of the following parts :—

(1) Field magnet—In small motors, the field magnet may be either a permanent or an electro-magnet but wherever a large power is required electro-magnets are used. (2) Armature—It consists of a large number of coils wound on an iron-core. (3) Split-rings. (4) Metal brushes or carbon rods.

Principle of motor: The fundamental principle of motor is explained with reference to the adjoining diagram. A rectangular coil of wire ABCD is placed between the pole-pieces N, S of a powerful magnet and has its two ends connected to two *split-rings*. Two metal brushes press lightly against the split-rings.

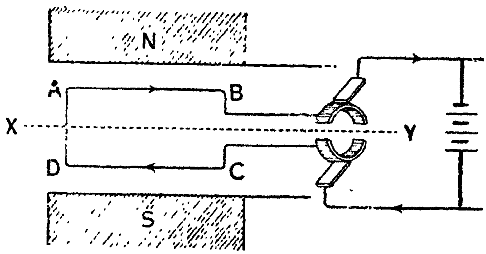


Fig. 131

If the terminals of a battery be connected to the metal brushes a current will pass through the coil in the direction indicated in Figure 131 and a couple will act on the coil tending to rotate it.

As the coil rotates, the split-rings connected with its ends also rotate and the rotation will continue till the plane of the coil is perpendicular to the magnetic field. But due to momentum it has gained, the coil will move past this position and the upper brush comes in contact with the lower split-ring and lower brush with the upper split-ring. That is, the positive terminal of the battery is always connected to the split rings which comes to the bottom.

It is to be noticed that the direction of the current in the coil remains the same and the coil rotates continuously in one direction. With a single turn in the coil the rotation is somewhat jerky, but with several coils set in different planes a steady rotation is obtained.

Uses: In electric fans, blades are made to rotate by fixing them to the axle of the armature.

Motors are used in tram cars, cinema machines, printing press and in various other purposes for which rotation is required.

167. Power and Power Loss in a motor: The construction of a D. C. motor is similar to that of a D. C. Dynamo and the

rotation of armature produces in it an E.M.F. whose direction is opposed to the external or impressed E.M.F. It is called the **back E.M.F.**

The power obtainable from the motor is the product of the back E.M.F. and the current, the rest of the power supplied being used in overcoming the resistance of the circuit.

If E be the E.M.F. applied to the armature, E′, the back E.M.F. and R, the resistance of the machine, we have $E = E' + iR$

$$i = \frac{E - E'}{R}$$

Thus if E′ is the back E. M. F. and i the current, the Power **obtainable** $= E \times i = E'\left(\frac{E - E'}{R}\right)$

Power Loss (Joule heat loss) $= i^2R$ and **Power Supplied** $= Ei$.

We know that E′ depends on the speed of rotation of the motor, When the motor is at rest, $E' = 0$, As the speed increases, the back E. M. F. increases and the current $i = \frac{E - E'}{R}$ diminishes. The acceleration of the motor ceases when the current attains such value as to make the total force developed equal to the retarding force. The back E. M. F. regulates the flow of the current and acts as an automatic governor of the steam engine.

168. Alternators : Alternators are dynamos or generators which deliver an alternating current (A. C.) to the external circuit, whereas Direct Current Dynamos are generators which deliver continuous (direct) or unidirectional current (D. C.) to the external circuit.

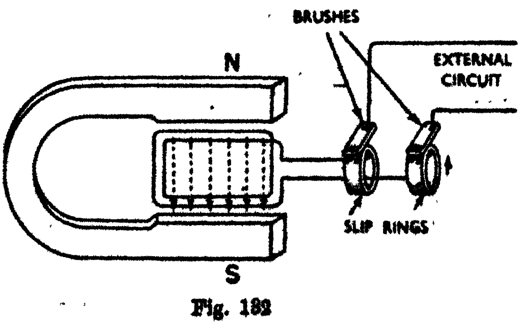


Fig. 182

In **alternators, the ends of the armature coils instead of being connected to two commutator segments as in D. C. dynamos, are**

joined to two seperate rings as in Figure 132 and the brushes of the external circuit press on these rings so that the current in the external circuit is *alternating*.

The alternating current (A. C.) dynamo consists of an armature containing a large number turns of wire rotating very rapidly within a strong magnetic field of a field magnet NS. The ends of the coils are connected to two slip-rings R_1, R_2 on which rest two brushes B_1, B_2 connected to the external circuit (Figure 133).

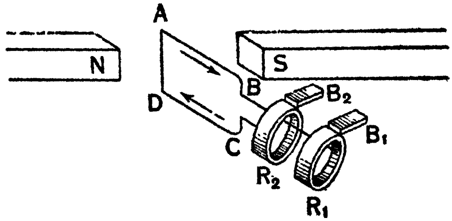


Fig. 133

Action : Let the plane of the coil ABCD be at first perpendicular to the direction of the lines of force in the field due to the field magnet with poles N, S. As the coil rotates from 0° to 90°, the lines of force through the coil change from maximum to minimum and then when it rotates from 90° to 180° the lines change from minimum to maximum. Thus during the first half of revolution of the coil from 0° to 180°, due to changes in the lines of force through it, the current induced in the coil circulates *in the same direction* as shown in Figures [134(*a*) & (*b*)]. During the other half of

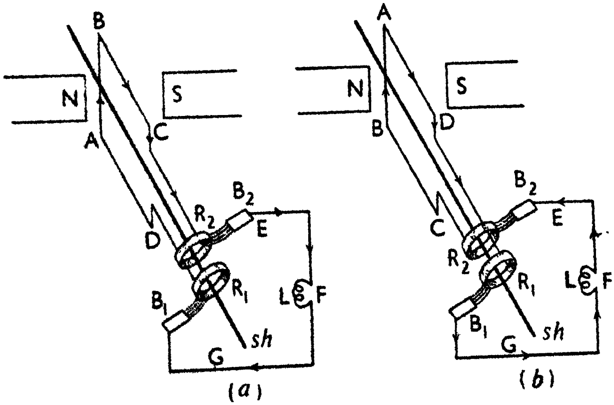


Fig. 134

revolution, the direction of the induced current will be reversed in direction when rotated from 180° to 360°. This induced current which changes or alternates its direction at each half revolution is called the

alternating current (A. C.). This alternating current induced in the coil circulates in the external circuit through the slip-rings R_1 and R_2 and brushes B_1 and B_2. The above facts will be evident from Figures 134 (*a*) and (*b*) which correspond to consecutive half-cycle in one complete rotation of the coil. One complete rotation of the coil is called the **cycle** and the number of cycles completed per second is **frequency** of the current. The machine producing such an alternating current is called an **alternating current dynamo** or an **alternator.**

Usually commercial generators have a stationary armature whose ends are permanently connected to the line wires and the field magnets rotate inside it. This enables very high voltage to be generated as there is no danger of sparking and the wires can be well insulated from each other.

169. Advantages of Alternating Current : The production of electric energy is most economical on a large scale. Large scale production is not generally possible near the place where the energy is to be consumed. The electric energy is, therefore, transmitted from the generating station to the consumer through power lines which are often many miles long. A large amount of electric energy is wasted in the lines in the form of heat. The heat produced is proportional to the square of the current. So the loss can be decreased by decreasing the strength of the current.

But, $\text{Power} = \frac{1}{\sqrt{2}}\ \text{max. volt} \times \frac{1}{\sqrt{2}}\ \text{max. current} \times \cos\theta$.

As the current is decreased the power is decreased. High pressure E. M. F. is dangerous to human life. Therefore the pressure must be lowered before supplying power to the consumer. So we require high pressure for transmission and low pressure for consumption.

The pressure of the alternating current (A. C.) can be easily raised or lowered by means of a *transformer*. But the pressure of direct current (D. C.) cannot be easily altered.

For *long distance transmission* it is always necessary to minimise the cost of transmission by using comparatively thin conductors. To do this, alternating currents generated at high volts (10,000 volts) are lowered down to alternating currents at low voltage by a step-down transformer and then safely used for domestic supply and other useful purposes.

170. Disadvantages of A. C. : **It can not be used in electrolysis and in charging batteries though it is as good as D. C. for heating and lighting purposes.**

171. Dangers from Electricity : Alternating currents (A. C.) at 220 (virtual volts) pressure are *more dangerous* than Direct Current (D. C.) at 220 volts pressure.

For A. C. pressure although it registers 220 virtual volts, it is really fluctuating between ± 311 volts.

Current at 220 volts may sometimes be safely handled if the skin is perfectly dry or if the person stands on an insulating substance but if the skin is wet fatal shocks might produce disastrous results.

The resistance of the human body is mainly confined to the skin and is about 30,000 ohms but when the skin is wet it is as low as 200 or 300 ohms.

172. Distinction Between D. C. and A. C. : When a *direct current* is passed into a conductor, magnetic effect is obtained outside it and produces a deflection when it is passed into moving coil instruments for measurement of current and potential. But when an *alternating current* is passed into a conductor, no deflection is produced in a moving coil instrument as reversals of the current take place rapidly.

A direct current causes electrolysis when it is passed through an electrolyte but with alternating current no electrolysis is produced as the A. C. reverses many times per second. So the net electrolysis is zero and hence no change is produced at the electrodes.

For heating effect D. C. is as good as A. C. as heat is produced in the wire in whichever direction the current flows through it.

173. Transformer : Any device of raising or lowering the voltage in an alternating current-supply is called a transformer. It is intended to work with alternating or varying currents and does not work with steady currents.

There are two forms of transformers (1) the *Step-up*, (2) *Step-down*.

The transformer when used to raise the voltage is called the **step-up transformer**, and when used to lower the voltage it is called the **step-down transformer.**

The earliest form of a transformer consists of a ring made of soft iron on which two coils known as the primary and the secondary are wound.

Now-a-days, the transformer consists of a rectangular frame of laminated iron strips round the two arms of which the primary coil and the secondary coil are wound (Fig. 135).

The alternating current passing through the primary circuit produces magnetic flux of varying intensity within the soft iron core. As the core is also embraced by the secondary coil the core induces alternating current in the secondary circuit.

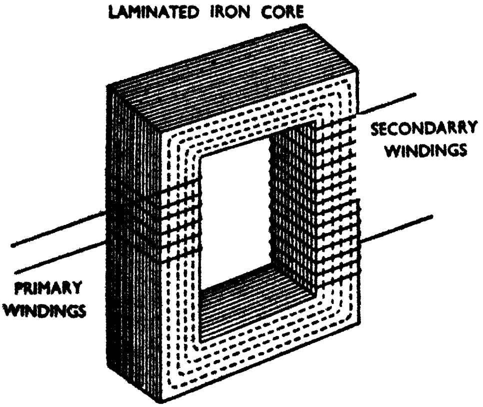


Fig. 135

If E_1 be the E. M. F. at the ends of the primary and E_2, that at the ends of the secondary and n_1, the number of turns in the primary, and n_2 that in the secondary, then

$$\frac{E_1}{E_2} = \frac{n_1}{n_2}$$

173(a). Uses for transformers :

(a) The conversion of a high-voltage low-current A. C. supply of constant frequency into a low-voltage high-current A. C. supply of the same frequency and *vice-versa.*

(b) The conversion of alternating potentials of the same frequencies as in telephone system.

(c) On account of the transformer alternating voltage is easier to distribute than steady (D. C.) voltage.

(d) Economic transmission of electrical energy over long distance has been made possible by the use of transformer.

Note : **Audio-frequency transformer** (known as low frequency transformer) is employed in radio-receivers and has a laminated core to reduce eddy current losses and is used for alternating voltages ranging from 20 to 16000 cycles per sec.

Radio-frequency transformer (known as high frequency transformer) is used for alternating voltages of the order of million volts. At these high frequency eddy current losses in laminated iron core is considerable and so the latter is replaced by an air core.

174. Commercial Meters : Meters or 'measuring instruments' may be divided into standard instruments of precision and commercial meters.

Commercial meters are simpler in construction and also cheaper. They are more robust and portable.

Meters are used for measuring

(1)	Electric pressure or potential difference.	(Voltmeter)
(2)	Strength or intensity of current.	(Ammeter)
(3)	Quantity of electricity.	(Supply meter, Electrolytic meter)
(4)	Electric energy.	
(5)	Electric power.	

174(a). Motor Meter: The simplest form of a motor meter is identical in construction with the Barlow's wheel.

A fraction of the current in the circuit is passed through the meter and the wheel begins to rotate. Eddy currents are induced by the magnetic field in the metallic part of the wheel and the strength of those currents is proportional to the speed of rotation. The wheel therefore rotates with constant speed.

The driving force arising from the action of the magnetic field on the current passing through the wheel is equal to the retarding force due to the action of the magnetic field on the induced currents (Lenz's Law).

As the magnetic field is constant and the same in the two cases when the speed is constant the strength of the current in the circuit is proportionl to the strength of the induced current which again is proportional to the speed.

Therefore current, $i \propto$ speed

or $it \propto \text{speed} \times t$

or $Q \propto$ number of rotations.

The number of rotations of the wheel is recorded on dials as in a watch or clock and the quantity of electricity is thereby measured.

174(b). Energy Meter: As energy consumed in a circuit is the product of the supplied pressure and the quantity of electricity passed through the circuit, the value of the energy consumed can be obtained by multiplying the reading of a quantity meter by the supply pressure which is generally constant.

Quantity meters are graduated in ampere-hours and the energy meter in kilowatt-hours.

The unit used for charging the consumers is the K. W. H. or B. T. U.

If the supply pressure is not constant the reading in the quantity meter will not directly show the number of units consumed.

QUESTIONS

1. Describe briefly :—
   (1) Dynamo and how it is used? [C. U. 1937, '49, '56]
   (2) A. C. Generator and a D. C. motor.

2. Describe in general terms how a D. C. generator acts. [C. U. 1956]
Why are the coils of the armature wound on an iron cylinder? Why is the cylinder made of vanished discs? What is meant by hysteresis and eddy current losses? [C. U. 1956]

3. Describe the construction and action of a transformer. What are the uses of the transformer?

4. Write a short note on Commercial meters.

EXAMPLES

1. An electric motor whose armature resistance is 25 ohms takes a current of 10 amps. from a 100 volt main.

Find (a) the power loss in the armature, (b) the available horse-power. (1 H. P. = 746 watts).

(a) Power supplied $= i \times E = 10 \times 100 = 1000$ watts.
Power loss in armature $= i^2 R = 100 \times \cdot 25 = 25$ watts.
Power available $= 1000 - 25 = 975$ watts. $= \frac{975}{746} = 1 \cdot 31$ H. P.

2. A small electric motor is rated $\frac{1}{8}$th H. P. and is run off at 230 volts supply. How much current will it draw from the supply mains when worked at full capacity ? [D. U. 1947]

3. What is the efficiency of a dynamo driven by 1 H. P. engine, the dynamo delivering a current of 5 amps. when the *p. d.* across its terminals is 116 volts ?

Power delivered $= i. V = 5 \times 116 = 580$ watts.

Power supplied $= 1$ H. P $= 746$ watts
Efficiency $= \frac{580}{746} \times 100 = 77$ per cent.
Thus there is a loss of 23% of energy in the form of heat inside the dynamo.

4. A series motor has a resistance of 3 ohms and a *p. d.* of 150 volts is applied to it. Calculate the current when the motor runs at such a speed that the back e. m. f. developed by the armature is 114 volts. [Ans. 12 amps.]

CHAPTER XIV

CONDUCTION THROUGH GASES

175. Ionisation : The phenomenon of the separation of a molecule into oppositely charged ions is called **ionisation.** According to electron theory the ion of an element is an atom which has gained or lost one or more electrons.

When the atom has gained an electron it is negatively charged and is called a *negative ion* but when the atom has lost an electron it is positively charged and is called a *positive ion.*

The number of gaseous ions produced by any exciting cause is generally small in number.

176. Ionisation Current : When two electrodes, one positive and the other negative are placed in a space containing gaseous ions, the positive ions will move towards the negative electrode, and the negative ions to the positive electrode. This movement of the oppositely charged ions constitutes what is known as *ionisation current.* The minimum potential difference maintained between two electrodes for starting of ionisation process is called *ionisation potential.*

177. Passage of Electricity through Gases : A dry gas at ordinary pressure is a bad conductor of electricity. A spark can be produced between two conductors placed in air at normal pressure by applying suitable high P.D. between them, as in the case of spark gap in an induction coil. The potential necessary for production of the spark is called sparking potential. An electric discharge can be passed through a gas at a very low pressure. In all cases the passage of spark or discharge is due to the formation of conducting carriers, *i.e.*, ions by the process of ionisation.

A gas can be also ionised by the action of external agencies such as X-rays, ultra-violet rays, α_1 β and λ rays. These are called ionising agents.

177(a). Measurement of Ionisation Current : Figure 136 shows an experimental arrangement for measurement of ionisation current. Under ordinary conditions the plate A and both sets of quadrants of the electrometer will be at zero potential and there will be no deflection of the electrometer needle. If an ionising agent such as X-rays be made to act on the gas between A and B, the gas becomes ionised. Due to movement of $+ve$ and $-ve$ ions towards plates A and B an ionisation current will flow. The electrometer needle will be deflected and the deflection will increase gradually owing to gradually increasing potential of A caused by the ionisation current. By noting the deflection for a given time the rate of increase of deflection can be found. The rate of increase of deflection is proportional to the rate of change of potential of A.

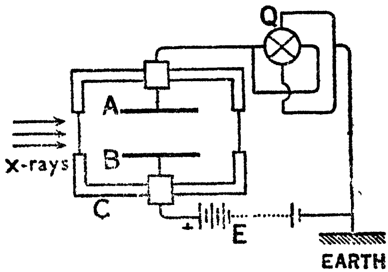


Fig. 136

Let C be the capacity of the plate A and of the electrometer and V the change of potential in a given time t. The quantity of electricity Q reaching A in time t is given by $Q=CV$. Hence average ionisation current i during this time t is given by $i=Q/t = CV/t$.

If θ be the observed change in deflection of the electrometer needle during the time t, then $V=K\theta$, where K is a constant of the electrometer which can be found by independent experiment.

$\therefore$ $i=CV/t=KC.\theta/t$. Thus ionisation current for any given P.D. between A and B can be measured.

The ionisation current varies with applied voltage between A and B in a manner shown by curve in Figure 137.

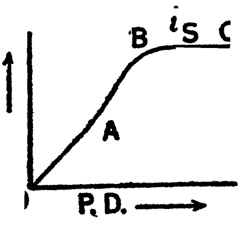


Fig. 137

From O to A, current increases with V at a slower rate, from A to B at a much quicker rate. The portion BC which is parallel to P. D. axis shows that current remains the same although P. D. is increased. This portion corresponds to maximum or what is called **saturation current**, when the number of ions reaching the collecting plate per second is equal to the number of ions produced in same time by the ionising agent.

Conduction current in a wire strictly obeys Ohm's law. Ionisation current obeys Ohm's law up to a certain stage. If AB be a metal wire, current through it, will decrease if wire be longer, *i.e.*, distance between A and B be greater; again current will increase if the wire be thicker. But ionisation current will increase if distance between plates A and B, or area of the plates A and B be increased. For in each case number of ions produced and received by plate A will be greater.

178. Phenomena of Discharge Tube: When an electric discharge is made to pass through a gas at atmospheric pressure, contained in a glass tube which is provided with two metal electrodes C (*Cathode*) and A (*Anode*), at its two ends, and a side tube with stop-cock S near one end, by connecting the electrodes to two terminals of an induction coil, a variety of changes is observed as the pressure of confined gas is gradually reduced by means of an air pump connected to the tube through the stop-cock. The more important changes in the tube are as follows.

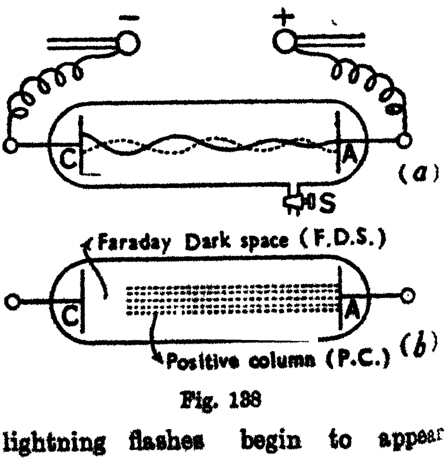


Fig. 138

(a) At about one atmospheric pressure, since the resistance of confined gas is exceedingly high no spark is found to take place.

(b) When the pressure is reduced to about 10 *mm* of mercury, irregular sinuous tracks of light resembling lightning flashes begin to appear [Fig. 138 (*a*)].

(c) At a preassure of 1 *mm* of mercury, a column of beautiful crimson colour, called **Positive Column**, is found to extend up to the anode [Fig. 138(*b*)]. Near the Cathode, however, a short dark space separates the Cathode from the positive column. This dark space is called **Faraday's Dark Space** (as it was discovered by Faraday). The Cathode itself again is found to be covered by a thin glow, called the **Negative glow** [Fig. 139(*c*)].

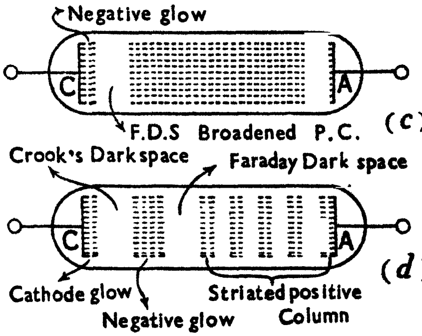


Fig. 139

(d) On further reduction of pressure, the negative glow is found to detach itself from the Cathode and a second dark space, called the **Crooke's Dark Space** appears between the negative glow and the Cathode which itself emitting a glow called **Cathode glow**. When the pressure is reduced to ·1 *mm* of mercury, the same appearance continues in the tube except that the positive column splits up into alternate dark and bright bands called **striations**, which become fewer when the pressure is still further decreased [Fig. 139(*d*)].

(e) With further reduction of pressure, the Crooke's dark space increases rapidly in size and the negative glow moves almost to the anode and the striated positive column almost disappears.

(f) At a pressure between ·01 and ·02 *mm* of mercury, the Crooke's dark space alone fills the whole tube and the glow inside the tube disappears completely. The portions of glass wall opposite to the cathode emits a glow, yellowish green or bluish according to the composition of the glass.

The glow emitted from the wall is due to the impingement on the wall, of an invisible stream of charged particles moving with high velocity. These moving charged particles emanating from the cathode are called Cathode rays.

Note: As the electrical resistance of the tube becomes considerably large at this stage, it is difficult to pass any discharge at all if the pressure be reduced still further.

179. Cathode Rays: When an electric discharge is passed through a gas at a pressure of about 10^{-1}*mm* of mercury, contained inside a glass tube having two electrodes connected to an induction coil, a stream of invisible negatively charged particles falling upon the

walls near the anode excites it into fluorescence. These streams of charged particles are called Cathode rays.

The particles are not solid, liquid or gaseous, but consists of ultra-atomic corpuscles much smaller than the atom and were considered by Crooke as a fourth state of matter. These particles were subsequently however given the name **electrons** which are atoms or smallest entities of electricity and quite independent of gross matter.

179(a). Properties of Cathode Rays :

(1) Cathode rays travel in straight lines and cast shadows of objects when placed in their path.

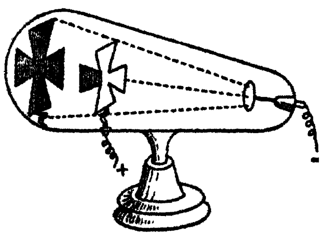


Fig. 140

This is demonstrated by placing a mica cross in the vacuum tube in a direction at right angles to the direction of the Cathode rays. A well-defined shadow of the cross will be formed on the fluorescent wall at the extreme end of the tube.

(2) Cathode rays are deflected by a magnetic field.

A magnet is held near the beam of the Cathode rays travelling along the length of the tube. The beam is seen to be deflected by a magnet according to Fleming's Left-Hand Rule. The direction of deflection is perpendicular both to the field and to the rays.

(3) Cathode rays are deflected by an electrostatic field.

The rays are seen to be deflected if they are made to pass through two horizontal metal plates supported within the tube and maintained at a constant potential difference thus creating a vertical electrostatic field.

(4) Cathode rays proceed normally from the Cathode surface.

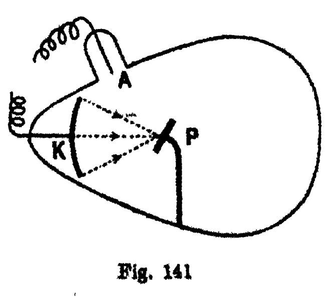


Fig. 141

If the Cathode be the part of a spherical surface (concave), the rays are seen to pass along the normals to the surface and converge to the centre of curvature of the surface and due to their concentration, an aluminium plate placed at the centre becomes red hot.

(5) Cathode rays exhibit heating and mechanical effects. If the Cathode be concave, a piece of platinum placed at the centre of curvature will be heated to redness.

Again if the mica vanes of a rotating wheel be placed in the path of the rays the wheel is set in rotation and can move from one side to another by interchanging the electrodes of the vacuum tube.

(6) Cathode rays consist of negatively charged particles. The Cathode rays are made to pass into a metal vessel placed within the vacuum tube and the vessel acquires a negative charge.

If the vessel be connected to an electrometer or to an electroscope the vessel is found to be charged negatively.

(7) Cathode rays can pass through thin metal foils, or plate without puncturing them, and a bluish stream comes out of the foil or the plate. The emerging rays first observed by Lenard, are known as **Lenard rays**.

(8) Cathode rays can produce phosphorescence on certain bodies. If the vanes of the mica wheel (mentioned under 5 above) be coated with suitable substances, variously coloured phosphorescent light can be seen, when the wheel rotates.

(9) Cathode rays can affect a photographic plate.

(10) Cathode rays produce X-rays when they strike any material substance.

(11) Cathode rays can impart conductivity to the gases, through which they pass, or in other words Cathode rays can ionise a gas.

179(b). Negative Charge conveyed by the Cathode Rays: The fact that Cathode rays carry negative charge is experimentally demonstrated by Thomson by an apparatus described below. His experiment is a modification of the original experiment of Perrin.

In Figure 142, C is the Cathode and the solid brass anode A is earth-connected and has a slit in it. B and D are two metal cylinders insulated from each other and each cylinder is provided with a narrow transverse slit. The outer cylinder B is earth-connected and D is connected to an electrometer.

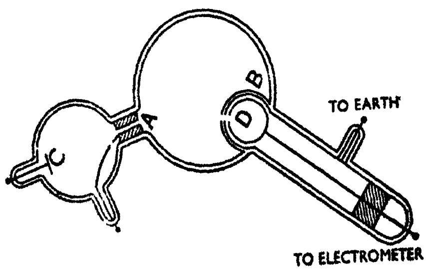


Fig 142

When the metal electrodes of the apparatus are connected to the terminals of an induction coil, the Cathode rays are emitted from the surface of the Cathode and pass straight along lines normal to the surface of the Cathode and no charge is acquired by D through the slit in the anode

when the rays are undeflected. But if the rays are deflected by means of a magnet held in such a way that the rays strike the slit in B, the cylinder D rapidly receives a negative charge as is shown by the electrometer deflection. A limit is however soon reached owing to the gas in the discharge tube becoming conducting.

180. Determination of v and e/m of electron : J. J. Thomson's Experiment : The apparatus used by Thomson is a highly evacuated discharge tube in which C is the Cathode, A the anode in a side tube. The wall S of the tube remote from the Cathode has a coating of some fluorescent compound. When the terminals of the induction coil are connected to C and A Cathode rays are produced in the tube (Fig. 143).

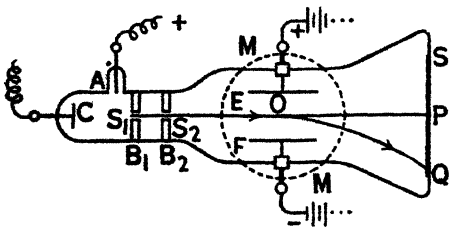


Fig. 143

Cathode rays proceeding from the Cathode pass through the slits S_1 and S_2 in two metal blocks B_1 and B_2 respectively, and emerge out as a very narrow beam. The narrow cathode beam then passes through a region in which a vertical electric field and a transverse magnetic field can be produced. The electric field is set up by two plates E, F maintained at a potential difference by connecting them to the terminals of a battery, while the magnetic field restricted to the circular space MM, is set up by electromagnet or by means of external coils of wire carrying a current.

In the absence of any field, electric or magnetic, the narrow cathode beam proceeds straight and producas a luminous patch at P on the screen S. As the electric and magnetic fields are mutually perpendicular to each other and to the direction of motion of electrons in the cathode beam, the electric deflection and magnetic deflection occur in the same plane.

If e be the charge on an electron and X the intensity of the electric field, then the force acting on an electron is equal to Xe. Again the magnetic force on each electron is Hev, where H is the intensity of the magnetic field, and v the velocity of the electrons [Compare $F=Hil$].*

[* **Note:** If a wire of length l traversed by a current i be placed in a magnetic field H acting at right angles to the wire, it is urged to move under a force $F=Hil$. But $i=q/t$ where q is charge flowing in t sec,

$\therefore F=H\frac{q}{t}.l=Hq.\frac{l}{t}=Hq.v$. In the present case $q=e$; hence $F=Hev$, where v=velocity of electron.]

Then, when both the two fields are applied and adjusted so that their effect nutralise each other and the patch of light remains in the central undisturbed position P.

$$\text{we have} \quad Xe = Hev \quad \text{or.} \quad v = \frac{X}{H} \quad \ldots (1)$$

The cathode beam is then subjected to magnetic field only. The beam is deflected and on falling on the fluorescent screen produces a patch of light at Q. The path of the beam under magnetic field is an arc of a circle of radius r, since the direction of the force is perpendicular to the direction of motion of the electron.

If m be the mass of electron, then the centripetal force acting on it is equal to $\frac{mv^2}{r}$

$$\text{Hence we have} \quad \frac{mv^2}{r} = Hev \quad \text{or,} \quad \frac{e}{m} = \frac{v}{Hr} \qquad (2)$$

$$\text{Substituting value of } v \text{ from (1)} \quad \frac{e}{m} = \frac{X}{H^2 r} \quad \ldots (3)$$

The value of r can be found from a knowledge of the distance OP and PQ, or in other words form the geometry of the apparatus.

Hence knowing X, H and r, e/m can be determined. It is to be noted that value of H in two experiments above—one under combined fields and another under magnetic field only—must be kept the same.

Thomson found that the value of $e/m = 1.77 \times 10^8$ Coulombs/*gm* $= 1.77 \times 10^7$ *e. m. u.* per gram or 5.21×10^{17} *e. s. u./gm.* The value of velocity of electron "v" was found to be of the order of 10^9 cms./sec. [between $(2 \rightarrow 3) \times 10^9$ cms./sec.].

Note : The ratio e/m may be found by making observations under (1) combined fields and (2) under electric field, as follows.

When electric field alone is applied, a force Xe acts on an electron of mass m in a vertical direction producing an acceleration Xe/m acting during the time t spent in traversing the field.

This time t is obviously equal to l/v, where l is the length of the field and v the velocity of the electron. The vertical velocity which is acceleration $\times$ time $= \frac{Xel}{mv}$. On leaving the field, the

or $(mg)^2 = \dfrac{162\pi^2\eta^3 v_1}{(\rho-\sigma)g}$ $\qquad mg = \left\{\dfrac{162\pi^2\eta^3 v_1{}^3}{(\rho-\sigma)g}\right\}^{\frac{1}{2}}$

or $mg = \dfrac{9\sqrt{2}.\pi\eta^{\frac{3}{2}}v_1{}^{\frac{3}{2}}}{(\rho-\sigma)^{\frac{1}{2}}.g^{\frac{1}{2}}}$

Then putting mg in (3)

$$XE = \frac{9\sqrt{2}.\pi\eta^{\frac{3}{2}}v_1{}^{\frac{3}{2}}}{(\rho-\sigma)^{\frac{1}{2}}.g^{\frac{1}{2}}}.\frac{v_1+v_2}{v_1} = \frac{9\sqrt{2}.\pi\eta^{\frac{3}{2}}v_1{}^{\frac{1}{2}}(v_1+v_2)}{(\rho-\sigma)^{\frac{1}{2}}g^{\frac{1}{2}}}$$

$$\therefore E = \frac{9\sqrt{2}.\pi\eta^{\frac{3}{2}}v_1{}^{\frac{1}{2}}(v_1+v_2)}{X.\ (\rho-\sigma)^{\frac{1}{2}}.g^{\frac{1}{2}}}. \qquad \ldots \quad (5)$$

The values of v_1 and v_2 are obtained by noting the times of transit of the oil drop through a fixed distance along the scale of the eye-piece. Then as all quantities of right hand side of (5) can be found, the charge E on the drop can be measured.

Discussion : As the oil drop may have captured one, two or more electrons during observation by the microscope, the charge E does not therefore necessarily correspond to electronic charge itself, but may be twice, thrice or even higher multiples of it. The experiment was conducted a large number of times using various drops at different intervals of time. It was found that different values of E obtained in different observations, were all integral multiples of some smallest entity or unit. This smallest entity or unit is therefore equal to the electronic charge.

Millikan found that the smallest value of E, *i. e.*, the charge of electon "e" was equal to $4{\cdot}77\times10^{-10}$ e.s.u.

Note : In the above experiment fresh ions both +ve and negative were produced by X-rays. The oil drop in that case could capture one, two or more +ve or −ve ions. If the oil drop having already a negative charge captured +ve ion, then its motion under electric field will be retarded when plate P was +ve. By observing mode of variation of the velocities of the oil drop under electric field, Millikan could know whether charged oil drop has captured more electrons or positive ions. From this experiment Millikan was therefore able to determine also the smallest unit of +ve charge which was found to be the same as that of electron.

182(a). Rough method of determination of e : The oil drop was allowed to fall under gravity and its steady terminal velocity v_1 was found by necessary observation with microscope. Then by Stoke's law

$mg = 6\pi\eta a v_1$ where different symbols are same as in (1) of Article 182

But $mg = \frac{4}{3}\pi a^3(\rho - \sigma).\ g, \ldots, \frac{4}{3}\pi a^3(\rho - \sigma).\ g = 6\pi\eta a v_1$

or $a^2 = \frac{9}{2}\ \dfrac{\eta . v_1}{g(\rho - \sigma)}$ or $a = \left\{\dfrac{9}{2} . \dfrac{\eta v_1}{g(\rho - \sigma)}\right\}^{\frac{1}{2}}$

Keeping the plate P positive, the electric field was so adjusted that the drop remained stationary due to equal and opposite forces of gravity and electric field. Then if X be electric intensity, E the charge on the oil drop.

$$XE = mg = \frac{4}{3}\pi a^3(\rho - \sigma).\ g = \frac{4}{3}\pi . \left(\frac{9}{2} . \frac{\eta v_1}{g(\rho - \sigma)}\right)^{\frac{3}{2}} . (\rho - \sigma) . g$$

$$= \frac{9\sqrt{2} . \pi \eta^{\frac{3}{2}} v_1^{\frac{3}{2}}}{g^{\frac{1}{2}}(\rho - \sigma)^{\frac{1}{2}}}$$

$$\therefore\ E = \frac{9\sqrt{2} . \pi \eta^{\frac{3}{2}} v_1^{\frac{3}{2}}}{X . g^{\frac{1}{2}}(\rho - \sigma)^{\frac{1}{2}}},\ \text{whence E can be found out.}$$

Discussion same as before.

183. Some Applications of discharge of electricity through gases and vapours : (1) The formation of positive column (BC, Fig. 146) in a discharge tube has been utilised in the construction of **Helium**, **Hydrogen** and **Neon** tubes. These tubes are used as spectroscopic source of light and for advertisements.

Fig. 146

(2) **Fluorescent Lamp** : It is a discharge lamp in which electrical discharge through a gas is employed in producing bright fluorescent light at a fairly low cost. It consists of a long glass tube two to four feet long and having a diameter of 1 to 1·5 inches. At each end of the tube there is a small coil filament of tungsten wire coated with suitable electron emitting material and the terminals of each coil are brought outside for external connection. The tube contains a little argon and nitrogen at a moderately low pressure and there is a small quantity of mercury inside it. The inner wall of the tube is coated with fluorescent powder such as zinc silicate, Calcium tungstate etc. to produce the desired coloration.

The mode of connection of a fluorescent tube is shown in Figure 147. M, M′ represent the terminals to be connected to the mains which may supply an alternating or a direct current. A resistance R is usually connected in series to drop the voltage of the mains to a value necessary to run the lamp. A choke Ch, called ballast

choke which consists of a large coil of wire wound on a soft iron core is connected in series in mains circuit The two filaments F_1, F_2 are connected with a push button switch, called the starter switch, S.

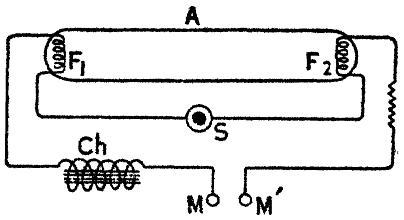


Fig. 147

Keeping main switch on, as the switch S is pressed the circuit is completed and the current which is set up heats the filaments and electrons are emitted. These electrons make the path in the tube between the filaments partially conducting. On releasing the starter switch S, the self induction of the choke generates a momentary high voltage impulse which causes an arc discharge inside the tube. As the discharge commences, the electrons previously ejected move with a high velocity and colliding with argon atoms ionise them so that the latter give out their characteristic colour. In the mean time mercury also vaporises due to heat and the combination of mercury vapour and argon gives out, luminous radiations consisting of blue, violet and ultraviolet.

As violet and ultraviolet are injurious to the eyes and other healthy tissues of the human body, they are absorbed by the inner coating of fluorescent substances which re-emit the bluish green radiation.

The efficiency of a fluorescent lamp is much higher than that of a glow lamp, as in it 50% of the electrical energy is converted into light energy, whereas in a glow lamp only 20% of electrical energy is converted into light energy.

N. B. Instead of push-button type, the starter switch of special automatic type is also incorporated with the lamp. When cold the switch remains "on". So when main switch is closed, current can flow through filament. But in a short time, due to heating by current the starter switch is cut out and a change of magnetic flux in the choke being produced the above effects are obtained.

184. Cathode Ray Oscillograph : An important use to which Cathode rays are put is in a device known as the Cathode Ray Oscillograph.

The essential parts of a Cathode ray oscillograph are schematically shown in Figure 148.

Inside the tube T, F is the tungsten filament coated with alkaline earth which also serves as Cathode, A the anode, B a

metal block with a central slit and S the fluorescent screen remote from cathode. Besides these, there are two horizontal plates H_1, H_2 and two vertical plates V_1, V_2 fitted inside the tube.

Cathode rays emitted from the electrically heated Cathode filament F travel straight through the slit in B, horizontal and vertical plates, and produce a luminous patch on the screen.

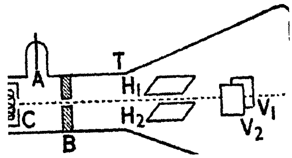


Fig. 148

If an alternating field be applied to the vertical plates electrons are deflected up and down by the electrostatic field and trace a vertical line on the fluorescent screen. Again if an alternating field is applied to horizontal plates electrons will trace out horizontal line on the same screen.

When two simple harmonic or alternating fields having different frequencies or phases are simultaneously applied on the horizontal and vertical plates, the patch of light will draw out a resultant graph which will be visible due to persistence of vision.

With the help of this oscillograph the nature of the resultant wave form of two Simple Harmonic or alternating fields can be studied.

185. Canal rays or Positive particles: We know that electrons or negatively charged particles are liberated from electrically neutral atoms of gases under the action of an intense electric field. Hence, the remainder of the atom will be positively charged particles and will travel towards the cathode. If the cathode is perforated, with one or more holes, the positive particles will pass through these holes or *canals* into the space and form faintly luminous streams called positive or canal rays. These positive rays are reflected by a powerful magnet in a direction opposite to those of the Cathode rays when similarly treated.

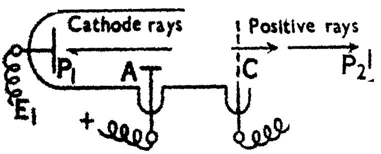


Fig. 149

In Figure 149 is shown a discharge tube in which A is the anode, C a perforated cathode, P_1, P_2 two metal plates attached at two ends. If the discharge tube be put into action by an indication coil and the plates are connected to two quadrant

electro-meters E_1, E_2, then electro-meter E_1, indicates a negative charge, while the electro-meter E_2 indicates a positive charge.

185(a). Properties of Positive rays :

(1) Positive rays can produce phosphorescence.

(2) Positive rays are deflected by electric and magnetic fields in directions opposite to those of cathode rays showing that they consist of streams of +vely charged particles.

(3) Positive rays can disintegrate metals.

(4) The specific charge (e/m) of positive ray particles is much smaller than for cathode ray particles, its value again depending on the nature of the gas in the tube.

186. X-rays or Rontgen Rays : Prof. Rontgen while studying Cathode Rays, discovered an unknown form of radiation which like ordinary light affected a photographic plate. This happened even if the plate were covered. He termed these radiations as **X-rays** and they are also known as **Rontgen Rays.**

186(a). Vacuum X-ray tube : For the production of X-rays, a vacuum tube of a special shape (Fig. 150) is fitted with two electrodes inside the tube by means of short lengths of platinum wires sealed into its walls. One of the electrodes which is connected to the negative terminal of an induction coil, *i.e.*, the Cathode, consists of a concave disc of aluminium and the other called the anti-cathode which is connected to the positive terminal of the induction coil is a round plate of tungsten mounted on a support of solid copper and placed at the centre of curvature of the Cathode and inclined at 45° to its axis. The anti-cathode is joined externally to another electrode, the anode, the function of which is not clearly understood.

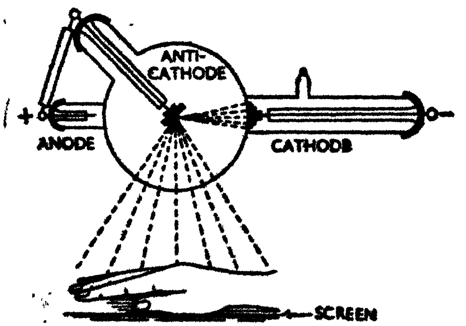


Fig. 150

As the discharge passes through the tube, X-rays are emitted by the platinum anti-cathode in all directions and the whole of the wall of the tube between the Cathode and the anti-cathode becomes brightly fluorescent.

These radiations, *i.e.*, X-rays are considered as *ether* waves and are emitted only when the flying electrons, *i.e.*, the Cathode rays are suddenly stopped by the anti-cathode, a metal target.

186(b). Collidge tube: In this form (Fig. 151) designed by Coolidge, the exhaustion of tube is carried to such an extent (10^{-4} *mm* of mercury) that no discharge can pass through it. To provide

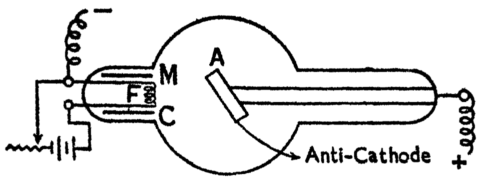


Fig. 151

carriers for the discharge, the Cathode C is made of a tungsten filament heated independently by a low voltage current. Thermo-electrons emitted by the filament serve as carriers in the discharge tube, the speed of which can be increased to any required value by applying corresponding potential difference by means of an induction coil. The anti-cathode A which is made of tungsten and placed as before serves also the function of the anode. The filament being surrounded by a cylinder M of molybdenum, the thermions or electrons are nearly focussed into a beam which falling on the anti-cathode produces X-rays.

187. Properties of X-rays:

(1) They pass through many solid substances with comparatively little absorption.

The opacity of metals to these rays is approximately proportional to their atomic weights. The flesh in the hand is more transparent than the bones and consequently if the hand be placed in front of a photographic plate and exposed to these rays, a beautiful shadowgraph will be obtained showing the details of the bones.

(2) They travel in straight lines.

(3) They are not deflected by magnetic or electric field, and hence they are not charged particles.

(4) They excite fluorescence in many substances, *e.g.*, Barium-plantinocyanide.

(5) They impart conductivity to air or any other gas through which they pass, *i.e.*, they can ionise a gas.

(6) They affect photographic plates.

(7) They produce photo-electric effect, since when they fall on certain bodies electrons are emitted.

(8) They are radiations of the same nature as light but they are of much shorter wave-length than visible light.

When X-rays fall upon any substance, other X-rays known as secondary X-rays are emitted. The quality of the secondary X-rays does not depend on the quality of the primary X-rays which cause their radiation, but on the nature of the substance emitting them. There are two principal types of these secondary X-rays ; one of them, the K-series is more penetrating and the other, known as L-series, is less penetrating.

187(a). Hard and Soft X-rays : The rays produced in a tube having different degrees of vacuum differ in their penetrating powers, the penetrating power being greater when the vacuum is higher.

The highly penetrating rays are called *hard rays* as they can penetrate a larger thickness of matter and the relatively less penetrating rays are called *soft rays* as they can penetrate comparatively smaller thickness of matter.

X-rays are absorbed by any substance and the absorption decreases with decreasing wave-length.

X-rays are also diffracted by crystals having fine and regular grouping of atoms.

187(b). Applications of X-rays : The following are the purposes for which X-rays have been applied.

(1) To determine with precision the position of any foreign matter lodged in different parts of the human body from an X-ray photograph.

(2) To detect the fracture of the bones.

(3) To detect the diseases of tuberculosis, abdominal ulcers, cancer and several other diseases.

(4) To detect crimes.

(5) To detect the genuineness of diamonds and other valuable stones.

(6) To ionise gases.

(7) To get an intimate knowledge of the nature of the crystalline structure of a substance.

(8) To determine the atomic number or the positive charge in the atom.

(9) To justify the quantum theory.

QUESTIONS

1. Describe how cathode rays are produced and enumerate their chief characteristics. Explain how their nature has been experimentally established.
[C. U. 1938, '41, '43, '45, '47, '50, '51, '53, '54, '55]

2. Describe briefly experiments from which the charge and mass of an electron are determined. [C. U. 1937. '48. '54]

3. Write a short note on the production and properties of X-rays. [C. U. 1947, '52, '53, '55]
Explain how soft X-rays differ from hard X-rays. [C. U. 1949]
4. Write short notes on :—
(1) Ionisation. [C. U. 1936, '38]
(2) Ionisation current. [C. U. 1936, '43, '47]
(3) Positive Rays. [C. U. 1953, '56]
(4) Glow and Discharge Lamps. [C. U. 1956]
5. Write notes on the following :—
(a) Positive rays, (b) Nature of X-rays. [C. U. 1957]

CHAPTER XV

ELECTRON THEORY AND RADIO-ACTIVITY

188. Electron : While studying the nature of the cathode rays, it has been found that electrons consist of particles having an extremely small mass equal to $\frac{1}{1850}$ of that of an atom of Hydrogen ($8{\cdot}8 \times 10^{-28}$ grm.) and with each particle is associated a negative charge of $1{\cdot}55 \times 10^{-20}$ e.m. units.

The mass and charge of the electron are constant whatever may be the material of the electrode or the gas in the tube through which electric discharge takes place.

These particles known as **electrons** are said to be the ultimate and indivisible unit of electricity.

Thus each electron carries a definite negative charge and the ratio of its charge to the mass is also constant.

The following are the various sources from which these electrons are obtained.

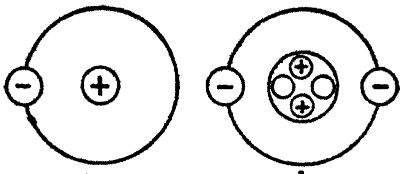
(1) Cathode Rays. (2) X-Rays, impinging on metal target. (3) Thermionic tube—electrons are emitted in a copious stream when a piece of metal is heated in a vacuum tube. (4) Photo-electric cell—electrons are obtained when light rays especially violet or ultra-violet act on metals or alkalies in a vacuum tube. (5) Radio-active substances—of which β-radiation consists of electrons.

·189. Electron Theory of Matter : The electron theory has been built up by J. J. Thomson, Rutherford, Bohr and others, and according to this theory the atom of an element is made up of a positively charged central nucleus called **proton** round which rotate one or more electrons like planets revolving round the sun.

The total negative charge of the electrons in an atom is equal to the positive charge of the nucleus of the atom and the atom as a whole is neutral.

The number of electrons outside the nucleus is represented by the **atomic number** and the number of protons in the nucleus is represented by the atomic weight of the element.

In Hydrogen, the atomic number is 1 and the atomic weight is 1. So in Hydrogen there is only one proton in the nucleus with one electron revolving round it [Fig. 152 (*a*)]. But in Helium the atomic number is 2 and the atomic weight is 4. So in an atom of Helium there are two protons and two neutrons (Art. 190) in the nucleus with two electrons revolving round is [Fig. 152 (*b*)]. The complete atom is neutral. In Figure 152(*a*) and Figure 152(*b*) the positive circle and the negative circle represent respectively a *proton* and an *electron* while the white circle represents a *neutron*.



(*a*) (*b*)

Fig. 152

With the increase in the atomic number, the distribution of moving electrons becomes complex dividing into two or more groups each group having its characteristic orbit or a definite period.

It is due to the instability of highly complex nuclei that give rise to the phenomenon of *radio-activity*.

The electrons move round the nucleus because of the centrifugal force due to their motion in their orbits.

But by the action of external forces such as heat, light, electricity, etc. one or more of the freely moving electrons can be expelled from the atom leaving it with an excess of positive charge and the atom is called a **positive ion**. When the atom receives one or more electrons it becomes negatively charged and the atom is then called a **negative ion**.

Besides the electrons lodged in the atoms of a substance, there are free electrons moving within the inter-spaces between the atoms at random.

When an electro-motive force acts on the conductor, the free electrons tend to move forward in the direction in which the E.M.F. acts. This flow of electrons under the action of the E.M.F. is called the electronic current in the conductor.

These electrons during their motion frequently collide with the molecules of the substance and the kinetic energy of motion of the electrons is transformed into **heat.**

According to the Electron Theory, the electro-magnetic waves are generated by the motion of the electrons under certain conditions and according to the quantum theory they are produced when the electrons inside an atom suddenly jump off from one orbit to another.

These electro-magnetic waves constitute light and heat rays, and also X-rays.

190. Certain Terms :

(a) Proton : It is the positively charged central nucleus of a hydrogen atom with unit mass. It is a stable particle and has charge equal to $+4.776 \times 10^{-10}$ e. s. u. and mass equal to 1850 times that of an electron.

(b) Positron : It is said to be the positive counterpart of an electron and also called positive electron having a positive charge comparable to that of an electron.

Anderson using a strong magnetic field in Wilson chamber photographs, showed that trucks due to particles liberated by cosmic rays are of a new type whose curvatures correspond to + particles.

Curie and Joliot have observed that positrons also appear when aluminium is disintegrated by α particles.

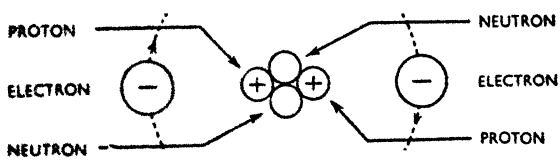


Fig. 153

(c) Neutron : It is an elementary particle having zero charge and unit mass. It has been observed that when *berylium* is struck by α-particles a highly penetrating radiation is emitted. This radiation consists of particles having unit mass (*i.e.*, the mass of a proton) but no charge. These particles are called **neutrons.** According to Chadwick, it is a stable compound of proton and electron. But Curie and Joliot got the opposite result. In the

disintegration of aluminium not only do protons appear but also neutrons with positrons are detected. Figure 153 shows the distribution of neutrons in helium atom.

[Note: Recent researches, especially regarding the discovery of neutron have altered the notion regarding the structure of the nucleus.

According to the present theory, the nucleus of an atom does not contain electrons embedded in it but contains protons and neutrons.

If W be the atomic weight of an element and N be the atomic number, then the number of neutrons in the nucleus is equal to (W – N).

The conception of attaching neutrons to the protons in the nucleus of the atom of an element so as to form groups of stable pair as there is a strong attraction between a portion and neutron.]

(d) **Dueteron:** It is the nucleus of the heavy hydrogen which is an isotope of hydrogen of atomic weight 2·0136. Its charge is equal to $+4{\cdot}77\times10^{-10}$ e. s. unit.

191. Becquerel Rays: Becquerel discovered that compounds of uranium emit spontaneously invisible rays or radiations which are able to pass through black paper and a sheet of thin glass and can affect a photographic plate. Other substances have been found to emit rays similar to those emitted by uranium and all salts containing this metal, and the name of Becquerel rays has been given to these invisible rays.

These rays have the power of ionising a gas, *i.e.*, of rendering the gas through which they pass conducting.

P. Curie and Madam Curie afterwards succeeded in isolating radium from pitch blende which was found to spontaneously emit Becquerel rays in an intense degree.

192. Radio-activity: It is the property of certain heavy elements either in free state or as a component of compound, of emitting rays which affect photographic plates, cause Phosphorescense on certain compounds and ionise gases.

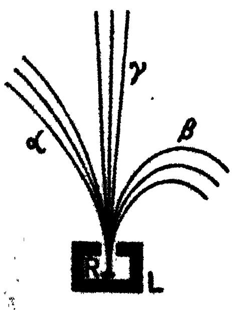


Fig. 154

The substances like Uranium, Radium, Thorium, etc. and their compounds which exhibit radio-activity are called radio-active bodies.

192 (a). Radiations given out by Radio-active bodies: Radio-active bodies give out three distinct types of rays or radiations called (a) α-rays, (b) β-rays and (c) γ-rays.

Rutherford distinguished and identified them by observing their different penetrating powers and their different behaviours with strong electric and

magnetic fields. The following simple experiment may be done to study some properties of these rays.

A small quantity of some radio-active compound R is taken within a hollow lead cylinder L (Fig. 154). The radiations or rays come out vertically upwards through the small opening of the cylinder. On applying a strong magnetic field these rays were found to separate into three distinct components. One component was deflected to the left to a small extent, another component was deflected to the right to a much greater extent while the third component was not deflected at all. The three components were respectively α-, β- and γ-rays.

The deflections and the directions in which they occur, indicate that the α- and β-rays are made of charged particles, α-particles being positively charged and β-particles negatively charged. Small deflections of α-ray particles show, that they are heavy or bulky, whereas very large deflections of β-particles show that they are extremely light particles. The fact that γ-rays are not deflected shows that they carry no charge at all.

193(a). Properties of α-rays and its nature:

(1) They can produce fluorescence on a suitable fluorescent screen.

(2) They are easily absorbed by matter. An aluminium foil ·01 cm. thick is enough to absorb α-particles.

(3) They can produce intense ionisation of a gas.

(4) They are deflected by a magnetic field in a direction opposite to that of Cathode or β-rays.

(5) They are positively charged particles, each α-particle having two units of positive charge, *i.e.*, numerically twice the charge of an electron. [Charge of electron $= 4{\cdot}7 \times 10^{-10}$ e.s.u., charge on an α-particle $= 9{\cdot}4 \times 10^{-10}$ e.s.u.].

(6) The mass of an α-particle is four times the mass of a hydrogen atom, which is also the mass of a helium atom.

It follows from (5) and (6) that an α-particle is an atom of helium from which two electrons have been expelled, or in other words, an α-particle is an ionised atom of helium.

This can be verified by the following experiment.

Experiment: An evacuated glass tube AB was fused inside another evacuated glass tube C (Fig. 155) having two electrodes P and Q. A wire sealed into AB carries at its top a speck of radium. The end B of tube AB, inside C is made of thin glass so that α-particles could pass through it into the tube C.

On passing an electric discharge between P and Q for a fairly long time, spectrum of helium could be seen. α-particles coming into the tube C from AB have captured electrons produced by the electric discharge and have formed helium (He) gas. Thus we can conclude that α-particles are simply nuclei of Helium atom.

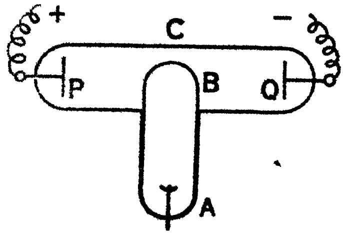


Fig. 155

(7) They are scattered while passing through metal sheets. They can produce weak action on photographic plates.

(8) The value of e/m for an α-particle is $1 \cdot 45 \times 10^{14}$ e.s.u./gm. and its mass $= 6 \cdot 6 \times 13^{-24}$ grm.

193(b). Properties of β-rays :

(1) They are negatively charged particles and the magnitude of their charge and the value of $\frac{e}{m}$ are the same as those of electrons.

(2) The mass of a β-particle is very small compared with that of an electron. The mass of β-particle changes with its velocity.

(3) They move with very high velocity comparable with that of light.

(4) They have far more penetrating power but less ionising power.

(5) They can produce action on photographic plates.

(6) They are deflected by electric and magnetic fields.

193(c). Properties of γ-rays :

(1) They are similar to X-rays and consists of *ether pulses*.

(2) They have a very great penetrating and ionising power.

(3) They are not material in nature.

(4) They are not deflected by electric or magnetic fields and therefore carry no electric charge.

(5) They are scattered and absorbed by material bodies.

(6) They can be diffracted, refracted, polarised and can interfere with each other.

(7) They excite fluorescence.

Note : α- and β-rays are analogous to positive and cathode rays. But γ-rays are very similar to X-rays as they possess practically all the properties of X-rays except this that X-rays are not natural radiation, γ-rays have shorter wave-length than X-rays.

The property of radio-activity is not influenced by the physical or chemical states of the element but the physical and chemical

properties of an element are determined solely by the nuclear charge of its atom. According to the theory of Rutherford and Soddy the phenomenon of radio-activity is due to the disintegration of α- and β-particles.

Taking the charge on an electron as the unit of charge and the mass of H_2 nucleus as the unit of mass, the α-particle has a resultant charge of $+2$ and a mass equal to that of Helium atom and the β-particle has a charge of -1 and a mass which is negligible.

194. Expulsion of α- and β-particles from an atom : The atomic number of the element is its numerical position in the Periodic table and is equal to the nuclear charge and is indicated by the number of electrons outside the nucleus.

When an α-particle is expelled from an atom, the atomic weight decreases by 4 units and the atomic number by 2 since the loss of an α-particle diminishes the nuclear charge by two units and a new element is formed.

Again when a β- particle is expelled, there is no change of mass but the atomic number is increased by 1 since the loss of β-particle (*i.e.*, an electron) raises the positive nuclear charge by one unit. A new element is again formed having the same atomic weight as the parent element but the atomic number is greater by 1. Such elements are called **Isobars.**

If again the nucleus of an element emits an α-particle and two β-particles in succession the nuclear charge remains unchanged but the atomic weight is changed by 4 units. So the resulting element has the same atomic number as the parent element but with different atomic weights. Such elements are called **Isotopes.**

195. Wilson Cloud Chamber Experiment : The simplest experimental device for the investigation of the rudimentary particles

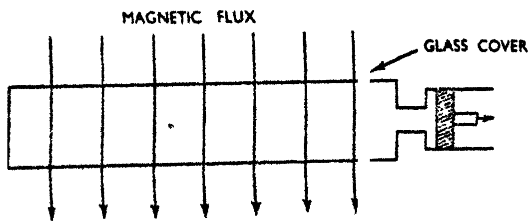


Fig. 156

is the cloud chamber. It consists of a flat cylindrical box with a light glass cover, containing water or alcohol vapour. A piston pump or a rubber diaphragm is connected to the inside of the

chamber so that the pressure of the vapour may be suddenly reduced, and the dust-free air is saturated with vapour. If the degree of saturation is not too great, condensation will occur on suitable nuclei such as gaseous ions when a charged particle is allowed to pass into the chamber. The track of the charged particle is then revealed in the photograph by the string of water drops formed around the ions by the particles along its path.

The box is placed between the poles of a magnet so that a magnetic flux is sent through the box. If a charged particle, say proton passes into the chamber it will curve to the right, electron to the left but a neutron being uncharged, will leave no track or path but will often manifest itself by striking some other particle and splitting it into charged particles which will leave their characteristic tracks or paths on the photo plate.

196. Isotopes: Elements having the same physical and chemical properties but differing in atomic weights are called *isotopes*. The existence of the isotopes was indicated during the process of disintegration.

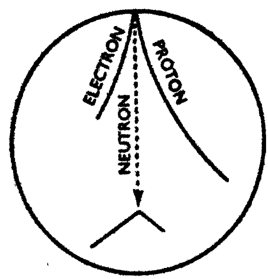


Fig. 157

The chemical and physical properties of an element are determined solely by the nuclear charge of the atom.

If the complex nucleus of an element loses α-particles and also β-particles and as a result leave residual nucleus with the same charge as the element possessed originally, the residue and the original element are called *isotopes* and they have the same chemical and physical properties but with different atomic weights.

The isotopes of an element were determined by Aston by his instrument known as the mass-spectrograph. Hydrogen is found to consist of atomic weights 1 and 2, Neon, Oxygen, Mercury, Xenon have been found to contain isotopes having different atomic weights.

Uranium—1 and *Uranium*—2 have the same nuclear charge and different atomic weights but they have the same chemical properties.

197. Heavy Hydrogen is an isotope of hydrogen of atomic weight 2·0136 and **heavy water** is the oxide of heavy hydrogen as ordinary water is the oxide of ordinary hydrogen.

198. Selenium Cell: The element selenium resembles sulphur in its chemical properties and in addition it has a special property ... very sensitive to light. When it is exposed to light its

remains localised. This charge travels round and as it passes near a set of pointed spikes attached to the inside of the hollow metal sphere M, it is neutralised by the negative charge leaking from pointed spikes leaving the hollow metal sphere positively charged.

In course of time as more and more positive charge developes on the hollow conductor M it acquires a very large charge and potential. Discharge through air is reduced by placing the conductor M in an earth-connected tank of air at high pressure provided with stop-cocks T_1 and T_2 (Fig. 159).

The Van de Graff generator is used for obtaining high-speed particles for bombarding the atom and for generating very penetrating X-rays.

With this generator voltage ranging from 2 to 5 million volts may be obtained.

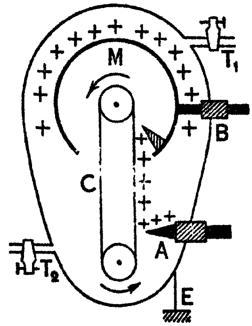


Fig. 159

(2) Cyclotron : Lawrence and Livingstone invented the cyclotron for developing high-speed particles using only a few thousand volts.

It consists of two semicircular hollow metal boxes, called "*dees*" separated by a gap as shewn in Figure 160. They are placed within and insulated from a tight circular box so that the "*dees*" may be immersed in any gas at any degree of vacuum.

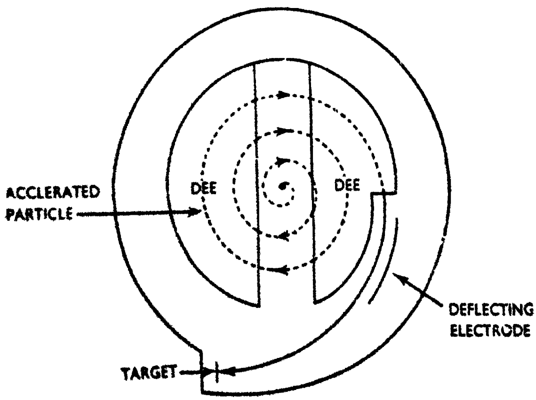


Fig. 160

An alternator with a voltage of about 10,000 volts and a frequency of 10^7 cycles per second is connected to the "dees". The entire apparatus is mounted horizontally in a vertical magnetic field produced by a huge electromagnet.

An incandescent filament placed at the centre of the '*dees*' acts as a source of electrons which ionise the gas and produce positive ions. These ions under the action of the magnetic field commence to wave round a circular path.

The frequency of the applied alternating voltage is so arranged that the voltage reverses every time the ions cross from the right-hand dee to the left-hand dee and *vice-versa*, with the result that they are continuously accelerated along a spiral-shaped curve until they approach a high-voltage negatively charged deflecting plate and then pass through an opening at an exceedingly high velocity to the target. The nuclei of the atoms of any gas placed near the opening will be desrupted by this bombardment so that the original atoms will be transmuted into others with the escape of neutrons in the surrounding space.

The disrupting energy associated with the high-velocity particles is about 100 *mev.* (million electron-volts).

QUESTIONS

1. What is an electron? Enumerate the different sources from which the electrons are obtained. [C. U. 1948]

Write an essay on Electron theory of matter.

2. Write short notes on (1) Positron, (2) Proton, (3) Neutron and (4) Deuteron. [C. U. 1936, '54]

3. Write a short note on Radio-activity. [C. U. 1940, '44, '49, '54, '57]

4. What are α-, β and γ-rays, and how do they differ from one another? What evidence is there that α-rays are the nuclei of Helium atoms? [C. U. 1936, '38, '49, '55, '58]

5. Write short notes on (1) Istope (2) heavy hydrogen (3) heavy water. [C. U. 1936]

6. Write short notes on (1) Selenium cell (2) Photo-electric cell. [C. U. 1938, '41, '42, '43, '50, '58]

7. Describe the experiments which have shown that the radiations emitted by radioactive substances are of three distinct types. Give an account of the nature and properties of these. [C. U. 1957]

CHAPTER XVI

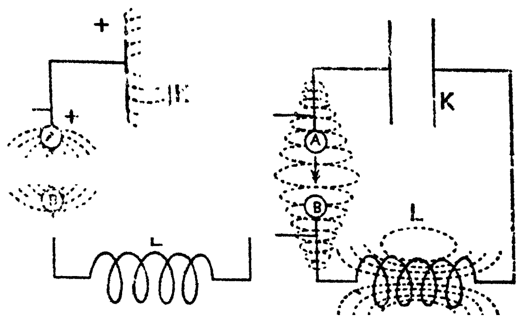
ELECTRO-MAGNETIC RADIATION AND WIRELESS COMMUNICATION

202. Oscillatory Discharge: When a parallel plate condenser is charged, the dielectric between the plates is in a state of strain. If the plates of the condenser are now connected up by a poor conductor the discharge takes place slowly until they are brought to the same potential.

But if the plates of the charged condenser are connected by a conductor of low resistance, the discharge is **oscillatory**, *i.e.*, the charge surges to and fro along the conductor several times and rapidly diminishes in magnitude, the whole process being completed in a short interval of time.

The effect is better illustrated if the plates of the condenser are connected up to a closed circuit with a capacity and inductance and charged by an induction coil. An oscillating current is generally set up in the circuit and the condenser is rapidly charged and discharged first in one direction and then in another.

These rapid process of charging and discharging produces rapid changes in electric and magnetic fields in the surronding medium and are known as oscillatory discharge of the condenser. Let the condenser K be connected to an inductance coil L through a narrow spark gap and let the condenser K be charged, the left plate having a + charge and the right one, a − charge.



(a) Fig. 161 (b)

As soon as the condenser is charged, lines of force pass from A to B [Fig. 161(*a*)] and when the potential difference between the plates, *i.e.*, between the knobs becomes too high, sparks pass from A to B causing a current to pass from A to B and magnetic lines of force encircle the line joining A and B [Fig. 161(*b*)].

As current passes, charges on the plates decrease and electric lines of force dwindle away while the magnetic lines of force due to the current are established mainly within the coil L [Fig. 161(*b*)].

Due to self-induction in the coil, current does not stop when the potentials of the plates of the condenser become the same but the charge overshoots the mark and recharges the plates of the condenser with charges of reversed sign and the electric lines run from B to A.

Again the spark passes when the potential difference between the plates becomes high and the magnetic lines of force encircle the line joining B and A and are also established in the coil L but in opposite directions to the lines in the last case.

Again due to self-induction the plates of the condenser are oppositely charged and the same state of affairs repeated over and over again the current surging to and fro between A and B with a period depending on the *inductance* and *capacity* of the circuit as

given by the expression $t = 2\pi\sqrt{LC}$, where t is the period, L, the coefficient of self-inductance of the coil and C, the capacity of the condenser.

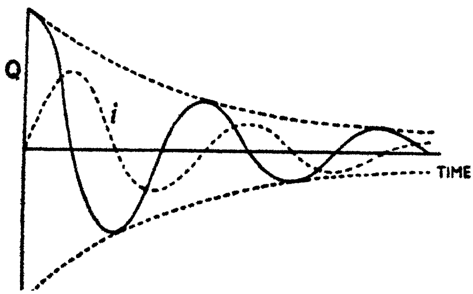


Fig. 162

The discharge of the condenser is now oscillatory and the current (i) is alternating and of decreasing amplitude (Fig. 162).

The oscillatory character of the discharge is observed when a photograph of the spark is taken and the spark is found to consist of a rapid sequence of separate discharges.

Immediately before the discharge takes place, the energy is entirely electrostatic, but during the discharge it is partly electrostatic and partly electro-magnetic and is gradually dissipated away in the form of heat and transmitted in a direction perpendicular to the plane containing the electric and magnetic forces.

Thus during this discharge (oscillatory) two sets of strains, electric and magnetic in perpendicular directions constitute electro-magnetic waves which travel outwards with the velocity of light and with a period equal to that of oscillations in the circuit and in a direction at right angles to the direction of propagation.

203. Wireless Telegraphy or Radio-Telegraphy : *Historical Development :* The development of wireless telegraphy or radio-telegraphy dates from the publication of Maxwell's paper "A Dynamical Theory of Electro-magnetic field" written in 1864. In this paper Maxwell proved theoretically the existence of electro-magnetic waves and shewed that they must be propagated with the velocity of light.

In 1878 Hertz published the result of his experiments on the production and detection of electro-magnetic waves. The possibility of using these waves for signalling purposes was demonstrated by Lodge in 1894. In 1899 Marconi succeeded in sending a message across the English Channel.

204. Electro-magnetic Radiation : To understand the principle of electro-magnetic radiation in an oscillating circuit let the two plates of the condenser be represented by two discs A and B at the ends of the rods separated by sparking knobs. Such an arrangement is called an **Oscillator.**

When a spark passes between the discs a high frequency current surges to and fro between the plates of the condenser and the upper side is alternately positively and negatively charged, the lower one being always charged oppositely.

With each charge a wave of electric strain in one direction or the other will be sent out from the oscillator in all directions. Between each +ve and −ve charging of the oscillator there will be a large rush of current of a very short duration. Each rush sets up a magnetic field which surrounds the oscillator. These fields spread out from the oscillator. Each line of electric strain shrinks and gives rise to lines of magnetic strain, the rise and fall of which again produce lines of electric strain Thus two sets of strain, electric and magnetic, are propagated outwards. The result will be that Faraday tubes of force between the two conductors will gradually form into loops and will ultimately be detached from the conductors. Since the discharge is oscillatory the loops deatch alternately in groups, the direction of the electric strain in successive loops being opposite (Fig. 163).

These loops travel outwards with the velocity of light and give rise to lines of magnetic strain which are in opposite directions through consecutive loops.

The two sets of strain, electric and magnetic are propagated outwards and constitute what is known as electro-magnetic waves.

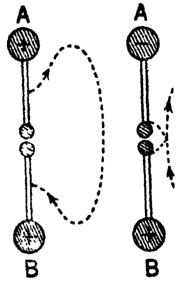


Fig. 163

If the electric and magnetic fields are quite local, the circuit does not radiate much energy but if the capacity and inductance are distributed over the circuit, the electric and magnetic fields spread out into the surrounding space and energy is radiated in the form of electro-magnetic waves.

One form of such a circuit called the *Aerial* or *Antenna* consists of a number of elevated wires joined together and connected through an inductive coil with the earth.

205. Open Aerial: For long distance transmission Marconi raised a part of the oscillating (transmitting) circuit high up in the air and connected the other part to the earth thus forming a condenser with the earth and the aerial as the plates.

The higher the aerial above the surface of the earth the greater is the distance over which the electro-magnetic waves can be transmitted.

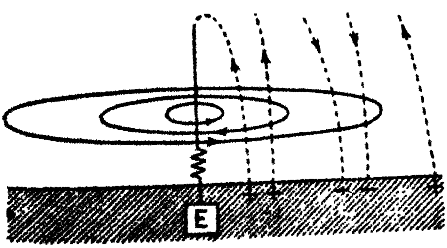


Fig. 164

The radiation of electro-magnetic waves from an aerial connected to the earth is shown in Figure 164.

Here the vertical dotted lines represent half the lines of electrostatic strain and the circular lines represent the lines of magnetic strain.

The free ends of the opposite groups of electrostatic lines of force terminate on the surface of the earth and travel outwards with the velocity of light.

When these lines of force fall on a bare wire at the receiving station, an alternating current is set up in a suitably tuned circuit.

206. Telegraphy: In the spark system, an induction coil I produces sparks between two balls S and S which are joined to the aerial A and the earth E (Fig. 165).

Each spark consists of a number of oscillations which gradually die down and give rise to a train of electro-magnetic waves whose amplitude gradually decreases. The frequency is generally of the order of one million.

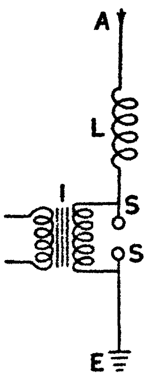


Fig. 165

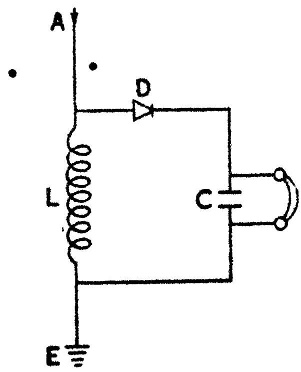


Fig. 166

At the receiving station in which a simple radio receiving set is situated, the damped waves on striking the aerial A whose natural frequency is equal to that of the incoming waves give rise to resonant oscillations which are detected by a suitable arrangement. A simple telephone is not used for the detection of the oscillations as their frequency is too high to affect it.

In modern practice, the detector D is a rectifier (which makes current unidirectional) connected with a telephone T with a condenser in parallel. The rectifier allows only half of the oscillating current to pass through it and stops the other half (Fig. 166).

The crystal detector D consists of an adjustable contact maker touching a small crystal of silicon or galena placed in a brass cup, ... has the special property of conducting current only in one ...on.

. Each train of waves will, therefore, produce the same effect as a steady current of short duration and will give rise to a click in the telephone.

If the frequency of the sparks be n, musical note of frequency n will be heard in the telephone, so long as sparks are being produced in the sending station.

By keeping a key in the primary circuit of the induction coil closed for a longer or shorter time, *longs* and *shorts* of the Morse code are produced.

The above is the description of a simple receiving set. For telephony, the description of the receiving set is given in a subsequent article.

In Figure 165 a tuning inductance L is placed between the balls S, S and aerial A. The utility of this inductance is to alter the frequency, *i.e.*, the wave-length of the radiating wave as given by the relation $n \propto \frac{1}{\sqrt{LC}}$, where L is the inductance and C, the capacity of the aerial.

In Figure 166 a tuning inductance L is placed between the aerial and the earth for tuning the aerial to the incoming waves. A condenser C is placed across the telephone to act as a reservoir of current and to give up the energy stored in it to the telephone when required.

207. Crystals as rectifires : When certain crystalline mineral substances are placed in contact with certain metals, the resistance at the point of contact does not behave according to Ohm's Law but is altered according to the direction of the current due to the local heating effect of the current. In addition, Peltier effect at the point of contact will assist the voltage in one direction and oppose it in the other.

It is due to this property of the crystals that an oscillating current when received by the crystals is helped in one direction and opposed in the other direction and the resulting current is unidirectional.

The voltage current curve of the combination is a curved line instead of a straight one as for ordinary conductors obeying Ohm's Law. This curve is known as the *characteristic curve* of the combination.

Simple crystal Detectors cannot be used for long distance transmission because the current received will not be strong to operate the telephone.

Figure 167 shows that the damped waves after passing through the crystal come out as rectified pulses which then pass into the

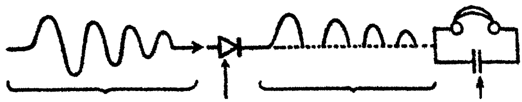


Fig. 167

telephone coupled with a condenser and produce clicks in the telephone.

208. Thermionic Valve: An important device was made by Edison after the discovery of the behaviour of electrons (thermions) emitted from a heated metal inside an evacuated bulb provided with a metal filament electrode. This device known as thermionic valve is now almost universally used in Wireless Telephony for the production of rectification and amplification.

208(a). Diode or Two Electrode Valve: It consists of an evacuated glass bulb having a metal electrode P and a wire filament F. If the filament be heated by a low voltage, *i.e.*, low tension (L. T.) battery and the electrode P be raised to a high positive potential by means of a high voltage or tension (H. T.) battery, electrons emitted by the filament will move towards P and constitute a current, which can be detected by a galvanometer G placed in the circuit. If the filament is not giving off electrons or the plate is made $-ve$ relative to filament, no current is found to flow in this circuit. The arrangement thus enables current to flow in the plate-filament circuit only in one direction and thus acts like valve, and as there are two electrodes, the plate and filament, it is called a two electrode or diode valve (Fig. 168).

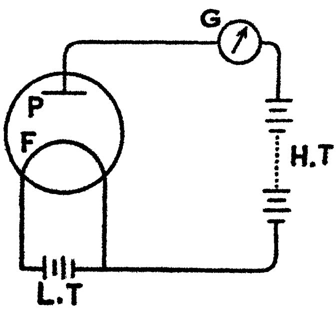


Fig. 168

If an A. C. potential be applied between the filament and the plate of a diode valve it will allow only unidirectional flow of current in the filament-plate circuit only when the plate P is $+ve$. Hence a diode valve can convert alternating current into direct current, *i.e.*, it acts as rectifier of alternating current and voltage.

• **208(b). Diode as a rectifier :** The action of diode as a rectifier is shown by Figure 169. A device which changes something from alternating to unidirectional is a rectifier. The valve rectifies electric current or potential differences. Suppose it is required to convert the alternating current in the secondary of a transformer into a unidirectional one. For this, the plate P and the filament F are connected with the secondary, the filament being also separately connected with a few turns of the secondary for emission of electrons. As the p. d. of the secondary alternates, the electrons from F move to P only when P is positive with respect to F. Hence electron current flows through the valve only during this time. When during the other half of the cycle, P becomes negative with respect to F, electrons are repelled by P and no current flows through the valve. Thus current through the secondary circuit travels in pulses in one direction only.

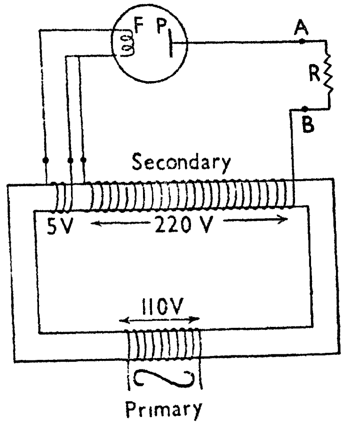


Fig. 169

The other half of the cycle which has been suppressed may be utilised by using a second diode. Two diodes are often incorporated

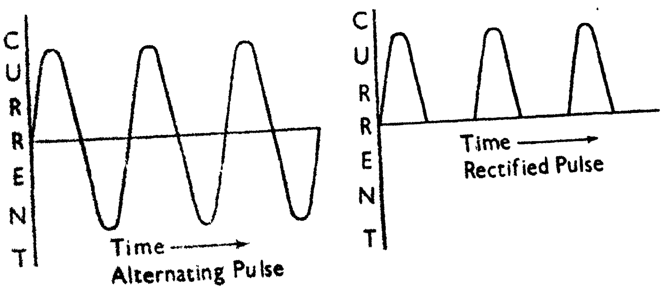


Fig. 170

in the same valve so that there are two filaments and two plates in it. It is known as **duo-diode**. The duo-diode is a **full-wave rectifier** while the diode is a half-wave one. Figure 170 shows current-time curves for alternating current before and after rectification.

208(c). Triode Valve : It consists of an (Fig. 171) evacuated bulb with a tungsten *filament* F surrounded by a cylinder of perforated metal or wire gauze, called grid G. The filament and grid are surrounded by a second cylnider P made of thin sheet of copper and called the *plate*. The plate and the grid are connected to the terminals P′ and G′ and the filament to F′, F′ respectively. The grid is placed nearer to the filament than the plate. The valve is called triode or three electrode valve as it has three electrodes—the filament, the plate and the grid.

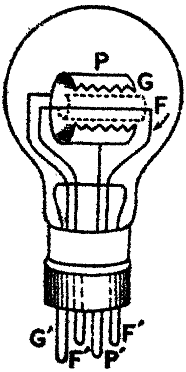


Fig. 171

When the filament is heated it gives off electrons (thermions) some of which pass though the grid and strike the plate. Negative electricity flows from the plate to the filament along external circuit. This constitutes a current called the plate current. A change in the grid potential will in general change the plate current.

208(d). Characteristic Curve of a Three Electrode Valve : The characteristic curve (Fig. 172) of a valve shews the variation of the plate current with the variation of the grid potential.

The curve shows the relation between the plate current (i_p) and the grid voltage (V_g) for a constant plate potential.

With the highly negative grid voltage the plate current is nil.

As the grid voltage is made less negative the plate current increases and finally attains a saturation value for a certain *+ve* value of the grid voltage as the top portion of the curve becomes nearly parallel to axis representing grid potential.

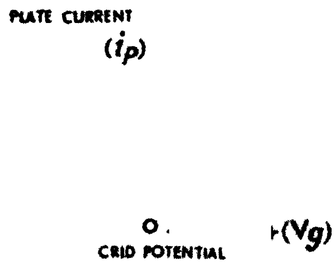


Fig. 172

The curve as shown in Figure 173 is drawn such that the electron current is near saturation. When the grid potential is zero, the plate current is OC. If a potential of −2 is given to the grid, increase of current due to the positive half of each oscillation will be greater than the diminution of current due to the negative half.

Thus if the valve operates at the point A where the bending is great the mean plate current during the oscillations will be increased.

The valve is then said to act as a rectifier, *i.e.*, it allows the ate current to flow in e direction only. This called *anode bend ectification*.

Again if the grid potential is -1 and the valve operates at the point B where the curve is practically a *straight line*, the mean current during oscillations will remain unaltered.

The valve will then act as an *amplifier*, *i.e.*, it has amplified or magnified the variations impressed on the grid.

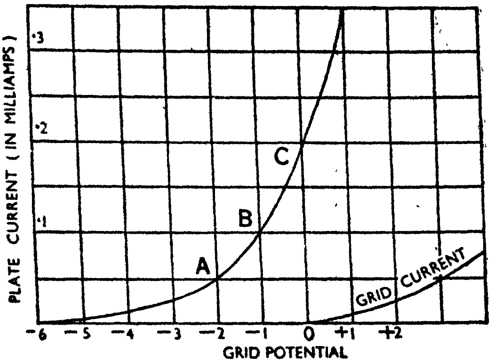


Fig. 173

209. Triode Valve as a Rectifier: In Figure 174 V is the valve, P, the plate, G, the grid, A, the aerial and T, the telephone.

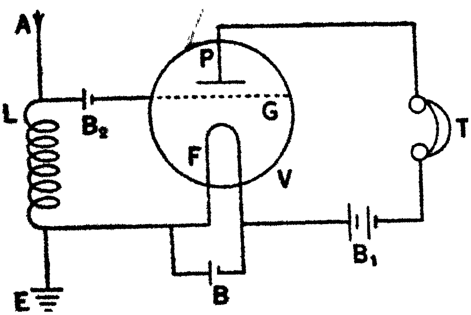


Fig. 174

The filament F, the plate P and the grid G are connected with different batteries B, B_1 and B_2 respectively as shown in the figure.

Let the grid be charged negatively. If the filament current be now turned on, no current can flow in the plate circuit as the electrons from the filament will not be able to reach the plate by the repulsive action of the negative charge on the grid. But if the grid be positively charged a current will flow in the plate cir it.

the grid circuit instead of applying a steady current an ing current is allowed to flow, the current in the plate in a series of unidirectional pulses.

e grid be coupled by an aerial circuit A the oscillations aerial when the waves are falling on it will serve to close the valve permitting little spurts of current at the on spurts a second.

210. Triode Valve as an Amplifier : To use the valve as amplifier for magnifying a weak incoming signal, the potential of the plate or the grid is adjusted to work on the straight part of the characteristic curve and with the normal grid potential the position is at the middle of the straight portion of the curve.

As the grid potential swings to and fro (due to the arriving waves) increasing and decreasing alternately by equal amounts, the swing of the plate current will be much bigger. Thus the valve has amplified or magnified the variations impressed on the grid.

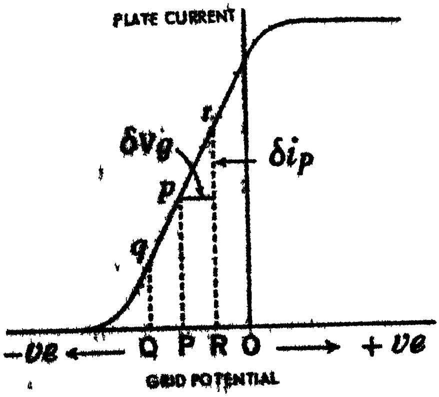


Fig. 175

For amplification, a negative voltage OP called grid bias is applied to the grid so that it corresponds to the middle point P of the straight portion of the characteristic curve. When the incoming electro-magnetic waves fall on the grid, potential goes on fluctuating between OQ and OR about the mean voltage OP. As a result of this, there is a corresponding variation of the plate current between Q_q and R_r. It will be easily seen from Figure 175 that for a very small variation (δV_g) of grid potential, the variation of the plate current is large, specially in case of a very steep characteristic curve. This is the underlying principle of amplification.

Amplification Factor : A certain change in grid potential is a certain number (μ) of times as effective in influencing the anode current, as it is for the same change in plate potential. The quantity μ is called the amplification factor.

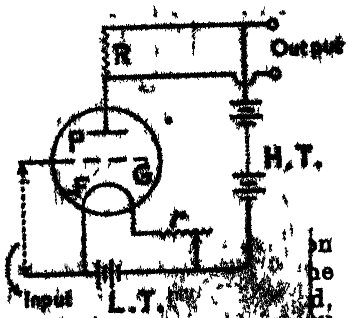


Fig. 176

To see whether any amplification has been produced by the valve, we are to convert change of plate current into a change of potential and then compare it with the original potential change of the [illegible] ...n ...he ...d, ...l grid. This can be done by using a resistance R in [illegible] circuit as shown in Figure 176. It can be [illegible]